Chapter 5 Circuit Theorems

Exercises

Ex 5.3-1 R = 10 Ω and i_s = 1.2 A.

Ex 5.3-2 R = 10 Ω and i_s = -1.2 A.

Ex 5.3-3 R = 8 Ω and v_s = 24 V.

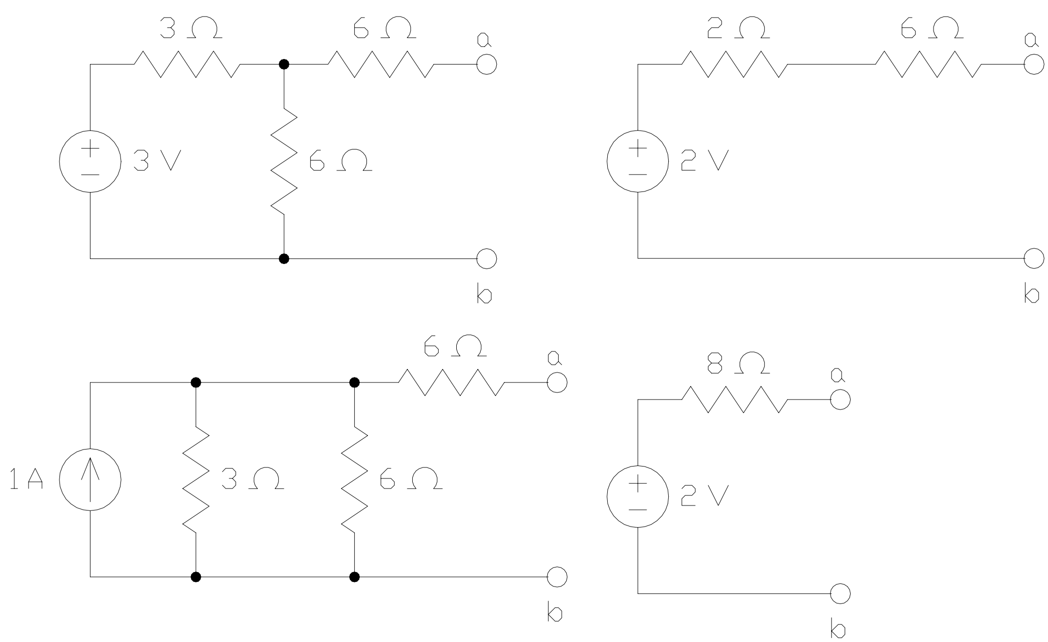
Ex 5.3-4 R = 8 Ω and v_s = -24 V.

Ex 5.4-1 $v_m = \frac{20}{10+20+20} 15 + 20\left(-\frac{10}{10+(20+20)} 2\right) = 6 + 20\left(-\frac{2}{5}\right) = -2 \text{ V}$

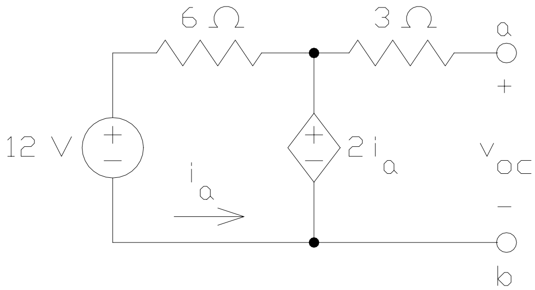
Ex 5.4-2 $i_m = \frac{25}{3+2} - \frac{3}{2+3} 5 = 5 - 3 = 2 \text{ A}$

Ex 5.4-3 $v_m = 3\left(\frac{3}{3+(3+3)} 5\right) - \frac{3}{3+(3+3)} 18 = 5 - 6 = -1 \text{ A}$

Ex 5.5-1

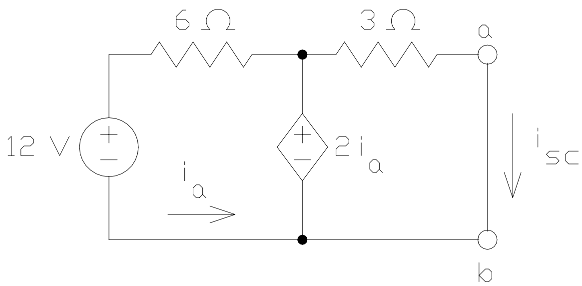


Ex 5.5-2



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$v_{oc} = 2\,i_a = -6 \text{ V}$$

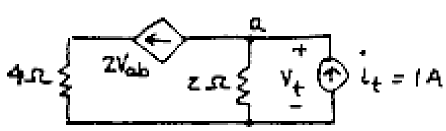


$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$3 i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2 \text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

Ex. 5.5–3 No independent sources $\therefore$ $v_{oc} = i_{sc} = 0 \Rightarrow$ apply 1A test source



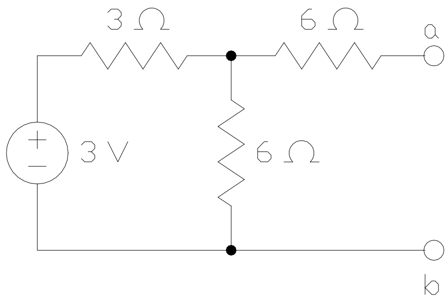
$v_{ab} = v_t$

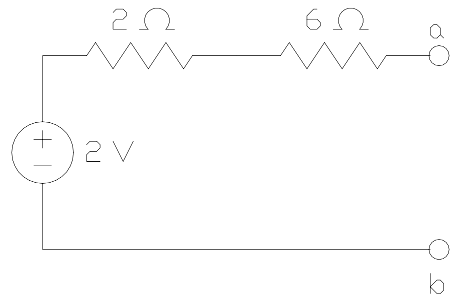
KCL at a: $2v_t + \frac{v_t}{2} - 1 = 0 \Rightarrow v_t = 2/5$ V

