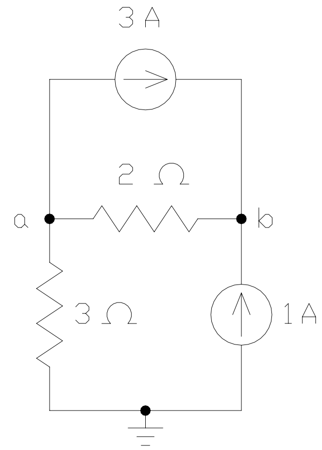
Chapter 4 – Methods of Analysis of Resistive Circuits

Exercises

Ex 4.3-1



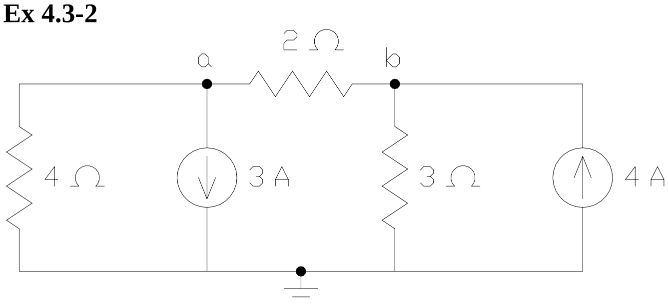
KCL at a: $\dfrac{v_a}{3}+\dfrac{v_a-v_b}{2}+3=0 \;\Rightarrow\; 5\,v_a-3\,v_b=-18$

KCL at b: $\dfrac{v_b-v_a}{2}-3-1=0 \;\Rightarrow\; v_b-v_a=8$

Solving these equations gives:

v_a = 3 V and v_b = 11 V

Ex 4.3-2



KCL at a:

$\dfrac{v_a}{4}+\dfrac{v_a-v_b}{2}+3=0 \;\Rightarrow\; 3\,v_a-2\,v_b=-12$

KCL at a:

$\dfrac{v_b}{3}-\dfrac{v_a-v_b}{2}-4=0 \;\Rightarrow\; -3\,v_a+5\,v_b=24$

Solving:

v_a = -4/3 V and v_b = 4 V

Ex. 4.4-1 $2+\dfrac{v_b+10}{20}+\dfrac{v_b}{30}=5 \;\Rightarrow\; v_b=30\text{ V and }v_a=40\text{ V}$

Ex. 4.4-2 $\dfrac{(v_b+8)-(-12)}{10}+\dfrac{v_b}{40}=3 \;\Rightarrow\; v_b=8\text{ V and }v_a=16\text{ V}$

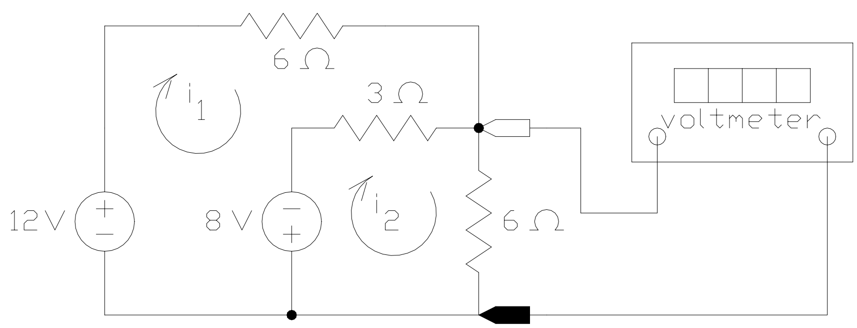
Ex 4.5-1 Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b=4\,i_a$ and solve for v_b.

$$v_b=4\left(\frac{9+v_b}{12}\right) \;\Rightarrow\; v_b=4.5\text{ V}$$

Ex. 4.5-2 : The controlling voltage of the dependent source is a node voltage so it is already expressed as a function of the node voltages. Apply KCL ate node a.

$$\frac{v_a - 6}{20} + \frac{v_a - 4\,v_a}{15} = 0 \quad \Rightarrow \quad v_a = -2 \text{ V}$$

Ex. 4.6-1



Mesh equations:

$$-12 + 6\,i_1 + 3\left(i_1 - i_2\right) - 8 = 0 \quad \Rightarrow \quad 9\,i_1 - 3\,i_2 = 20$$

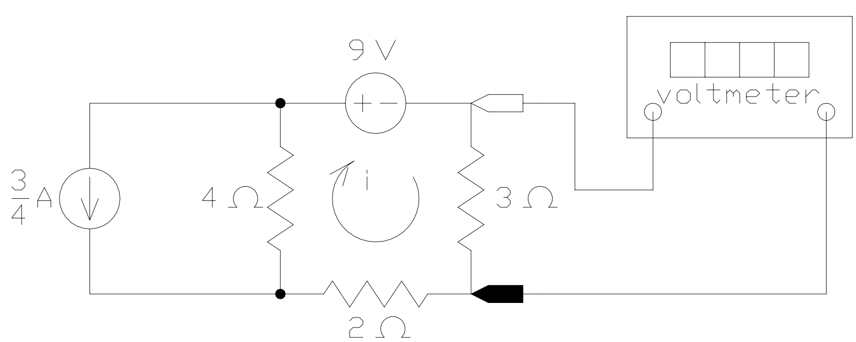
$$8 - 3\left(i_1 - i_2\right) + 6\,i_2 = 0 \quad \Rightarrow \quad -3\,i_1 + 9\,i_2 = -8$$

Solving these equations gives:

$$i_1 = \frac{13}{6} \text{ A and } i_2 = -\frac{1}{6} \text{ A}$$

The voltage measured by the meter is 6 i_2 = -1 V.

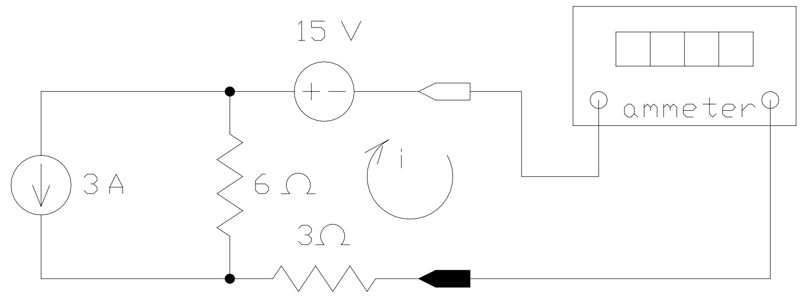
Ex 4.7-1



Mesh equation: $9 + 3\,i + 2\,i + 4\left(i + \frac{3}{4}\right) = 0 \quad \Rightarrow \quad \left(3 + 2 + 4\right) i = -9 - 3 \quad \Rightarrow \quad i = \frac{-12}{9} \text{ A}$

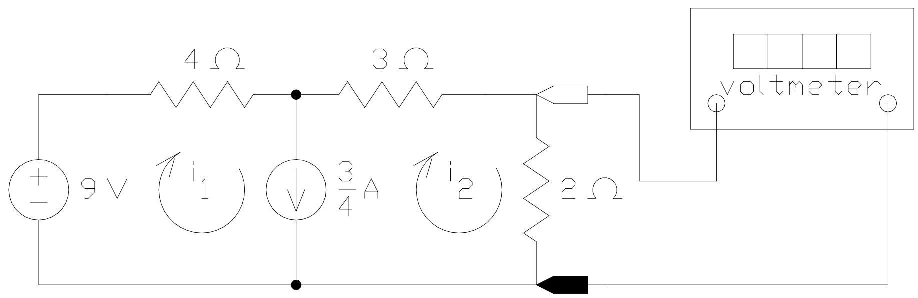
The voltmeter measures $3\,i = -4$ V

Ex 4.7-2



Mesh equation: $15+3\,i+6\left(i+3\right)=0 \quad\Rightarrow\quad \left(3+6\right)i=-15-6\left(3\right) \quad\Rightarrow\quad i=\dfrac{-33}{9}=-6\dfrac{2}{3}\text{ A}$

Ex 4.7-3

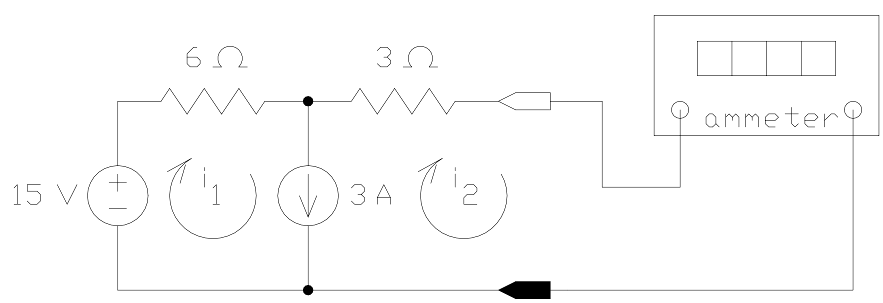


Express the current source current in terms of the mesh currents: $\dfrac{3}{4}=i_1-i_2 \quad\Rightarrow\quad i_1=\dfrac{3}{4}+i_2$.

