Introduction

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1 Resistance

1.1 Basics

1.1.1 Integral Form of Ohm’s Law

- Ohm’s Law: $I = V/R$
- Integrated Circuit Resistor (fig. 1)

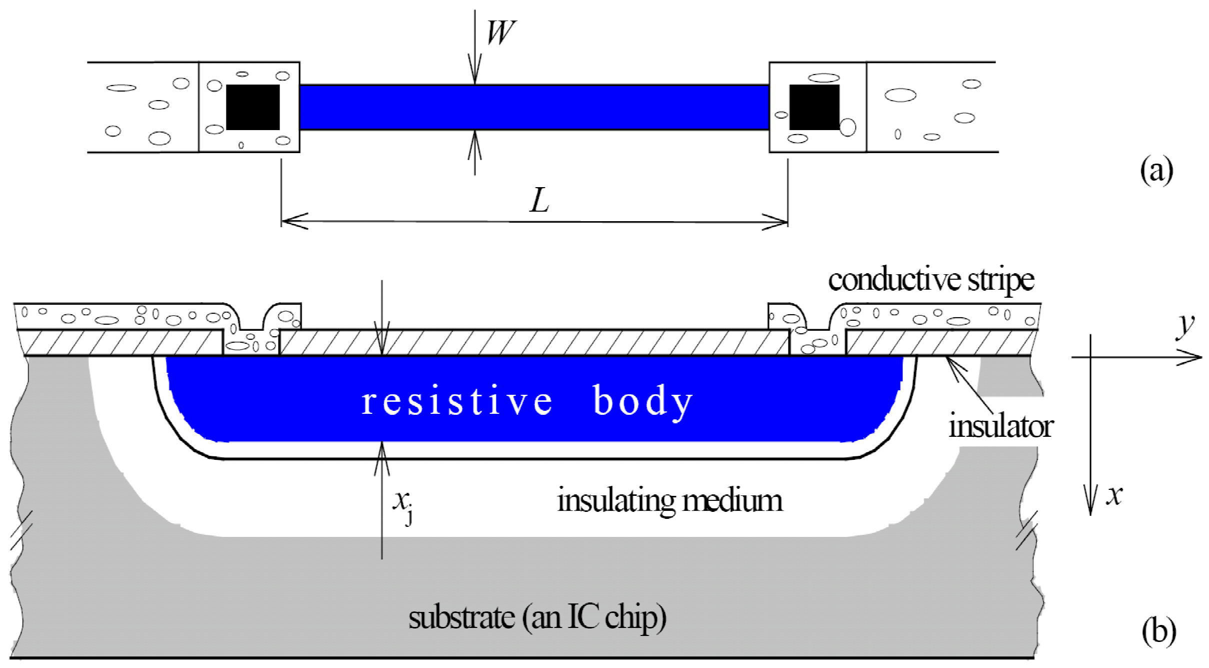


Figure 1: Textbook figure 1.2.

$$R = \rho \frac{L}{x_j W}$$

- ρ is the material's resistivity
- conductivity = $\sigma = \frac{1}{\rho}$
- Sheet resistance: $R_S = \frac{1}{\sigma x_j}$

$$R = R_S \frac{L}{W}$$

Sheet resistance is expressed in $\Omega/\square$.

- Example: For $R = 3.5k\Omega$, and a sheet resistance of $R_S = 200\Omega/\square$ and a feature size $W = 1\mu m$, use $L = 17.5\mu m$.
- The smaller W the better.

1.1.2 Integral vs. Differential Physical Quantities

- Integral = Average → like above form of Ohm's Law
- Differential = local → more accurate model.
- Conductivity varies with depth. See figure 2.
- Average value of conductivity is used as an approximation.

$$\bar{\sigma} = \frac{1}{x_j} \int_0^\infty \sigma(x) dx$$

- Average conductivity → no information about current distribution in resistor.

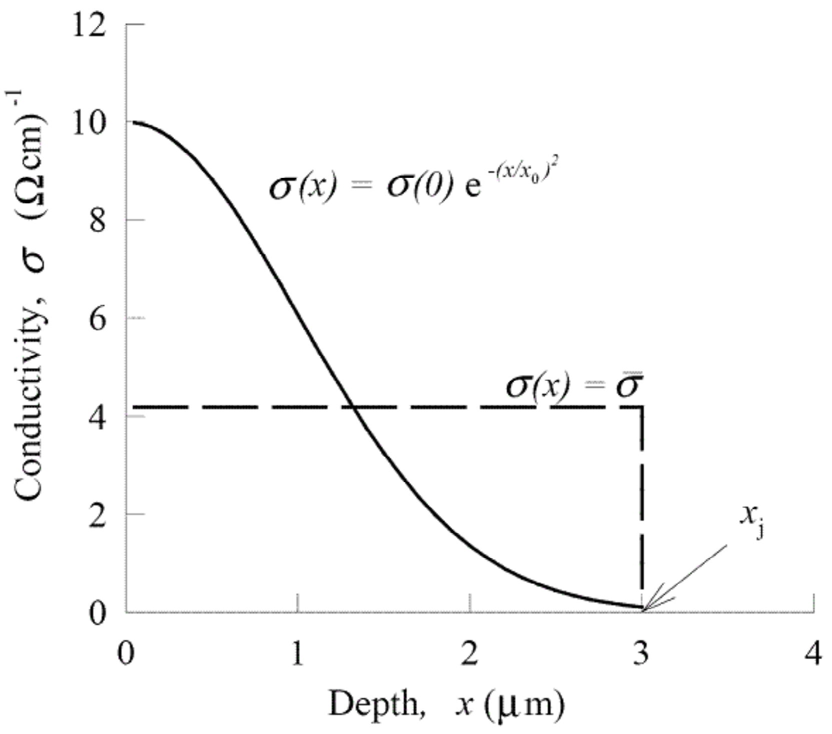


Figure 2: Typical conductivity profile.

- For fig. 2, using $x_0 = \sqrt{2}$, we can find the average conductivity:

$$\begin{aligned} \sigma(x) &= \sigma(0)e^{-(x/x_0)^2} \\ \bar{\sigma} &= \frac{10}{x_j}\int_0^{\infty} e^{-(x^2/2)}dx \\ &= \frac{10(\Omega - cm)^{-1}}{3\mu m} \times \sqrt{\pi/2}\mu m \\ &= 4.18(\Omega - cm)^{-1} \end{aligned}$$

  Integration was performed using what is known as Laplace integral.

- Using this device, to build a $100k\Omega$ resistor

$$\begin{aligned} R_S &= \frac{1}{\bar{\sigma}x_j} \\ &= \frac{1}{4.18(\Omega - cm)^{-1} \times 3\mu m} \end{aligned}$$

$$
\begin{aligned}
&= 797.4\Omega/\square \\
\frac{L}{W} &= \frac{R}{R_S} = \frac{100,100\Omega}{797.4\Omega} = 125
\end{aligned}
$$

For a feature size of $W = 1\mu m$, $L = 125\mu m$ is required.

1.1.3 Differential form of Ohm's Law

- The differential form of Ohm's Law is

$$\mathbf{j} = \sigma \mathbf{E}$$

  where $\mathbf{j}$ and $\mathbf{E}$ are vectors.

- $\mathbf{j}$ = *drift current* = current induced by an electric field.
- In terms of the *electric potential*, φ,

$$\mathbf{E} = -\nabla\varphi = -\frac{\partial\varphi}{\partial x}\hat{x} + \frac{\partial\varphi}{\partial y}\hat{y} + \frac{\partial\varphi}{\partial z}\hat{z}$$

- Three-dimensional drift current:

$$\mathbf{j} = -\sigma\nabla\varphi$$

- To handle multi-dimensional aspects of the problem (like the corner effect) requires knowledge of the electric potential distribution.

- Semiconductor devices are based on externally influencing the conductivity, σ.
- Two things directly influence the conductivity:
  - concentration of carriers.
  - mobility of carriers.
- The conductivity is proportional to the carrier concentration and the mobility.
- Example: Find the maximum current density and the terminal current for the resistor designed in the previous example if a voltage of 5V is applied. Neglect corner effects.

