

P-N Junctions

Lecture Notes for INEL 6055

Manuel Toledo Quiñones
Assistant Professor
ECE Department
University of Puerto Rico
Mayaguez, Puerto Rico

14th March 2005

1 Fermi Energy

It was established previously that a crystal will exhibit bands of allowed energy states. For semiconductors these bands are separated by energy gaps in which no states exist. The Fermi-Dirac distribution

$$f = \frac{1}{1 + e^{\frac{E - E_f}{kT}}}$$

relates the density of states N with the filled state density, n by

$$n = N \times f$$

at a given energy E for an isolated system.

The density of vacant states is given by

$$v = N \times (1 - f)$$

Now consider what happens if two such systems are brought together and become a single system. Initially, a non-equilibrium situation exists and electrons from one material transfer to the other material. Eventually this process is balanced by another that has the opposite effect. Labeling the systems with the subscripts 1 and 2, we expect the transfer probability from material 1 into material 2 to be proportional to the density of filled states in 1 and the density of

vacant states in 2 at a given energy E

$$n_1 \times v_2$$

At equilibrium this process is balanced by the equivalent process going from 2 into 1;

$$n_1 \times v_2 = n_2 \times v_1$$

which can be expressed as

$$N_1 \times f_1 \times N_2(1 - f_2) = N_2 \times f_2 \times N_1(1 - f_1)$$

Canceling the density of states, which are common to both sides, and rearranging we get that

$$\frac{f_1}{1 - f_1} = \frac{f_2}{1 - f_2}$$

This requires the Fermi energy to assume the same, constant, value throughout the whole system.

2 Reverse-biased Junctions

A p-n junction is built by forming adjacent p- and n-type regions. The interface between regions is called the *pn junction*. At the junction there are carrier concentration gradients for both electrons and holes. This gives place to a diffusion current across the junction. The holes that

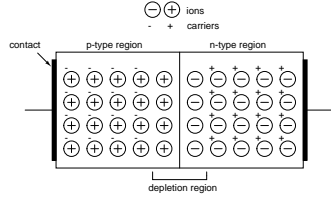


Figure 1: Sketch of a junction diode.

diffuse across the junction become excess minority carriers in the n-type region. Similarly, diffused electrons are excess minority carriers in the p-type region. These minority carriers recombine as they reach deeper into the other type region, leaving the device depleted of majority carriers near the junction. This gives place to the *depletion region*, shown in figure 1.

When the excess minority carriers recombine, fixed ions remain in the depletion region. These ions generate an electric potential called the *build-in voltage*. Diffusion across the junction continues until the build-in voltage becomes large enough to prevent it.

2.1 Build-in voltage

As excess carriers recombine near the junction, fixed ions remain. The electric charge represented by these ions give place to an electric field that cause the energy bands to bent. When the bending is enough to produce a drift current that cancels the diffusion current, the system reaches equilibrium. The Fermi level is constant throughout a system in thermal equilibrium, as shown in figure 2.

The voltage drop across the depletion region is said to form a *barrier*. This internal voltage gives place to a drift current I_S across the junction. An equilibrium is reached when the diffusion current due to the concentration gradient equals the drift current due to the build-in voltage. At equilibrium

$$I_D = I_S$$

From chapter 1 we know that for $e^{-\frac{E_F - E_V}{kT}} \gg$

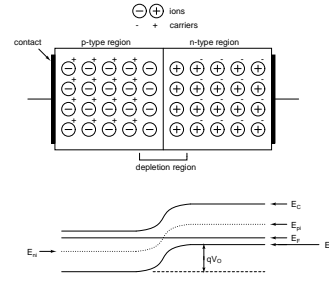


Figure 2: Electronic bands at equilibrium.