Apply KVL to the supermesh: $-9+4i_1+3\,i_2+2\,i_2=0 \quad\Rightarrow\quad 4\left(\dfrac{3}{4}+i_2\right)+5\,i_2=9 \quad\Rightarrow\quad 9\,i_2=6$

so $i_2=\dfrac{2}{3}$ A and the voltmeter reading is $2\,i_2=\dfrac{4}{3}$ V

Ex 4.7-4

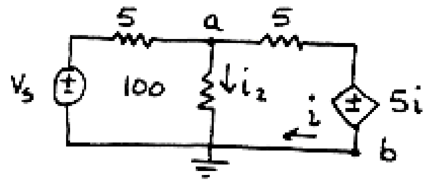


Express the current source current in terms of the mesh currents: $3=i_1-i_2 \quad\Rightarrow\quad i_1=3+i_2$.

Apply KVL to the supermesh: $-15+6\,i_1+3\,i_2=0 \quad\Rightarrow\quad 6\left(3+i_2\right)+3\,i_2=15 \quad\Rightarrow\quad 9\,i_2=-3$

Finally, $i_2=-\dfrac{1}{3}$ A is the current measured by the ammeter.

Ex. 4.7-5



$v_{ab} = 5i + 5i = 10i$

$\therefore P_{motor} = (v_{ab})i = 10i^2 = 150$

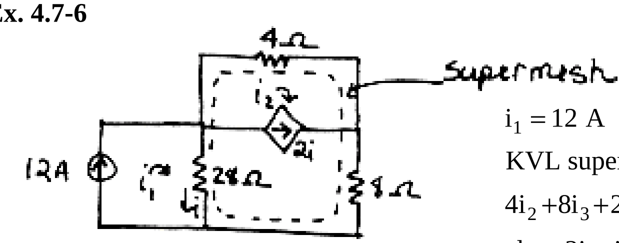
$\Rightarrow i = \sqrt{15}$ A

$$\therefore v_{ab} = (10)\left(\sqrt{15}\right) \text{ V}$$

$$\Rightarrow i_2 = \frac{v_{ab}}{100} = \frac{\sqrt{15}}{10} \text{ A}$$

KCL at a : $(v_{ab} - v_s)/5 + i_2 + i = 0 \quad \Rightarrow \quad \underline{v_s = \frac{31}{2}\sqrt{15} = 60.03 \text{ V}}$

Ex. 4.7-6



$i_1 = 12$ A (1)

KVL supermesh:

$4i_2 + 8i_3 + 28(i_3 - 12) = 0$ (2)

also: $2i = i_3 - i_2$ (3)

Solving (1) → (3) yields $i_3 = 9$A

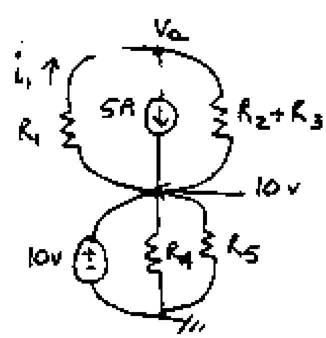
$\therefore i = \underline{12 - 9 = 3 \text{ A}}$

Ex. 4.8-1

(a) Nodal analysis since the other node is known (= v_s); thus only need one node equation at a.

(b) Nodal analysis since when the circuit is redrawn (shown below), only one node equation at v_a is required

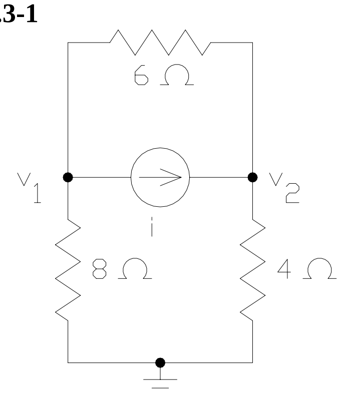
Mesh analysis would require
4 mesh currents
⇒ 4 unknows.



Problems

Section 4-3 Node Voltage Analysis of Circuits with Current Sources

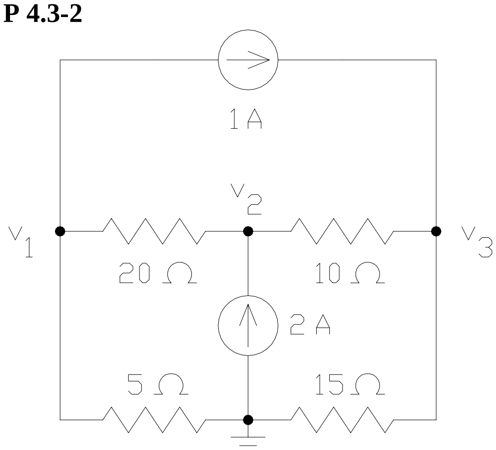
P4.3-1



KCL at node 1:

$$0=\frac{v_1}{8}+\frac{v_1-v_2}{6}+i=\frac{-4}{8}+\frac{-4-2}{6}+i=-1.5+i \quad\Rightarrow\quad i=1.5\text{ A}$$

P 4.3-2



KCL at node 1:

$$\frac{v_1-v_2}{20}+\frac{v_1}{5}+1=0 \quad\Rightarrow\quad 5\,v_1-v_2=-20$$

KCL at node 2:

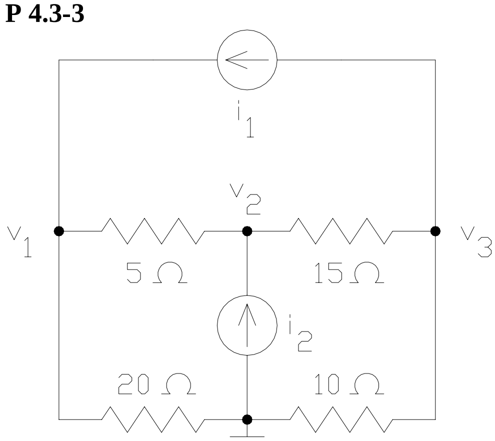
$$\frac{v_1-v_2}{20}+2=\frac{v_2-v_3}{10} \quad\Rightarrow\quad -v_1+3\,v_2-2\,v_3=40$$

KCL at node 3:

$$\frac{v_2-v_3}{10}+1=\frac{v_3}{15} \quad\Rightarrow\quad -3\,v_2+5\,v_3=30$$

Solving gives v_1 = 2 V, v_2 = 30 V and v_3 = 24 V.

P 4.3-3



KCL at node 1:

$$\frac{v_1-v_2}{5}+\frac{v_1}{20}=i_1 \quad\Rightarrow\quad i_1=\frac{4-15}{5}+\frac{4}{20}=-2\text{ A}$$

KCL at node 2:

$$\frac{v_1-v_2}{5}+i_2=\frac{v_2-v_3}{15} \quad\Rightarrow\quad i_2=-\left(\frac{4-15}{5}\right)+\frac{15-18}{15}=2\text{ A}$$

P4.3-4

$$-.003+\frac{v_1}{R_1}+\frac{v_1-v_2}{500}=0$$

$$-\frac{v_1-v_2}{500}+\frac{v_2}{R_2}-.005=0$$

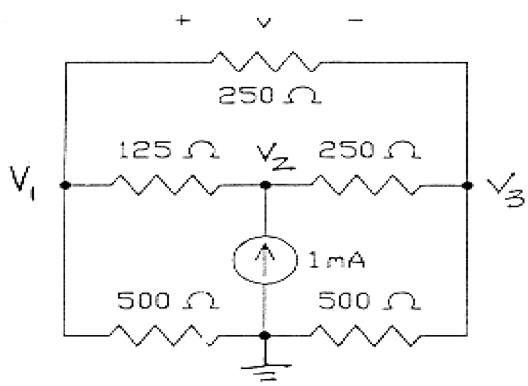
When

$$v_1=1 \text{ and } v_2=2$$

$$-.003+\frac{1}{R_1}+\frac{-1}{500}=0\Rightarrow R_1=\frac{1}{.003+\frac{1}{500}}=\underline{200\Omega}$$

$$-\frac{-1}{500}+\frac{2}{R_2}-.005=0\Rightarrow R_2=\frac{2}{.005-\frac{1}{500}}=\underline{667\Omega}$$

