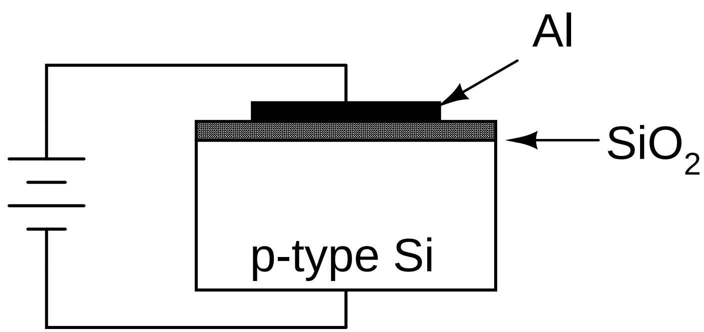
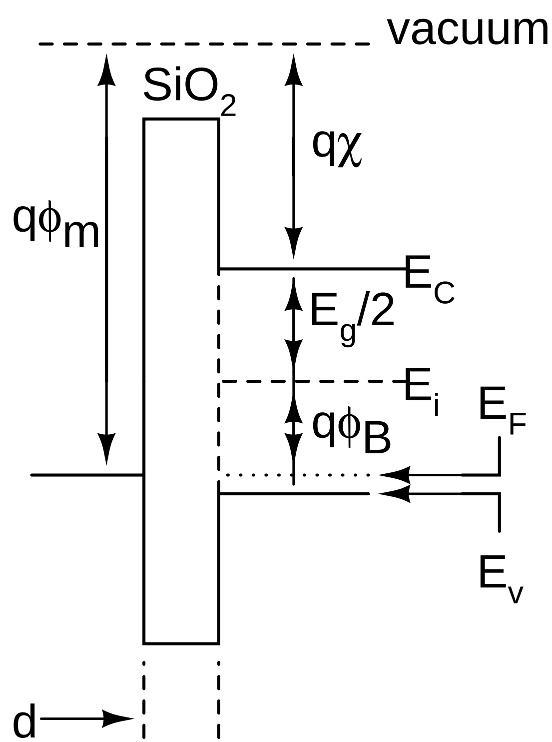
Ideal MOS Capacitor





Work function: difference between Fermi and vacuum levels

$q\phi_S, q\phi_m$: semiconductor and metal work functions, respectively.

Electron affinity $q\chi$: difference between the conduction band edge and the vacuum level.

Fermi potential $q\phi_F$: difference between the mid-gap and the Fermi level. Proportional to doping type and level.

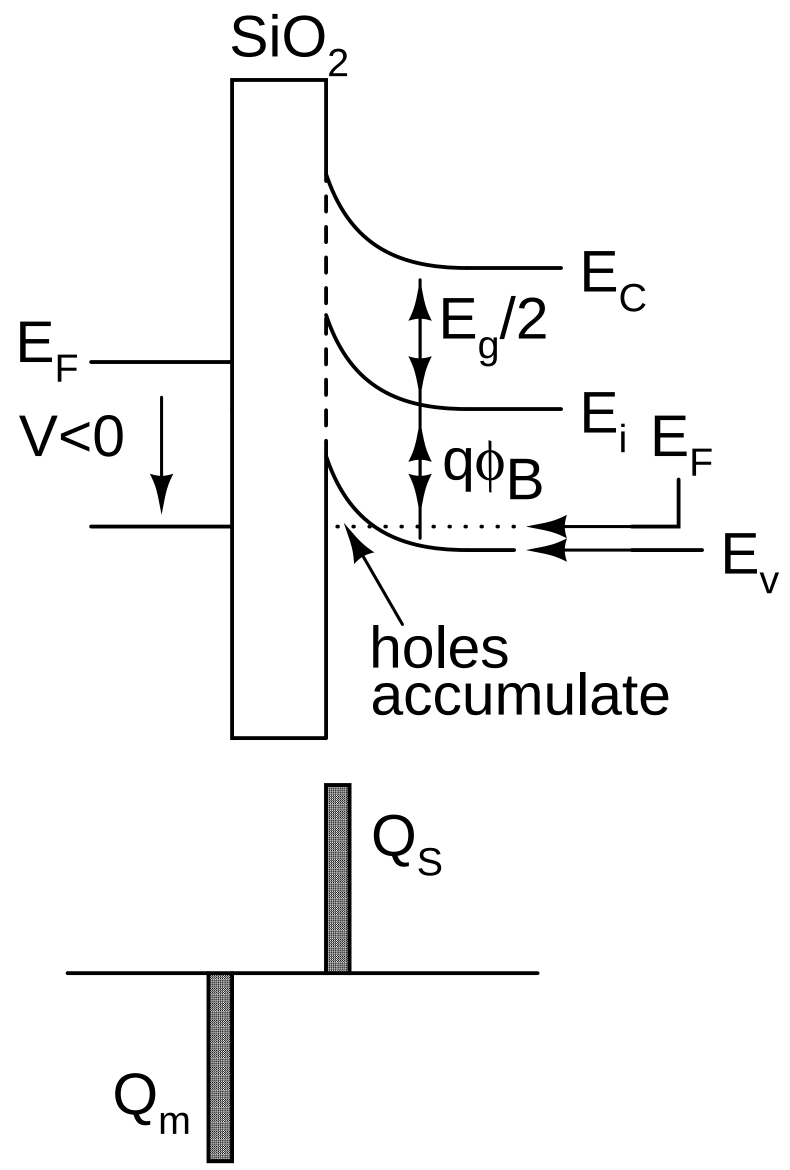
- At zero bias, $\phi_m = \phi_S$. Work function difference is zero:

$$q\phi_m - \left[q\chi + \frac{E_G}{2} + q\phi_B \right] = 0$$

- charges accumulate in the $Si - SiO_2$ interface and in the metal.
- no carrier transport in the oxide under dc-bias conditions.

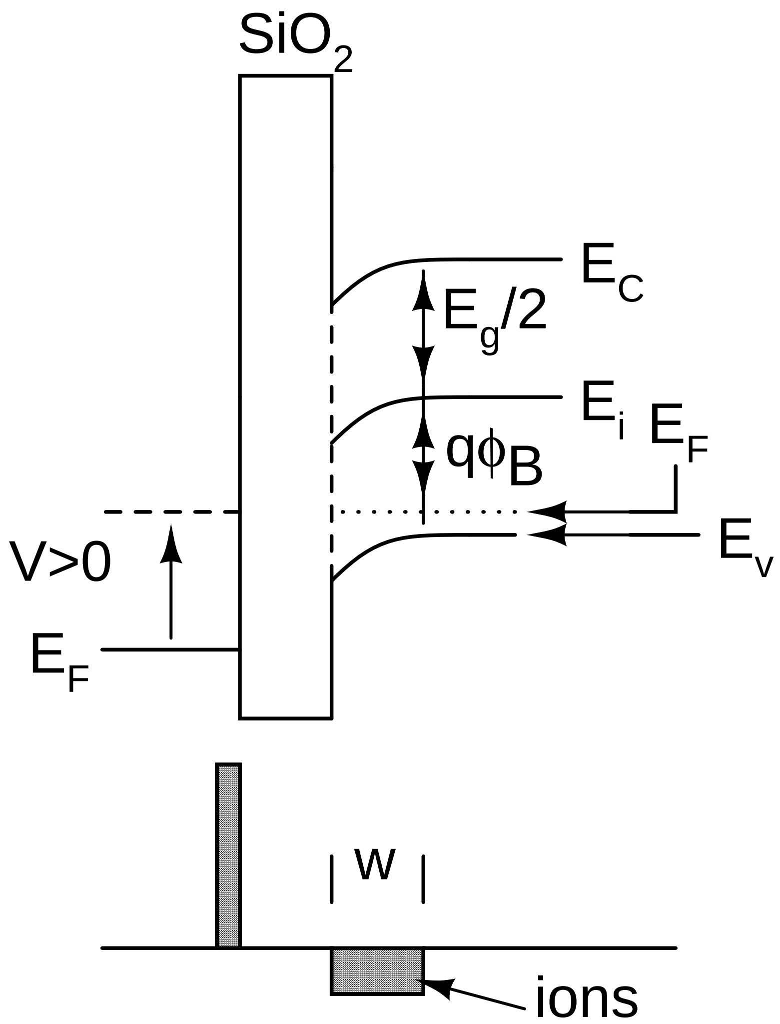
MOS Biasing p-type Si

- Accumulation mode: If a negative voltage is applied to the metal


SiO2
EF
V<0
EC
Eg/2
Ei
EF
qϕB
Ev
holes accumulate
QS
Qm

- majority carriers (holes) will be attracted to the $Si - SiO_2$ interface
- bands will bent near the surface.
- $p_p = n_i e^{\frac{E_i - E_F}{kT}}$