1,

$$p = N_V e^{-\frac{E_F - E_V}{kT}}$$

Let the hole concentration for intrinsic material be $p = p_i$, and the corresponding Fermi energy E_{pi} . Using the formulae developed in chapter 1 for intrinsic and extrinsic materials,

$$\begin{aligned} p_i &= N_V e^{-\frac{E_{pi} - E_V}{kT}} \\ N_A &= N_V e^{-\frac{E_F - E_V}{kT}} \\ \frac{N_A}{p_i} &= \frac{e^{-\frac{E_F - E_V}{kT}}}{e^{-\frac{E_{pi} - E_V}{kT}}} \\ &= e^{\frac{E_{pi} - E_F}{kT}} \end{aligned}$$

$$E_{pi} = E_F + kT \ln \left(\frac{N_A}{p_i} \right) \quad (1)$$

Similarly, for the n-type side, calling the intrinsic material electron concentration $n = n_i$ and the Fermi energy E_{ni} , we get that

$$\begin{aligned} n_i &= N_C e^{-\frac{E_C - E_{ni}}{kT}} \\ N_D &= N_C e^{-\frac{E_C - E_F}{kT}} \\ \frac{N_D}{n_i} &= \frac{e^{-\frac{E_C - E_F}{kT}}}{e^{-\frac{E_C - E_{ni}}{kT}}} \\ &= e^{\frac{E_F - E_{ni}}{kT}} \end{aligned}$$

and

$$E_{ni} = E_F - kT \ln \left(\frac{N_D}{n_i} \right) \quad (2)$$

At both sides of the junction, the bands will bent by qV_0 to make the Fermi level the same, as shown in figure 2. To find the build-in potential V_0 , observe that

$$qV_0 = E_{pi} - E_{ni}$$

After subtracting equation 2 from 1, dividing by q and equating $\frac{kT}{q}$ to V_T , the build-in voltage is found to be

$$V_0 = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

2.2 Externally-biased Junction

An external voltage can force carriers to cross the junction. For that the positive terminal must be connected to the p-type region and the negative terminal to the n-type region. The diode is said to be *forward-biased*. Because of this, the n- and p-type regions are called *cathode* and *anode*, respectively.

If the external voltage makes the cathode more positive than the anode, then the diode is *reverse-biased*.

2.3 Reverse-biased Junction Capacitance

The capacitance of a reverse-biased junction can be determined by modeling the situation as a parallel capacitor, with the depletion region playing the role of the dielectric. This *depletion-layer capacitance* can thus be found from

$$C_d = \epsilon_s \frac{A}{w_d}$$

where ϵ_s is the semiconductor permittivity ($11.8 \times \epsilon_0 = 11.8 \times 8.85 \times 10^{-12}$ F/m for Silicon), A is the junction area and w_d is the depletion-layer width.

The width depends on the dopant profile. Two limiting cases admit a closed solution and are discussed in the next two sections.

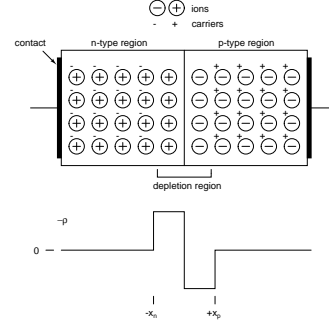


Figure 3: Sketch of an abrupt junction.

2.3.1 Abrupt Profile

If the dopant profile is such that at one side of the junction there are no donors and at the other side there are no acceptors, the junction is said to be **abrupt**. A sketch is shown in figure 3. The charge accumulated in each side of the depletion region can be expressed as

$$q_p = qN_A x_p A = q_n = qN_D x_n A$$

Poisson's equation,

$$\frac{d^2 \varphi}{dx^2} = -\frac{\rho}{\epsilon_s}$$

must be integrated twice to find the electric potential φ . It assumes the form

$$\frac{d^2 \varphi}{dx^2} = \begin{cases} 0 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_s} & \text{if } -x_n < x \leq 0 \\ +\frac{qN_A}{\epsilon_s} & \text{if } 0 < x < x_p \\ 0 & \text{if } x \geq x_p \end{cases}$$

Integrating once yields,

$$\frac{d\varphi}{dx} = \begin{cases} C_1 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_s} x + C_2 & \text{if } -x_n < x \leq 0 \\ +\frac{qN_A}{\epsilon_s} x + C_3 & \text{if } 0 < x < x_p \\ C_4 & \text{if } x \geq x_p \end{cases}$$

The solution must satisfy the following boundary conditions:

- The electric field E must be continuous
- $E(-x_n) = -\frac{d\varphi}{dx} = 0$
- $E(x_p) = -\frac{d\varphi}{dx} = 0$

Thus, after some algebra,

$$\frac{d\varphi}{dx} = \begin{cases} 0 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}(x + x_n) & \text{if } -x_n < x \leq 0 \\ +\frac{qN_A}{\epsilon_S}(x - x_p) & \text{if } 0 < x < x_p \\ 0 & \text{if } x \geq x_p \end{cases}$$