P 4.3-5



$$\frac{v_1}{500}+\frac{v_1-v_2}{125}+\frac{v_1-v_3}{250}=0$$

$$-\frac{v_1-v_2}{125}-.001+\frac{v_2-v_3}{250}=0$$

$$-\frac{v_2-v_3}{250}-\frac{v_1-v_3}{250}+\frac{v_3}{500}=0$$

$$\Rightarrow v_1=0.261 \text{ V}$$

$$v_2=0.337 \text{ V}$$

$$v_3=0.239 \text{ V}$$

Finally, $v=v_1-v_3=\underline{0.022 \text{ V}}$

Section 4-4 Node Voltage Analysis of Circuits with Current and Voltage Sources

P4.4-1

$$\frac{v_a-10}{100}+\frac{v_a}{100}+\frac{v_a-2}{100}=0\Rightarrow 3v_a=12$$

$$\Rightarrow \underline{v_a=4 \text{ V}}$$

P4.4-2

$$-.003+\frac{v_a+8}{500}+\frac{v_a}{500}-.005=0$$

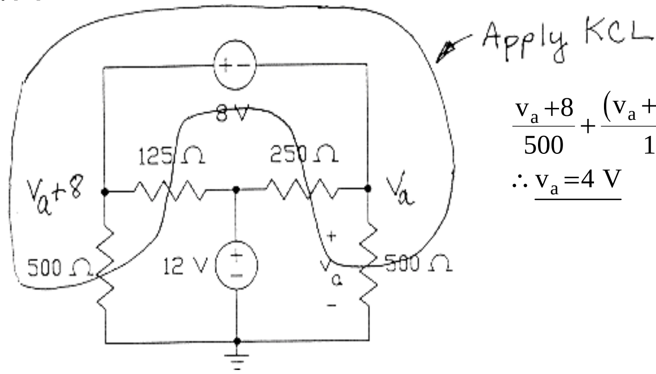
$$\therefore \underline{v_a=-2 \text{ V}}$$

P4.4-3

$$\frac{v_a-10}{100}+\frac{v_a}{100}+\frac{v_a-8}{100}-.03=0$$

$$\therefore \underline{v_a=7 \text{ V}}$$

P4.4-4



$$\frac{v_a+8}{500}+\frac{(v_a+8)-12}{125}+\frac{v_a-12}{250}+\frac{v_a}{500}=0$$

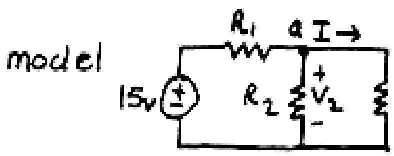
$$\therefore \underline{v_a = 4\text{ V}}$$

P4.4-5

$$-\frac{10-6}{100}+\frac{6}{R_3}+\frac{6-4}{100}=0 \Rightarrow \underline{R_3 = 300\,\Omega}$$

P4.4-6

a)



need to keep v_2 across R_2 as $4.8 \le v_2 \le 5.4$

I=.3 or .1 A depending on whether activated or not

↑ ↑

display is active / not active

KCL at a: $\frac{v_2-15}{R_1}+\frac{v_2}{R_2}+I=0$

$\Rightarrow v_2{}' = 4.8\text{v}\ (I' = .3\text{A})\ \ \&\ \ v_2{}'' = 5.4\text{V}\ (I''=.1\text{A})$

assumed that maximum I results in minimum v_2 and Visa-Versa

Now plug in $v_2{}'$ & $v_2{}'$ into KCL eqn. to generate 2 eqns and then solve for

R_1 & $R_2 \Rightarrow \underline{R_1 = 7.89\ \Omega}$, $\underline{R_2 = 4.83\ \Omega}$

b)

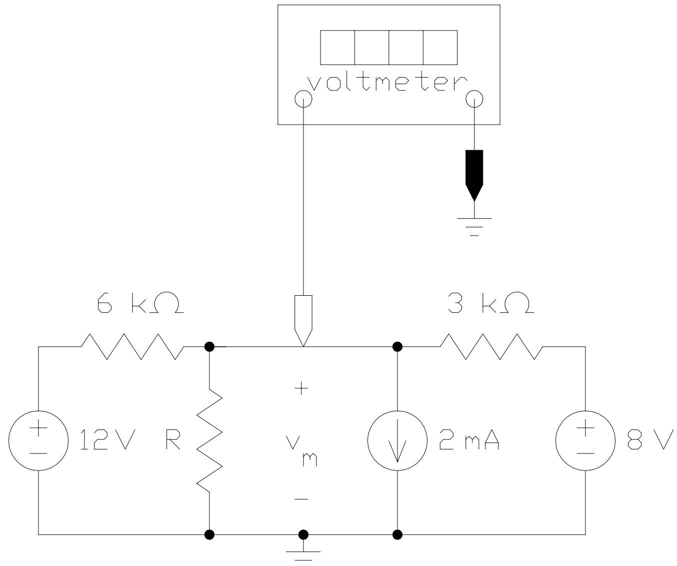
$$I_{R_{1\max}} = \frac{15-4.8}{7.89} = 1.292\text{ A} \Rightarrow P_{R_{1\max}} = (1.292)^2(7.89) = 13.17\text{ W}$$

$$I_{R_{2\max}} = \frac{5.4}{4.83} = 1.118\text{ A} \Rightarrow P_{R_{2\max}} = \frac{(5.4)^2}{4.83} = 6.03\text{ W}$$

$$I_{15v_{\max}} = 1.292\text{ A}$$

c) No; if the supply voltage (15V) were to rise or drop, the voltage at the display would drop below 4.8V or rise above 5.4V.
The power dissipated in the resistors is excessive. Most of the power from the supply is dissipated in the resistors, not the display.

P4.4-7 Label the voltage measured by the meter. Notice that this is a node voltage.



Write a node equation at the node at which the node voltage is measured.

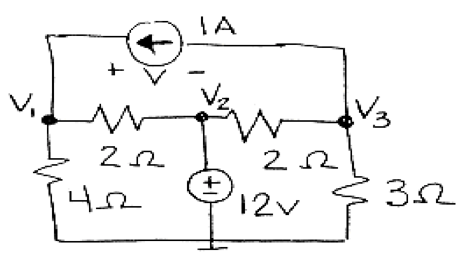
$$-\left(\frac{12-v_m}{6\,\text{k}\Omega}\right)+\frac{v_m}{R}+2\,\text{mA}+\frac{v_m-8}{3\,\text{k}\Omega}=0$$

That is

$$\left(3+\frac{6\text{ k}\Omega}{R}\right)v_m=16 \;\Rightarrow\; R=\frac{6\text{ k}\Omega}{\frac{16}{v_m}-3}$$

(a) The voltage measured by the meter will be 4 volts when R = 6 kΩ.
(b) The voltage measured by the meter will be 2 volts when R = 1.2 k Ω.

P4.4-8



$$v = v_1 - v_3$$
$$v_2 = 12\text{ V}$$

KCL at v_1: $\frac{v_1}{4}+\frac{v_1-v_2}{2}-1=0$

KCL at v_3: $\frac{v_3}{3}+\frac{v_3-v_2}{2}+1=0$

Solving for v_1 & v_3 : $v_1 = 9.33$ V

$v_3 = 6$ V

$v = 9.33-6 = \underline{3.33\text{ V}}$

Section 4-5 Node Voltage Analysis with Dependent Sources

P4.5-1

$$v_a = 9 - v_b - \frac{9-v_b}{100} + (.02)(9 - v_b) + \frac{v_b}{200} = 0$$

$$\therefore \underline{v_b = +18\ V};\ v_a = -9\ V$$

P4.5-2

$$-.002 + \frac{v_a}{10{,}000} + \frac{v_a - 4000\left(\frac{v_a}{10{,}000}\right)}{6000} = 0$$

$$-.002 + \frac{v_a}{10{,}000} + \frac{.6v_a}{6000} = 0 \Rightarrow \underline{v_a = 10\ V}$$

P4.5-3

$$\frac{v_a - 6}{1000} + \frac{v_a}{2000} + \frac{v_a - 4v_a}{3000} = 0 \Rightarrow v_a = 12\ V$$

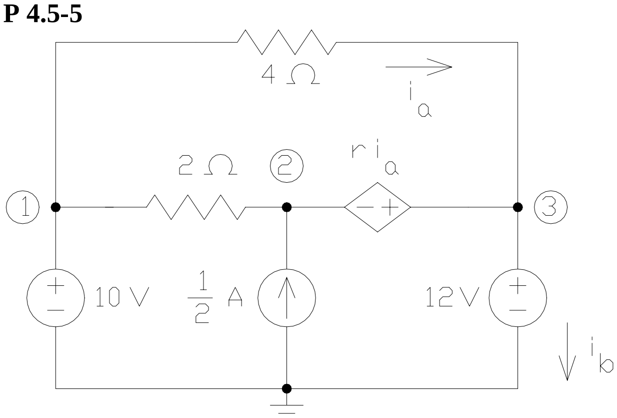
$$i_b = \frac{v_a - 4v_a}{3000} = \underline{-12\ mA}$$

P4.5-4

$$i_a = \frac{2 - v_b}{4000}$$

$$-\frac{2-v_b}{4000} + \frac{v_b}{2000} - 5\left(\frac{2-v_b}{4000}\right) = 0 \Rightarrow \underline{v_b = 1.5\ V}$$

