Ubung 1. TSAI Calibration Algorithm

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Camera calibration is the process of determining the internal camera geometric and optical characteristics (intrinsic parameters) and the 3D position and orientation of the camera frame relative to a certain world coordinate system (extrinsic parameter). The fundamental task of calibration is to find the parameters of projection model that linking the known 3D points and their projections.

1 Parameters

Two groups of parameters:

- **Exterior parameter**: three rotation angles and three translation parameters.
- **Interior parameter:** focal length, principal point and lens distortion.

2 The four steps of Transformation from 3D World coordinate to Computer image coordinate

Step1: The rigid transformation from the object *world coordinate system* (x_w, y_w, z_w) to the 3D *camera coordinate system* (x, y, z). The parameters to be calibrated are R and T.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T \tag{1}$$

where R is the 3×3 rotation matrix.

$$R = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}$$

and T is the translation vector

$$T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

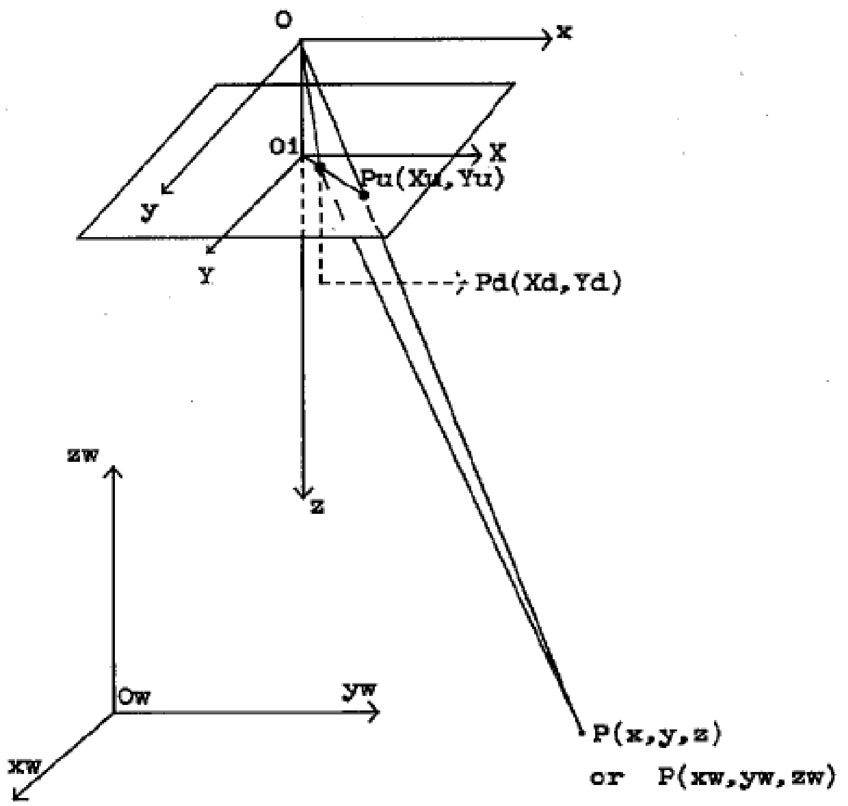

Figure 1: Coordinate systems for calibration. 1) World coordinate: (x_w, y_w, z_w). 2) Camera coordinate: (x, y, z). 3) Image coordinate: (X, Y). 4) Ideal image coordinate: (X_u, Y_u). 5) Distorted image coordinate: (X_d, Y_d)

Step 2: Transformation from 3D *camera coordinate* (x, y, z) to *ideal (undistorted) image coordinate* (X_u, Y_u) using perspective projection with pinhole camera geometry

$$X_u = f\frac{x}{z} \tag{2}$$

$$Y_u = f\frac{y}{z} \tag{3}$$

The parameter to be calibrated is the effective focal length f.

Step 3: Radial lens distortion. Between the *ideal image coordinates* (X_u, Y_u) and *distorted and measured image coordinates* $(X_d, Y_d)^T$ the following relation holds:

$$X_d(1 + \kappa_1 r^2) = X_u \tag{4}$$

$$Y_d(1 + \kappa_1 r^2) = Y_u \tag{5}$$

with $r = \sqrt{X_d^2 + Y_d^2}$. The parameter to be calibrated are distortion coefficients κ_1.

Step 4: The *distorted image coordinate* (X_d, Y_d) to *computer image coordinate* (X_f, Y_f) transformation. (Assume $s_x = 1$)

$$X_f = s_x \frac{X_d}{{d_x}'} + C_x \tag{6}$$

$$Y_f = \frac{Y_d}{d_y} + C_y \tag{7}$$

where

$${d_x}' = d_x \frac{N_{cx}}{N_{fx}} \tag{8}$$

(d_x, d_y)	width and height of sensor element
(C_x, C_y)	row and column numbers of the center of frame memory
N_{cx}	number of sensor elements in the X direction.
N_{fx}	number of pixels in a line as sampled by computer
s_x	uncertainty scale factor for x, due to camera scanning and acquisition timing error

For the sake of simplicity, we only consider the ideal case, in which camera and framegrabber are synchronized ($s_x = 1$).