Integrating once more,

$$\varphi = \begin{cases} C_5 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}\left(\frac{x^2}{2} + x_n x\right) + C_6 & \text{if } -x_n < x \leq 0 \\ +\frac{qN_A}{\epsilon_S}\left(\frac{x^2}{2} - x_p x\right) + C_7 & \text{if } 0 < x < x_p \\ C_8 & \text{if } x \geq x_p \end{cases}$$

The boundary conditions for the electric potential φ are

- φ is constant for $x \leq -x_n$ and $x \geq x_p$
- $\varphi(-x_n) = V_0$
- $\varphi(x_p) = 0$ (defined as ground)

After applying these boundary conditions,

$$\varphi = \begin{cases} V_0 & \text{if } x \leq -x_n \\ V_0 - \frac{qN_D}{2\epsilon_S}(x + x_n)^2 & \text{if } -x_n < x \leq 0 \\ +\frac{qN_A}{2\epsilon_S}(x - x_p)^2 & \text{if } 0 < x < x_p \\ 0 & \text{if } x \geq x_p \end{cases}$$

Requiring continuity at $x = 0$

$$V_0 - \frac{qN_D}{2\epsilon_S}x_n^2 = +\frac{qN_A}{2\epsilon_S}x_p^2 \quad (3)$$

Since the amount of ionic charge is equal on both sides of the junction,

$$qx_p N_A = qx_n N_D$$

This can be expressed as

$$x_p = \frac{N_D}{N_A}x_n$$

and

$$W_{dep} = x_n + x_p = \left(1 + \frac{N_D}{N_A}\right)x_n = \frac{N_A + N_D}{N_A}x_n$$

Substituting into equation 3 and rearranging,

$$\begin{aligned} V_0 &= \frac{q}{2\epsilon_S}(N_D x_n^2 + N_A x_p^2) \\ &= \frac{qN_D}{2\epsilon_S}\left(x_n^2 + \frac{N_A}{N_D}x_p^2\right) \\ &= \frac{qN_D}{2\epsilon_S}x_n^2\left(\frac{N_A + N_D}{N_A}\right) \end{aligned}$$

and

$$x_n = \sqrt{\frac{2\epsilon_S}{qN_D} \frac{N_A}{N_D + N_A} V_0}$$

The depletion region charge is given by

$$\begin{aligned} q_j &= N_D q A x_n \\ &= q A \sqrt{\frac{2\epsilon_S}{q} \frac{N_A N_D}{N_D + N_A} V_0} \end{aligned}$$

For a reverse-biased junction to which an external voltage V_{bias} is applied, replace V_0 with $V_0 + V_{bias}$

$$q_j = q A \sqrt{\frac{2\epsilon_S}{q} \frac{N_A N_D}{N_D + N_A} (V_0 + V_{bias})}$$

Now apply the definition of capacitance,

$$C = \frac{dq}{dV}$$

to the junction charge using the applied voltage, V_{bias} as the voltage. Thus,

$$C_j = \frac{dq_j}{dV_{bias}} \big|_{V_{bias}=V_R}$$

where V_R is the applied bias voltage.

$$C_j = \frac{A}{2} \sqrt{2q\epsilon_S \frac{N_A N_D}{N_D + N_A} \frac{1}{V_0 + V_R}}$$

In terms of

$$C_{j0} = A \sqrt{\frac{q\epsilon_S}{2} \left(\frac{N_A N_D}{N_A + N_D}\right) \frac{1}{V_0}}$$

the junction capacitance becomes

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

Notice that these results can be expressed as

$$C_j^2 = \frac{qA^2\epsilon_S}{2} \frac{N_A N_D}{N_D + N_A} \frac{1}{V_0 + V_R}$$

or

$$V_R = \frac{qA^2\epsilon_S}{2} \frac{N_A N_D}{N_D + N_A} \frac{1}{C_j^2} - V_0$$

Thus if we measure C_j as we change V_R and plot V_R versus $1/C_j^2$, we should get a straight line with intercept $-V_0$ and slope

$$\frac{qA^2\epsilon_S}{2} \frac{N_A N_D}{N_D + N_A}$$

This expression is, of course, valid only for an abrupt junction.