ANSWER: From our previous results, $L = 125\mu m$. Thus

$$E = \frac{V}{L} = \frac{5V}{125\mu m} = 40kV/m$$

Since

$$j_{max} = \sigma_{max} E$$

and from figure 2 $\sigma_{max} = 10(\Omega - cm)^{-1}$,

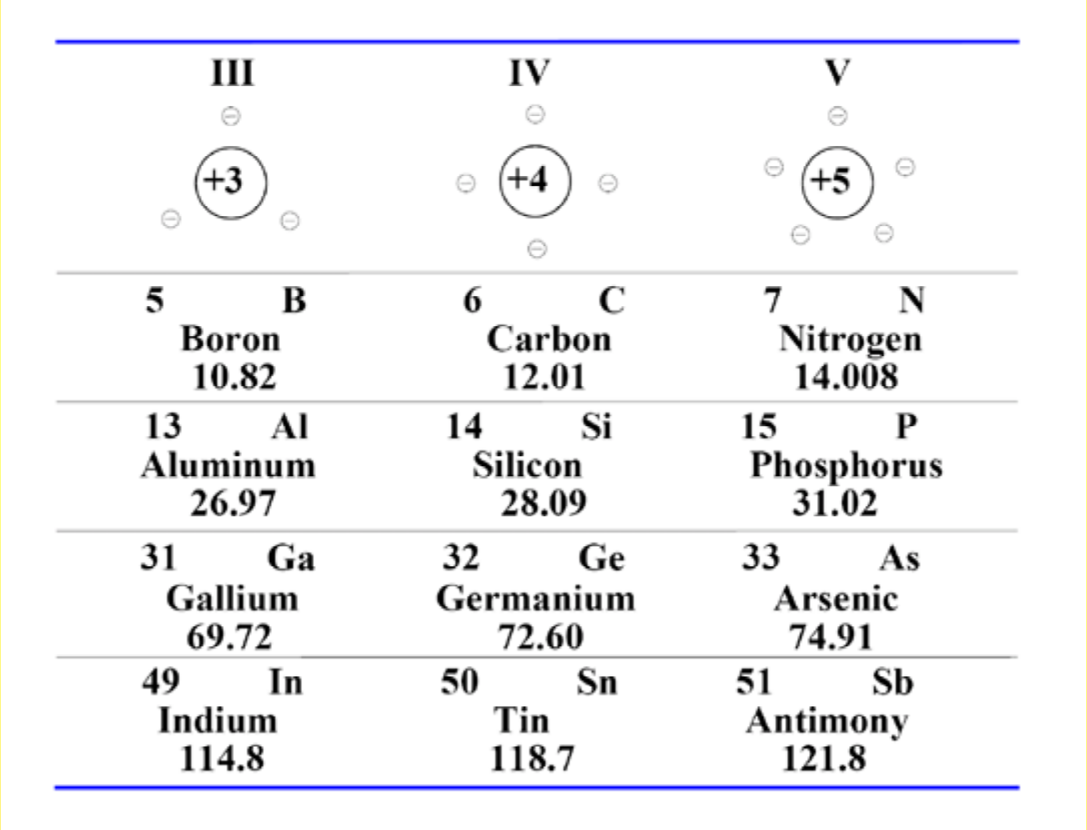
$$j_{max} = 4 \times 10^7 A/m^2$$

To find the terminal current, integrate j from 0 to x_j (or to ∞) and multiply by W,

$$I = W \int_0^\infty j(x)dx = W \frac{V}{L} \sigma(0) \int_0^\infty e^{(x/x_0)^2} dx$$

which yields a result of $50\mu A$.

2 Chemical-bond Model



III	IV	V
+3	+4	+5
5 B Boron 10.82	6 C Carbon 12.01	7 N Nitrogen 14.008
13 Al Aluminum 26.97	14 Si Silicon 28.09	15 P Phosphorus 31.02
31 Ga Gallium 69.72	32 Ge Germanium 72.60	33 As Arsenic 74.91
49 In Indium 114.8	50 Sn Tin 118.7	51 Sb Antimony 121.8

Figure 3: Elements used in semiconductor devices.

- Periodic table groups III to V are important. See figure 3.
- Two types of carriers: electrons and holes.
- Doping:
  - intrinsic: pure semiconductor.
    - Hole and electron concentrations are equal:

$$n = p = n_i$$

    - Conductivity:

$$\sigma = qn\mu_n + qp\mu_p$$

    - See typical numbers in table 1

	n_i (cm^{-3})	μ_n $(cm^2/(V-s))$	μ_p $(cm^2/(V-s))$
Si	1.02×10^{10}	1450	500
GaAs	2.1×10^{5}	8500	400

Table 1: Mobility and carrier concentration at 300K.

  - n-type - donors (group V atoms or impurities) are added.

$$n \approx N_D \gg p; \sigma \approx q\mu_n N_D$$

  - p-type semiconductors - acceptors (group III atoms) are added.

$$p \approx N_A \gg n; \sigma \approx q\mu_p N_A$$

- At **thermal equilibrium**, recombination rate equals carrier generation rate.

- The *Mass-action Law* is a consecuence of thermal equilibrium, and is expressed as

$$np = n_i^2$$

- If donors (acceptors) are added to the semiconductor, more holes (electrons) recombine with the majority carriers, i.e. the extra electrons (holes). As a consequence the mass-action law holds also for extrinsic semiconductors.

- Minority-carrier concentration:
  - N-type

$$p = n_i^2/N_D$$

  - P-type

$$n = n_i^2/N_A$$

- Example: Consider N-type Silicon sample for which $N_D = 10^{16} cm^{-3}$. Find: (i) the minority carrier concentration, and (ii) the samples conductivity.

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  ANSWER: (i) $p = \frac{n_i^2}{N_D}$; (ii) $\sigma = q\mu_n N_D$

- Example: Determine the resistivity of a Silicon crystal if $N_D = 10^{17} cm^{-3}$ and $N_A = 10^{16} cm^{-3}$. Use $\mu_n = 770 cm^2/V - s$.

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  ANSWER: The N-type semiconductor as been partially compensated by adding acceptors. As a consecuence the effective donor density is $n = N_D - N_A = 9 \times 10^{16} cm^{-3}$ and

$$\rho = \frac{1}{q\mu_n n} = 0.09\, \Omega - cm$$

- Example: A P-type Silicon sample with resistivity of $0.5\,\Omega - cm$ is illuminated by a flash of light that 2×10^{16} electron-hole pairs per cm^3. Determine (i) the electron concentration, and (ii) the change in resistivity caused by the flash of light.

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  ANSWER: (i)

  $$p = N_A = \frac{1}{\rho q \mu_p} = 2.5 \times 10^{16}$$

  $$n = \frac{n_i^2}{N_A}$$

  (ii) Excess carriers can not be neglected. Add them to find n and p and use

  $$\rho = \frac{1}{q(\mu_n n + \mu_p p})$$

3 Carrier Mobility

- Mobility directly determines conductivity.
- Model of hole and electron gas: charged spherical particles performing random thermal motion. Used to describe transport phenomena.
- Mass of isolated electron: constant $m_0 = 9.1 \times 10^{-31} kg$.
- Electron in a solid: interact with semiconductor atoms.
  - dopants
  - traps
  - phonons
- To use electron gas model an *effective mass* m^* is used. The same thing is true for holes. Usually, $m^* < m_0$. Notice that m^* is not a constant, but depends on the semiconductor material

and other properties and conditions, such as doping level, temperature, etc.

- Particle's momentum:

$$\mathbf{p} = m^* \mathbf{v}$$

- Particle's energy:

$$E_{kin} = \frac{\mid \mathbf{p} \mid^2}{2m^*}$$

- The kinetic energy of the carriers due to the crystal temperature is

$$E_{kin} = \frac{1}{2} m^* v_{th}^2$$

and depends on the dimensionality of the model being use; for three-dimensions

$$E_{kin} = \frac{3}{2}kT$$

- v_{th} is the *thermal velocity* and is random due to *carrier scattering* - collisions with crystal imperfections.
- If no electric field is applied, the *drift velocity* of the carriers is equal to zero.
- When an electric field is applied, the effective carrier motion is expressed as the drift velocity.