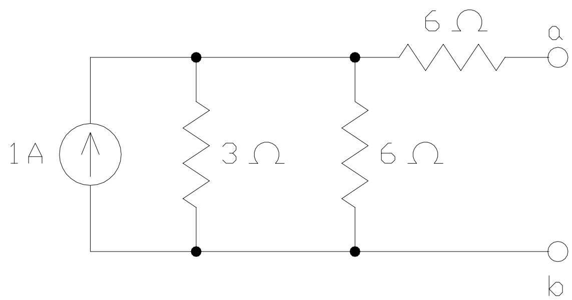
$\therefore$ $R_T = \frac{v_t}{i_t} = 2/5\ \Omega$

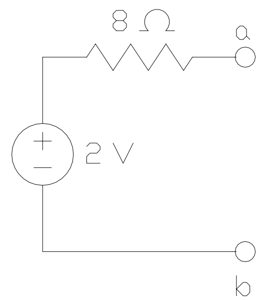
Thev. equiv. ckt

Ex 5.6-1

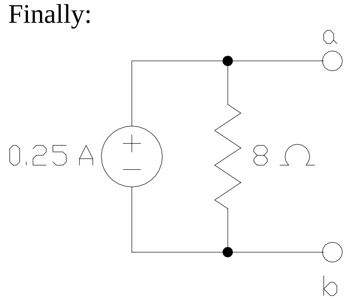




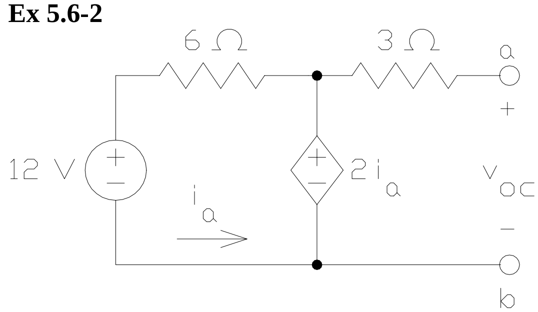




Finally:

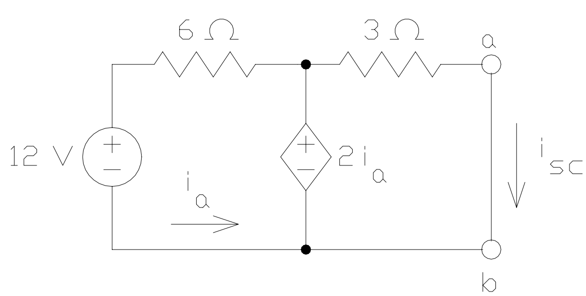


Ex 5.6-2



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3\text{ A}$$

$$v_{oc} = 2\,i_a = -6\text{ V}$$

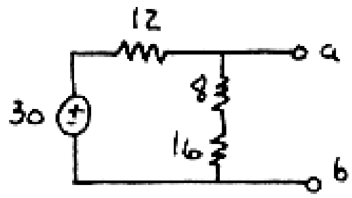


$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3\text{ A}$$

$$3\,i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2\text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

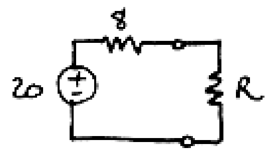
Ex. 5.6-3



$$R_T = \frac{12\times24}{12+24} = \frac{12\times24}{36} = 8\Omega$$

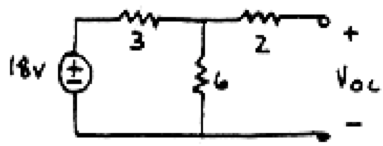
$$v_{oc} = \frac{24}{12+24}\ 30 = 20V$$

So we have



$$i = \frac{20}{8+R}\ A$$

Ex. 5.7-1 Find v_{oc}



From voltage divider

$$v_{oc} = 18V\left(\frac{6}{6+3}\right) = 12V$$

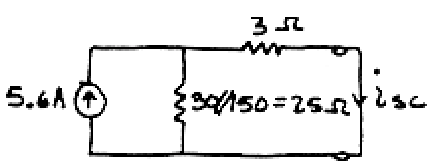
Find R_T (short 18V source)

$= 3\|6 + 2 = 4\Omega$

$\therefore$ Thev. equiv ckt $\Rightarrow$

For max power to $R_L \Rightarrow R_L = R_T = 4\Omega$ $\quad \therefore P_{\substack{max\\ to R_L}} = \frac{(v_{RL})^2}{R_L} = \frac{(6)^2}{4} = \underline{9W}$

Ex. 5.7-2 Find i_{sc}

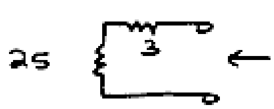


From current divider

$$i_{sc} = 5.6A\left(\frac{25}{25+3}\right)$$

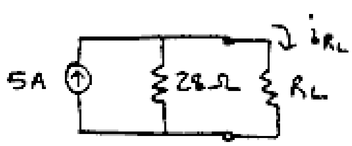
$$\underline{i_{sc} = 5A}$$

Find R_T (open 5.6A source)



