Chapter 1 – Electric Circuit Variables

Exercises

Ex. 1.3-1

$$i(t) = 8t^2 - 4t \text{ A}$$

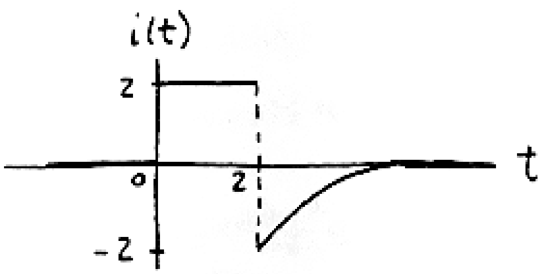
$$q(t) = \int_0^t i\,dt = \int_0^t (8t^2 - 4t)dt = \frac{8}{3}t^3 - 2t^2\Big|_0^t = \underline{\frac{8}{3}t^3 - 2t^2 \text{ C}}$$

Ex. 1.3-3

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 4\sin 3t\,d\tau + 0 = \frac{4}{3}\cos 3\tau\Big|_0^t = \frac{4}{3}\cos 3t - \frac{4}{3} \text{ C}$$

Ex. 1.3-4

$$i = dq/dt \quad i(t) = \begin{cases} 0 & t<0 \\ 2 & 0<t\le 2 \\ -2e^{-2(t-2)} & t>2 \end{cases}$$



Ex. 1.4-1 i_1 = 45 μA = 45 × 10^{-6} A < i_2 = 0.03 mA = .03 × 10^{-3} A = 3 × 10^{-5} A < i_3 = 25 × 10^{-4} A

Ex. 1.4-2 $\Delta q = i\Delta t = (4000 \text{ A})(0.001\text{s}) = \underline{4 \text{ C}}$

Ex. 1.4-3

$$i = \frac{45\times10^{-9}}{5\times10^{-3}} = 9\times10^{-6} = 9\ \mu\text{A}$$

Ex. 1.4-4

$$\text{billion} = 10^9$$

$$i = [10 \text{ billion elect/s}]\, q$$

$$= [10\times10^9 \text{ elect/s}]\, q$$

$$= 10^9 \text{ elect/s} \times 1.602\times10^{-19}\ \text{C/elect} = \underline{1.602\text{nA}}$$

Ex. 1.6-1 Energy $= \Delta W = q\Delta V = (2C)(4v) = \underline{8J}$

$P = vi = (10V)(20A) = \underline{200\ W}$

$\Delta W = P\Delta t = (200W)(10s) = \underline{2000J=2kJ}$

Ex. 1.6-2 $P = vi = \left(50e^{-10t}\right)\left(5e^{-10t}\right) = \underline{250\,e^{-20t}\ W}$

$$W = \int P\,dt = \int_0^{10} 250e^{-20t} = \frac{25}{2}e^{-20t}\Big|_0^{10}$$

$$= \underline{\frac{25}{2}\left(1-e^{-200}\right) \approx \frac{25}{2}\,J}$$

Ex. 1.6-3 $P = vi = (100\,kV)(120\,A) = 12000\,kW = \underline{12\ MW}$

$$\Delta W = P\Delta t = \left(12\times10^6\ W\right)(24\,hrs)\left(\frac{3600\,s}{hr}\right) = 1.04\times10^{12}\ J$$

$$= \underline{1.04\,TJ}$$

Problems

Section 1-3 Electric Circuits and Current Flow

P1.3-1 $i = \dfrac{\Delta q}{\Delta t} \Rightarrow \Delta q = i\Delta t = \left(10\times10^{-3}\ A\right)(20\,s)$

$= \underline{0.2\ C}$

P1.3-2 $i(t) = \dfrac{d}{dt}4\left(1-e^{-5t}\right) = 20\,e^{-5t}$ A

P1.3-3

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 4\left(1-e^{-5\tau}\right)d\tau + 0 = \int_0^t 4\,d\tau - \int_0^t 4e^{-5\tau}\,d\tau = 4t + 20\,e^{-5t} - 20\ \text{C}$$

