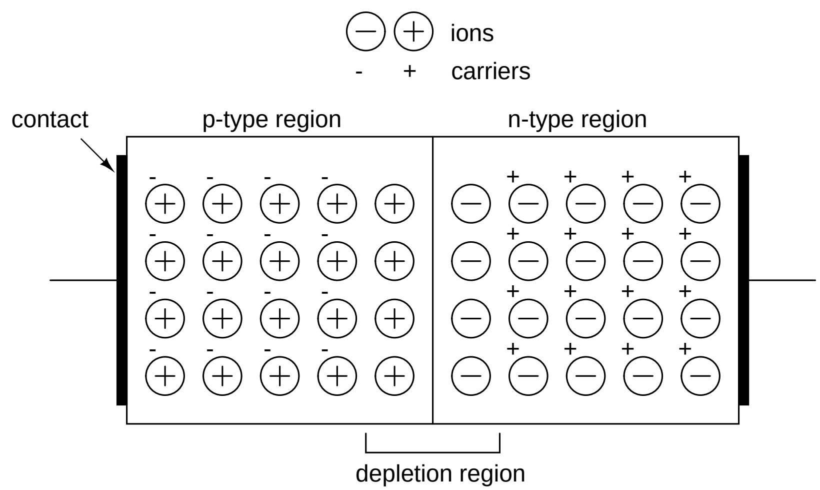
Junction Diode

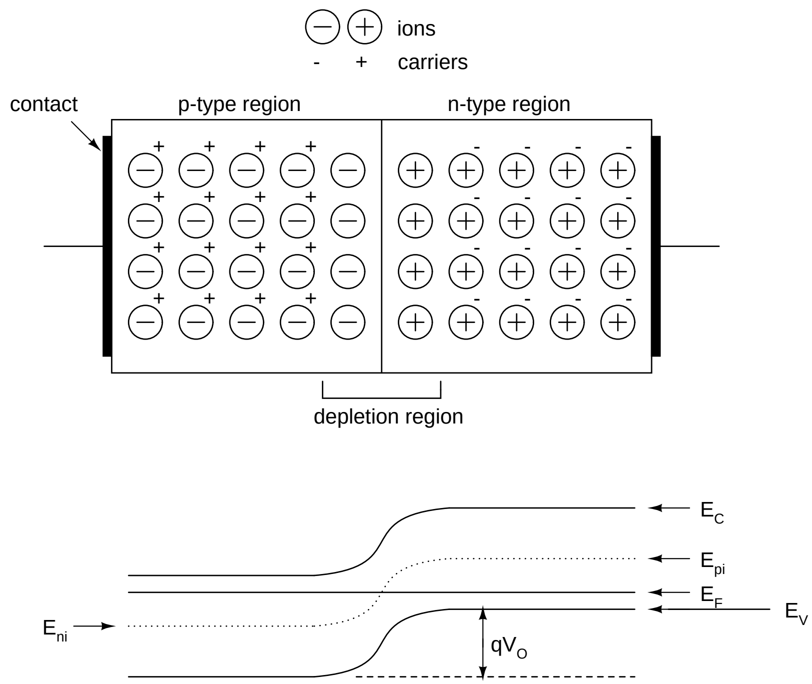
- Build by forming adjacent p- and n-type regions. Interface between regions is called the *pn junction*
- There are concentration gradients for free electrons (n-type side) and (p-type side). This gives place to diffusion currents across the junction.
- The holes that diffuse across the junction become excess minority carriers in the n-type region. Similarly, diffused electrons are excess minority carriers in the p-type region.
- Near the junction, device is depleted of majority carriers - *depletion region*.

- fixed ions remain in the depletion region. These ions generate an electric potential called the *build-in voltage*.
- Diffusion across the junction continues until the build-in voltage is large enough to prevent.



Build-in voltage

- As carriers diffuse across the junction, fixed ions remain. The electric charge represented by these ions give place to an electric field that cause the energy bands to bent.
- The Fermi level is constant throughout a system in thermal equilibrium.



- The voltage drop across the depletion region is said to form a *barrier*.This internal voltage gives place to a drift current I_S across the junction.
- An equilibrium is reached when the diffusion current due to the concentration gradient equals the drift current due to the build-in voltage. At equilibrium

$$I_D = I_S$$

- Since the Fermi level is the same at both sides of the junction, the bands will bent by qV_0. To find the build-in potential V_0, observe that

$$qV_0 = E_{pi} - E_{ni}$$

where E_{pi} and E_{ni} correspond to the middle of the forbidden band in the p- and n-type regions, respectively.