$R_T = 25+3 = \underline{28\Omega}$

$\therefore$ Norton equiv. ckt $\Rightarrow$



For max power $R_L = R_T = 28\Omega$ $\quad \therefore P_{L_{max}} = (i_{R_L})^2 R_L = (5/2)^2\ (28) = \underline{175W}$

Ex. 5.7-3

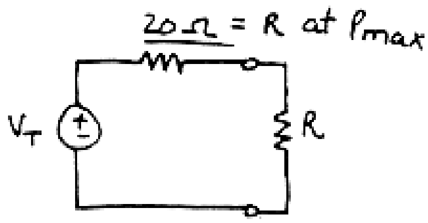
$$P_{L_{max}} = \frac{(v_{L_{max}})^2}{R_L} = \frac{\left[10V\left(\frac{5}{5+R_t}\right)\right]^2}{R_L}$$

Now for V_L to be maximized, R_t must be minimized

$\therefore$ choose $R_t = 1\Omega$

$$\therefore\ P_{L_{max}} = \frac{\left[10\left(\frac{5}{6}\right)\right]^2}{5} = \underline{13.9W}$$

Ex. 5.7-4



$$P_{max} = 5 = \left(\frac{v_T}{40}\right)^2 20 = \frac{{v_T}^2}{80}$$

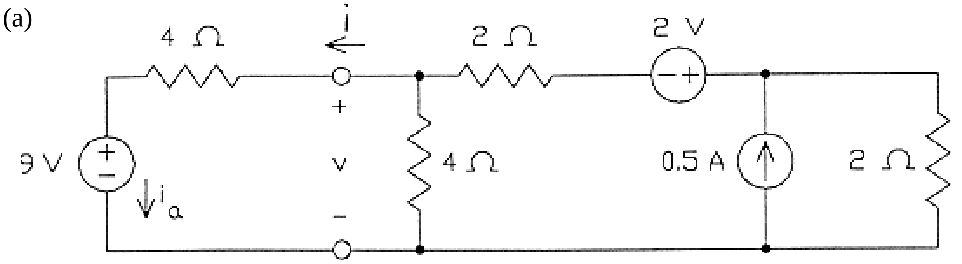
$$v_T = \sqrt{400} = \underline{20V}$$

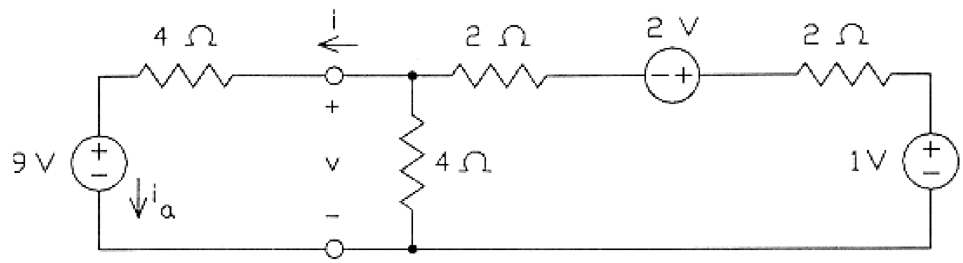
PROBLEMS

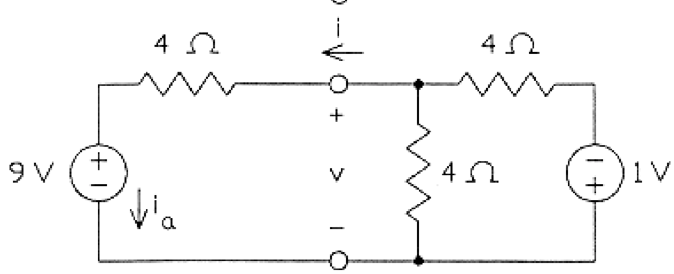
<u>Section 5-3:</u> Source Transformations

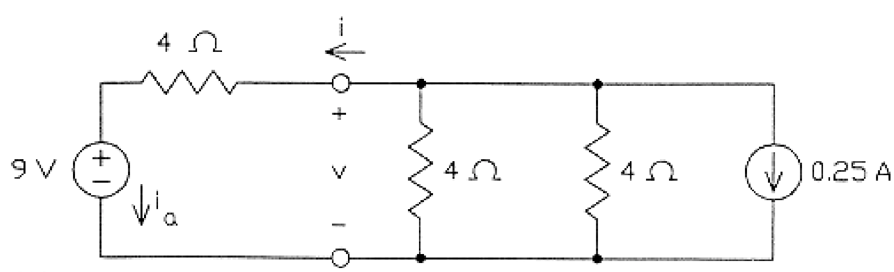
P5.3-1

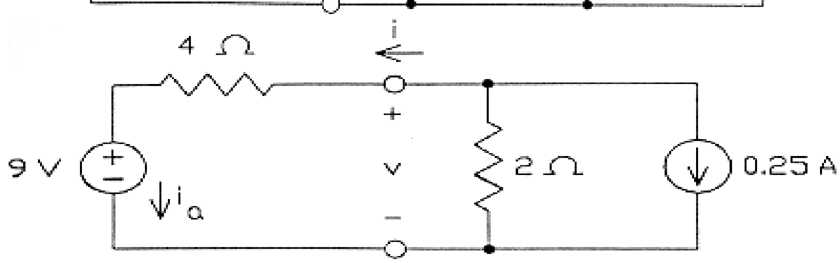
(a)