P1.3-4

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}\left(2k_1t + k_2t^2\right) = 2k_1 + 2k_2t$$

now $i(0) = 4 = 2k_1 + 2k_2(0) \Rightarrow \underline{k_1 = \frac{4}{2} = 2}$

also $i(t = 3s) = 2k_1 + 2k_2(3) = -4$

$\Rightarrow 2(2) + 6(k_2) = -4 \Rightarrow \underline{k_2 = -4/3}$

P1.3-5

$$\text{billion} = 10^9$$
$$i = [10\,\text{billion elect/s}]\, q$$
$$= [10 \times 10^9 \text{ elect/s}]\, q$$
$$= 10^9 \text{ elect / s} \times 1.602 \times 10^{-19}\, {}^{\text{C}}\!\!/\!_{\text{elect}} = \underline{1.602\,\text{nA}}$$

P1.3-6

$$q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau = \int_{-\infty}^{t} 0\, d\tau = 0 \text{ C} \quad \text{for } t < 2 \text{ so } q(2) = 0.$$
$$q(t) = \int_{2}^{t} i(\tau)\, d\tau + q(2) = \int_{2}^{t} 2\, d\tau = 2\tau\Big|_2^t = 2t - 4 \text{ C} \quad \text{for } 2 < t < 4. \text{ In particular, } q(4) = 4 \text{ C}.$$
$$q(t) = \int_{4}^{t} i(\tau)\, d\tau + q(4) = \int_{4}^{t} -1\, d\tau + 4 = -\tau\Big|_4^t + 4 = 8 - t \text{ C} \quad \text{for } 4 < t < 8. \text{ In particular, } q(8) = 0 \text{ C}.$$
$$q(t) = \int_{8}^{t} i(\tau)\, d\tau + q(8) = \int_{8}^{t} 0\, d\tau + 0 = 0 \text{ C} \quad \text{for } 8 < t.$$

P1.3-7

$$I = 600\,\text{A} = 600\,\text{C/s}$$
$$\text{Silver deposited} = 600 \frac{\text{C}}{\text{s}} \times 20\,\text{min} \times \frac{60\text{s}}{\text{min}} \times 1.118\,\text{mg/C}$$
$$= 8.05 \times 10^5 \text{ mg} = \underline{805\,\text{g}}$$

Section 1-6 Power and Energy

P1.6-1

$$P = iv = (2\,\text{mA})(1.5\text{V}) = 3\,\text{mW}$$
$$\Delta t = \frac{\Delta W}{P} = \frac{150\text{J}}{0.003\text{ W}} = 5 \times 10^4 \text{ s}$$
$$\#\text{ of days} = (5 \times 10^4 \text{ s})\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)\left(\frac{1\,\text{day}}{24\text{ hr}}\right) = \underline{0.58\,\text{day}}$$

P1.6-2

a) $$q = \int i\, dt = i\Delta t = (10\,\text{A})(2\,\text{hrs})(3600\,\text{s/hr})$$
$$\underline{= 7.2 \times 10^4 \text{ C}}$$

b) $$P = vi = (110\text{V})(10\,\text{A}) = \underline{1100\,\text{W}}$$

c) $$\text{Cost} = \frac{6¢}{\text{kWhr}} \times 1.1\,\text{kW} \times 2\,\text{hrs} = \underline{13.2¢}$$

P1.6-3

$$P = (6\text{V})(10\,\text{mA}) = 0.06\ \text{W}$$

$$\Delta t = \frac{\Delta W}{P} = \frac{200\,\text{W}\cdot\text{s}}{0.06\,\text{W}} = \underline{3.33\times 10^3\ \text{s}}$$

