

Lecture Notes for INEL 6055: Chapter 8

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1 Semiconductor-Photon Interaction

- Absorption: photo-detectors, solar cells, radiation sensors.
- Radiative transitions: light emitting diodes, displays.
- Stimulated emission: lasers.
- Above effects are determined by energy bands/states.
- Opto-electronics (photonics): enabling technology in many technologically important areas. See page 3 of *MIT News*.
- Interaction with light also provides an important experimental tool to study semiconductors.

Excellent reference at a low cost:

Optical Processes in Semiconductors, by Jacques I. Pankove, Dover Publications, Inc, New York, 1971.

1.1 Review of Energy Bands in Semiconductors

- Formation of energy bands.
- Optical transitions must conserve both energy and momentum.
- Allowed states are also distributed in *momentum space*.

- Classical energy-momentum relationship:

$$E = \frac{p^2}{2m^*}$$

where m^* is the electron's effective mass.

- From quantum mechanics

$$p = k\hbar$$

where \hbar represents *Dirac's constant* and equals $h/2\pi$, h being Planck's constant, and k is the *wave vector*.

- crystal: model as a square quantum well with infinite barriers and a bottom of width L . QM tells us that k can have discrete values

$$k = n \frac{\pi}{L}$$

where n is a nonzero integer.

- L is an integral number N of unit lattice cells having a periodicity a . When $n = N$,

$$k = N \frac{\pi}{L} = \frac{\pi}{L/N} = \frac{\pi}{a}$$

is the maximum significant value of k .

- A *Brillouin zone* is the volume of k -space containing all the values of k up to π/a .
- Since the crystal is not homogeneous, a varies with direction.
- From *Academic Press Dictionary of Science and Technology*:

Brillouin zone Solid-State Physics. a fundamental polyhedron in wave vector space (k -space) whose geometry plays an important role in band theory and the specification of diffraction condition; it is bounded by a Wigner-Seitz primitive cell in the reciprocal lattice.

- See <http://cst-www.nrl.navy.mil/bind/kpts/>
- See <http://www.sjsu.edu/faculty/watkins/brillouin.htm>
- In terms of k , the kinetic energy of an electron can be expressed as

$$E = \frac{k^2 \hbar^2}{2m^*}$$

- If the crystal is a cubic potential well of side L , the allowed energies are

$$E = \frac{\hbar^2}{2m^* L^2} (n_x^2 + n_y^2 + n_z^2)$$

- E varies discretely, but steps are so small that E appears as a band.
- Above relationship between E and k is parabolic. Distribution is called a *parabolic valley*. In three dimensional momentum-space, the constant energy surfaces form closed shells. With every increment in momentum, the energy of successive shells increase quadratically.
- Interatomic distance varies with direction. Therefore the shape of a constant-energy surface must deviate from that of a perfect sphere. Furthermore, due to the influence of nearest neighbors, next nearest neighbors and higher-order neighbors, the minimum of the valley may not occur at $k_x = k_y = k_z = 0$. If such is the case, a transition between conduction and valence bands involve a change of both energy and momentum. generally in addition of the emission/absorption of a photon, a phonon with the right momentum needs to be emitted or absorbed.

2 LEDs and Carrier Recombination

- See slide 1.
- Forward bias injects excess minority carriers
- When excess carriers recombine, a photon is emitted.
- Light intensity is proportional to current flowing through the diode.
- A photon is a quantum of light with energy determined by its wavelength (color) by:

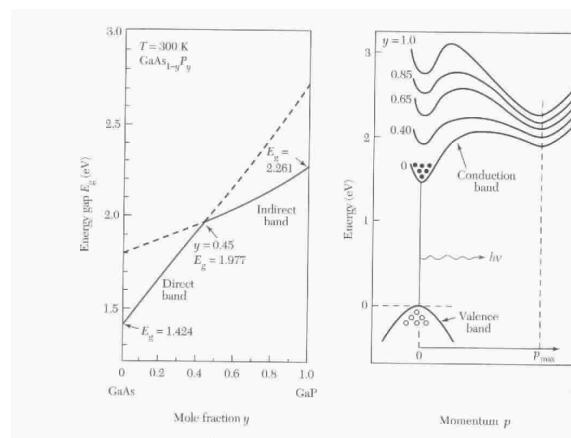
$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda}$$

where $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$, ν is the frequency, c is the speed of light ($c = 3 \times 10^8 \text{ m/s}$) and λ is the light wavelength.