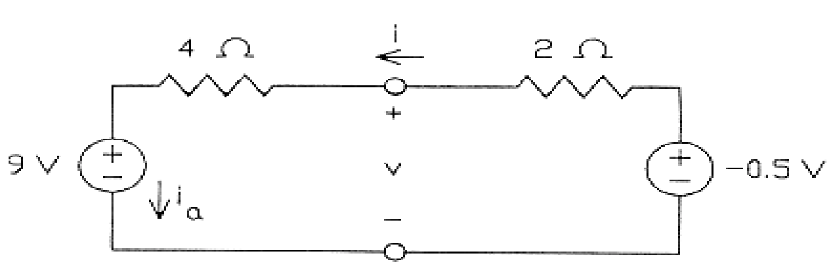












$\therefore\ R_t = 2\Omega$

$v_t = -0.5\text{V}$

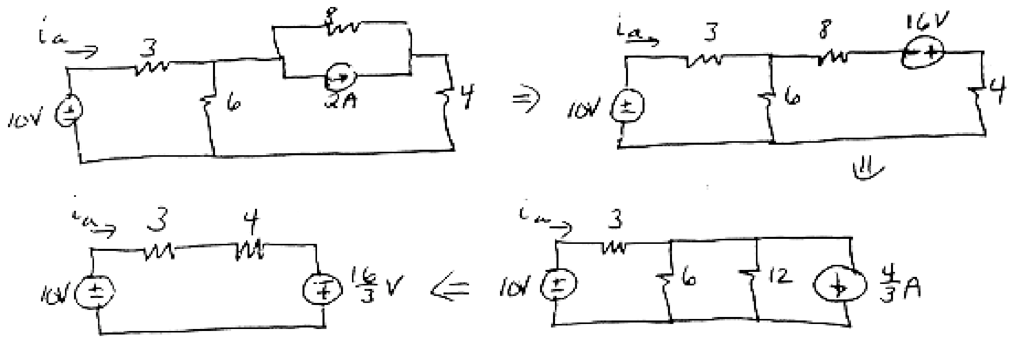
(b) $-9-4i-2i+(-0.5)=0$

$$i = \frac{-9+(-0.5)}{4+2} = -1.58A$$

$$v = 9+4i = 9+4(-1.58) = 2.67V$$

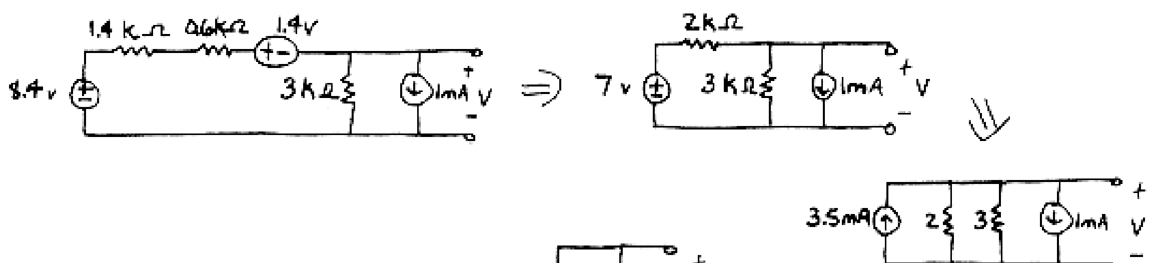
(c) $i_a = i = -1.58A$

P5.3-2

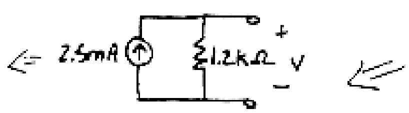


KVL: $-10+3i_a+4i_a-\frac{16}{3}=0 \quad \therefore \ \underline{i_a = 2.19A}$

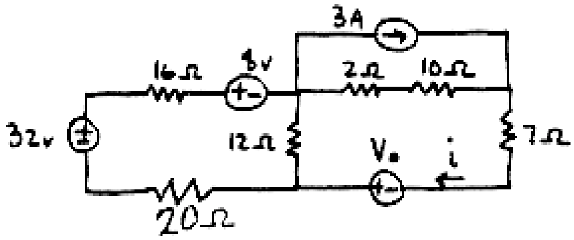
P5.3-3

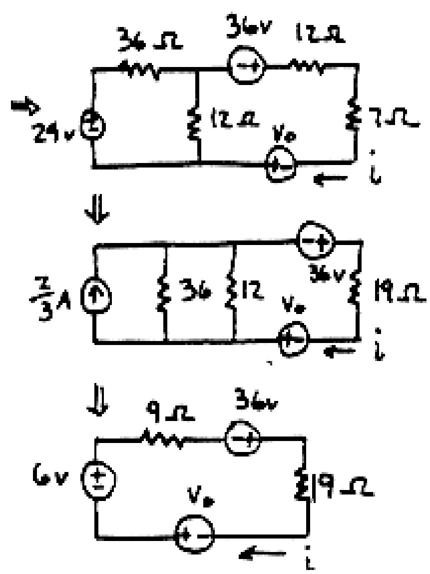


$\underline{V = (1.2k\Omega)(2.5mA) = 3V}$