From chapter 1,

$$p = N_V e^{-\frac{E_F - E_V}{kT}}$$

For intrinsic material, $p = p_i$ and $E_F = E_{pi}$. Thus

$$\begin{aligned} p_i &= N_V e^{-\frac{E_{pi} - E_V}{kT}} \\ N_A &= N_V e^{-\frac{E_F - E_V}{kT}} \\ \frac{N_A}{p_i} &= \frac{e^{-\frac{E_F - E_V}{kT}}}{e^{-\frac{E_{pi} - E_V}{kT}}} \\ &= e^{\frac{E_{pi} - E_F}{kT}} \\ E_{pi} &= E_F + kTln\left(\frac{N_A}{p_i}\right) \end{aligned}$$

Similarly, for the n-type side,

$$n = N_C e^{-\frac{E_C - E_F}{kT}}$$

For intrinsic material, $n = n_i$ and $E_F = E_{pi}$. Thus

$$\begin{aligned}
n_i &= N_C e^{-\frac{E_C - E_{ni}}{kT}} \\
N_D &= N_C e^{-\frac{E_C - E_F}{kT}} \\
\frac{N_D}{n_i} &= \frac{e^{-\frac{E_C - E_F}{kT}}}{e^{-\frac{E_C - E_{ni}}{kT}}} \\
&= e^{\frac{E_F - E_{ni}}{kT}} \\
E_{ni} &= E_F - kTln\left(\frac{N_D}{n_i}\right)
\end{aligned}$$

- From these, after subtracting, dividing by q and equating $\frac{kT}{q} = V_T$, the build-in voltage is found to be

$$V_O = V_T ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Width of the Depletion Region

- For charge equality $qx_pN_A = qx_nN_D$, where x_p and x_n represent the depletion region width in the p- and n-type regions, respectively. This can be expressed as

$$\frac{x_p}{x_n} = \frac{N_D}{N_A}$$

The total width of the depletion region, given by $x_p + x_n$, can be shown to be given by

$$W_{dep} = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_O}$$

Biased Diodes

- An external voltage can force carriers to cross the junction. For that the positive terminal must be connected to the p-type region and the negative terminal to the n-type region. The diode is said to be *forward biased*. Because of this, the n- and p-type regions are called *cathode* and *anode*, respectively.
- If the external voltage makes the cathode more positive than the anode, then the diode is *reversed biased*.

Reverse Bias Junction Capacitance

Abrupt Profile

- Charge accumulated in each side of the depletion region is

$$q_p = q_n = qN_D x_n A$$

- Poisson's Equation:

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\epsilon_s}$$

Must be integrated twice to find φ.

$$\frac{d^2\varphi}{dx^2} = \begin{cases} 0 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S} & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S} & \text{if } x \geq x_p \\ 0 & \text{if } x \geq -x_p \end{cases}$$

- Solving problem for equilibrium conditions.

- First integration:

$$\frac{d\varphi}{dx} = \begin{cases} C_1 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}x + C_2 & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S}x + C_3 & \text{if } x \geq x_p \\ C_4 & \text{if } x \geq -x_p \end{cases}$$

- Boundary conditions for first integration:
  - E must be continuous
  - $E(-x_n) = -\frac{d\varphi}{dx} = 0$
  - $E(-x_p) = -\frac{d\varphi}{dx} = 0$
- First integration after using b.c.

$$\frac{d\varphi}{dx} = \begin{cases} 0 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}(x + x_n) & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S}(x - x_p) & \text{if } x \geq x_p \\ 0 & \text{if } x \geq -x_p \end{cases}$$

- Second integration:

$$\varphi = \begin{cases} C_5 & \text{if } x \leq -x_n \\ -\frac{qN_D}{\epsilon_S}(\frac{x^2}{2} + x_n x) + C_6 & \text{if } x \leq -x_n \\ +\frac{qN_A}{\epsilon_S}(\frac{x^2}{2} - x_p x) + C_7 & \text{if } x \geq x_p \\ C_8 & \text{if } x \geq -x_p \end{cases}$$

- Boundary conditions for second integration:
  - φ is constant for $x \leq -x_n$ and $x \geq x_p$
  - $\varphi(-x_n) = V_0$

- $\varphi(x_p) = 0$ (defined as ground)

- Second integration after using b.c.:

$$\varphi = \begin{cases} V_0 & \text{if } x \leq -x_n \\ V_0 - \frac{qN_D}{2\epsilon_S}(x + x_n)^2 & \text{if } -x_n \leq x \leq 0 \\ +\frac{qN_A}{2\epsilon_S}(x - x_p)^2 & \text{if } 0 \leq x \leq x_p \\ 0 & \text{if } x \geq x_p \end{cases}$$