- Recombination leads to the emission of photon with energy approx. equal to wavelength. See slide 3.
- Materials with a band-gap larger than 1.8 eV are used to produce visible light.
- Common III-V materials used to produce LEDs and their emission wavelengths (taken from S.M. Sze, *Semiconductor Devices*, Second Ed., 2002).

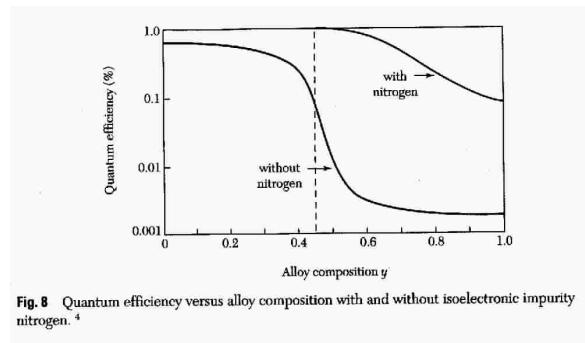
Material	Wavelength (nm)
InAsSbP/InAs	4200
InAs	3800
GaInAsP/GaSb	2000
GaSb	1800
$Ga_xIn_{1-x}As_yP_y$	1100-1600
$Ga_{0.47}In_{0.53}As$	1550
$Ga_{0.37}In_{0.73}As_{0.63}P_{0.37}$	1300
GaAs:Er,InP:Er	1540
Si:C	1300
GaAs:Yb,InP:Yb	1000
$Al_xGa_{1-x}As : Si$	650-940
GaAs:Si	940
$Al_{0.11}Ga_{0.89}As : Si$	830
$Al_{0.4}Ga_{0.6}As : Si$	650
$GaAs_{0.6}P_{0.4}$	660
$GaAs_{0.4}P_{0.6}$	620
$GaAs_{0.15}P_{0.85}$	590
$(Al_xGa_{1-x})_{0.5}In_{0.5}P$	655
GaP	690
GaP:N	550-570
$Ga_xIn_{1-x}N$	340,430,590
SiC	400-460
BN	260,310,490

- Scanned figure - energy gap for $GaAs_{1-y}P_y$. REF: M.G. Craford, "Recent Developments in LED Technology," *IEEE Trans. Electron Devices*, **ED-24**, 935 (1977).



- $y > 0.45$: indirect band-gap semiconductor. Also GaP.
- Special recombination centers (N) added to enhance recombination. This produces traps called *iso-electronic centers*. Recombination probability is greatly enhanced.

Scanned figure. REF: W.O. Groves, A.H.Herzong, and M.G.Crawford, "The Effect of Nitrogen Doping on GaAsP Electroluminescent Diodes," *Appl. Phys. Lett*, **19**, 184 (1971).



- Impurity states: can produce several types of interactions.
 - Acceptors
 - Donors
 - Interstitial - donor
 - Vacancy - acceptor
 - Often a vacancy and an interstitial form a molecular impurity that can be a donor or acceptor.
 - Compound semiconductors: deviation from stoichiometry can form a donor or acceptor depending of whether the cation or the anion are in excess.
 - extra electron in a donor - attracted most strongly to positive ion - acts as an electron of a hydrogen atom immersed in the high dielectric constant of the semiconductor. Orbit becomes very large.
 - A free hole and a free electron are attracted to each other. Since the hole is "heavier" than the electron, the latter orbits around the first and forms an *exciton*. Usually its binding energy is lower than the donor's or acceptors'.
 - Complex excitonic interaction, involving more than two free carriers, have also been observed.
 - Donor-acceptor pairs, in which the two ions attract each other, are also observed.
- Light intensity of LED:

$$\frac{P_{opt}}{h\nu} A_I = \eta_Q \frac{I_D}{q}$$

where

- P_{opt} is the optical power density in W/m^2
- A_J is the junction area
- $h\nu$ is the energy of a single photon

- I_D/q is the number of injected minority carriers, and
- η_Q expresses the efficiency of the LED, and is called the *radiative recombination efficiency*.
- The *recombination rate* is proportional to the concentration of excess minority carriers. The *effective recombination rate* is the difference between recombination and thermal generation rates, will be labeled r_n and r_p for electrons and holes, respectively, and is proportional to the excess minority carrier concentration:

$$r_n = \frac{n_p(x) - n_{p0}}{\tau_n}$$

where τ_n is the excess electron lifetime. A similar expression applies for holes.