* Depletion mode: If a small positive voltage is applied to the metal,

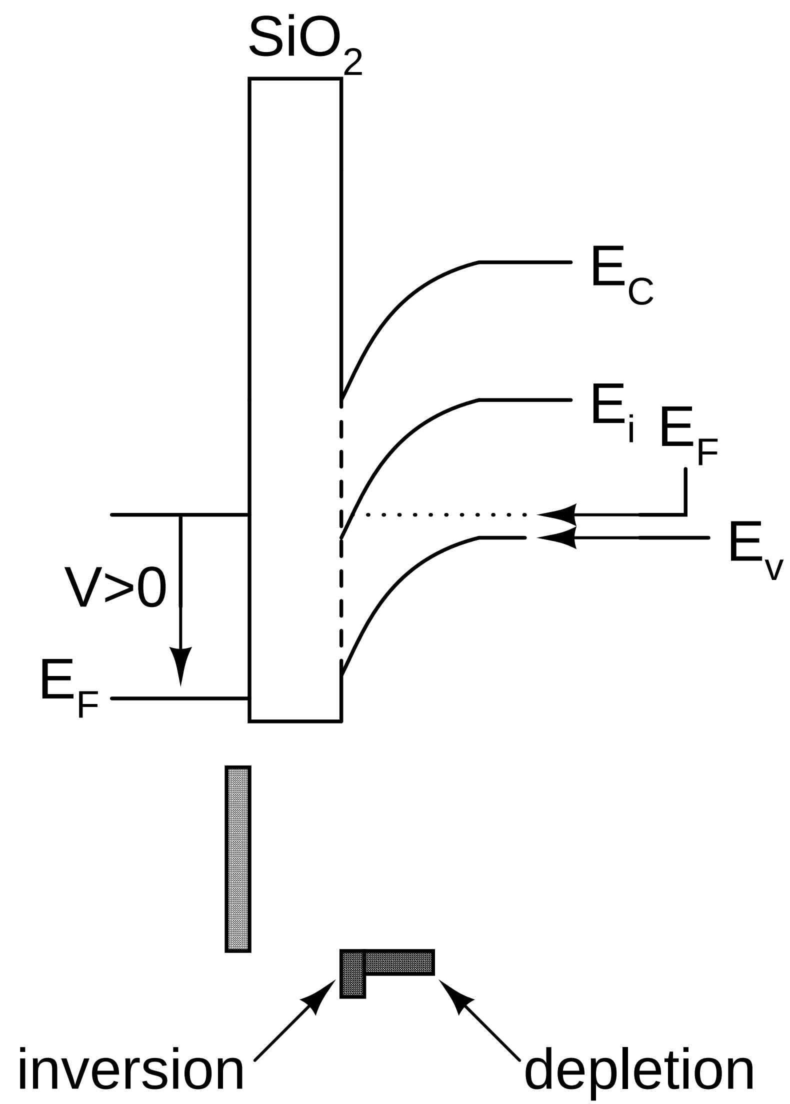


– bands bent downward near the surface.

  - majority carriers are driven away from interface.
  - a depletion region is formed.

$$Q_{SC} = -qN_A w$$

- Weak inversion mode: A larger positive voltage is applied to the metal.