2.4 Linear Junction

The junction is said to be linear if the dopant concentration profile varies linearly from one side of the junction to the other. The charge density or for this case can be expressed as

$$\rho = -ax$$

The Poissons equation becomes

$$\frac{d^2\varphi}{dx^2} = \frac{a}{\epsilon_S}x$$

Integrating once,

$$E = -\frac{d\varphi}{dx} = \frac{a}{\epsilon_S} \frac{1}{2}x^2 + C_1$$

This solution must be continuous. Also at $x = x_p = +\frac{w}{2}$, $E = 0$. Thus

$$C_1 = -\frac{a}{\epsilon_S} \frac{1}{2}x_p^2$$

and

$$\frac{d\varphi}{dx} = \frac{a}{2\epsilon_S}(x^2 - x_p^2)$$

Integrating once more,

$$\varphi = \frac{a}{2\epsilon_S} \left(\frac{1}{3}x^3 - x_p^2x + C_2 \right)$$

Since φ must vanish at x_p ,

$$\begin{aligned} C_2 &= -\frac{a}{2\epsilon_S} \left(\frac{1}{3}x_p^3 - x_p^2x_p \right) \\ &= \frac{a}{3\epsilon_S}x_p^3 \end{aligned}$$

and

$$\varphi = \frac{a}{2\epsilon_S} \left(\frac{1}{3}x^3 - x_p^2x + \frac{2}{3}x_p^3 \right)$$

At $x = -x_n = -\frac{w}{2}$, $\varphi = V_0 + V_{bias}$. Using $x_p = \frac{w}{2}$,

$$\begin{aligned} V_0 + V_{bias} &= \frac{a}{2\epsilon_S} \left(-\frac{1}{3}x_n^3 - x_p^2x_n + \frac{2}{3}x_p^3 \right) \\ &= \frac{a}{2\epsilon_S} \left(-\frac{1}{3}\left(\frac{w}{2}\right)^3 + \left(\frac{w}{2}\right)^3 + \frac{2}{3}\left(\frac{w}{2}\right)^3 \right) \\ &= \frac{a}{2\epsilon_S} \left(-\frac{1}{3}\frac{w^3}{8} + \frac{w^3}{8} + \frac{2}{3}\frac{w^3}{8} \right) \\ &= \frac{a}{12\epsilon_S}w^3 \end{aligned}$$

$$w^3 = \frac{12\epsilon_S}{a} (V_0 + V_{bias})$$

The junction capacitance is thus given by

$$\begin{aligned} C_j &= \frac{A\epsilon_S}{w} \\ &= A\epsilon_S \sqrt[3]{\frac{a}{12\epsilon_S}} \frac{1}{\sqrt[3]{V_0 + V_{bias}}} \\ &= A \sqrt[3]{\frac{a\epsilon_S^2}{12V_0}} \frac{1}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}} \\ &= \frac{C_{j0}}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}} \end{aligned}$$

where

$$C_{j0} = A \sqrt[3]{\frac{a\epsilon_S^2}{12V_0}}$$

3 Parameter Estimation

3.1 General Problem Formulation

Given a mathematical description of a process, or model

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, \mathbf{k})$$

where

- $\mathbf{x} = [x_1 x_2 x_3 \dots x_n]^t$ represent n manipulable or independent variables, independently adjustable by the experimenter.
- $\mathbf{y} = [y_1 y_2 y_3 \dots y_m]^t$ represent an m -dimensional vector of dependent variables; being measured in the experiment.
- $\mathbf{k} = [k_1 k_2 k_3 \dots k_p]^t$ represent a p -dimensional vector of parameters that are unknown.
- $\mathbf{f} = [f_1 f_2 f_3 \dots f_m]^t$ is an m -dimensional function of known form. These are the actual model equations.

find \mathbf{k} such that the error between l measurements and model predictions is minimum in some well defined sense.

3.1.1 Pseudo-inverse

Consider a model consisting of a linear equation such as

$$y = p_1 x_1 + p_2 x_2$$

where y is a measurable quantity, x_1 and x_2 are manipulable variables, and p_1 and p_2 are parameters.

We want to obtain estimates of the parameters based on measurements. Assume that n experiments are performed, such that results $y^1, y^2 \dots y^n$ are available. For such experiments the manipulable variables assumed values $(x_1^1, x_2^1), (x_1^2, x_2^2), \dots, (x_1^n, x_2^n)$.