- The continuity equation must take into account the carrier recombination:

$$\frac{\partial n_p}{\partial t} = \frac{1}{q} \frac{\partial j_n}{\partial x} - r_n = \frac{1}{q} \frac{\partial j_n}{\partial x} - \frac{n_p(x) - n_{p0}}{\tau_n}$$

- Steady-state: $\frac{\partial n_p}{\partial t} = 0$

$$\frac{1}{q} \frac{\partial j_n}{\partial x} - \frac{n_p(x) - n_{p0}}{\tau_n} = 0$$

- From chapter 3,

$$j_n = qD_n \frac{\partial n_p(x)}{\partial x}$$

- Continuity equation becomes:

$$D_n \frac{\partial^2 n_p(x)}{\partial x^2} - \frac{n_p(x) - n_{p0}}{\tau_n} = 0$$

- Equivalently,

$$\underbrace{\tau_n D_n}_{L_n^2} \frac{\partial^2 \delta n_p}{\partial x^2} - \delta n_p = 0$$

L_n : diffusion length for electrons.

- Try exponential solutions of the form Ae^{sx} .

$$\begin{aligned}\frac{\partial}{\partial x} Ae^{sx} &= sAe^{sx} \\ \frac{\partial^2}{\partial x^2} Ae^{sx} &= s^2 Ae^{sx} \\ \frac{\partial^2 \delta n_p(x)}{\partial x^2} &= s^2 \delta n_p(x)\end{aligned}$$

and

$$(L_n^2 s^2 - 1) \delta n_p(x) = 0$$

Thus, $s = \pm 1/L_n$ and the solution has two terms

$$\delta n_p(x) = A_1 e^{x/L_n} + A_2 e^{-x/L_n}$$

- Boundary condition: $x \rightarrow \infty, \delta n_p(x) \rightarrow 0$; thus

$$0 = A_1 e^{\infty/L_n} + 0$$

and $A_1 = 0$.

- At $x = w_p$, from eq. 3.8,

$$n_p(w_p) = n_{pe} e^{V_D/V_t}$$

and

$$\delta n_p(w_p) = n_{pe} (e^{V_D/V_t} - 1)$$

- In terms of $n_p(w_p)$,

$$A_2 e^{-w_p/L_n} = n_p(w_p) - n_{pe}$$

and

$$A_2 = (n_p(w_p) - n_{pe}) e^{w_p/L_n}$$

- The solution becomes:

$$\delta n_p(x) = \delta n_p(w_p) e^{\frac{w_p - x}{L_n}}$$

where

$$\delta n_p(w_p) = n_{pe} (e^{V_D/V_t} - 1)$$

- From this result we can write an expression for the current density due to diffusion,

$$\begin{aligned}j_n &= qD_n \frac{\partial n_p(x)}{\partial x} \\ &= qD_n \frac{\partial}{\partial x} \delta n_p(x) \\ &= -\frac{qD_n}{L_n} \delta n_p(w_p) e^{\frac{w_p - x}{L_n}}\end{aligned}$$

- The diode current is found by multiplying this expression by the junction area.

$$I_D = \frac{qD_n A_j}{L_n} \delta n_p(w_p) e^{\frac{w_p - x}{L_n}}$$

- As x increases, the excess minority carrier current is reduced. The diode current remains constant because this reduction is compensated by the flow of holes that recombine with the electrons. Thus, we can find the diode current by evaluating the above expression at $x = w_p$:

$$I_D = \frac{qD_n A_j}{L_n} \delta n_p(w_p)$$

- The average excess charge in the p region can be estimated from

$$Q_s = -q \underbrace{A_j L_n}_V + \underbrace{\frac{n_p(w_p) - n_{pe}}{2}}_{\bar{n}_p}$$

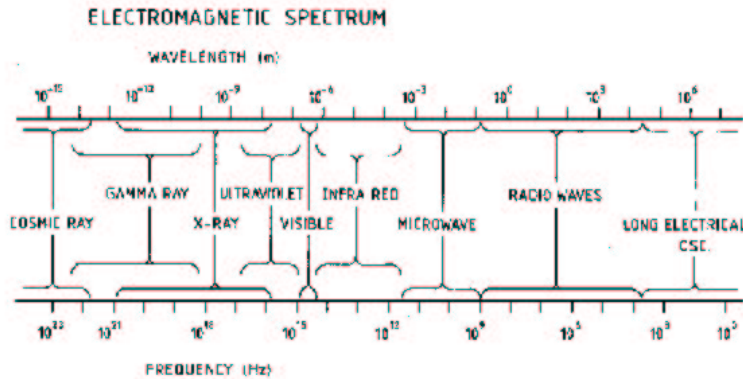
where V represents volume and \bar{n}_p stand for the average concentration of excess electrons.