SiO2
EC
Ei
EF
Ev
V>0
EF
inversion
depletion

- bands bent downward near the surface.
- intrinsic level reach the Fermi level.
- depletion region still exists.
- minority carriers accumulate under metal
- the electron concentration under the metal is

$$n_p = n_i e^{\frac{E_F - E_i}{kT}}$$

- electron concentration is small

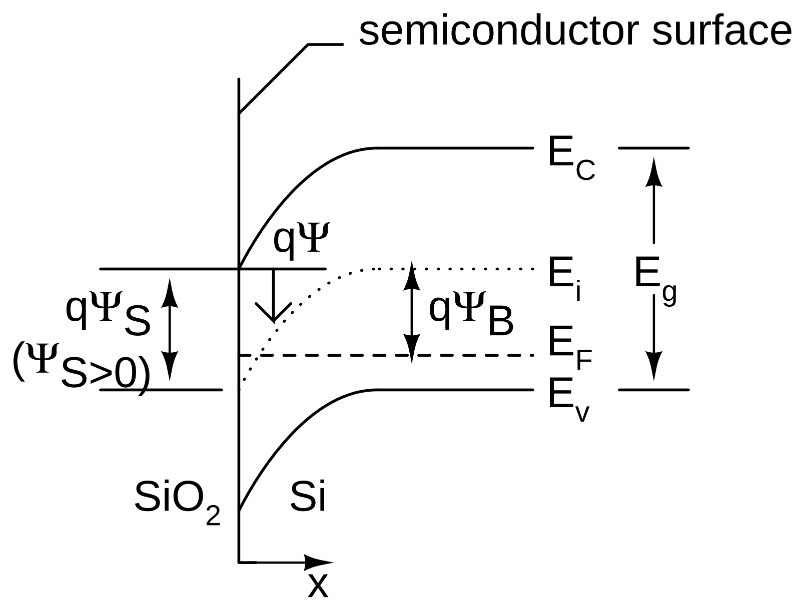
• Strong inversion mode: A still larger positive voltage is applied to the metal.
  - band bent further; conduction band edge comes close to the Fermi level.
  - for strong inversion

$$n_p \approx N_A$$

– In the strong inversion mode, small increases in band bending yield large increases in the electron charge in the inversion layer; the surface depletion region reaches a maximum width w_m

$$Q_S = Q_n + Q_{SC} = Q_n - qN_A w_m$$

Detailed Analysis



- $\Psi = 0$ in the semiconductor bulk, by definition.
- Ψ_S is called the surface potential.
- electron and hole concentration as a function of Ψ

$$n_p = n_i e^{\frac{q(\Psi - \Psi_B)}{kT}}$$

$$p_p = n_i e^{\frac{q(\Psi_B - \Psi)}{kT}}$$

  where Ψ is positive when the bands bent downward.
- At the surface,

$$n_S = n_i e^{\frac{q(\Psi_S - \Psi_B)}{kT}}$$

$$p_S = n_i e^{\frac{q(\Psi_B - \Psi_S)}{kT}}$$

- $\Psi_S < 0$: bands bent upward; accumulation of holes
- $\Psi_S = 0$: flat-band condition
- $\Psi_B > \Psi_S > 0$: bands bent downward; depletion of holes
- $\Psi_B = \Psi_S$: midgap with $n_s = n_p = n_i$ (intrinsic

concentration)

- $\Psi_S > \Psi_B$:inversion
- $\Psi(x)$ can be obtained from Poisson's equation

$$\frac{d^2\Psi}{dx^2} = -\frac{\rho_s(x)}{\epsilon_s}$$

- If the semiconductor is depleted to width w, the charge density in the depletion region is

$$\rho_s = -qN_A$$

- Setting $d\Psi/dx = 0$ and $\Psi = 0$ in the bulk, integration of Poisson's equation yields

$$\Psi(x) = \Psi_S \left(1 - \frac{x}{w}\right)^2$$

where the surface potential is

$$\Psi_S = \frac{qN_A w^2}{2\epsilon_S}$$

– For strong inversion, we can use the condition $n_S = N_A$. From

$$p_p = n_i e^{\frac{q(\Psi_B - \Psi)}{kT}}$$

evaluated at the bulk where $\Psi = 0$ and $p \approx N_A$,

$$N_A = n_i e^{\frac{q\Psi_B}{kT}} = n_S = n_i e^{\frac{q(\Psi_S - \Psi_B)}{kT}}$$

we get $\Psi_B = \Psi_S - \Psi_B$. For strong inversion w reaches a maximum w_m and

$$\begin{aligned} \Psi_{S,inv} &= 2\Psi_B \\ &\approx \frac{2kT}{q} ln\left(\frac{N_A}{n_i}\right) \end{aligned}$$

