

## Introduction

- A reverse junction as a current source. See slide 2.
- Minority electrons and holes can freely fall down or bubble up.
- We can control the current by adjusting the quantity of minority carriers. This can be done by increasing the temperature or by exposure to light.
- A *forward-biased* pn junction build next to the reverse-biased junction can provide the additional carriers.
- The resulting two-junction device is a BJT. The voltage across the forward-biased junction controls the supply of carriers and thus reverse-biased junction current.
- Consider an NPN BJT operating in the active region:
  - See slide 3.
  - Forward-biased base-emitter junction and reverse-biased

base-collector junction.

- Electrons flow from the emitter into the base, and holes from the base into the emitter, producing currents  $I_{nE}$  and  $I_{pE}$ , respectively.
- See slide 4.
- The total emitter current is  $I_E = I_{nE} + I_{pE}$ .
- Electrons entering the emitter can flow into the collector.
- Holes flowing from base to emitter can not do “useful” work.
- The *emitter efficiency* is defined by

$$\gamma_E = \frac{I_{nE}}{I_E} = \frac{1}{1 + \frac{I_{pE}}{I_{nE}}}$$

- $I_{nE}$  and  $I_{pE}$  depend on  $V_{BE}$  and on the majority carrier concentrations. To increase efficiency, dope the emitter as

much as possible and the base as little as possible.

- The ratio of successfully-collected to emitted electrons is the *transport factor*:

$$\alpha_T = \frac{I_{nC}}{I_{nE}}$$

- Electrons can recombine at the base; thus to maximize  $\alpha_T$ , make the base region as thin as possible.
- A reverse current  $I_{CB0}$  flows through the collector-base junction. This leakage current can be neglected in active operation, but is important in cutoff.
- $I_E$  depends exponentially on  $V_{BE}$ . For operation as a voltage-controlled current source, the *transconductance*  $g_m$  that relates  $I_C$  to  $V_{BE}$  is the device gain. The exponential relationship makes  $g_m$  high.
- Both base and emitter are used as amplifier inputs (CE and CB configurations). The corresponding current gains are

$$\beta = \frac{I_C}{I_B} \text{ and } \alpha = \frac{I_C}{I_E}:$$

$$\alpha \approx \frac{I_{nC}}{I_E} = \frac{I_{nC}}{I_{nE}} \frac{I_{nE}}{I_E} = \alpha_T \gamma_E$$

–  $\alpha$  and  $\beta$  are related:

$$\beta = \frac{\alpha}{1 - \alpha}$$

$$\alpha = \frac{\beta}{\beta + 1}$$

- Slide 5 shows the four modes of operating a BJT.

## Bipolar IC Technologies

- Structure of an IC BJT. See slide 7.
- The N+ buried layer provides a low-resistance path for electrons to flow out of the collector.
- Bipolar technology heavily relies on the quality of the N-epi layer.
- Layer deposition techniques:

- See

- <http://kottan-labs.bgsu.edu/teaching/workshop2001/chapter5.htm>

- <http://www.uoregon.edu/hutch-lab/semilab/thinfilm.html>

- <http://www.filebox.vt.edu/users/sprice/semiconductorprocess.htm>

- <http://www.postech.ac.kr/mse/semicon/facilities.htm>

- evaporation
- sputtering
- chemical vapor deposition (CVD)
- molecular beam epitaxy (MBE)

# BJT Modeling

## Ebers-Moll Model

### Injection Version

- See slide 14b.
- Forward active mode

$$I_F = I_{ES} \left( e^{V_{BE}/V_T} - 1 \right)$$

- Reverse active mode

$$I_R = I_{CS} \left( e^{V_{BC}/V_T} - 1 \right)$$

- The terminal currents are:

$$I_C = \alpha_F I_F - I_R$$

$$\begin{aligned}
 &= \alpha_F I_{ES} \left( e^{V_{BE}/V_T} - 1 \right) - I_{CS} \left( e^{V_{BC}/V_T} - 1 \right) \\
 I_E &= -I_F + \alpha_R I_R \\
 &= -I_{ES} \left( e^{V_{BE}/V_T} - 1 \right) + \alpha_R I_{CS} \left( e^{V_{BC}/V_T} - 1 \right)
 \end{aligned}$$