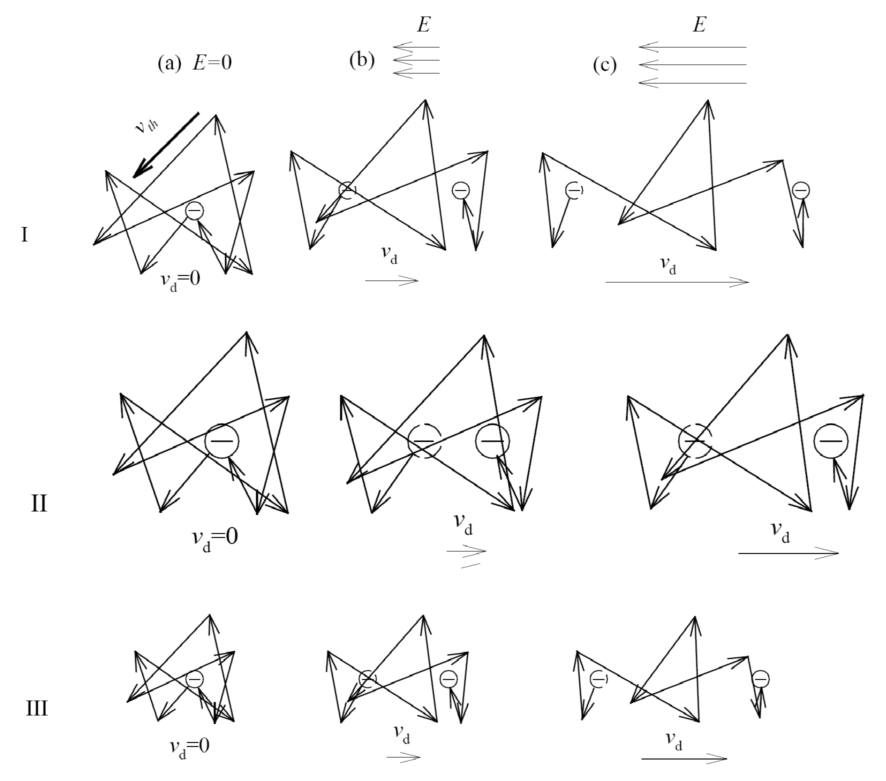

Figure 4: Drift velocity concept.

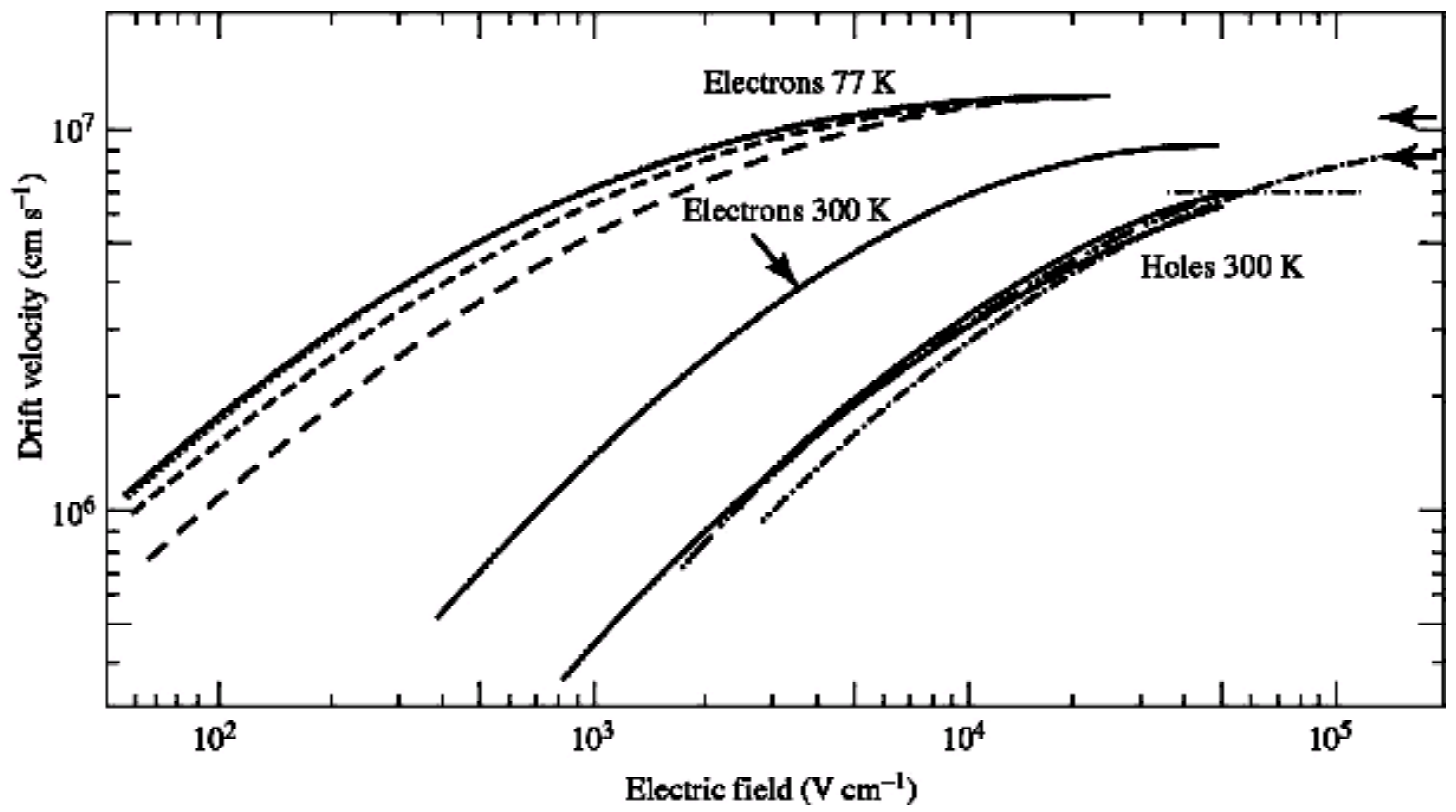

Figure 5: v_d versus E for Si. Published by Jacoboni, Canali, Ottaviani and Quaranta, *Solid-State Electronics* **20**, 77 (Feb. 1977).

- See figures 4 and 5.

3.1 Mobility

- If an electric field is applied,

  For electrons: $j = -qnv_d = q\mu_n nE$

  For holes: $j = qpv_d = q\mu_p pE$

- If a ”small” electric field is applied (column b in figure 4), drift velocity represents a small perturbation on random thermal velocity and the mean scattering time τ_{cn} is not altered appreciably.

- For steady state, the momentum gained between collisions is lost to the lattice in the collisions, and equals the impulse (force times time) applied by the electric field. The force on the electron is $F = -qE$. Thus

$$-qE\tau_{cn} = m_n^* v_d$$

or

$$v_d = -\left(\frac{q\tau_{cn}}{m_n^*}\right) E = -\mu_n E$$

- *mobility* μ: relates drift velocity and electric field.

  For holes: $v_d = \mu_p E$.

  Units of mobility: $< \mu >= m^2/V - s$.

- On figure 5 we see that the mobility is constant only at low electric fields.

- Electrons and holes have different mobilities due in part to their different effective masses. Also, mobility depends on the material since the effective mass also changes.

3.1.1 Dependence of Mobility on T and N_D/N_A

- Phonons = crystal vibrations.
- As temperature increases, phonons increase in magnitude. This increases *phonon scattering*. As T is increased, the collision probability increases and the *mean-free path* decreases. Mobility is reduced as a consequence.
- Impurity atoms (ions) also interact with carrier, attracting and repealing electrons and holes. This is called *Coulomb scattering*.
- At low temperatures, Coulomb scattering is more effective because carriers spend more time close to the ions. As temperature increases, Coulomb scattering is reduced and mobility increases at low temperatures.
- At higher temperatures, phonon scattering dominates and the mobility decreases with temperature.