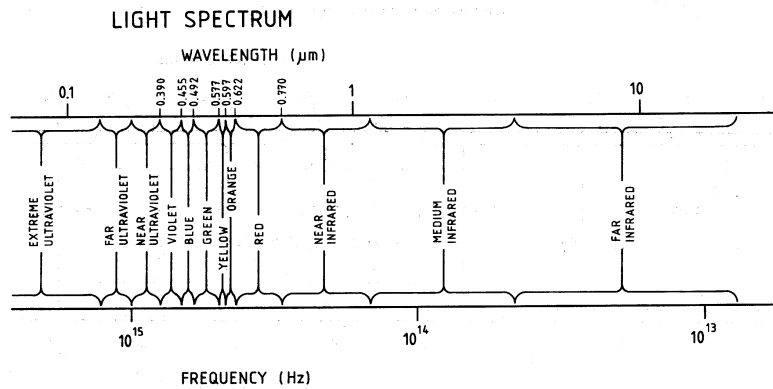
- For an n^+p junction, in which the hole current can be neglected, the SPICE parameter τ_T (transit time) can be found from the above to be

$$\tau_T = \frac{Q_s}{I_D} = \frac{L_n^2}{2D_n} = \frac{\tau_n}{2}$$

- For a regular np junction, τ_T depends on both τ_n and τ_p .
- Similar results apply for the p -side of the junction.

3 Radiant Signal Sensors





When light hits an interface between two materials with different indexes of refraction, it can:

- be reflected
- be refracted (transmitted)

depending on the angle of incidence.

Eye sensitivity to electromagnetic radiation (light).

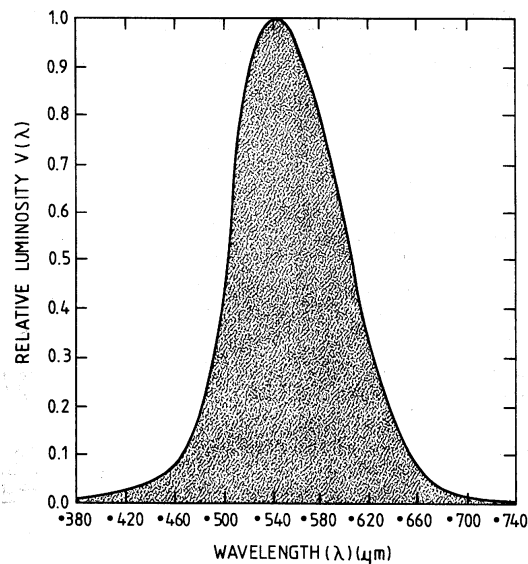


Fig. 2.4 The photopic-eye response [2.24].

3.1 Adsorption

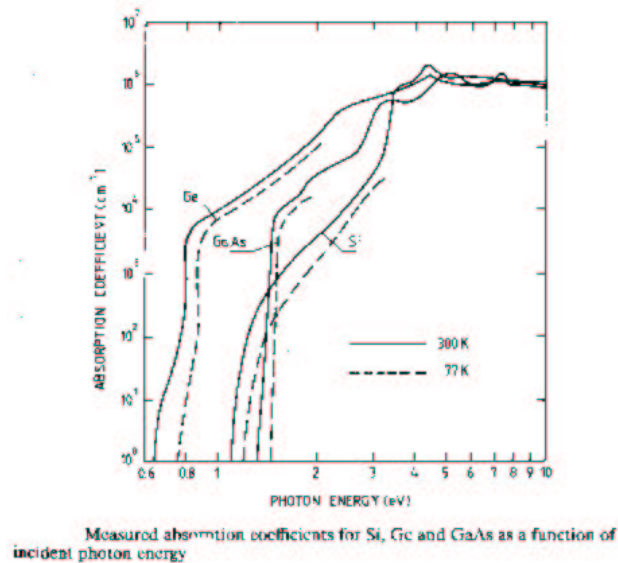
Once light is transmitted into the material,

- it can be absorbed to produce hole-electron pairs (assuming a semiconductor).