3 Equations Relating the 3D world Coordinates to the 2D Computer Image Coordinates

By combining the second and the third steps, (X_d, Y_d), the distorted image coordinate centered at O_i, is related (x, y, z), the 3D coordinate of the object point in camera coordinate system, by following equations:

$$X_d(1+\kappa_1 r^2) = f\frac{x}{z} \tag{9}$$

$$Y_d(1+\kappa_1 r^2) = f\frac{y}{z} \tag{10}$$

Substituting (1) into (9) and (10) gives

$$X_d(1+\kappa_1 r^2) = f\frac{r_1 x_w + r_2 y_w + r_3 z_w + T_x}{r_7 x_w + r_8 y_w + r_9 z_w + T_z} \tag{11}$$

$$Y_d(1+\kappa_1 r^2) = f\frac{r_4 x_w + r_5 y_w + r_6 z_w + T_y}{r_7 x_w + r_8 y_w + r_9 z_w + T_z} \tag{12}$$

4 Tsai/Lenz Algorithm

Some parameters are known from the data sheet of camera:

(d_x, d_y)	width and height of sensor elements
(C_x, C_y)	row and column numbers of the center of frame memory
N_{cx}	number of sensor elements in the X direction.
N_{fx}	number of pixels in a line as sampled by computer

Some parameters need to be calibrated:

R	rotation matrix
T	translation vector
f	focus length
κ_1	distortion coefficients.

Tsai algorithm requires a calibration pattern, in which a set of 3D points (x_{iw}, y_{iw}, z_{iw}) and their projections X_{fi}, Y_{fi} are known. The aim of algorithm is to find the unknown parameters using such correspondences.

4.1 Stage 0—Compute the distorted image coordinate (X_d, Y_d)

Detect the row and column number of each calibration point i. Call it (X_{fi}, Y_{fi}). Compute the counterparts (X_{di}, Y_{di}) in the distorted image coordinate using (6) and (7):

$$X_{di} = {d_x}'(X_{fi} - C_x) \tag{13}$$

$$Y_{di} = d_y(Y_{fi} - C_y) \tag{14}$$

4.2 Stage 1—Compute 3D Orientation, Position (T_x, T_y)

Divide equation (11) by (12):

$$\frac{X_{di}}{Y_{di}} = \frac{r_1 x_w + r_2 y_w + r_3 z_w + T_x}{r_4 x_w + r_5 y_w + r_6 z_w + T_y} \tag{15}$$

Set up the following linear equation with $T_y^{-1}r_1$, $T_y^{-1}r_2$, $T_y^{-1}T_x$, $T_y^{-1}r_4$ and $T_y^{-1}r_5$ as unknowns:

$$\begin{bmatrix} Y_{di}x_{wi} & Y_{di}y_{wi} & Y_{di} & -X_{di}x_{wi} & -X_{di}y_{wi} \end{bmatrix} \begin{bmatrix} T_y^{-1}r_1 \\ T_y^{-1}r_2 \\ T_y^{-1}T_x \\ T_y^{-1}r_4 \\ T_y^{-1}r_5 \end{bmatrix} = X_{di} \tag{16}$$

With N (the number of object points) much larger than five, the five unknowns $T_y^{-1}r_1$, $T_y^{-1}r_2$, $T_y^{-1}T_x$, $T_y^{-1}r_4$ and $T_y^{-1}r_5$ can be solved.

Since rotation matrix R is a normalized orthogonal matrix with 3 Dof, all the elements r_i can be determined by the solution of (20). And T_y, T_x can also be computed if r_i is determined.

4.3 Stage 2—Compute focal length, distortion coefficients and position T_z

Rewrite (12) like

$$Y_d(1+\kappa_1 r^2) = f\frac{\hat{y}_i}{\hat{w}_i + T_z} \tag{17}$$

where

$$\hat{y}_i = r_4x_{wi} + r_5y_{wi} + r_6z_{wi} + T_y \tag{18}$$

$$\hat{w}_i = r_7x_{wi} + r_8y_{wi} + r_9z_{wi} \tag{19}$$

4.3.1 Step.1: Compute an approximation of f and T_z by ignoring lens distortion. ($\kappa_1 = 0$)

The resulting linear equation:

$$\begin{bmatrix} \hat{y}_i & -Y_{id} \end{bmatrix} \begin{bmatrix} f \\ T_z \end{bmatrix} = \hat{w}_i Y_{id} \tag{20}$$

With several object calibration points, this yields an over–determined system of linear equations that can be solved for the unknowns f and T_z.

4.3.2 Step.2: Compute the exact solution for f, T_z, κ_1

Use the approximation for f and T_z computed in previous step as initial guess, and zero as the initial guess for κ_1. Solve (17) with f, T_z, κ_1 as unknowns using standard optimization scheme such as steepest descent. Usually only one or two iterations are needed.

5 Summary

The algorithm given by Tsai is a two stage process designed to calibrate parameter using a calibration pattern. The algorithm is efficiently executed due to the absence of the large non–linear searches.

The first stage of the process determines the extrinsic parameters R and the first two components of the translation vector, T_x and T_y. This is achieved by solving a system of linear equations whose input is the coordinates of points in the calibration pattern.

The second stage of the process involves a steepest descent search to find T_z, f and κ_1. Before the optimization scheme, a reasonable initial guess is computed by solving a linear system. And such linear system is built up in the case that radial distortion is ignored.

References

[1] Tsai, R. *A versatile camera calibration technique for high-accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses*, IEEE Journal of Robotics and Automation. Vol. RA-3, No. 4., pp: 323-244, August 1987