- Continuity at $x = 0$

$$V_0 - \frac{qN_D}{2\epsilon_S}x_n^2 = +\frac{qN_A}{2\epsilon_S}x_p^2$$

- Expressing $x_p = \frac{N_D}{N_P}x_n$ and

$$W_{dep} = x_n + x_p = (1 + \frac{N_D}{N_A})x_n = \frac{N_A + N_D}{N_A}x_n$$

$$
\begin{aligned}
V_0 &= \frac{q}{2\epsilon_S}\left(N_D x_n^2 + N_A x_p^2\right) \\
&= \frac{qN_D}{2\epsilon_S}\left(x_n^2 + \frac{N_A}{N_D}x_p^2\right) \\
&= \frac{qN_D}{2\epsilon_S}x_n^2\left(\frac{N_A + N_D}{N_A}\right)
\end{aligned}
$$

$$
x_n = \sqrt{\frac{2\epsilon_S}{qN_D}\frac{N_A}{N_D + N_A}V_0}
$$

- The charge is given by

$$
\begin{aligned}
q_j &= N_D q A x_n \\
&= qA\sqrt{\frac{2\epsilon_S}{q}\frac{N_A N_D}{N_D + N_A}V_0}
\end{aligned}
$$

- For a reversed biased junction, replace V_0 with $V_0 + V_{bias}$

$$q_j = qA\sqrt{\frac{2\epsilon_S}{q}\frac{N_A N_D}{N_D + N_A}(V_0 + V_{bias})}$$

- Apply the definition of capacitance,

$$C = \frac{dq}{dV}$$

to the junction charge using the applied voltage, V_{bias} as the voltage. Thus,

$$C_j = \frac{dq_j}{dV_{bias}} \,|_{V_{bias}=V_R}$$

where V_R is the applied bias voltage.

$$C_j = \frac{qA}{2}\sqrt{\frac{2\epsilon_S}{q}\frac{N_A N_D}{N_D + N_A}\frac{1}{V_0 + V_R}}$$

- in terms of

$$C_{j0} = A\sqrt{\frac{q\epsilon_s}{2}\left(\frac{N_A N_D}{N_A + N_D}\right)\frac{1}{V_0}}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

- From our previous expression for C_j,

$$C_j^2 = \frac{qA^2\epsilon_S}{2}\frac{N_A N_D}{N_D + N_A}\frac{1}{V_0 + V_R}$$

$$V_R = \frac{qA^2\epsilon_S}{2}\frac{N_A N_D}{N_D + N_A}\frac{1}{C_j^2} - V_0$$

If we measure C_j as we change V_R and plot V_R versus $1/C_j^2$, we should get a straight line with intercept $-V_0$ and slope

$$\frac{qA^{2\epsilon_S}}{2}\frac{N_A N_D}{N_D + N_A}$$

This only works if the junction is abrupt.

Linear Junction

- Charge density: $\rho = -ax$

$$\frac{d^2\varphi}{dx^2} = \frac{a}{\epsilon_S} x$$

- First integration:

$$\frac{d\varphi}{dx} = \frac{a}{\epsilon_S}\frac{1}{2}x^2 + C_1$$

B.C.: $E = \frac{d\varphi}{dx}$ continuous.

At $x = x_p = +\frac{w}{2}$, $E = 0$. Thus

$$C_1 = -\frac{a}{\epsilon_S}\frac{1}{2}x_p^2$$

and

$$\frac{d\varphi}{dx} = \frac{a}{2\epsilon_S}(x^2 - x_p^2)$$

- Second integration:

$$\varphi = \frac{a}{2\epsilon_S}(\frac{1}{3}(x^3 - x_p^2 x) + C_2$$

B.C.: $\varphi = 0$ at x_p

$$\begin{aligned} C_2 &= -\frac{a}{2\epsilon_S}(\frac{1}{3}(x_p^3 - x_p^2 x_p) \\ &= \frac{a}{3\epsilon_S} x_p \end{aligned}$$

and

$$\varphi = \frac{a}{2\epsilon_S}(\frac{1}{3}(x^3 - x_p^2 x + x_p)$$

- At $x = -x_n = -\frac{w}{2}$, $\varphi = V_0 + V_{bias}$,

$$\begin{aligned} V_0 + V_{bias} &= \frac{a}{3\epsilon_S}(x_n^3 + x_p^3) \\ &= \frac{a}{12\epsilon_S} w^3 \end{aligned}$$

or

$$w^3 = \frac{12\epsilon_S}{a}(V_0 + V_{bias})$$

- The junction capacitance is given by

$$\begin{aligned}
C_j &= \frac{A\epsilon_S}{w} \\
&= A\epsilon_S \sqrt[3]{\frac{a}{12\epsilon_S}} \frac{1}{\sqrt[3]{V_0 + V_{bias}}} \\
&= A\sqrt[3]{\frac{a\epsilon_S^2}{12V_0}} \frac{1}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}} \\
&= \frac{C_{j0}}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}}
\end{aligned}$$

where

$$C_{j0} = A\sqrt[3]{\frac{a\epsilon_S^2}{12V_0}}$$