- See figure 6 and 7.

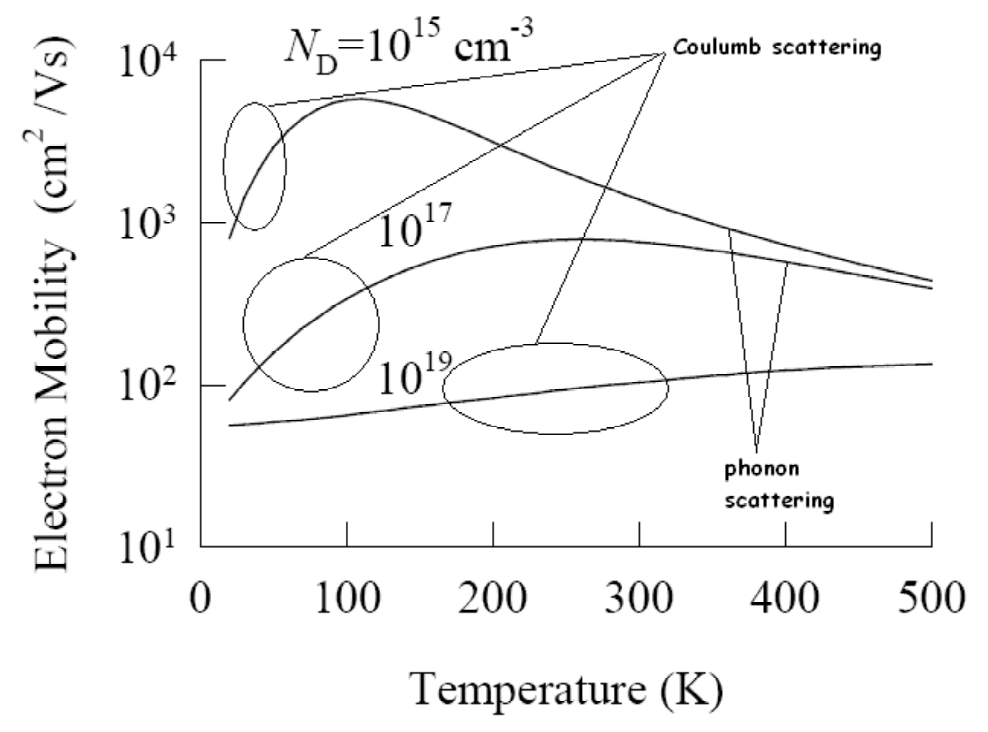

Figure 6: Temperature dependence of mobility in Si at different doping levels.

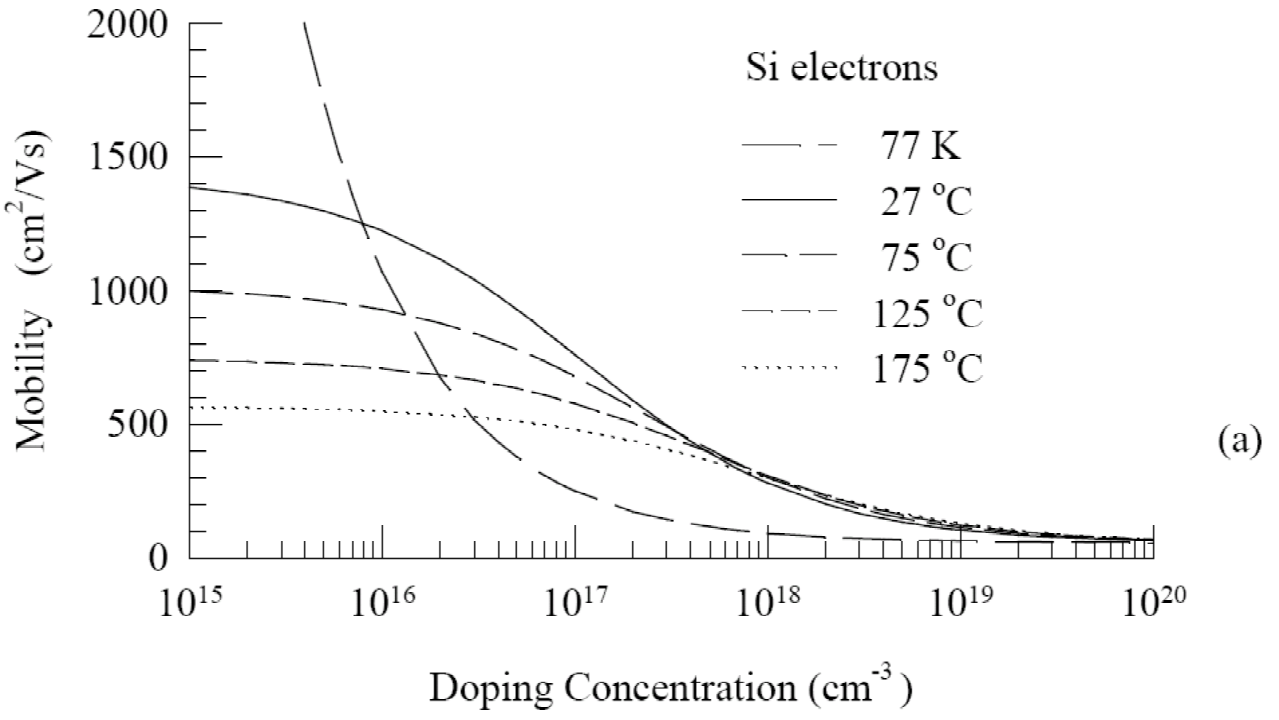

Figure 7: Temperature dependence of mobility in Si at low fields.

- The following empirical equations expressing the dependence on mobility on temperature $T_n = T/300$ and dopant concentration N were proposed by:

N.D. Arora, J.R. Hauser and D.J. Roulson, IEEE Trans. Electr. Dev. ED-30, 292 (Feb. 1982).

$$\mu_n = 88T_n^{-0.57} + \frac{1250T_n^{-2.33}}{1 + (N/(1.26 \times 10^{17}T_n^{2.4}))\, 0.88T_n^{-0.146}} \tag{1}$$

$$\mu_p = 54.3T_n^{-0.57} + \frac{407T^{-2.33}n}{1 + (N/(2.35 \times 10^{17}T_n^{2.4}))\, 0.88T_n^{-0.146}} \tag{2}$$

- At high electric fields the *hot electrons* interact with the lattice through an additional scattering process and mobility decreases. See figure 5.

- The following expression that approximates the data shown in figure 5 was provided by the authors of the publication.

$$|v_d| = v_l \frac{E}{E_C} \left(\frac{1}{1 + (E/E_C)^{\beta}} \right)^{1/\beta}$$

For electrons, the fitting parameters v_l, E_C and β are, at room temperature, equal to $1.07 \times 10^7 cm/s$, $6910V/cm$ and 1.11, respectively.

- Problem 1.21 from textbook. An $E = 1V/\mu m$ produces $j = 0.8 \times 10^9 A/m^2$ on an N-type sample with $N_D = 10^{17} cm^{-3}$. Find the current density if the electric field is increased 5 times so that the electrons reach velocity saturation $v_{sat} = 0.1 \mu m/ps$. Find the conductivity on both cases.

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- Problem 1.22 from textbook. A bar of Si with a uniform N-type doping concentration of $10^{15} cm^{-3}$ is 1 cm. long, 0.5 cm. wide and 0.5 mm thick, and has a resistance of 190 Ω. Find (i) the electron mobility, and (ii) the drift velocity of the electrons when 10V are applied to the ends of the bar.

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  ANSWER: (i) $R = \rho \frac{L}{t \times W}$; $\mu_n = \frac{1}{q N_D \rho}$. (ii) $v_d = \mu_n E$.