P5.3-4

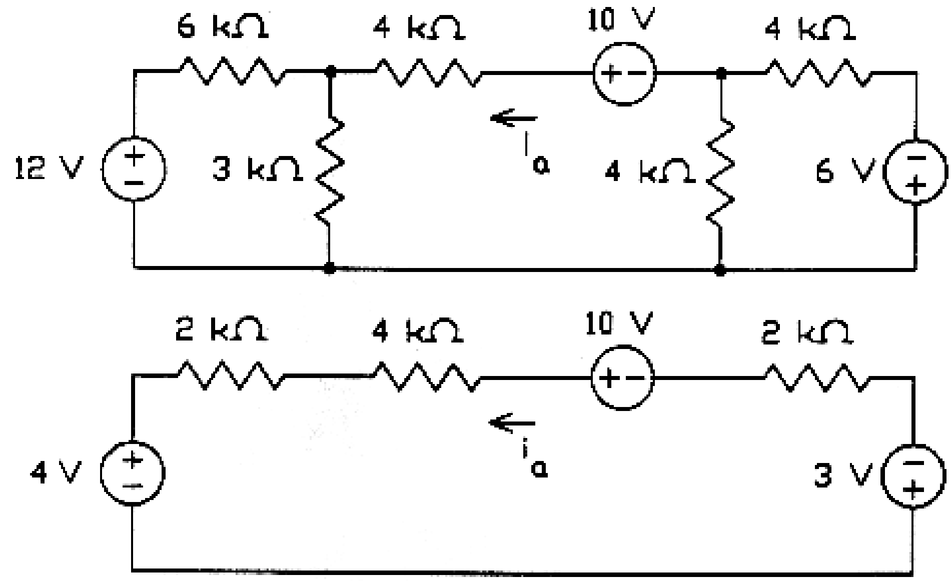




KVL a: $-6 + i(9+19) - 36 - v_0 = 0$

$\Rightarrow v_o = -42+28(5/2) = 28V$

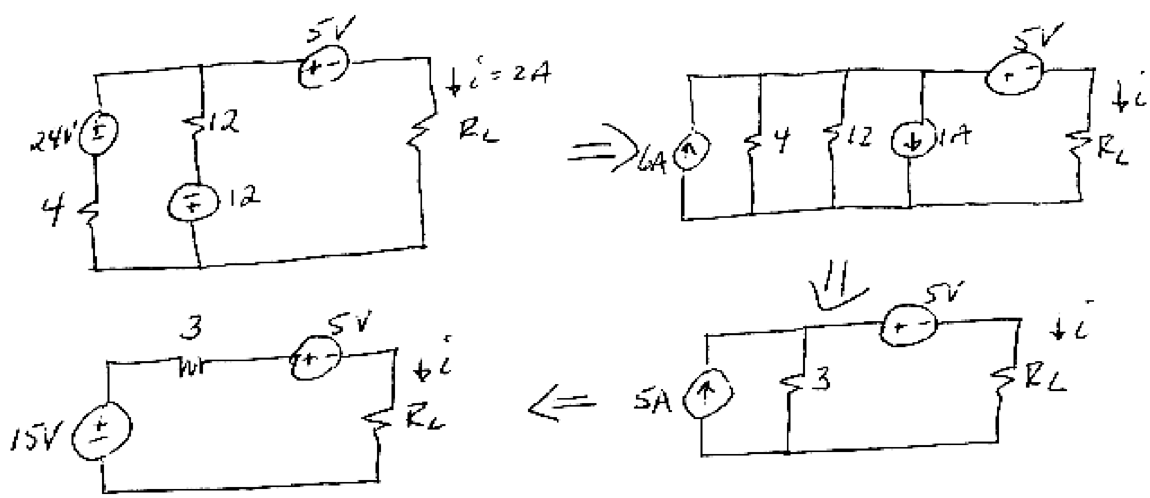
P5.3-5

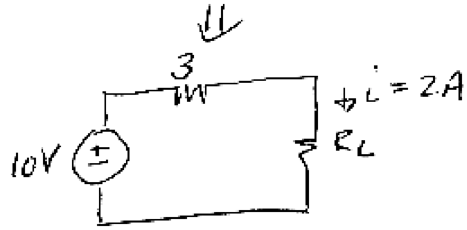


$$-\ 4\ -\ 2000\ i_a -\ 4000 i_a + 10\ - 2000 i_a - 3 = 0$$

$$\therefore\ i_a\ =\ 375\ \mu A$$

P5.3-6





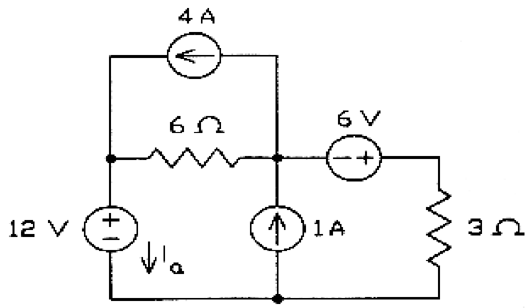
$$0V = (3 + R_L)\ (2A)$$

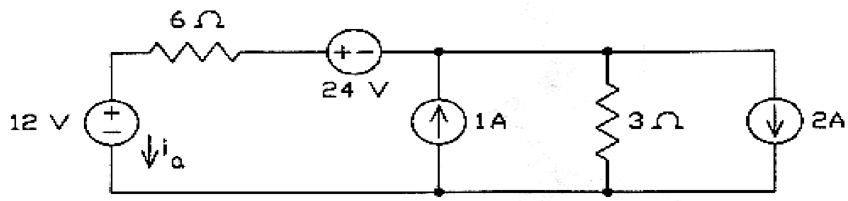
$$5 = 3 + R_L$$

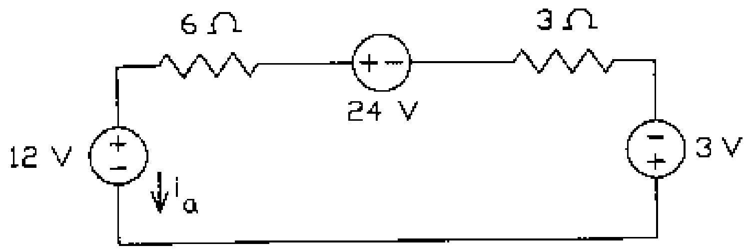
$$\therefore \underline{R_L\ = 2\Omega}$$