- to be absorbed, the photon energy must be larger than the energy gap.

$$\lambda_{max} = \frac{hc}{E_g}$$

where λ_{max} is the maximum wavelength, c is the speed of light, and E_g is the semiconductor energy gap. For Si, $E_g = 1.1\text{eV}$ and $\lambda_{max} = 1.1\mu\text{m}$.



- Below this wavelength light is absorbed progressively as the photons penetrate the material.

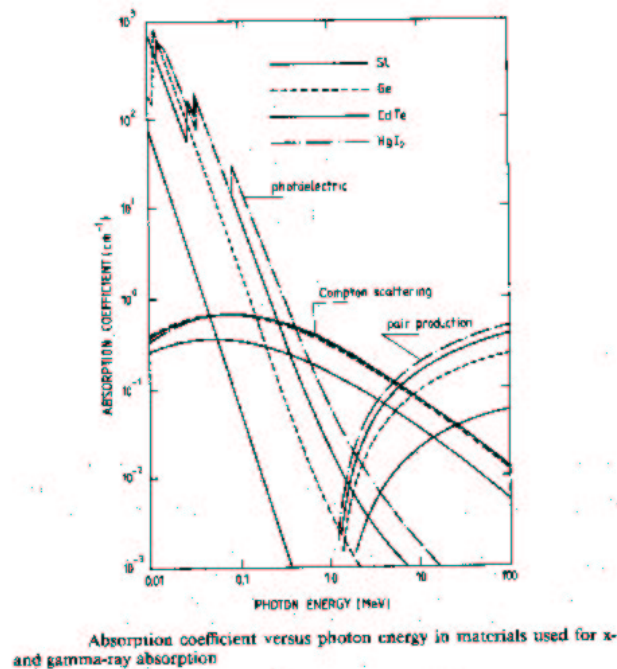
- absorption law:

$$\Phi(x) = \Phi_0 \exp(-\alpha x)$$

where Φx is the photon flux density ($photons/cm^2$), Φ_0 is the surface value and α is the absorption coefficient.

- If the photon energy is much larger than E_g , (nuclear radiation, x-rays) the interaction can be of three types
 - Photoelectric effect
 - * dominant at low photon energy, up to 100keV.
 - * photon is absorbed by a single electron.
 - * high-energy free electrons can be produced.
 - * excited electron can produce other hole-electron pairs (quantum efficiency larger than 1).
 - Compton process
 - * dominant from 100keV to 1 MeV.

- * original photons are deflected and decreased in energy
- * deflected photon can then be reabsorbed by the photoelectric effect, be deflected again or escape from the crystal.
- Electron-positron pair creation
 - * requires 1.022 MeV
 - * original photon disappears; excess energy is transferred to the electron-positron pair.
 - * electron and positron lose their energy through collisions, and eventually come to rest. Positron annihilates.



3.2 Photo-conductors

Photo-conductors consist of a slab of semiconductor with ohmic contacts at both ends. When photons interact with the semiconductor, hole-electron pairs are created either by direct transitions between valence and conduction band states, or by transitions that involve forbidden-band states. The change in conductivity produced by the extra carriers provide the signal that is measured.

3.3 Solar Cells and Photo-diodes

- A photo-diode is shown slide 7.
- Photo-diode's dark I-V characteristic is similar to a regular diode.
- Photo-diodes are operated in reverse bias; dark current is the leakage current.

- When exposed to light, the photo-diode's current, or *photo-current* I_{photo} is proportional to the intensity of the photon flux with energy larger than E_g .
- See load-line in slide 7. If $I_{photo}R > V_R$ the the diode becomes forward biased. Thus the maximum useful photo-current is V_R/R .
- Slide 8 shows a solar cell.
- Solar cells operate in forward bias.
- Build-in voltage collects excess carriers generated in the depletion region by incident photons. See slide 9.
- The voltage across the solar cell is positive (forward bias) but the photo-current is negative, since the excess electrons flow through the load to recombine with the excess holes. Thus the photo-current can never become positive.
- Photo-current sign indicated that the device works as a generator.
- Maximum delivered power is $P_d = |V_{DO} I_{DO}|$.
- To maximize photo-diode sensitivity, the depletion region should be as large as possible → use very low doping level. A region with very low doping level, almost intrinsic, is created between P and N regions. The diode is called *PIN photo-diode*.
- To provide fast response, the photo-diode's area is minimized while at the same time maximizing the width of the "I" region.
- Solar cells use a large area to maximize the current generated by the light.
- Silicon is used in solar cells; impurity states are created to facilitate the valence-band to conduction-band transition.