- Problem: An electron is moving in a piece of lightly doped Si under an electric field at room temperature. Its drift velocity is one-tenth of its thermal velocity. Find (i) the average number of collisions it will experience in traversing by drift a region $1\mu m$ wide. (ii) the electric field applied across the region.

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  ANSWER: From $v_{th} = \sqrt{\frac{3kT}{m^*}}$, find $v_d = \mu_n E$. Total traversing time $t = L/v_d$; use known μ_n to find time between collisions is $\tau = \frac{m^* \mu_n}{q}$ and then number of collisions t/τ. E can be obtained from v_d and μ_n.

- Problem: The voltage across a uniform 2 μm-long region of 1 Ω-cm , N-type Si is doubled, but the current only increases by 50%. Assume room temperature. Find the applied voltage.

3.1.2 Haynes-Shockley Experiment

- Light pulse creates *excess carriers* - in this case, excess holes, above thermal equilibrium.
- Excess carriers drift due to the electric field. This gives place to a second kind of current due to diffusion.

$$j_{diff} = \begin{cases} qD_n \frac{\partial n}{\partial x} & \text{electrons} \\ -qD_p \frac{\partial p}{\partial x} & \text{electrons} \end{cases}$$

- Excess carriers are collected at the terminals and a voltage pulse is observed.

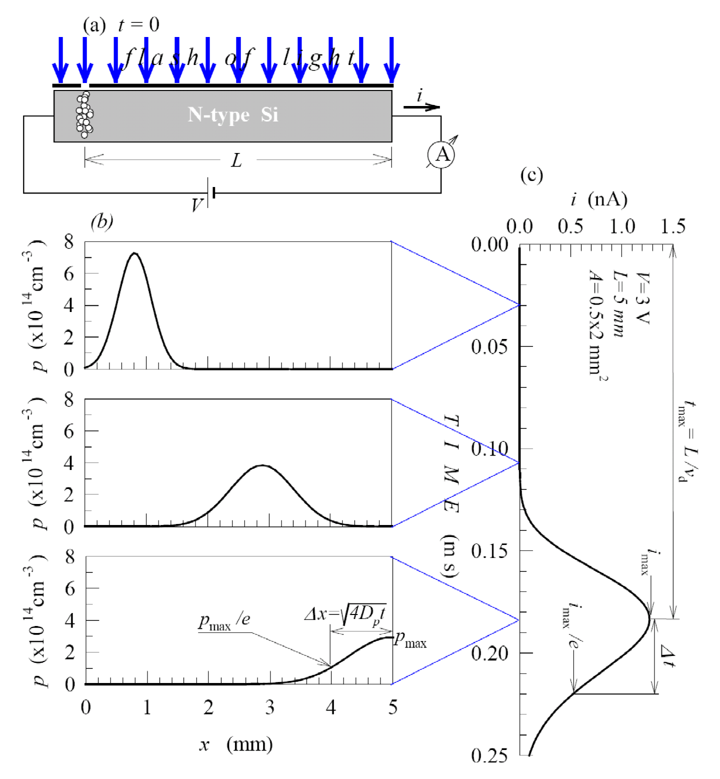


Figure 8: Haynes-Shockley experiment.

- Monitoring the pulse peak, we can measure the time it takes the pulse to arrive; from this we can figure out the carrier drift velocity.

$$v_d = L/t_{max}$$

and the mobility

$$\mu_p = \frac{v_d}{E} = \frac{L/t_{max}}{V/L} = \frac{L^2}{V \times t_{max}}$$

- Because of diffusion, the width of the pulse widens with time. The hole distribution can be expressed by a Gaussian form,

  $$p = p_{max} e^{-\frac{(x-x_{max})^2}{4D_p t}}$$

  For $(x - x_{max})^2 = 4D_p t$, $p = p_{max} \div e$. We can identify this level in the measured pulse and from it measure

  $$\Delta x = \sqrt{4D_p t} = \sqrt{4D_p(t_{max} + \Delta t)}$$

  Using $v_d = \frac{\Delta x}{\Delta t}$ and also $v_d = \frac{L}{t_{max}}$, and solving for D_p yields

  $$D_p = \frac{((\Delta t \times L)/t_{max})^2}{4(t_{max} + \Delta t)}$$

- The diffusion coeficient is related to mobility by *Einstein relation*:

$$D_{n,p} = \frac{kT}{q}\mu_{n,p}$$

4 Energy-band Model

4.1 Quantum Mechanics

- De Broglie Postulate

$$\lambda = \frac{h}{m_0 v}$$

  Implies the *wave-particle duality*.

- Atomic states or quantum numbers
  - *principal* quantum number n
  - *orbital, or angular* quantum number l
  - *magnetic* quantum number m
  - *spin* quantum number s
- Pauli Exclusion principle: In a given *system* (which may be an

atom, a molecule, or an entire crystal made of interacting atoms), no two electrons can occupy the same quantum state.

- Atomic states in Silicon:

  14 electrons in each Silicon atom.

  - First orbit ($n = 1$) have spherical symmetry and can not accommodate different angular nor magnetic quantum numbers. It holds 2 electrons with different spin. These are the $1s^2$ electrons.
  - Second orbit: ($n = 2$) have two "sub-orbitals," label s and p. The s sub-orbitals are spherically symmetric and can only hold 2 electrons, identified as $2s^2$. The p sub-orbitals have $x-$, $y-$ and $z-$directional symmetry, and can thus hold 6 electrons, identified as $2p^6$.
  - Third orbit: $3s^2$ and $3p^6$ sub-orbitals, but only 4 are filled.
- Electronic bands

Isolated atoms have electrons confined to single potential wells. A crystal represents a periodic potential well. The many atoms that form a crystal form a single system and thus must satisfy Pauli exclusion principle. Atomic states then must *split* in order to do so, and in the process they form electronic bands. See figure 9.