Section 5-4 Superposition

P5.4-1







$$-12-6i_a + 24 - 3i_a - 3 = 0 \Rightarrow i_a = 1\text{ A}$$

P5.4–2   Consider 6A source only (open 9A source)

From current divider:

$$v_1 / 20 = 6\left[\frac{15}{15+30}\right] \Rightarrow \underline{v_1 = 40\text{V}}$$

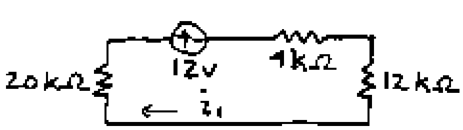
Consider 9A source only (open 6A source)

Current divider

$$v_2 / 20 = 9\left[\frac{10}{10+35}\right] \Rightarrow \underline{v_2 = 40\text{V}}$$

$$\underline{\therefore v = v_1 + v_2 = 40 + 40 = 80\text{V}}$$

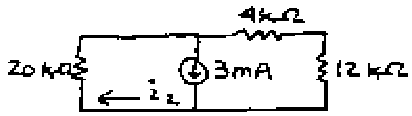
P5.4–3   Consider 12V source only (open both current sources)



KVL a: $20i_1 + 12 + 4i_1 + 12i_1 = 0$

$\Rightarrow \underline{i_1 = -1/3\text{mA}}$

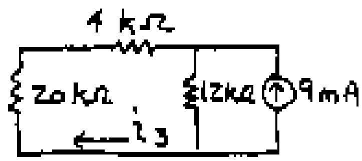
Consider 12mA source only (short 12V and open 6mA sources)



From current divider

$$i_2 = 3\left[\frac{16}{16+20}\right] = \underline{\frac{4}{3}mA}$$

Consider 9mA source only (short 12V and open 12mA sources)

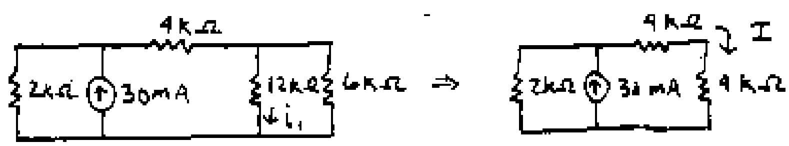


From current divider

$$i_3 = -9\left[\frac{12}{24+12}\right] = \underline{-3mA}$$

$$\therefore \underline{i = i_1 + i_2 + i_3 = -1/3 + 4/3 - 3 = -2mA}$$

P5.4–4 Consider 30mA source only (open 15mA and short 60V sources)

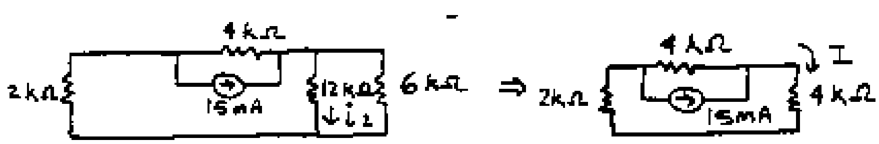


$$\text{Current divider} \Rightarrow I = 30\left(\frac{2}{2+8}\right) = 6mA$$

$$\therefore i_1 = I\left(\frac{6}{6+12}\right) = \underline{2mA}$$

Consider 15mA source only (open 30mA source and short 60V source)

Continued



$$\text{Current divider} \Rightarrow I = 15\left(\frac{4}{4+6}\right) = 6mA$$

$$\therefore i_2 = I\left(\frac{6}{6+12}\right) = \underline{2mA}$$

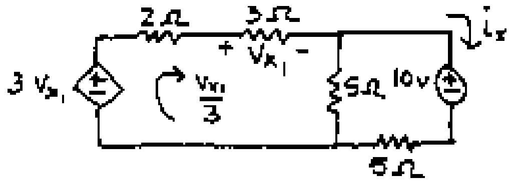
Consider 15V source only (open both current sources)

From current divider

$$i_3 = -2.5\left(\frac{6//6}{6//6+12}\right) = -10\left(\frac{3}{3+12}\right) = \underline{-.5mA}$$

$$\therefore \underline{i = i_1 + i_2 + i_3 = 2 + 2 - .5 = 3.5mA}$$

P5.4-5 Consider 10V source only (open 4A source)



KVL 1st mesh a:

$$-3v_{x_1}+5\left(\frac{v_{x_1}}{3}\right)+5\left(\frac{v_{x_1}}{3}-i_x\right)=0$$

$$\Rightarrow \underline{v_{x_1}=15i_x} \quad (1)$$

KVL 2nd mesh a: $5\,(i_x - v_{x_1}/3) + 10 + 5i_x = 0$ (2)

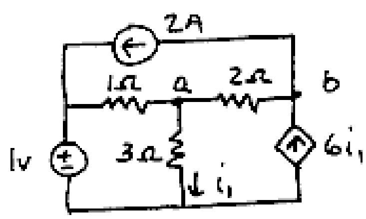
Solving (1) and (2) simultaneously $\Rightarrow$ $\underline{v_{x_1} = 10\text{V}}$