P1.6-4

a) $$P_S = (675\,\text{A})(12\text{V}) = \underline{8100\,\text{W}}$$

$$P_T = (20\,\text{A})(11\text{V}) = \underline{220\,\text{W}}$$

b) $$\begin{aligned} W &= P_S\Delta t + P_T\Delta t \\ &= (8100\,\text{W})(30\,\text{s}) + (220\,\text{W})(200\,\text{min}\times 60\,\text{s/min}) \\ &= \underline{2883\ \text{kJ}} \end{aligned}$$

P1.6-5

$\underline{\text{at t = 1ms}}$ $$P = \frac{\Delta W}{\Delta t} = \frac{8\,\text{mJ}}{2\,\text{ms}} = \underline{4\,\text{W}} = vi\Big|_{t=1\text{ms}}$$

$$\therefore i = \left.\frac{4}{12\cos(\pi t)}\right|_{t=1\text{ms}} = \underline{-1/3\,\text{A}}$$

$\underline{\text{at t = 3ms}}$ $$P = \frac{\Delta W}{\Delta t} = 0 \qquad \therefore \underline{i = 0\,\text{A}}$$

$\underline{\text{at t = 6ms}}$ $$P = \frac{\Delta W}{\Delta t} = \frac{20-8}{7-5} = 6\,\text{W}$$

$$\therefore\ i(6\,\text{ms}) = \frac{6}{12\cos 6\pi} = \underline{1/2\,\text{A}}$$

P1.6-6 P = vi, for $0 \le t \le 10$s $v = 30$ V

$$i = \frac{30}{15}t = 2t$$

$$\therefore\ P = 30(2t) = 60t$$

for $10 \le t \le 15$s

$$v = -\frac{25}{5}t + b \Rightarrow v(10) = 30 = -150 + b$$

$$\Rightarrow b = 80$$

$$v(t) = -5t + 80$$

$$i(t) = 2t \quad \Rightarrow \quad P = (2t)(-5t+80)$$

$$= -10t^2 + 160t$$

for $15 \le t \le 25$s

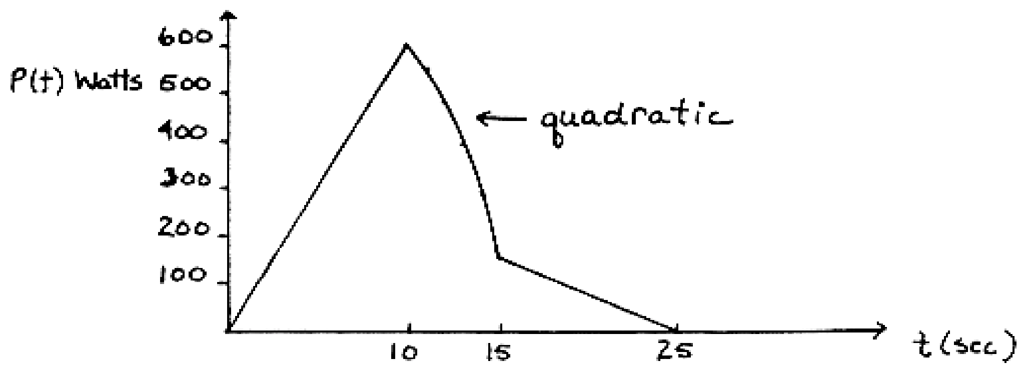
$v = 5$V

$$i(t) = -\frac{30}{10}t + b \Rightarrow i(25) = 0 = -3(25) + b$$

$$\Rightarrow b = 75$$

$$i(t) = -3t + 75$$

$$\Rightarrow P = (5)(-3t+75) = -15t + 375$$



$$\text{Energy} = \int P\,dt = \int_0^{10} 60t\,dt + \int_{10}^{15}\left(160t - 10t^2\right)dt + \int_{15}^{25}\left(375 - 15t\right)dt$$