If we combine the measures y 's with the estimates from our model,

$$\begin{aligned} y^1 &= p_1 x_1^1 + p_2 x_2^1 \\ y^2 &= p_1 x_1^2 + p_2 x_2^2 \\ &\dots \\ y^n &= p_1 x_1^n + p_2 x_2^n \end{aligned}$$

is obtained.

This can be expressed in matrix form,

$$\mathbf{Y} = \mathbf{X}\mathbf{p} \quad (4)$$

where

$$\mathbf{Y} = [y^1 y^2 \dots y^n]^t$$

\mathbf{X} is a matrix given by

$$\mathbf{X} = \begin{bmatrix} x_1^1 & x_2^1 \\ x_1^2 & x_2^2 \\ \dots & \dots \\ x_1^n & x_2^n \end{bmatrix}$$

and \mathbf{P} is a vector of parameters,

$$\mathbf{p} = [p_1 p_2 \dots p_n]^t$$

To solve equation 4, multiply from the left by \mathbf{X}^t ,

$$\mathbf{X}^t \mathbf{Y} = \mathbf{X}^t \mathbf{X} \mathbf{p} = \mathbf{M} \mathbf{p}$$

to get a square matrix \mathbf{M} . Now we can multiply from the left by the inverse of \mathbf{M} , called the *pseudo-inverse* of \mathbf{X} , and get

$$\mathbf{p} = \mathbf{M}^{-1} \mathbf{X}^t \mathbf{Y}$$

It can be shown that a minimal-least-square-error estimate of \mathbf{p} obtained in this way.

3.1.2 Linear Regression

For the n experimental data described above, we can write an equation for the square of the difference between model predictions and measured data:

$$S = \sum_{i=1}^n (y^i - p_1 x_1^i + p_2 x_2^i)^2$$

A *maximum likelihood estimate* of the parameters p_1 and p_2 is obtained if S is minimized with respect to the parameters. This means that

$$\frac{\partial S}{\partial p_1} = -2 \sum_{i=1}^n ((y^i - p_1 x_1^i + p_2 x_2^i)) x_1^i = 0$$

$$\frac{\partial S}{\partial p_2} = -2 \sum_{i=1}^n ((y^i - p_1 x_1^i + p_2 x_2^i)) x_2^i = 0$$

Equivalently,

$$\begin{aligned} p_1 \sum_{i=1}^n (x_1^i)^2 + p_2 \sum_{i=1}^n x_2^i x_1^i &= \sum_{i=1}^n n y^i x_1^i \\ p_1 \sum_{i=1}^n x_1^i x_2^i + p_2 \sum_{i=1}^n (x_2^i)^2 &= \sum_{i=1}^n y^i x_2^i \end{aligned}$$

By defining

$$\begin{aligned} a_{11} &= \sum_{i=1}^n (x_1^i)^2 \\ a_{12} &= \sum_{i=1}^n x_2^i x_1^i \\ a_{21} &= \sum_{i=1}^n x_1^i x_2^i \\ a_{22} &= \sum_{i=1}^n (x_2^i)^2 \\ b_1 &= \sum_{i=1}^n y^i x_1^i \\ b_2 &= \sum_{i=1}^n y^i x_2^i \end{aligned}$$

and

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\mathbf{p} = (p_1 p_2)^t$$

$$\mathbf{b} = (b_1 b_2)^t$$

we can express the solution to the above matrix equation as

$$\mathbf{p} = \mathbf{A}^{-1} \mathbf{b}$$

3.1.3 Junction Capacitance Parameter

Both the abrupt and linear junction models yield results that can be expressed as

$$C_j = C_{j0} \left(1 + \frac{V_R}{V_0} \right)^{-m} \quad (5)$$

with $m = 1/3$ for the linear junction and to $1/2$ for the abrupt. If the capacitance C_j is measured several times, estimates for C_{j0} , V_0 and m could be obtained.