Consider 4A source only (short 10V source)

Using current divider: $\frac{v_{x_2}}{3}=\left(3/2v_{x_2}-4\right)\left(\frac{2}{2+3+5/2}\right) \Rightarrow \underline{v_{x_2}=16\text{V}}$

$$\therefore \underline{v_x = v_{x_1}+v_{x_2} = 10+16 = 26\text{V}}$$

P5.4-6



KCL at b : $i + 6i_1 - 2 = 0$

$\Rightarrow i_1 = 1/3 - 1/6\ i$ (1)

KVL around left lower mesh:

$1(i_1+i)+3i_1 - 1 = 0$ (2)

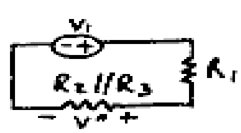
Plugging (1)into (2) $\Rightarrow \underline{i = -1\text{A}}$

P5.4-7 Consider v_2 source only

Voltage divider : $v' = -v_2\left[\frac{R_2}{R_2+R_1\|R_3}\right]$

$$v' = -v_2\left[\frac{R_2(R_1+R_3)}{R_1R_2+R_1R_3+R_2R_3}\right]$$

Consider v_1 source only



Voltage divider $v'' = v_1\left[\frac{R_2\|R_3}{R_2\|R_3+R_1}\right]$

$$v'' = v_1\left[\frac{R_2\ R_3}{R_1R_2+R_1R_3+R_2R_3}\right]$$

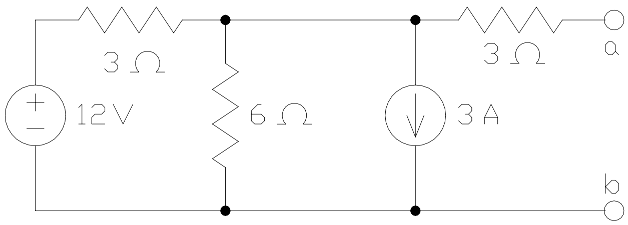
Consider i_1 source only

$v''' = 0$ since no current flows through R_2, R_3 and R_1

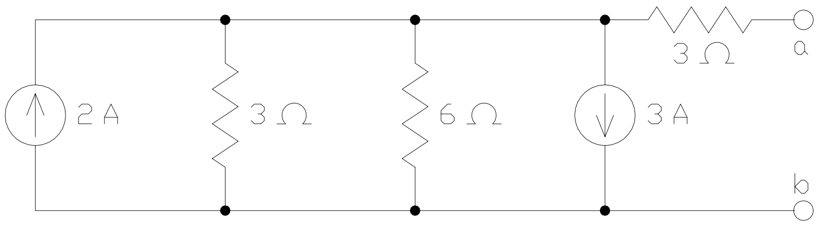
$$\therefore \underline{v = v'+v''+v''' = \frac{v_1R_2R_3 - v_2(R_2(R_1+R_3))}{R_1R_2+R_1R_3+R_2R_3}}$$

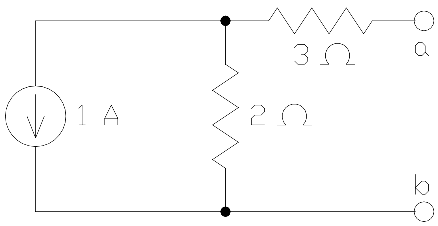
<u>Section 5-5:</u> Thèvenin's Theorem

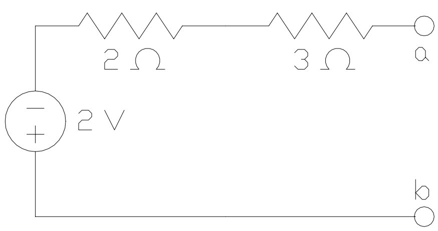
Ex 5.5-1

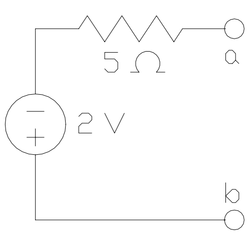


(a)