P 4.5-5



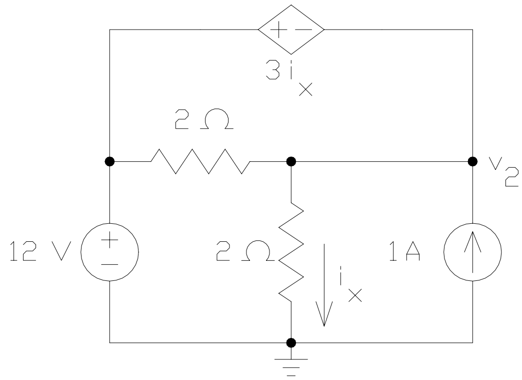
Apply KCL to the supernode of the CCVS to get

$$\frac{12-10}{4} + \frac{14-10}{2} - \frac{1}{2} + i_b = 0 \quad \Rightarrow \quad i_b = -2\ A$$

Next

$$\left.\begin{array}{l} i_a = \frac{10-12}{4} = -\frac{1}{2} \\ r\,i_a = 12 - 14 \end{array}\right\} \Rightarrow \quad r = \frac{-2}{-\frac{1}{2}} = 4\ \frac{V}{A}$$

P4.5-6



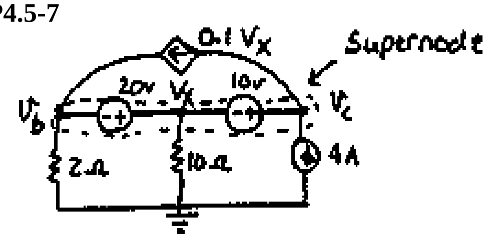
First, express the controlling current of the CCVS in terms of the node voltages: $i_x = \frac{v_2}{2}$

Next, express the controlled voltage in terms of the node voltages:

$$12 - v_2 = 3 i_x = 3\frac{v_2}{2} \quad \Rightarrow v_2 = \frac{24}{5} \text{ V}$$

so i_x = 12/5 A = 2.4 A.

P4.5-7



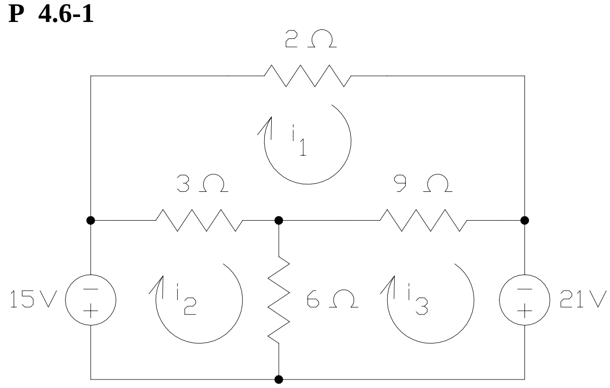
KCL: $\frac{v_b}{2} + \frac{v_x}{10} + 4 = 0$ (1)

also $v_b = v_x - 20$ (2)

(1) & (2) yields $v_x = 10$ V

<u>Section 4-6</u> Mesh Current Analysis with Independent Voltage Sources

P 4.6-1



$$2\, i_1 + 9\,(i_1 - i_3) + 3(i_1 - i_2) = 0$$
$$15 - 3\,(i_1 - i_2) + 6\,(i_2 - i_3) = 0$$
$$-6\,(i_2 - i_3) - 9\,(i_1 - i_3) - 21 = 0$$

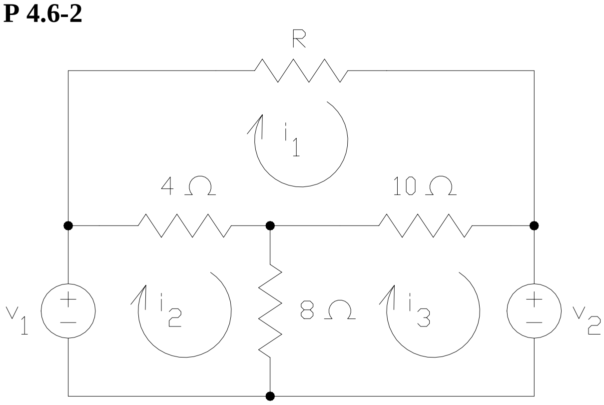
or

$$14\, i_1 - 3\, i_2 - 9\, i_3 = 0$$
$$-3\, i_1 + 9\, i_2 - 6\, i_3 = -15$$
$$-9\, i_1 - 6\, i_2 + 15\, i_3 = 21$$

so

i_1 = 3 A, i_2 = 2 A and i_3 = 4 A.

P 4.6-2



Top mesh:

$$4\,(2-4) + R(2) + 10\,(2-4) = 0$$

so R = 12 Ω.

Bottom, left mesh:

$$8\,(4-3) + 10\,(4-2) + v_2 = 0$$

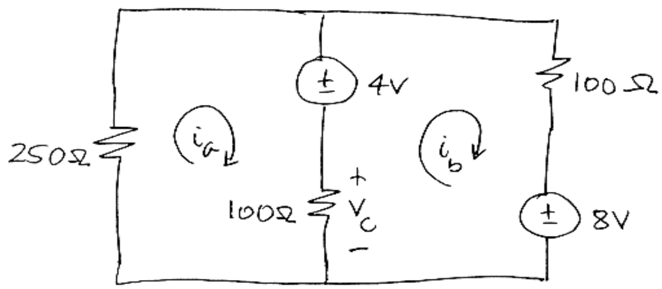
so v_2 = -28 V.

Bottom right mesh

$$-v_1 + 4\,(3-2) + 8\,(3-4) = 0$$

so v_1 = -4 V.

P4.6-3



loop a $\quad 250\ i_a + 4 + 100\,(i_a - i_b) = 0$

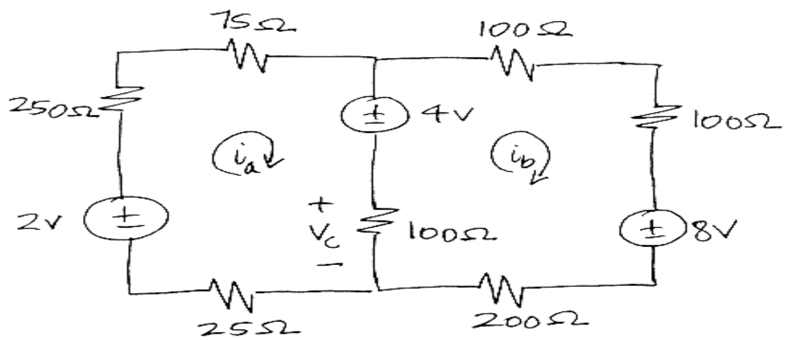
$350\ i_a - 100\ i_b = -4$

loop b $\quad -100(i_a - i_b) - 4 + 100\ i_b + 8 = 0$

$-100\ i_a + 200\ i_b = -4$

$\underline{i_a = -20\text{ mA}}$, $\underline{i_b = -30\text{ mA}}$

P4.6-4



loop 1 $\quad 25\ i_a - 2 + 250\ i_a + 75\ i_a + 4 + 100\,(i_a - i_b) = 0$

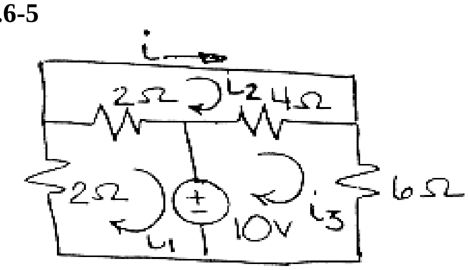
$450\ i_a - 100\ i_b = -2$

loop2 $\quad -100(i_a - i_b) - 4 + 100\ i_b + 100\ i_b + 8 + 200\ i_b = 0$

$-100\ i_a + 500\ i_b = -4$

$\underline{i_a = -6.5\text{ mA}}$, $\underline{i_b = -9.3\text{ mA}}$

P4.6-5



KVL1 : $2i_1 + 2\,(i_1 - i_2) + 10 = 0$

KVL2 : $2(i_2 - i_1) + 4\,(i_2 - i_3) = 0$

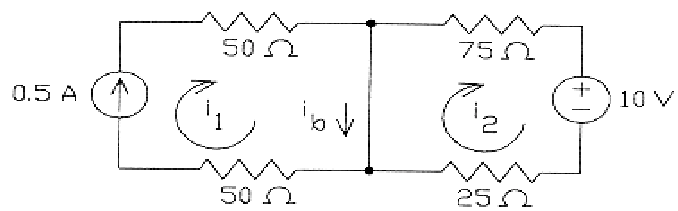
KVL3 : $-10 + 4\,(i_3 - i_2) + 6\,i_3 = 0$