To use the previously described methods, it is necessary to modify the above expression for C_j so that it becomes linear. Recognizing the fact that any capacitance measurement will include a parasitic capacitance C_p , we can take logarithms on both sides of equation 5 to get

$$\log(C_j - C_p) = \log(C_{j0}) - m \times \log \left(1 + \frac{V_R}{V_0} \right)$$

This is still not a linear equation due to the presence of V_0 and C_p . To apply the estimation methods described above, it is necessary to assume values for these parameters and then perform the estimation. Rather than relying on a single guess, the fitting procedure can be done several times with different values of V_0 and C_p . The square of the difference between experimental and calculated values can be found each time a new pair of values is tried, and then the pair that yields the lowest difference be chosen.

Notice that both the pseudo-inverse and linear regression methods can be used on a the model of the form

$$p_1 x_1 + p_2$$

such as the one obtained for the junction, simply by replacing all x_2 by 1 in the above formulae.

4 Diode Current

See slides 5 and 6.

Forward voltage injects excess minority carriers at the depletion region boundaries. Excess minority carriers diffuse due to the concentration gradient.

Electrons drift through the depletion region, then diffuse through the neutral regions.

Assumptions:

- No recombination in the depletion region.
- Negligible voltage drop in the neutral regions.

Diffusion equation for electrons: linear approximation.

$$J_n = qD_n \frac{\partial n_p(x)}{\partial x} \approx \frac{n_{pe} - n_p(w_p)}{L_n}$$

where

- n_{pe} is equilibrium concentration of electrons in p-type side.
- L_n is the *diffusion length* of electrons.

Fermi-Dirac Distribution:

$$f = \frac{1}{1 + e^{(E-E_F)/kT}} \approx e^{-\frac{E-E_F}{kT}}$$

Energy barrier when voltage V is applied: $q(V_0 - V_D)$. V_0 is the build-in voltage.

Excess electron concentration at w_p

$$n_p(w_p) \propto \int_{q(V_0-V_D)}^{\infty} e^{-\frac{E-E_F}{kT}} dE$$

After integration, we get that

$$n_p(w_p) \propto e^{\frac{qV_D}{kT}}$$

and thus

$$n_p(w_p) = C e^{\frac{qV_D}{kT}}$$

Since for $v_D = 0$, $n_p(w_p) = C = n_{pe}$,

$$n_p(w_p) = n_{pe} e^{\frac{qV_D}{kT}}$$

The electron current density becomes

$$\begin{aligned} J_n &= qD_n \frac{n_{pe} - n_p(w_p)}{L_n} \\ &= qD_n \frac{n_{pe}(1 - e^{\frac{qV_D}{kT}})}{L_n} \\ &= \frac{qD_n n_{pe}}{L_n} (1 - e^{\frac{qV_D}{kT}}) \end{aligned}$$

Using $n_{pe} = \frac{n_i^2}{N_A}$, replacing L_n by w_{anode} because $L_n \gg w_{anode}$, and multiplying by the junction area A , we get the current due to electrons:

$$i_n = -\frac{qAD_n n_i^2}{N_A w_{anode}} (e^{\frac{qV_D}{kT}} - 1)$$

where the fact that conventional current is opposite to electron flow was used.

The junction current due to both holes and electrons is

$$i_D = I_S (e^{\frac{qV_D}{kT}} - 1)$$

where

$$I_S = \frac{qAD_n n_i^2}{N_A w_{anode}} + \frac{qAD_p n_i^2}{N_D w_{cathode}}$$

4.1 Second-order Corrections

There is recombination in the depletion region. Carrier with energy below $q(V_0 - V_D)$ can penetrate a fraction of the depletion region, and can recombine there. Therefore, it is possible for an electron and a hole, each with energy $q\frac{V_0-V_D}{2}$ to reach the middle of the d.r. and recombine, producing a current proportional to $\frac{V_0-V_D}{2}$.

To take into account for recombination in the depletion-region, the diode equation is modified to

$$i_D = I_S (e^{\frac{qV_D}{\eta kT}} - 1)$$

where η is a parameter that take values between 1 and 2.

See slide 7.

The resistance due to the metal-semiconductor contacts and neutral regions are taken into account by adding a resistor r_S to the diode's model.

See slide 8.

Measurement of diode static parameters: see slide 10.

4.2 Stored-charge capacitance

For reverse-biased junctions, the depletion-layer capacitance dominates the dynamic response.

For forward-bias voltages large than about $\frac{V_0}{2}$ the stored-charge capacitance C_S is more important.

C_S is due to excess minority-carrier accumulation at the edge of the depletion region, which represent a stored charge Q_S .

See slide 11.