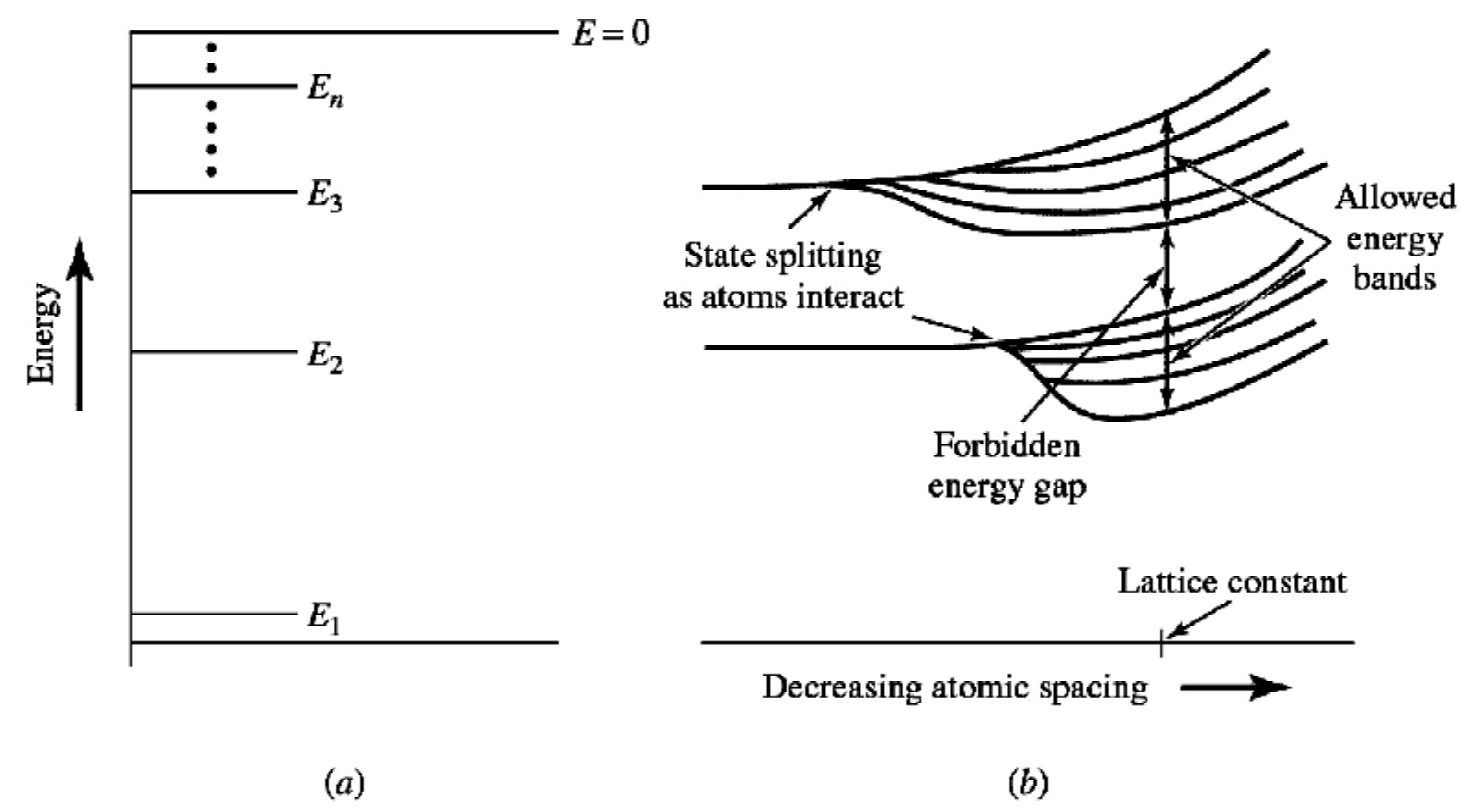
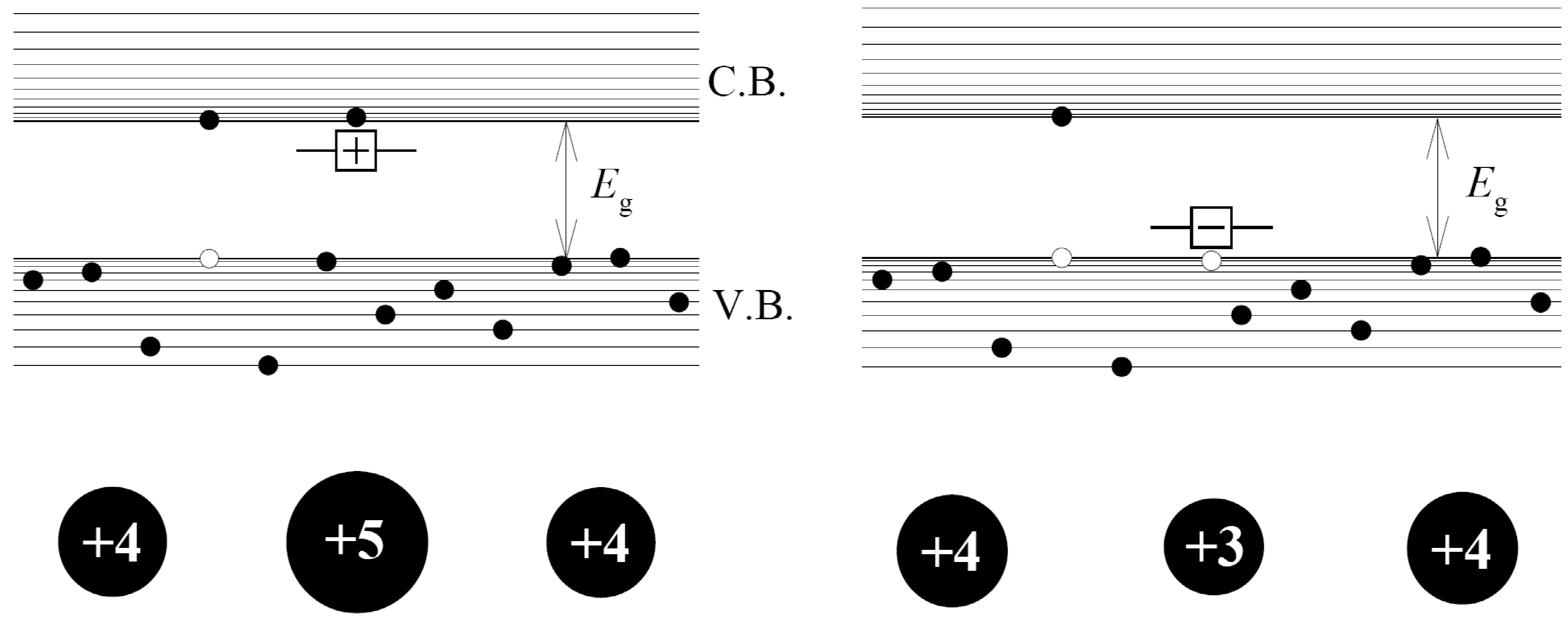


Figure 9: Splitting of energy states into allowed bands separated by a forbidden energy gap as the atomic spacing decreases.

- Energy gap values for some semiconductors

Material	E_g (eV)
Silicon	1.12
Gallium arsenide	1.42
Germanium	0.66
Silicon dioxide (SiO_2)	9
Silicon nitride (Si_3N_4)	5
Carbon	5.47

- P- and N-type doping introduce states in the energy gap. See figure 10.


(a) N-type doping

(b) P-type doping

Figure 10: Effects of doping in energy-band model presentation.

4.2 Population of Energy Bands

- The probability that an electron occupies an electronics state with energy E is given by the *Fermi-Dirac distribution*,

  $$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

  where k is the Boltzmann constant, T is the absolute temperature, and E_F is the energy of the *Fermi level*. The *Fermi energy* is the energy at which the probability of occupation by an electron is one-half.

- If E_C is the conduction-band energy, and if $\exp\left(\frac{E_C - E_F}{kT}\right) >> 1$, then the mathematically simpler *Maxwell-Boltzmann* distribution function can be used;

  $$f(E_C) \approx e^{-\frac{E_C - E_F}{kT}}$$

- The MB distribution function can be obtain independently if the Pauli exclusion principle limitations are ignored (i.e. any number of electrons can occupy an energy state). At energy well above E_f very few states are occupied and exclusion-principle limitations have little effect.
- The concentration of free electrons can be expressed as