$i = i_2 \Rightarrow \underline{i = -0.294\text{ A}}$

Section 4-7 Mesh Current Analysis with Voltage and Current Sources

(a) Independent Sources

P4.7-1

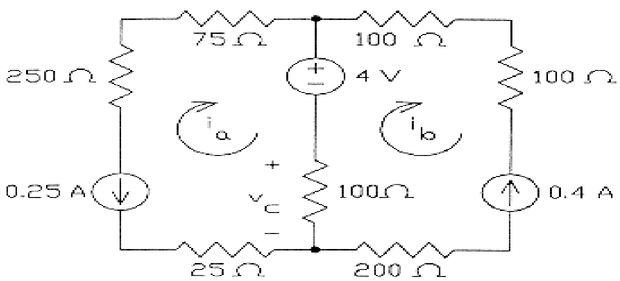


loop 1 $i_1 = \frac{1}{2}$ A

loop 2 $75\, i_2 + 10 + 25\, i_2 = 0 \Rightarrow i_2 = -0.1$ A

$i_b = i_1 - i_2 = \underline{0.6 \text{ A}}$

P4.7-2

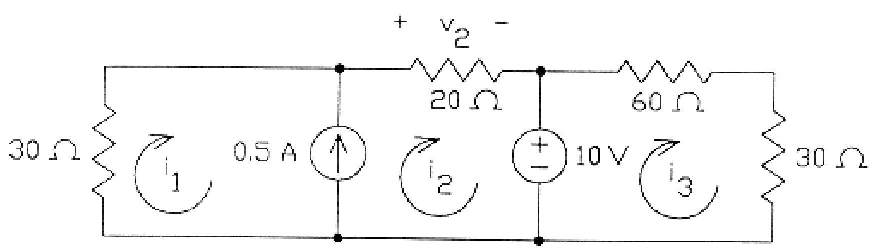


loop a $i_a = -0.25$ A

loop b $i_b = -0.4$ A

$v_c = 100(i_a - i_b) = 100(0.15) = \underline{15 \text{ V}}$

P4.7-3



loop1 $i_1 - i_2 = -0.5 \Rightarrow i_1 = i_2 - 0.5$

loop1,2 $30\, i_1 + 20\, i_2 + 10 = 0$

$30(i_2 - 0.5) + 20 i_2 = -10$

$50 i_2 - 15 = -10$

$i_2 = \frac{5}{50} = .1$ A

$i_1 = -.4$ A

$v_2 = 20 i_2 = \underline{2 \text{ V}}$

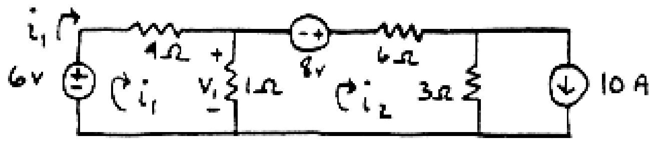
P4.7-4 $i_b = i_a - 0.02$

$250\, i_a + 100\,(i_a - 0.02) + 9 = 0$

$\therefore i_a = -.02 \text{ A} = -20 \text{ mA}$

$v_c = 100(i_a - 0.02) = \underline{-4 \text{ V}}$

P4.7-5

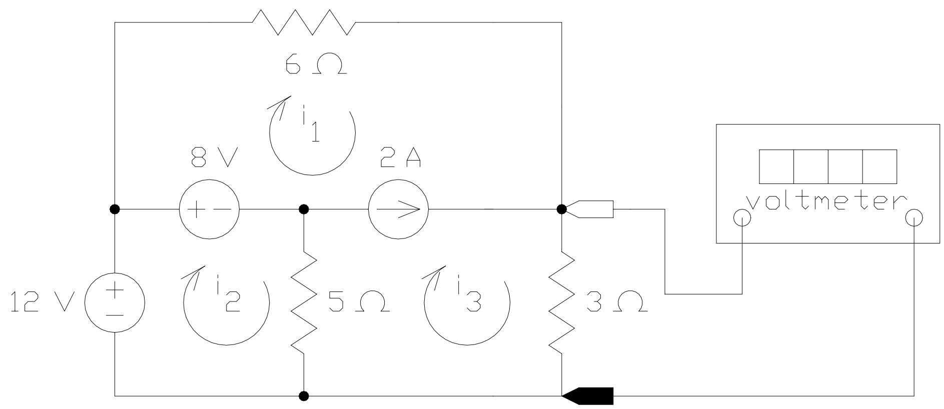


KVL mesh i_1: $-6+4i_1+(i_1-i_2)=0 \;\Rightarrow\; \underline{-6+5i_1-i_2=0}$ (1)

KVL mesh i_2: $(i_2-i_1)-8+6i_2+3(i_2-10)=0 \;\Rightarrow\; \underline{-38-i_1+10i_2=0}$ (2)

Solving (1) & (2) simultaneously yields $\underline{i_1=2\text{ A}}$ and $i_2=4\text{ A}$ $\therefore$ $\underline{v_1=1(i_1-i_2)=-2\text{ V}}$

P4.7-6



Express the current source current in terms of the mesh currents:

$$i_3-i_1=2 \quad\Rightarrow\quad i_1=i_3-2$$

Supermesh: $6\,i_1+3\,i_3-5\left(i_2-i_3\right)-8=0 \quad\Rightarrow\quad 6\,i_1-5\,i_2+8\,i_3=8$

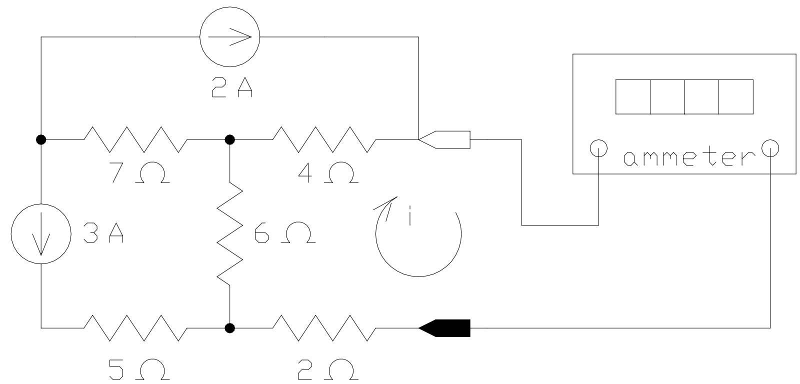
Lower, left mesh: $-12+8+5\left(i_2-i_3\right)=0 \quad\Rightarrow\quad 5\,i_2=4+5\,i_3$

Eliminating i_1 and i_2 from the supermesh equation:

$$6\left(i_3-2\right)-\left(4+5\,i_3\right)+8\,i_3=8 \quad\Rightarrow\quad 9\,i_3=24$$

The voltage measured by the meter is: $3\,i_3=3\,\dfrac{24}{9}=8\text{ V}$

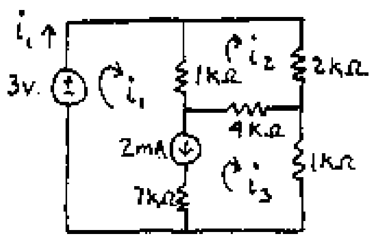
P4.7-7



Mesh equation for right mesh:

$$4(i-2)+2i+6(i+3)=0 \quad \Rightarrow \quad 12i-8+18=0 \quad \Rightarrow \quad i=\frac{10}{12} \text{ A}$$

P4.7-8



KVL around mesh i_1 & i_3 combined

$-3+(i_1-i_2)+4(i_3-i_2)+i_3=0$

yields $\underline{3-i_1+5i_2-5i_3=0}$ (1)

KVL around mesh i_2

$i_2-i_1+2i_2+4(i_2-i_3)=0$

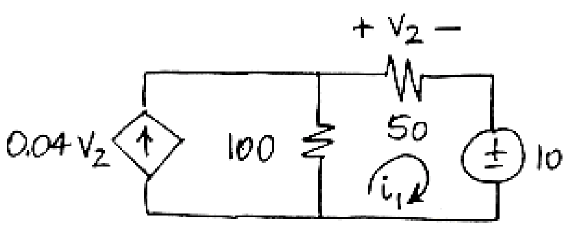
yields $\underline{i_1-7i_2+4i_3=0}$ (2)

also $\underline{i_1-i_3=2}$ (3)

Solving (1), (2), & (3) simultaneously yields $\underline{i_1 = 3 \text{ mA}}$

Section 4-7 Mesh Current Analysis with Voltage and Current Sources
b) Independent and Dependent Sources

P4.7-9



$v_2 = 50i_1$

$-100\,(0.04(50i_1)-i_1)+50i_1+10 = 0 \Rightarrow i_1 = 0.2 \text{ A}$

$v_2 = 50\,i_1 = 10 \text{ V}$

P4.7-10 $i_b = 4i_b - i_a \Rightarrow i_b = \frac{1}{3}\,i_a$

$-100\left(\frac{1}{3}i_a\right)+200i_a+8 = 0 \Rightarrow \underline{i_a = -0.048 \text{ A}}$