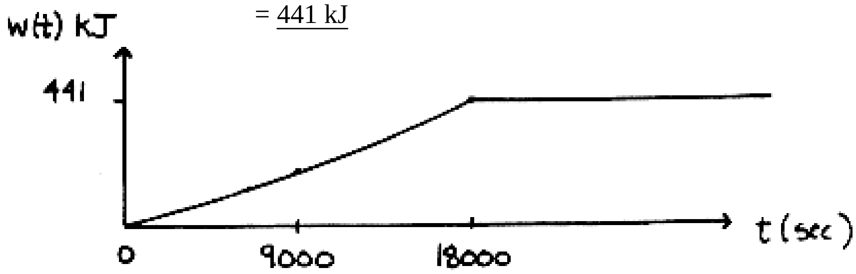
$$= 30t^2\Big|_0^{10} + 80t^2 - \frac{10}{3}t^3\Big|_{10}^{15} + 375t - \frac{15}{2}t^2\Big|_{15}^{25} = 5833.\bar{3}\text{ J}$$

P1.6-7

a) $$W = \int P dt = \int_0^t vi\,dt = \int^{5(3600)\sec} 2\left(11+\frac{.5t}{3600}\right)dt$$

$$= 22t+\frac{0.5}{3600}t^2\Big|_0^{5(3600)} = 441\times10^3 \text{ J}$$

$= \underline{441 \text{ kJ}}$



* Assuming no more energy is delivered to the battery after 5 hours (battery is fully charged).

b) $$\text{Cost} = 441\text{kJ}\times\frac{1\text{ hr}}{3600\text{s}}\times\frac{10¢}{\text{kWhr}} = \underline{1.23¢}$$

P1.6-8 a) Break up into time intervals

t	i	v	P=vi	W=∫P dt
$0 \le t \le .5$	$2t$	3	$6t$	$3t^2$
$0.5 \le t < 1$	$-2t+2$	3	$-6t+6$	$W_2(t)$
$1 \le t \le 2$	0	$-3t+6$	0	$W_2(t=1)$
$2 < t$	0	0	0	$W_2(t=1)$

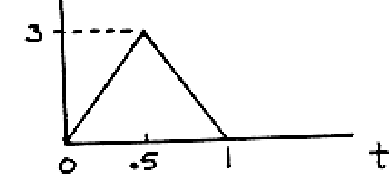
where $$W_2(t) = \int_0^{.5} 6t\,dt + \int_{.5}^{t}(-6t+6)dt$$

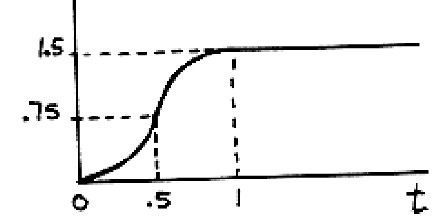
$$= .75+\left(\frac{-6t^2}{2}+6t\right)\Bigg|_{.5}^{t}$$

$\underline{W_2(t) = -3t^2+6t-1.5}$

$\underline{W_2(t = 1) = 1.5}$

b)





P 1.6-9

$$p(t)=\frac{1}{3}(\cos 3t)(\sin 3t)=\frac{1}{6}\sin 6t$$

$$p(0.5)=\frac{1}{6}\sin 3=0.0235 \text{ W}$$

$$p(1)=\frac{1}{6}\sin 6=-0.0466 \text{ W}$$

Here is a MATLAB program to plot *p*(*t*):

```
clear

t0=0;                      % initial time
tf=2;                      % final time
dt=0.02;                   % time increment
t=t0:dt:tf;                % time

v=4*cos(3*t);              % device voltage
i=(1/12)*sin(3*t);         % device current

for k=1:length(t)
   p(k)=v(k)*i(k);         % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```

P 1.6-10

$$p(t)=16(\sin 3t)(\sin 3t)=8(\cos 0-\cos 6t)=8-8\cos 6t \text{ W}$$

Here is a MATLAB program to plot *p*(*t*):