If a forward-bias is suddenly turned off, removing Q_S takes time. Current i_D does not drop to zero until Q_S is removed.

See slide 12.

This process of C_S discharge determines the time-response and high-frequency behavior of the diode.

SPICE model incorporates this effect by using a parameter called the *transit time* τ_t . $C_S = \tau_t \frac{di_D}{dV_{D0}}$.

4.3 Temperature Effects

Exponent in diode equation shows temperature dependence.

I_S is proportional to n_i^2 which have a strong temperature dependence.

Slide 13 shows the temperature dependence of a Silicon diode. This dependence can be approximated by a temperature coefficient of $-2mV/^\circ C$. When this happens a tunneling current is established.

See table 3.6 in textbook for temperature dependence of C_d .

5 Breakdown

5.1 Avalanche Breakdown

See Slides 14, 15 and 16.

Electrons move through the depletion region under reverse bias, Kinetic energy gained between collisions: $E_{kin} = -qEx$

- E : applied field
- x : distance between collisions

Average distance between collisions = scattering length = l_{sc}

Because $l_{sc} \ll w_d$, the width of the depletion region, many collisions happen before the electron crosses the depletion region.

Avalanche breakdown happens when $E_{kin} = -qEl_{sc} > E_{ionization}$, the *ionization energy* needed to create an electron-hole pair.

Temperature increases phonon scattering, reducing l_{sc} and thus increasing the field necessary for avalanche breakdown. The breakdown voltage exhibits a positive temperature coefficient.

Avalanche breakdown voltage depends on doping. Higher doping increases built-in voltage and reduces breakdown voltage.

Avalanche occurs at voltages larger than 6V.

5.2 Tunneling Breakdown

Electron wave-function penetrates across the depletion region.

Reverse bias produces band bending, which reduces depletion region width. See slide 16 right-hand side figure (a) and (b).

As bending increase, conduction band in the n-type side falls below valence band in the p-type side.

When this happens a tunneling current is established.

Tunneling is effective due to the large concentration of electrons in the valence band of the p-type side of the junction. Even a very small probability can cause a significant tunneling current.

Very narrow depletion regions are required for tunneling to be effective. This require high doping levels.

For tunneling diodes the t.c. is negative.

Tunneling occurs at lower voltages than avalanche, typically. $\approx 6V$.

6 Metal-Semiconductor Contact

Used to construct *Schottky diodes*.

See slide 17.

Energy-band diagrams are drawn so that the vacuum level match.

Electrons in solid have negative energy with respect to that of an electron in vacuum.

The energy needed to remove an electron from the metal is the *work function*, $q\phi_m$, is the difference between the vacuum energy and the Fermi level.

See slide 18.

The Fermi level of an n-type semiconductor is higher than the metal's. Conduction band electrons are at a higher energy in the semiconductor than in the metal.

When the metal-semiconductor is formed, electrons from the semiconductor flow into the metal leaving behind ions and forming a depletion region.

Thermal equilibrium is reached when the Fermi levels are equal at the interface.

See slide 19.

A potential barrier equal to the difference between the Fermi levels is formed.

$$qV_0 = q\phi_m - q\phi_s$$

The barrier in the metal is

$$q\phi_B = q\phi_m - q\xi_s$$

where ξ_s is the *electron affinity* (difference in energy between bottom of the conduction band and vacuum) of the semiconductor.

The reverse bias current in a Schottky diode is

$$I_S = I_{S0} e^{-\frac{q\phi_B}{kT}}$$

where I_{S0} is a constant that depend on temperature. This quantity plays the same role than the saturation current in the junction diode.

The forward-bias current in the Schottky diode can be modeled with the same equation used for the junction diode:

$$I_D = I_S e^{\frac{V_D}{kT}}$$

the *build-in voltage* V_0 is smaller for Schottky diodes than for junction diodes. See slide 20 and 21..

In Schottky diodes all carriers are electrons. Thus there is no charge storage and consequently Schottky diodes are faster than junction diodes. The cost is a larger I_S .

In the case of heavily-doped n-type semiconductor, a very narrow energy barrier is formed. This barrier is not high enough to prevent electrons from crossing. Also the depletion region is very narrow and tunneling takes place. Since the barrier is not effective to stop electrons from crossing across the interface, an *Ohmic contact* is formed.

See slides 22 and 23.