P4.7-11 $i_b = .06 - i_a$

$$-100\,(0.06 - i_a) + 50\,(0.06 - i_a) + 250\,i_a = 0$$

$$\therefore\ i_a = 10\text{ mA}$$

$$v_o = 50 i_b = 50(.06 - .01) = \underline{2.5\text{ V}}$$

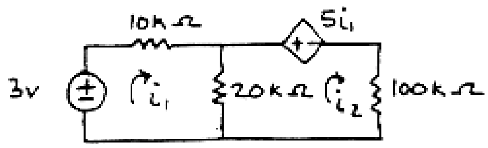
P4.7-12

$$v_b = 100\,(.006 - i_a)$$

$$-100\,(.006 - i_a) + 3\left[100(.006 - i_a)\right] + 250 i_a = 0$$

$$\therefore\ \underline{i_a = -24\text{ mA}}$$

P4.7-13



$$\text{KVL}i_1:\ -3 + 10 i_1 + 20(i_1 - i_2) = 0 \Rightarrow \underline{30 i_1 - 20 i_2 = 3} \qquad (1)$$

$$\text{KVL}i_2:\ 5 i_1 + 100 i_2 + 20(i_2 - i_1) = 0 \Rightarrow \underline{i_1 = 8 i_2} \qquad (2)$$

$$\text{Solving } (1)\ \&\ (2) \text{ simultaneously} \Rightarrow i_1 = \frac{6}{55}\text{ mA},\ i_2 = \frac{3}{220}\text{ mA}$$

$$\therefore\ P_{\text{deliv.}} \text{ to } 5i \text{ and } 100\text{k}\Omega = (5 i_1)(i_2) + 100 (i_2)^2$$

$$= 5\left(6/55\right)\left(3/220\right) + 100\left(3/220\right)^2 = 0.026\text{ mA}$$

$$= 2.6\times10^{-5}\text{W}$$

$$\therefore\ \text{Energy in 24 hr.} = Pt = \left(2.6\times10^{-5}\right)(24\text{ hr})\left(3600\text{s}/\text{hr}\right)$$

$$\underline{= 2.25\text{ J}}$$

P4.7-14 (a)

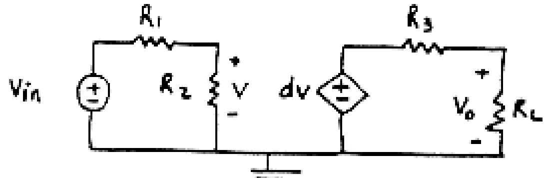
$$v_o = -g R_L v \text{ and } v = \frac{R_2}{R_1 + R_2} v_i \Rightarrow \frac{v_o}{v_i} = -g \frac{R_L R_2}{R_1 + R_2}$$

(b)

$$\text{So have } \frac{v_o}{v_i} = -g \frac{\left(5\times10^3\right)\left(10^3\right)}{1.1\times10^3} = -170 \Rightarrow \underline{g = 0.0374\text{ S}}$$

P4.7-15

d=10



$$\text{from voltage divider } v = v_{in} \frac{R_2}{R_1 + R_2}$$

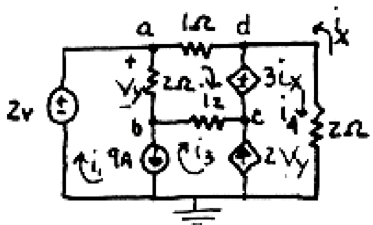
$$\text{from voltage divider } v_o = dv \frac{R_L}{R_L + R_3}$$

$$= (10) v_{in} \frac{R_2}{R_1 + R_2} \cdot \frac{R_L}{R_L + R_3}$$

$$\therefore\ \underline{\frac{v_o}{v_{in}} = \frac{10 R_L R_2}{(R_L + R_3)(R_1 + R_2)}}$$

Section 4-8 Node Voltage Method and Mesh Current Method Compared

P4.8.1



a) Mesh Analysis

Combine loop 1, 3, & 4 into supermesh

$\Rightarrow -2+(i_1-i_2)2+(i_3-i_2)-3i_x+2i_4=0$ (1)

loop i_2: $-2i_1+4i_2-i_3+3i_x=0$ (2)

also : $i_1 - i_3 = 9$ (3)

$i_4-i_3=2v_y$ (4)

$2(i_1-i_2)=v_y$ (5)

and $v_c = -2i_x - 3i_x$ (6)

$i_4 = -i_x$ (7)

Solving (1) − (7) simultaneously yields

$i_x = -1$ A, $\underline{v_c = 5 \text{ V}}$

b) Nodal Analysis

$v_a = 2V$

KCL at b: $9+\frac{v_b-v_c}{1}+\frac{v_b-2}{2} = 0 \Rightarrow 3v_b-2v_c = -16$ (1)

form super node around nodes c & d, then KCL yields

$$\frac{v_d-2}{1}+\frac{v_d}{2}-2v_y+\frac{v_c-v_b}{1} = 0$$

with $v_y = 2-v_b$ and multiplying above through by 2 yields

$2v_b+2v_c+3v_d = 12$ (2)

also: $v_d-v_c = 3i_x = 3\left(-v_d/2\right) \Rightarrow v_c+2.5v_d = 0$ (3)

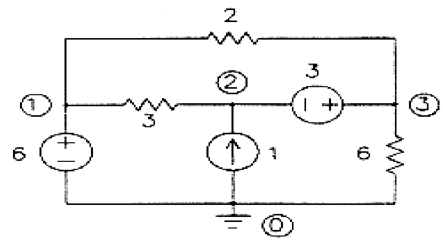
Solving (1)–(3) yields

$\underline{v_c = 5 \text{ V}}$ and $v_d = 2v \Rightarrow \underline{i_x = -\frac{v_d}{2} = -1 \text{ A}}$

PSpice Problems

SP 4-1 Spice deck corresponding to Problem SP 4-1

```
V1     1     0     dc    6
R2     1     2           3
I3     0     2     dc    1
V4     3     2     dc    3
R5     3     0           6
R6     3     1
.END
```

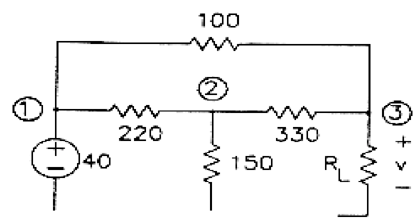



```
   NODE  VOLTAGE        NODE    VOLTAGE
(    1)   6.0000    (      2)    4.0000
(    3)   7.0000
```

```
    VOLTAGE SOURCE CURRENTS
    NAME           CURENT
    V1           -1.667E-01
    V4           -1.667E+00
```

SP 4-2 Spice Deck Corresponding to Problem SP 4-2

```
VI     1     0     dc   40
R2     1     2         220
R3     2     0         150
R4     2     3         330
R5     3     0          75
R6     1     3         100
.END
```



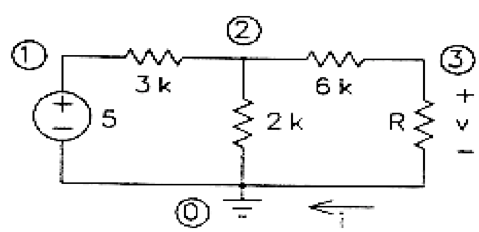

```
   NODE  VOLTAGE       NODE    VOLTAGE      NODE    VOLTAGE
(    1)  40.0000   (     2)    16.3950   (    3)    17.0570
```

```
   VOLTAGE SOURCE CURRENTS
   NAME           CURRENT
   V1            -3.367E-01
```

SP 4-3 Spice Deck corresponding to Problem SP 4-3

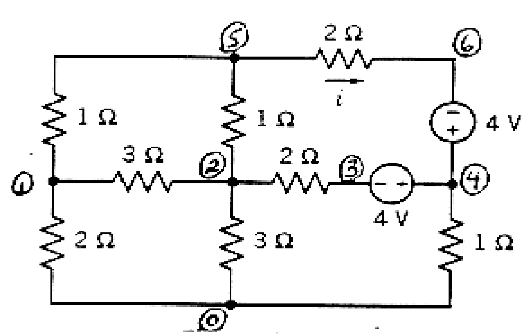
```
V1    1   0   dc     5
R3    2   0         2K
R4    2   3         6K
R5    3   0         3K
.END
```

V=0.588V



SP 4-4 Spice deck corresponding to Problem SP 4-4

```
R1  1  0 2
R2  1  2 3
R3  1  5 1
R4  2  0 3
R5  2  3 2
R6  2  5 1
V7  4  3 4
R8  4  0 1
V9  4  6 4
R10 5  6 2
. END
```



```
  NODE   VOLTAGE        NODE   VOLATAGE       NODE   VOLTAGE
(    1)  -1.2332      (    2)  -1.6364      (    3)  -2.8379
(    4)   1.1621      (    5)  -1.7154      (    6)  -2.8379
```

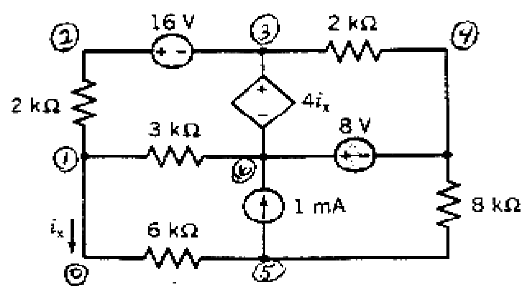
```
VOLTAGE SOURCE CURRENTS
NAME        CURRENT
V7         -6.008E-O.1
V9         -5.613E-0.1
```