```
clear

t0=0;                      % initial time
tf=2;                      % final time
dt=0.02;                   % time increment
t=t0:dt:tf;                % time

v=8*sin(3*t);              % device voltage
i=2*sin(3*t);              % device current

for k=1:length(t)
   p(k)=v(k)*i(k);         % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```

P1.6-11 $$p(t) = 4\left(1-e^{-2t}\right)\times 2e^{-2t} = 8\left(1-e^{-2t}\right)e^{-2t}$$

Here is a MATLAB program to plot *p*(*t*):

```
clear

t0=0;                      % initial time
tf=2;                      % final time
dt=0.02;                   % time increment
t=t0:dt:tf;                % time

v=4*(1-exp(-2*t));         % device voltage
i=2*exp(-2*t);             % device current

for k=1:length(t)
   p(k)=v(k)*i(k);         % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```

P1.6-12

$$P(t) = iv = (10e^{-t})(12e^{-t}) = \underline{120e^{-2t}W}$$

b) $$W = \int_0^t P\,dt = \int_0^t 120e^{-2t}dt$$

$$= -60e^{-2t}\Big|_0^t = \underline{60(1-e^{-2t})\text{ J}}$$

P1.6-13

a) $$P = vi = (10-20e^{-50t})(4e^{-50t}) \qquad (i = \text{mA})$$

$$= 40e^{-50t} - 80e^{-100t}$$

$$\text{at } t = 10\text{ms} = 0.01\text{s} \Rightarrow \underline{P = -5.17/\text{mW}}$$

b) $$W = \int_0^\infty P(t)dt = \int_0^\infty (40e^{-50t} - 80e^{-100t})dt$$

$$= (-0.8e^{-50t} + 0.8e^{-100t})\Big|_0^\infty$$

$$= 0-(-0.8+0.8)$$

$$= \underline{0\text{J}}$$

P1.6-14 a) $P = vi = (12V)(1A) = \underline{12W}$

b) $\sum P = 0 \quad \therefore P_{\text{absorbed by headlights}}$

$= P_{\text{supplied by battery}} = \underline{12W}$

c) $W = \int P dt = P\Delta t = (12W)(10\text{min})(60\text{s/min})$

$= \underline{7200 \text{ J} = 7.2\text{kJ}}$

P1.6-15 a) $P_{\text{deliv.}} = (18V)(5A) = \underline{90W}$

b) $P_{\text{absorbed}} = (8V)(8A) = \underline{64W}$

P1.6-16

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta W_t}{\Delta t} h \qquad W_t = \text{Weight}$$

$$= \left(5\times10^5 \frac{\text{tons}}{\text{min}}\right)\left(\frac{1\text{min}}{60\text{ sec}}\right)\left(\frac{2000\text{ lb}}{\text{ton}}\right)(168\text{ ft})$$

$$= 2.8 \frac{\text{ft}\cdot\text{1b}}{\text{sec.}}\left(\frac{1.356\text{W}}{\text{ft}\cdot\text{lb/sec}}\right) = 3.8\times10^9\text{ W} = \underline{3.8\text{ GW}}$$

P1.6-17

$P = VI = 3\times0.2 = \underline{0.6\text{ W}}$

$W = P\cdot t = 0.6\times5\times60 = \underline{180\text{ J}}$

P1.6-18

$$W = P\Delta t \qquad \therefore P = \frac{2\times10^6\text{ J}}{10\times60\times60}$$