$$n = N_C f$$

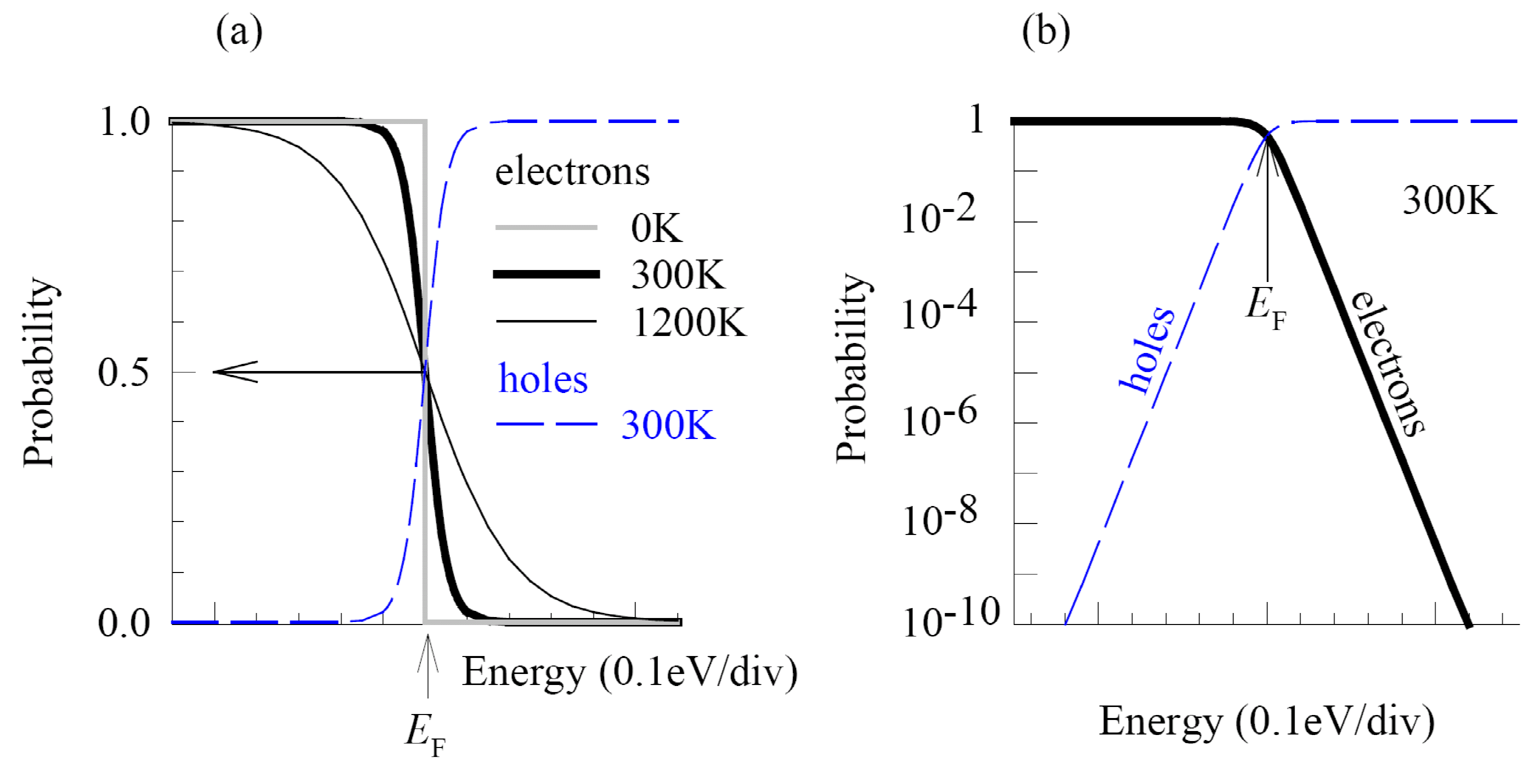


Figure 11: Fermi-Dirac distributions, plotted on (a) linear and (b) logarithmic scales.

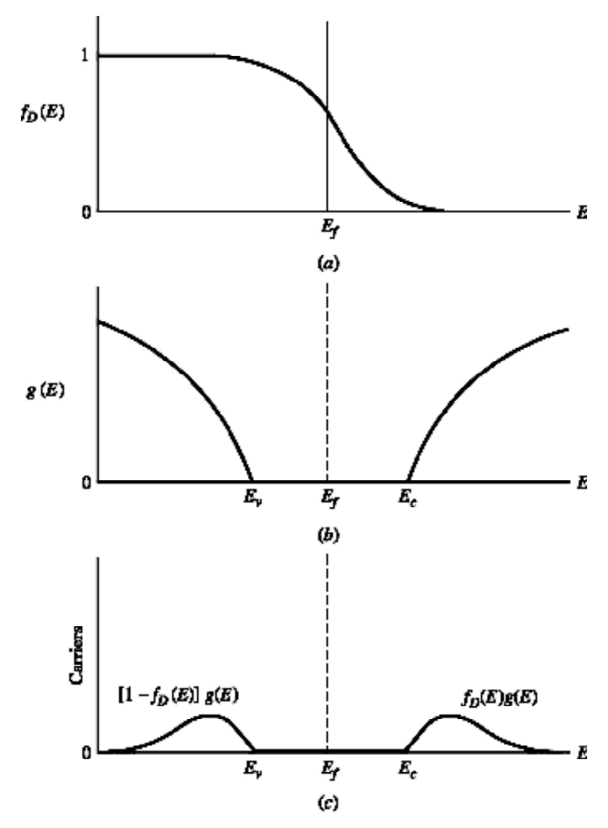


Figure 12: (a) Fermi-Dirac distribution function; (b) density of states; and (c) product.

- Density of States for electrons (N_C) and holes (N_V)

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

At room temperature $N_C = 2.86 \times 10^{19} cm^{-3}$ and $4.7 \times 10^{17} cm^{-3}$ for Si and GaAs, respectively.

The *effective density of states in the valence band* is

$$N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

for both Si and GaAs.

At room temperature $N_V = 2.86 \times 10^{19} cm^{-3}$ and $7.0 \times 10^{18} cm^{-3}$ for Si and GaAs, respectively.

- If the exponential approximation to the Fermi-Dirac distribution is valid,

$$n \approx N_C e^{-\frac{E_C - E_F}{kT}}$$

$$p \approx N_V e^{-\frac{E_F - E_V}{kT}}$$

- For intrinsic semiconductors, E_F is in the middle of the energy gap.
- For n-type semiconductors,

$$E_F = E_C - kTln\frac{N_C}{N_D}$$

- For p-type semiconductors,

$$E_F = E_V + kTln\frac{N_V}{N_A}$$

- Example: Find the equilibrium and hole concentrations and the location of E_f at 300K if Silicon is doped with $8 \times 10^{16}\ cm^{-3}$ arsenic (As) atoms and $2 \times 10^{16}\ cm^{-3}$ boron (B) atoms.

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  ANSWER: Arsenic is penta-valent; boron is tri-valent. Thus the effective donor concentration is

  $$N_{D,eff} = N_D - N_A = 6 \times 10^{16}\, cm^{-3} \approx n$$

  and

  $$p = \frac{n_i^2}{n} = 3.5 \times 10^3\, cm^{-3}$$

  With respect to the botton of the energy band, the Fermi level is at

  $$E_C - E_F = kTln\left(\frac{N_C}{N_D}\right) \approx 0.026eV\, ln\left(\frac{2.86 \times 10^{19}}{6 \times 10^{16}}\right) = 0.16eV$$

4.3 Bands under an Electric Field

- The bottom of the conduction band represents the potential energy of the electrons in the conduction band.
- The top of the valence band represents the potential energy of the holes in the valence band.
- Potential energy is related to an electric potential φ applied to the semiconductor by

$$E_{pot} = -q\varphi$$

- A constant applied electric field results in a linear variation in the energy-levels in the crystal. Electrons move down the hill on the conduction band when a field is applied.

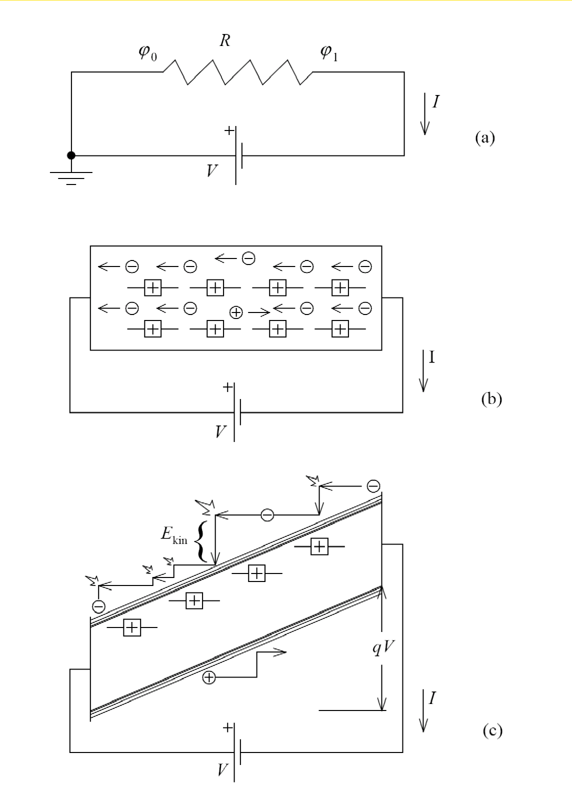


Figure 13: Energy band diagram, showing carrier potential energy E_C and E_V versus position $(E - x)$.