SP 4-5

Input File:

```
R1      0    5         6000
R2      1    6         3000
R3      1    2         2000
R4      3    4         2000
R5      4    5         8000
V1      2    3    dc   16
V2      6    4    dc   8
I1      5    4    dc   1m
Vsc1    1    0         0
H1      3    6    Vsc1 4

.dc V1 16 16 1
.print dc I(Vsc1)
.end
```



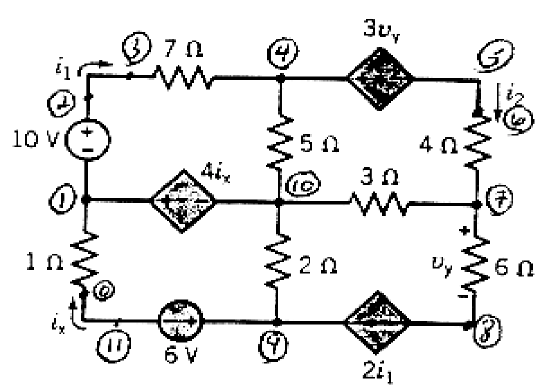
Output:

```
  V1         I(Vsc1)
  1.600E+1   1.684E-03
```

SP 4-6

Input File:

```
R1       0      1            1
R2       3      4            7
R3       4     10            5
R4      10      7            3
R5       6      7            4
R6      10      9            2
R7       7      8            6
V1       9     11      dc    6
V2       2      1      dc   10
Vsc1     2      3            0
Vsc2    11      0            0
Vsc3     5      6            0
H1       1     10      Vsc2  4
F1       8      9      Vsc1  2
E1       4      5     7   8  3
.dc V1 6 6 1
.print dc I Vsc2
.end
```

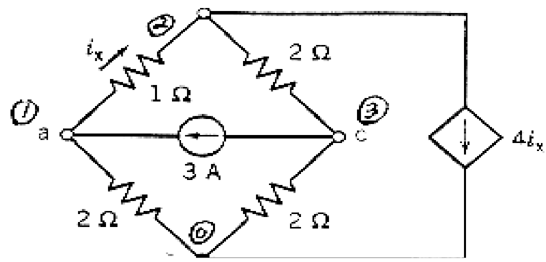


Output:

```
V1          I(Vsc2)
6.000+E00 -6.706E-01
```

SP 4-7 Spice deck corresponding to Problem SP 4-7

```
R1     1 0 2
R2     4 2 1
VSC3   1 4 0
F3     2 0 VSC3   4
R4     2 3 2
R5     3 0 2
I6     3 1 DC     3
.END
```



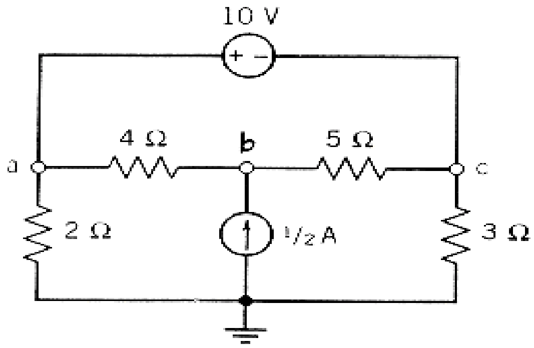

```
 NODE   VOLTAGE     NODE     VOLTAGE      NODE     VOLTAGE
(    1)    8.6667   (    2)     10.0000    (    3)        2.0000
(    4)    8.6667

VOLTAGE SOURCE CURRENTS
NAME         CURRENT

VSC3        -1.333E+00
```

Verification Problems

VP 4-1



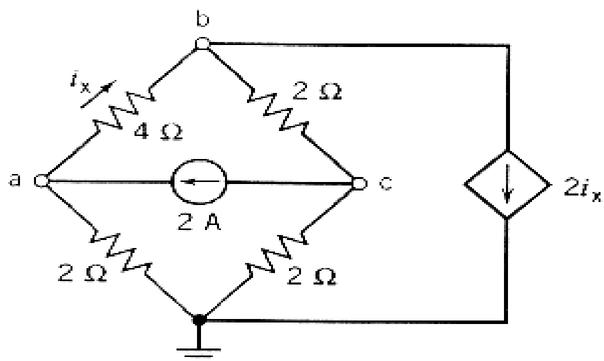
Apply KCL at node b

$$\frac{v_b - v_a}{4} - \frac{1}{2} + \frac{v_b - v_c}{5} = 0$$

$$-\frac{4.8-5.2}{4} - \frac{1}{2} + \frac{-4.8-3.0}{5} \neq 0$$

The given voltages do not satisfy the KCL equation at node b. They are not correct.

VP 4-2



Apply KCL at node a:

$$-\left(\frac{v_b + v_a}{4}\right) - 2 + \frac{v_a}{2} = 0$$

$$-\left(\frac{20-4}{4}\right) - 2 + \frac{4}{2} = -4$$

$$\neq 0$$

The given voltages do not satisfy the KCL equation at node a. They are not correct.

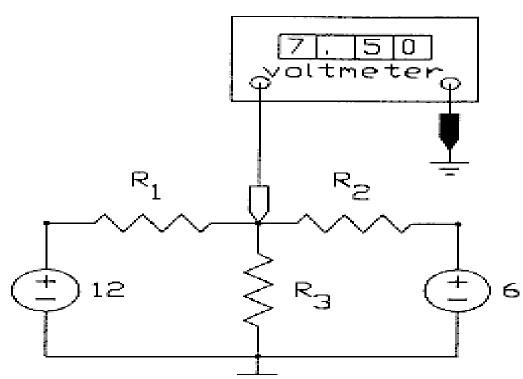
VP 4-3

Writing a node equation:

$$-\left(\frac{12-7.5}{R_1}\right) + \frac{7.5}{R_3} + \frac{7.5-6}{R_2} = 0$$

so

$$-\frac{4.5}{R_1} + \frac{7.5}{R_3} + \frac{1.5}{R_2} = 0$$

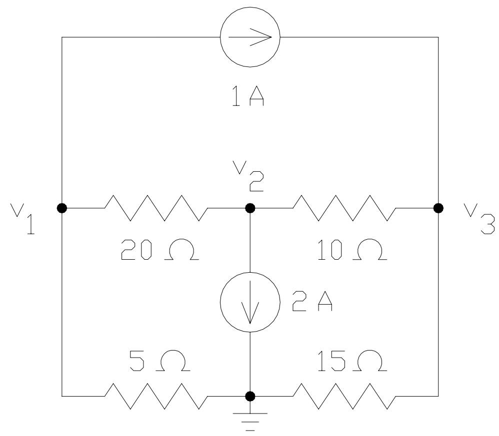


There are only three cases to consider. Suppose $R_1 = 5\text{k}\Omega$ and $R_2 = 10\text{k}\Omega$. Then

$$-\frac{4.5}{R_1} + \frac{7.5}{R_3} + \frac{1.5}{R_2} = -0.9 + 0.75 + 0.15 = 0$$

This choice of resistance values corresponds to branch currents that satisfy KCL. Therefore, it is indeed possible that two of the resistance are 10kΩ and the other resistance is 5kΩ. The 5kΩ is R .

VP 4-4



KCL at node 1:

$$0 = \frac{v_1 - v_2}{20} + \frac{v_1}{5} + 1 = \frac{-8-(-20)}{20} + \frac{-8}{5} + 1$$

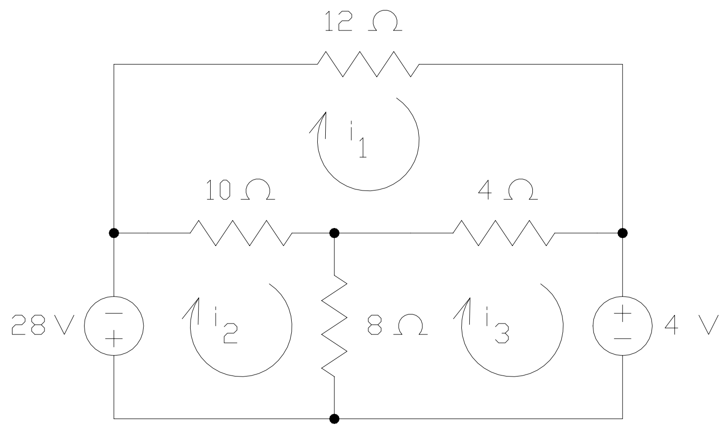
KCL at node 2:

$$\frac{-8-(-20)}{20} = 2 + \frac{-20-(-6)}{10} \quad \Rightarrow \quad \frac{12}{20} = \frac{6}{10}$$

KCL at node 3: $\frac{-20-(-6)}{10} + 1 = \frac{-6}{15} \quad \Rightarrow \quad \frac{-4}{10} = \frac{-6}{15}$

KCL is satisfied at all of the nodes so the computer analysis is correct.