$$\therefore I = \frac{P}{V} = \frac{2\times10^6}{12(10\times60\times60)} = \underline{4.63\text{ A}}$$

P1.6-19

$Q = 10^{20} e^- \times 1.6\times10^{-19}\ C/e^- = 16\text{ C}$

$\Delta W = v\Delta Q = (10^5\times16)\text{ J}$

$$P = \frac{\Delta W}{\Delta t} = \frac{10^5\times16\text{J}}{0.1\text{s}} = \underline{16\text{ MW}}$$

P1.6-20

$V = 168\text{ kV}$

$I = 2.5\text{ mA} \qquad \therefore P = VI = 168\text{kV}\times2.5\text{ mA}$

$= 2.5\times10^6\text{ A} \qquad \therefore W = P\Delta t = (168\times10^3\times2.5\times10^6)(5)$

$\underline{W = 2.1\times10^{12}\text{ J}}$

energy to launch 10 gms to 10 km above the earth is (assuming $g = 9.8\, m/s^2$ is constant)

$W = mgh = (0.01\text{ kg})(9.8\, m/s^2)(10^4\text{ m}) = 980\text{ J}$

Thus the 10 gm mass will surely leave the earth's orbit!

Verification Problems

VP 1-1
Notice that the element voltage and current of each branch adhere to the passive convention. The sum of the powers absorbed by each branch are:

(-2 V)(2 A)+(5 V)(2 A)+(3 V)(3 A)+(4 V)(-5 A)+(1 V)(5 A) = - 4 W + 10 W + 9 W - 20 W + 5 W
= 0 W

The element voltages and currents satisfy conservation of energy and may be correct.

VP 1-2
Notice that the element voltage and current of some branches do not adhere to the passive convention. The sum of the powers absorbed by each branch are:

$$-(3\text{ V})(3\text{ A})+(3\text{ V})(2\text{ A})+(3\text{ V})(2\text{ A})+(4\text{ V})(3\text{ A})+(-3\text{ V})(-3\text{ A})+(4\text{ V})(-3\text{ A}) \neq 0\text{ W}$$

The element voltages and currents do not satisfy conservation of energy and cannot be correct.

Design Problems

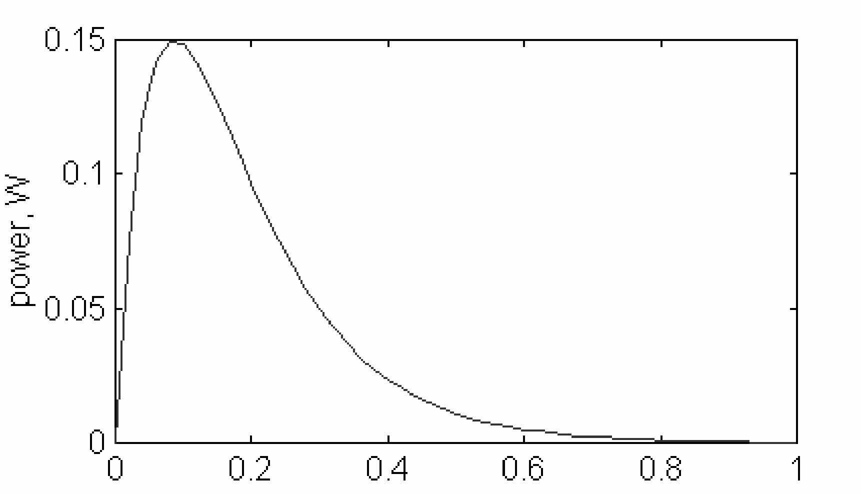
DP 1-1
The voltage may be as large as 20(1.25) = 25 V and the current may be as large as (0.008)(1.25) = 0.01 A. The element needs to be able to absorb (25 V)(0.01 A) = 0.25 W continuously. A Grade B element is adequate, but without margin for error. Specify a Grade B device if you trust the estimates of the maximum voltage and current and a Grade A device otherwise.

DP1-2 $$p(t) = 20\left(1-e^{-8t}\right)\times 0.03e^{-8t} = 0.6\left(1-e^{-8t}\right)e^{-8t}$$

Here is a MATLAB program to plot $p(t)$:

```
clear

t0=0;                    % initial time
tf=1;                    % final time
dt=0.02;                 % time increment
t=t0:dt:tf;              % time

v=20*(1-exp(-8*t));       % device voltage
i=.030*exp(-8*t);           % device current

for k=1:length(t)
   p(k)=v(k)*i(k);       % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```

Here is the plot:



The circuit element must be able to absorb 0.15 W.