- The base current is the difference between the emitter and collector currents;

$$I_B = -I_E - I_C$$

### Transport Version

- See slide 14c.
- For the two models to be equivalent:

$$I_{EC} = \alpha_R I_R = I_S \left( e^{V_{BC}/V_T} - 1 \right)$$

$$I_{CC} = \alpha_F I_F = I_S \left( e^{V_{BE}/V_T} - 1 \right)$$



where, for normal active mode,

$$I_S = \alpha_F I_{ES}$$

and for inverse active mode

$$I_S = \alpha_R I_{CS}$$

- The terminal currents are:

$$\begin{aligned} I_C &= I_{CC} - I_{CE}/\alpha_R \\ &= I_S \left( e^{V_{BE}/V_T} - 1 \right) - I_S/\alpha_R \left( e^{V_{BC}/V_T} - 1 \right) \\ I_E &= -I_{CC}/\alpha_F + I_{CE} \\ &= -I_S/\alpha_F \left( e^{V_{BE}/V_T} - 1 \right) + I_S \left( e^{V_{BC}/V_T} - 1 \right) \\ I_B &= -I_E - I_C \end{aligned}$$

### SPICE Version

- See slide 15.

- Terminal currents:

$$\begin{aligned}
 I_C &= I_{CC} - I_{CE}/\alpha_R \\
 &= -I_{?1} + I_{CT} \\
 &= -I_{?1} + I_{CC} - I_{CE}
 \end{aligned}$$

$$\begin{aligned}
 I_{?1} &= I_{CC} - I_{CE} - I_{CC} + I_{CE}/\alpha_R \\
 &= I_{CE} \left( \frac{1 - \alpha_R}{\alpha_R} \right) \\
 &= I_{CE}/\beta_R
 \end{aligned}$$

$$I_E = -I_{CC}/\alpha_F + I_{CE} = -I_{?2} - I_{CC} + I_{CE}$$

$$\begin{aligned}
 I_{?2} &= I_{CC}/\alpha_F - I_{CE} - I_{CC} + I_{CE} \\
 &= I_{CC} \left( \frac{1 - \alpha_F}{\alpha_F} \right) \\
 &= I_{CC}/\beta_F
 \end{aligned}$$

- In terms of  $I_S$ ,

$$I_C = I_S \left( e^{V_{BE}/V_T} - 1 \right) - \left( 1 + \frac{1}{\beta_R} \right) I_S \left( e^{V_{BC}/V_T} - 1 \right)$$

$$I_E = - \left( 1 + \frac{1}{\beta_F} \right) I_S \left( e^{V_{BE}/V_T} - 1 \right) + I_S \left( e^{V_{BC}/V_T} - 1 \right)$$

$$\begin{aligned} I_B &= -I_E - I_C \\ &= \frac{1}{\beta_F} I_S \left( e^{V_{BE}/V_T} - 1 \right) + \frac{1}{\beta_R} I_S \left( e^{V_{BC}/V_T} - 1 \right) \end{aligned}$$

- For the normal active mode,

$$I_C = I_S e^{V_{BE}/V_T}$$

$$I_E = - \left( 1 + \frac{1}{\beta_F} \right) I_S e^{V_{BE}/V_T}$$

$$I_B = \frac{1}{\beta_F} I_S e^{V_{BE}/V_T} = \frac{I_C}{\beta_F}$$

## Second Order Effects

### Early Effect

- See slide 16.
- Also called the *base modulation effect*.
- Base-collector depletion layer changes width, causing an increase in  $I_S$  (because more hole-electron pairs are generated in the depletion region) and therefore in  $I_C$ .
- Represented in terms of

$$I_S = I_{S0} \left( 1 + \frac{|V_{BC}|}{|V_A|} \right)$$

Parasitic resistances  $r_E$ ,  $r_B$  and  $r_C$ .