VP 4-5



Top mesh: $10\,(2-4)+12(2)+4\,(2-3)=0$

Bottom right mesh $8\,(3-4)+4\,(3-2)+4=0$

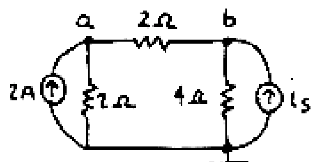
Bottom, left mesh: $28+10\,(4-2)+8\,(4-3)\neq 0$ (Perhaps the polarity of the 28 V source was entered incorrectly.)

KVL is not satified for the bottom, left mesh so the computer analysis is not correct.

<u>Design Problems</u>

DP 4-1

Simplify to



KCL at node a : $v_a\left(\frac{1}{2}+\frac{1}{2}\right)-\frac{1}{2}v_b-2=0$ (1)

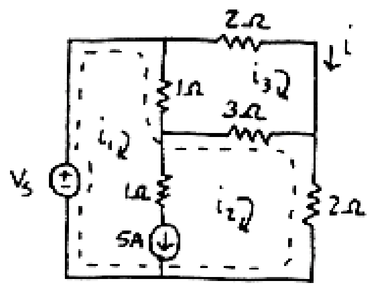
at node b: $-\frac{1}{2}v_a+\left(\frac{1}{2}+\frac{1}{4}\right)v_b-i_s=0$ (2)

now $v_{ba}=3=v_b-v_a \Rightarrow v_b=3+v_a$ (3)

Plugging (3) into (1) yields: $\underline{v_a=7\text{ V}}$ & $\underline{v_b=10\text{ V}}$

thus from (2) get: $\underline{i_s=4\text{ A}}$

DP 4-2



$i_3=i = 3\text{A}$

Supermesh: $-v_s+1(i_1-i_3)+3(i_2-i_3)+2i_2=0$

$-v_3+i_1+5i_2=12$ (1)

mesh 3:1 $(i_3-i_1)+2i_3+3(i_3-i_2)=0$

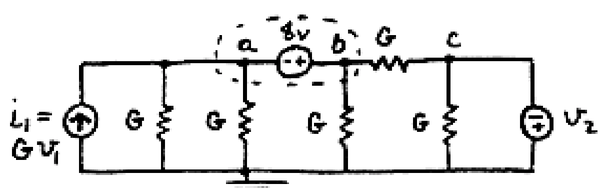
$i_1+3i_2=18$ (2)

also $i_1-i_2=5$ (3)

Combining (2) & (3) yields $i_2=3.25\text{ A},\ i_1=8.25\text{ A}$

from (1) $\underline{v_s=12.5\text{ V}}$

DP 4-3



$G = .5\text{ S}$

$i_1 = Gv_1$

$v_c = -v_2$

Supernode : $v_a(G+G) + v_b(G+G) - Gv_c = i_1$ (1)

also : $v_b-v_a=8$ (2)

Combining (1) & (2) yields $v_a + (8+v_a)+.5v_2 = .5v_1$

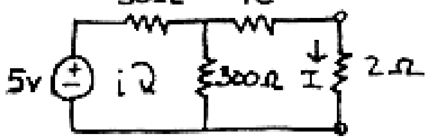
$2v_a = .5v_1 - .5v_2 - 8$

$\Rightarrow v_a = \dfrac{.5v_1 - .5v_2 - 8}{2} = 0$

$\therefore .5v_1 - .5v_2 = 8$

so let $\underline{v_2 = 2\text{ V}}$ and $\underline{v_1 = 18}$ V (one solution)

DP 4-4



a) KVL left mesh : $-5+50i + 300(i-I) = 0$ (1)

right mesh : $(R+2)I + 300(I-i) = 0$ (2)

Solving (1) & (2) for I $\Rightarrow$ $I = \dfrac{150}{1570 + 35R}$ (3)

Desire $50mA \le I \le 75mA$

so if R = 100, then <u>I = 29.59 mA</u> $\Rightarrow$ <u>1amp will not light</u>

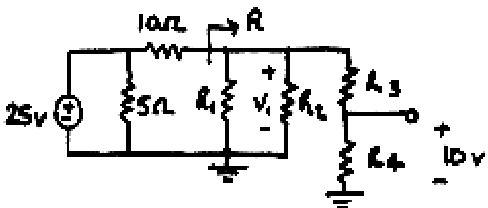
b) from (3) note that as $R\downarrow\ I\uparrow$, so try $R = 50\ \Omega \Rightarrow I = 45$ mA (won't light)

try $R = 25\Omega \Rightarrow I = 61$ mA $\Rightarrow$ <u>will light</u>

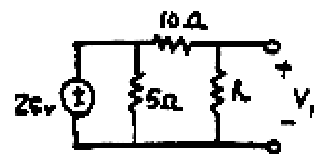
Now check if $R \pm 10\%$ will light and not burn out

$$\left.\begin{array}{l} -10\% \rightarrow 22.5\Omega \rightarrow I = 63.63 \text{ mA} \\ +10\% \rightarrow 27.5\Omega \rightarrow I = 59.23 \text{ mA} \end{array}\right\} \begin{array}{l}\text{lamp will} \\ \text{stay on}\end{array}$$

DP 4-5



$R = R_1 // R_2 // (R_3 + R_4)$



$$v_1 = 25\frac{R}{10+R}$$

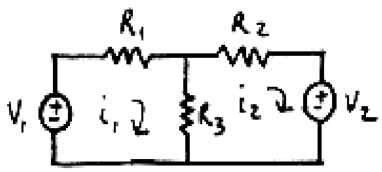
Using voltage divider

$$10 = \frac{R_4}{R_3+R_4} v_1 = \frac{R_4}{R_3+R_4} \frac{\left(R_1 \| R_2 \| (R_3+R_4)\right)}{10 + \left(R_1 \| R_2 \| (R_3+R_4)\right)} 25$$

one solution: choose $R_1 = R_2 = 25\Omega \rightarrow 10 = \dfrac{R_4}{20} \dfrac{(12.5\|20)}{10 + (12.5\|20)} 25 \Rightarrow \underline{R_4 = 18.4\Omega}$

$\&\ R_3 + R_4 = 20 \Rightarrow \underline{R_3 = 1.6\Omega}$

DP 4-6



mesh i_1: $(R_1+R_3)\, i_1 - R_3 i_2 - v_1 = 0$ (1)

mesh i_2: $-R_3 i_1 + (R_2+R_3)\, i_2 + v_2 = 0$ (2)

from (1) & (2) get : $i_1 = \dfrac{\begin{bmatrix} v_1 & -R_3 \\ -v_2 & (R_2+R_3) \end{bmatrix}}{\Delta}$ $\quad i_2 = \dfrac{\begin{bmatrix} (R_1+R_3) & v_1 \\ -R_3 & -v_2 \end{bmatrix}}{\Delta}$

where $\Delta = (R_1+R_3)(R_2+R_3) - R_3{}^2$

Now if $R_1 = R_2 = R_3 = 1K$ where K represents 1000

then $\Delta = 4-1 = 3K^2$

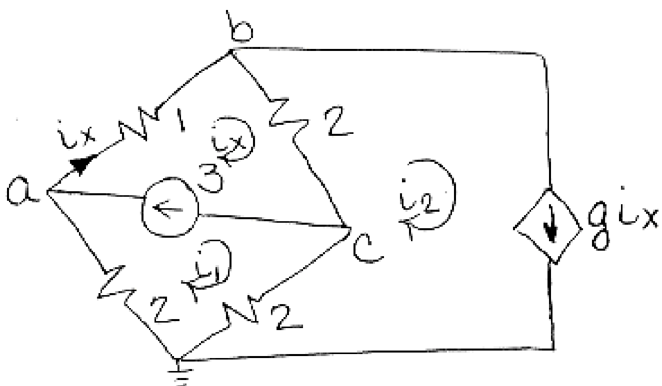
so we have $i_1 = \dfrac{[2v_1 - v_2]\, K}{3K^2}$, $i_2 = \dfrac{[-2v_2 + v_1]\, K}{3K^2}$

$\Rightarrow\ i = i_1 - i_2 = \dfrac{v_1 + v_2}{3K}$

if $v_1 = v_2 = 1\text{ V} \Rightarrow i = 2/3$ mA okay

if $v_1 = v_2 = 2\text{ V} \Rightarrow i = 4/3$ mA okay

DP 4-7



$v_a - v_c = \dfrac{20}{3}$

$i_x - i_1 = 3$ $\qquad v_a = -2i_1$

$i_2 = g i_x$ $\qquad v_c = 2i_1 - 2i_2$

$i_x + 2i_x - 2i_2 + 2i_1 - 2i_2 + 2i_1 = 0$

Substituting solve for g : $\underline{g=4}$