3.3.1 Continuity equation for excess minority carriers

- The continuity equation becomes:

$$\frac{\partial n_p}{\partial t} = \frac{1}{q} \frac{\partial j_n}{\partial x} + g_n(x) - r_n$$

where r_n is the recombination rate, $\frac{n_p(x) - n_{pe}}{\tau_n}$, and g_n is the *external generation rate* due to the absorption of light. Similarly for holes in the n-type side:

$$\frac{\partial p_n}{\partial t} = \frac{1}{q} \frac{\partial j_p}{\partial x} + g_p(x) - r_p$$

- Continuity equation for excess majority carriers:

n-type side:

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial j_n}{\partial x} + g_n(x) - \frac{n(x) - n_e}{\tau_n}$$

p-type side:

$$\frac{\partial p}{\partial t} = \frac{1}{q} \frac{\partial j_p}{\partial x} + g_p(x) - \frac{p(x) - p_e}{\tau_p}$$

- Carrier generation in the depletion layer:

$$I_{photo-d} = qA_J g_n w_d$$

- Carrier generation in the neutral regions

- far from the PN junction, generated carriers recombine:

$$g_n = r_n = \frac{n_p - n_{pe}}{\tau_n}$$

and the excess carrier concentration is $n_p - n_{pe} = \tau_n g_n$.

- near the reversed biased junction, $n_p(w_p) = n_{pe} \exp(V_D/V_t) \approx 0$. Thus a concentration gradient exist.
- for a dark photo-diode, this concentration gradient is small and leads to the leakage current I_s .
- carrier generation increases the carrier concentration in the neutral region to $n_p = n_{pe} + \tau_n g_n$, thus increasing the concentration gradient.
- a linear approximation to this gradient leads to the diffusion equation

$$\frac{\partial n_p}{\partial x} \approx \frac{n_p}{L_n} = \frac{n_{pe} + \tau_n g_n}{L_n}$$

- this leads to the following result:

$$\begin{aligned} I_n &= A_J j_{photo-n} \\ &= A_J q D_n \frac{\partial n_p(x)}{\partial x} \\ &= I_{S-n} + \underbrace{q A_J D_n \frac{g_n \tau_n}{L_n}}_{I_{photo-n}} \end{aligned}$$

- Using $L_n^2 = D_n \tau_n$,

$$I_{photo-n} = q A_J L_n g_n$$

- Similarly, for the p-type region,

$$I_{photo-n} = qA_J L_p g_p$$

- For uniform carrier generation, $g_n = g_p$ and

$$I_{photo} = qA_J g_n (w_d + L_n + L_p)$$

- The drift photo-current, $I_{photo-d}$, responds very fast to changes in illumination, while the diffusion photo-current, $I_{photo-n}$, is limited by the rate of establishing the concentration profiles.
- Because of this, for fast photo-detectors, it is desirable to have $w_d \gg L_n + L_p$; thus the PIN structure.

4 Lasers

- Photons obey Bose-Einstein statistics.
- The probability that an atom emits a photon with particular energy $h\nu$ is increased by the factor $(n + 1)$ if there are already n photons with this energy.
- If a PN junction is confined between two parallel mirrors and photons of appropriate energy are introduced, the probability of electron-hole recombination with the consequent emission of photons with the same energy is enhanced. This is called *stimulated emission*.
- To get light out of this rudimentary laser, one of the mirrors is made partially transparent.
- Concentration of electrons in the conduction band must be maintained at a high level. For this the forward bias current must be maintained above the *threshold level*.
- The fact that the concentration of injected minority electrons into the P-type region must be substantially higher than the equilibrium level is called *population inversion*.
- Necessary factors for stimulated recombination of an electron-hole pair:
 1. existence of photons with energy $h\nu$
 2. existence of an electron at an energy level E_2 in the conduction band
 3. existence of an electron at an energy level E_1 in the valence band, with $E_2 - E_1 = h\nu$.
- See slide.
- Emitted photons can be reabsorbed by the photoelectric effect. For stimulated emission, the rate of stimulated recombination must exceed the rate of photoelectric carrier generation.
- This condition can be translated in the requirement that

$$E_{FN} - E_{FP} > h\nu = E_2 - E_1$$