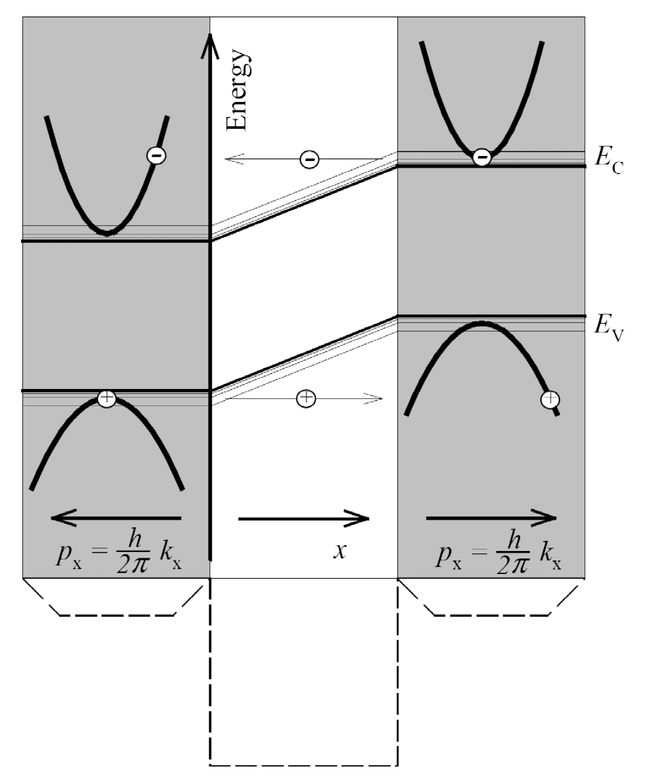


Figure 14: Relationship between $E - k$ and $E - x$ diagrams for the simplest case in which the relationship between kinetic energy and momentum is parabolic.

5 Diffusion and Lithography

Important to for understanding:

- Semiconductor doping
- device physics

Basic concept of diffusion - see figures 1.7 and 1.8.

6 Making an IC Resistor

See fabrication sequence in figures 1.10.

Figure 1.11 shows an example of a doping profile.

See MATLAB Animations for sec. 1.3

6.1 Diffusion Equation

In one-dimension:

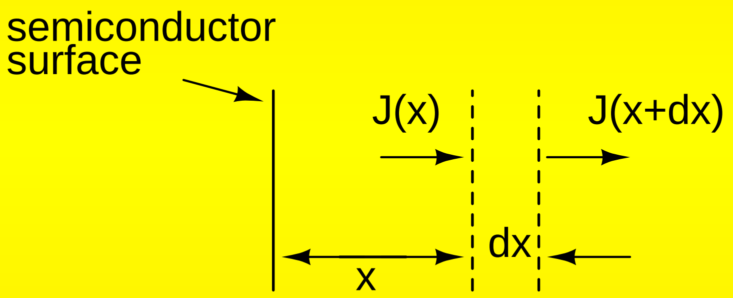
$$J_{diff} = -D\frac{\partial N}{\partial x}$$

In three-dimensions:

$$\mathbf{J}_{diff} = -D\nabla N$$

6.2 Continuity equation

Consider a narrow region of width dx at a depth x from the surface. Assume that dopands enter the region from the left and leave at the right, as shown in the following figure.



The change in concentration can be expressed as the difference between the influx and outflux of dopands,

$$\frac{\partial N}{\partial t} dx = J(x) - J(x + dx)$$

Expressing the last quantity by the first two terms of a Taylor series expansion,

$$J(x+dx) \approx J(x) + \frac{\partial J}{\partial x} dx$$

Substituting in the previous equation and canceling common terms we get that

$$\frac{\partial N}{\partial t} = -\frac{\partial J}{\partial x}$$

This is called the *Continuity equation*.

6.3 Fick's Equation

- Combining the Diffusion and Continuity equations, we get Fick's equation

$$\frac{\partial N(x,t)}{\partial t} = D\frac{\partial^2 N(x,t)}{\partial x^2}$$

- the *Diffusion Coefficient* depends on temperature and is a material property:

$$D = D_0 e^{\frac{-E_A}{kT}}$$

  where
  - T is the absolute temperature,
  - k is Boltzmann constant,
  - parameters E_A and D_0 are the *activation energy* and *frequency factor* depend on the semiconductor material and

the doping species.

This exponential dependance makes the diffusion process strongly temperature dependant.

- D plays the same role than conductivity but for diffusion: i.e. $D = -\frac{J}{\frac{\partial N}{\partial x}} =$ current/driving force.
- Diffusion for IC fabrication is done in two steps:
  - pre-deposition diffusion - constant-source diffusion, in which a constant surface dopant concentration is maintained.
  - drive-in diffusion - the substrate is heated while no dopant concentration is provided at the surface. Used to redistribute dopant atoms already in the substrate.
- The *solid-solubility limit* is the maximum surface concentration of dopant atoms that the semiconductor can absorb. Approximate values for Si are:

Dopant	Solid-solubility limit
boron	$4 \times 10^{20} cm^{-3}$
phosphorous	$8 \times 10^{20} cm^{-3}$
arsenic	$1.5 \times 10^{21} cm^{-3}$
antimony	$4 \times 10^{19} cm^{-3}$

- To find $N(x,t)$ Fick's Equation must be solved for the appropriate boundary conditions. For the pre-deposition step, the b.c. are
  - no dopands at $t = 0$ for all depths: $N(x,0) = 0$
  - constant concentration on the surface equal to the solubility limit: $N(0,t) = N_0$, and
  - very far from the surface the dopand concentration is zero, no matter how long we wait: $N(\infty,t) = 0$
- The solution to Fick's equation that satisfies these b.c.

can be expressed as

$$N(x,t) = N_0 erfc\left(\frac{x}{2\sqrt{Dt}}\right) = \frac{2N_0}{\sqrt{\pi}} \int_{\frac{x}{2\sqrt{Dt}}}^{\infty} e^{-v^2} dv$$

where $erfc$ is the *complementary error function*.

See textbook figure 1.14 and 1.15a for plots of $erfc$.

- Typical pre-deposition temperature and time are $950°C$ and 30 min.
- For the drive-in step, the boundary conditions are:
  - total quantity of dopants remain constant: $\int_0^\infty N(x,t)dx = \Phi$
  - very far from the surface the dopand concentration vanishes: $N(\infty, t) = 0$

The solution to Fick's equation that satisfies these b.c. is

$$N(x,t) = \frac{\Phi}{\sqrt{\pi D t}} e^{-\frac{x^2}{4Dt}}$$

where D and t refer to the value of these quantities during the drive-in step.

See textbook figure 1.15b for a plot of this expression.

- Φ is the *dose* of doping atoms incorporated into the substrate during pre-deposition, defined as the dopand density per unit area, given in cm^{-2}. Φ is a constant since no new atoms diffuse during the drive-in step, and is given by

$$\begin{aligned}\Phi &= N_0 \int_0^\infty erfc(\frac{x}{2\sqrt{Dt}})dx \\ &= 2N_0\sqrt{\frac{Dt}{\pi}}\end{aligned}$$

- For a two-step diffusion process (predeposition followed by drive-in),

$$N(x) = \overbrace{\frac{2N_0}{\pi}\sqrt{\frac{D_1 t_1}{D_2 t_2}}}^{N_S} e^{-\frac{x^2}{4D_2 t_2}}$$

  where D_1 and t_1 and D_2 and t_2 refer to the predeposition and drive-in steps, respectively, and $N_S = N(0)$ is the doping concentration at the surface after the drive-in.

- Example: Find D for boron at $1000°C$ and $1100°C$ assuming that $D_0 = 0.76cm^2/s$, $E_A = 3.46eV$. Use $k = 8.62 \times 10^5 eV/K$.

- Example: A constant-source diffusion is carried out at $1050°C$ on an N-type silicon substrate with background doping $N_B = 10^{16} cm^{-3}$. The surface concentration is kept at the solubility limit $N_0 = 4 \times 10^{20} cm^{-3}$. Determine the diffusion time necessary to obtain a junction depth of $1 \mu m$.

- Example: A drive-in diffusion step is performed after the constant-source diffusion described in the previous example. The time and temperature for the drive-in step are equat to those used for the constant-source diffusion. Determine the final junction depth and surface doping concentration after the drive-in diffusion is performed.