## Parasitic Capacitances

- See slides 18 and 19.
- For normal active mode,

$$C_{\mu} = C_{dC}$$

$$C_{\pi} = C_{dE} + C_{sE}$$

## Gummel-Poon Effect

- $\beta$ 's are not constant with current; see slide 20.
- Explained in terms of charge accumulation in the base at high voltage, in the capacitances  $C_{dE}$  and  $C_{sE}$ .
- Majority carrier concentration in the base: doping + depletion layer charge + stored charge;

$$\begin{aligned}
Q_{BT} = & Q_{B0} \\
& + C_{dE} V_{BE} + C_{dC} V_{BC} \frac{A_E}{A_C} \\
& + \frac{Q_{B0}}{Q_{BT}} \tau_F I_S \left( e^{V_{BE}/V_T} - 1 \right) + \frac{Q_{B0}}{Q_{BT}} \tau_R I_S \left( e^{V_{BC}/V_T} - 1 \right)
\end{aligned}$$

where  $Q_{B0} = qN_A A_E$ .

- Because  $I_S$  is inversely proportional to the carrier density, multiplying  $I_S$  by  $Q_{B0}/Q_{BT}$  effectively replaces the doping level by the total charge; in the normal active mode,

$$I_C \approx \frac{I_{S0}}{q_b} e^{V_{BE}/V_T}$$

where

$$q_b = \frac{Q_{BT}}{Q_{B0}}$$

$$\begin{aligned}
&= 1 + \frac{C_{dE}V_{BE}}{Q_{B0}} + \frac{C_{dC}V_{BC}}{Q_{B0}} \frac{A_E}{A_C} \\
&\quad + \frac{1}{q_b Q_{B0}} \tau_F I_S \left( e^{V_{BE}/V_T} - 1 \right) \\
&\quad + \frac{1}{q_b Q_{B0}} \tau_R I_S \left( e^{V_{BC}/V_T} - 1 \right) \\
q_b^2 &= q_b \left( 1 + \frac{C_{dE}V_{BE}}{Q_{B0}} + \frac{C_{dC}V_{BC}}{Q_{B0}} \frac{A_E}{A_C} \right) \\
&\quad + \frac{1}{Q_{B0}} \tau_F I_S \left( e^{V_{BE}/V_T} - 1 \right) + \frac{1}{Q_{B0}} \tau_R I_S \left( e^{V_{BC}/V_T} - 1 \right)
\end{aligned}$$

This can be rewritten as the quadratic:

$$q_b^2 - q_1 q_b - q_2 = 0$$

where

$$q_1 = 1 + \frac{C_{dE} V_{BE}}{Q_{B0}} + \frac{C_{dC} V_{BC}}{Q_{B0}} \frac{A_E}{A_C}$$

$$= 1 + \frac{V_{BE}}{|V_B|} + \frac{V_{BC}}{|V_A|}$$

$$V_B = \frac{Q_{B0}}{C_{dE}}$$

$$V_A = \frac{Q_{B0}}{C_{dC}} \frac{A_C}{A_E}$$

$$q_2 = \frac{1}{Q_{B0}} \tau_F I_S \left( e^{V_{BE}/V_T} - 1 \right) + \frac{1}{Q_{B0}} \tau_R I_S \left( e^{V_{BC}/V_T} - 1 \right)$$

$$= \frac{I_{S0}}{I_{KF}} \left( e^{V_{BE}/V_T} - 1 \right) + \frac{I_{S0}}{I_{KR}} \left( e^{V_{BC}/V_T} - 1 \right)$$

$$I_{KF} = \frac{Q_{B0}}{\tau_F}$$

$$I_{KR} = \frac{Q_{B0}}{\tau_R}$$



The solution is:

$$q_b = \frac{q_1}{2} + \frac{\sqrt{q_1 + 4q_2}}{2}$$

which in the SPICE model is approximated as

$$q_b \approx \frac{q_1}{2} \left( 1 + \sqrt{1 + 4q_2} \right)$$

- The complete BJT equations at the Gummel-Poon level in SPICE are:

$$I_C = \frac{I_{S0}}{q_b} \left( e^{V_{BE}/V_T} - e^{V_{BC}/V_T} \right) - \frac{I_{S0}}{\beta_{RM}} \left( e^{V_{BC}/V_T} - 1 \right) - C_4 I_{S0} \left( e^{V_{BC}/(n_{CL} V_T)} - 1 \right)$$

$$I_E = -\frac{I_{S0}}{q_b} \left( e^{V_{BE}/V_T} - e^{V_{BC}/V_T} \right) - \frac{I_{S0}}{\beta_{FM}} \left( e^{V_{BE}/V_T} - 1 \right) - C_2 I_{S0} \left( e^{V_{BE}/(n_{EL} V_T)} - 1 \right)$$

$$I_B = -I_E - I_C$$

$$\begin{aligned}
= & + \frac{I_{S0}}{\beta_{FM}} \left( e^{V_{BE}/V_T} - 1 \right) + C_2 I_{S0} \left( e^{V_{BE}/(n_{EL} V_T)} - 1 \right) \\
& + \frac{I_{S0}}{\beta_{RM}} \left( e^{V_{BC}/V_T} - 1 \right) + C_4 I_{S0} \left( e^{V_{BC}/(n_{CL} V_T)} - 1 \right)
\end{aligned}$$

- Terms associated with coefficients  $C_2$  and  $C_4$  in the above equations are added to account for the fact that, at low biasing levels, the recombination of carriers in the bulk and surface depletion layer, as well as other surface leakage mechanisms, lead to an increase in the base current and a corresponding decrease in  $\beta$ . See slide 20.
- The Ebers-Moll equations can be obtained from the Gummel-Poon level equations by
  - letting  $I_{KF} = I_{KR} \rightarrow \infty$  so that  $q_2 \rightarrow 0$
  - letting  $|V_A| = |V_B| \rightarrow \infty$  so that  $q_1 \rightarrow 1$
  - let  $C_2$  and  $C_4$  remain at their default value of 0

## Measurement of SPICE Parameters

$I_S$  and  $\beta$

In the normal active mode,

$$I_C \simeq I_S e^{V_{BE}/V_T}$$

and

$$I_B = I_C / \beta_F$$

so that

$$\ln I_C = \ln I_S + \frac{V_{BE}}{V_T}$$

and

$$\ln I_B = \ln I_S + \frac{V_{BE}}{V_T} - \ln \beta_F$$

See slide 21.

$V_A$

See slide 22.

$r_E$  and  $I_{KF}$

See slide 23.

To estimate  $r_E$ , observe that

$$\ln I_B = \ln I_S + \frac{V_{BE}}{V_T} - \ln \beta_F$$

fits the data well for small  $I_B$ . The fitting improves for intermediate values of  $V_{BE}$  if  $r_E \neq 0$  is selected.

For still larger values of  $V_{BE}$ , there will be a departure from linearity even if  $r_E \neq 0$  is selected because of the Gummel-Poon effect. In this region of the graph,  $q_b \gg 1$  and

$$I_C \approx \sqrt{I_{S0} I_{KF}} e^{V_{BE}/(2V_t)}$$

Thus,

$$\ln I_C \approx \ln \sqrt{I_{S0} I_{KF}} + \frac{1}{2V_t} V_{BE}$$

The parameter  $C_2$  could be estimated if the graph departs from a line with slope  $1/V_t$  for small values of  $V_{BE}$ . The change in slope of the line and its value at  $V_{BE} = 0$  can then be used to estimate  $n_{EL}$  and  $C_2 I_{S0}$ .