$$= \frac{qN_A w_m^2}{2\epsilon_S}$$

and

$$w_m = 2\sqrt{\frac{\epsilon_S kT ln\left(\frac{N_A}{n_i}\right)}{q^2 N_A}}$$

The charge in the depletion region is

$$Q_{SC} = -qN_A w_m$$

Capacitance

– In the absence of any work function differences, when a voltage V is applied across the MOS structure it appears partly across the oxide and partly across the semiconductor

$$\begin{aligned} V &= V_o + \Psi_S \\ V_o &= \mathcal{E} d \end{aligned}$$

$$= \frac{|Q_S|d}{\epsilon_{ox}}$$

$$= \frac{|Q_S|}{C_o}$$

– C can be found by considering two capacitors in series

$$C = \frac{C_o C_j}{C_o + C_j} F/cm^2$$

where $C_j = \epsilon_s/w$.

– From the above equations

$$\frac{C}{C_o} = \frac{1}{\sqrt{1 + \frac{2\epsilon_{ox}^2 V}{qN_A \epsilon_s d^2}}}$$

which predicts that the capacitance will decrease with applied voltage.

– Whe a negative voltage is applied, no depletion region is formed and $C = C_o = \epsilon_{ox}/d$.

- When strong inversion occurs, $\Psi_S = \Psi_{S,inv}$ and $Q_S = qN_A w_m$. The voltage at which this happens is called the *threshold voltage*,

$$V_T = \frac{qN_A w_m}{C_o} + 2\Psi_B = \frac{\sqrt{2\epsilon_s qN_A(2\Psi_B)}}{C_o} + 2\Psi_B$$

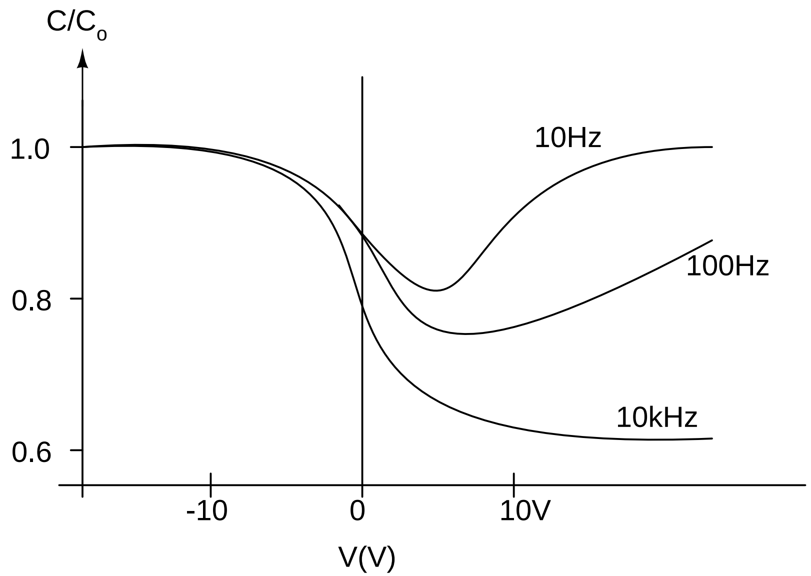
- Once strong depletion takes place, capacitance remains at a minimum value that can be obtained by letting $C_j = \epsilon_S/w_m$,

$$C = \frac{C_o C_j}{C_o + C_j}$$

$$= \frac{(\epsilon_{ox}/d) \times (\epsilon_S/w_m)}{\epsilon_{ox}/d + \epsilon_S/w_m}$$

$$= \frac{\epsilon_{ox}}{\frac{\epsilon_{ox}}{epsilon_S} w_m + d}$$

– The above equation is correct at high frquencies. It assumes that when the metal voltage changes, all incremental charge appears at the edge of the depletion region. At frequencies below 100 Hz, however, the depletion region generation-recombination mechanism is faster that the voltage variations. This leads to charge exchange with the inversion layer in step with the metal voltage variations. As a result the capacitance in strong inversion will be that of the inversion layer alone.


C/Co
1.0
0.8
0.6
-10
0
10V
V(V)
10Hz
100Hz
10kHz