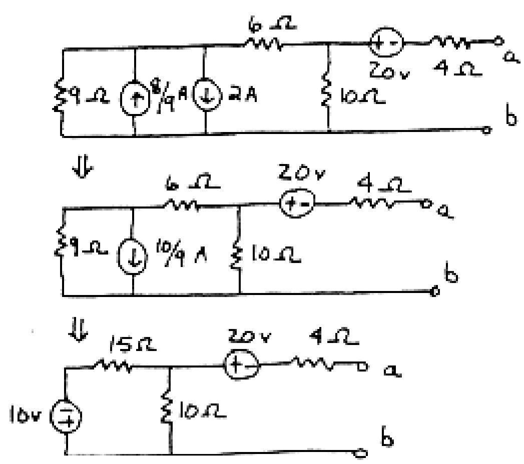


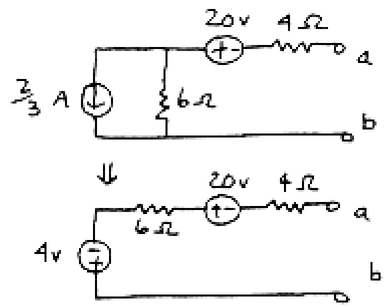


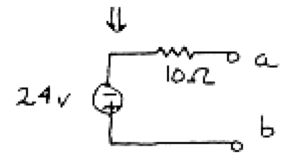




P5.5-2 Use source transformations

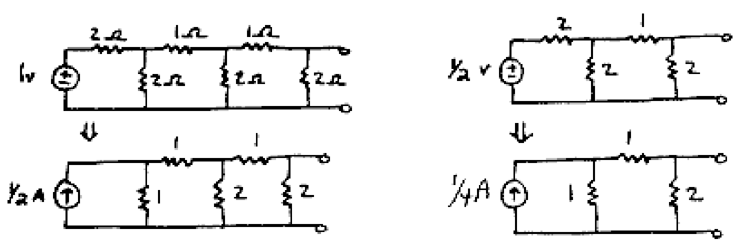


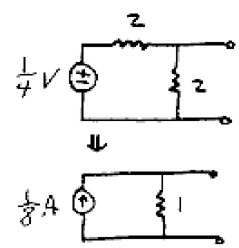




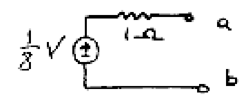
$$R_T = 10\Omega$$
$$v_{oc} = -24V$$

P5.5-3 Use source transformations

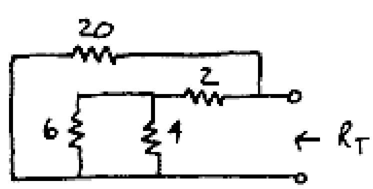


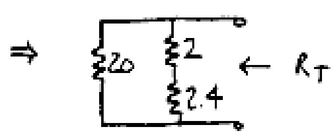


Thev. equiv.ckt



P5.5-4 Find R_T:

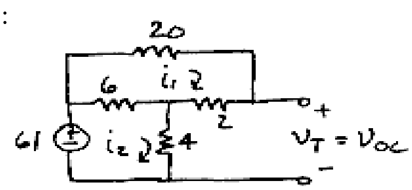




$$R_T = \frac{20(2+2.4)}{20+2+2.4} = \underline{3.61\Omega}$$

Continued

Find v_T:

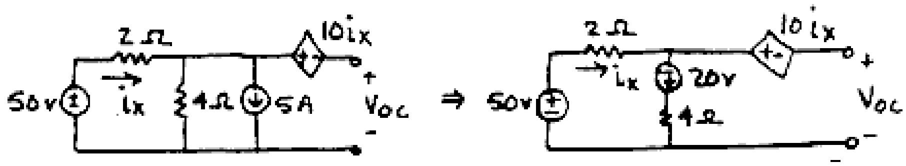


$$v_T = 2i_1 + 4i_2$$
$$\text{mesh } i_1: \quad 28i_1 - 6i_2 = 0 \qquad (1)$$
$$\text{mesh } i_2: \quad -6i_1 + 10i_2 - 61 = 0 \quad (2)$$

Solving (1) & (2) yields: $i_1 = 1.5\text{A}, \ i_2 = 7\text{A}$

$$\therefore \ \underline{v_T = 3+28 = 31\text{V}}$$

P5.5-5 Find v_{oc}

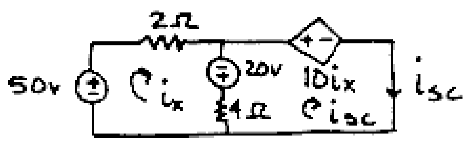


KVL around 1st mesh a: $-50+2i_x-20+4i_x = 0 \Rightarrow i_x = 70/6\text{ A}$

KVL around 2nd mesh a: $-4i_x+20+10i_x+v_{oc} = 0$

$$\Rightarrow \underline{v_{oc} = -90\text{V}}$$

Find i_{sc}



KVL i_x mesh a: $-50+2i_x-20+4(i_x-i_{sc}) = 0$

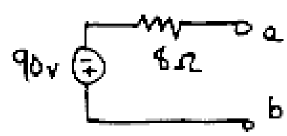
$$\underline{6i_x-4i_{sc}-70 = 0} \qquad (1)$$

KVL i_{sc} mesh a: $4(i_{sc}-i_x)+20+10i_x = 0$

$$\underline{6i_x+4i_{sc}+20 = 0} \qquad (2)$$

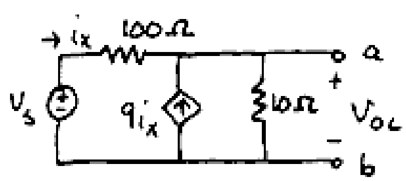
Solving (1) and (2) simultaneously $\Rightarrow \underline{i_{sc} = -45/4\text{ A}}$

$\therefore R_T = \dfrac{v_{oc}}{i_{sc}} = \underline{8\Omega}$ Thev. equiv.ckt :



P5.5-6

For v_{oc}:



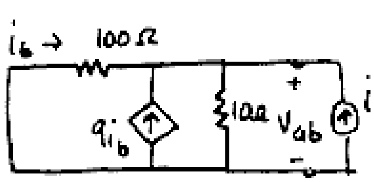
$$i_x = \frac{v_s-v_{oc}}{100}$$

KCL at terminal a:

$$\frac{1}{100}(v_{oc}-v_s)-9\left[\frac{1}{100}(v_s-v_{oc})\right]+\frac{1}{10}v_{oc} = 0$$

$$\Rightarrow \underline{v_{oc} = \frac{1}{2}v_s}$$

Use current source at a-b to find R_T:

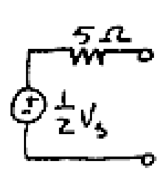


$$i_b = -\frac{v_{ab}}{100}$$

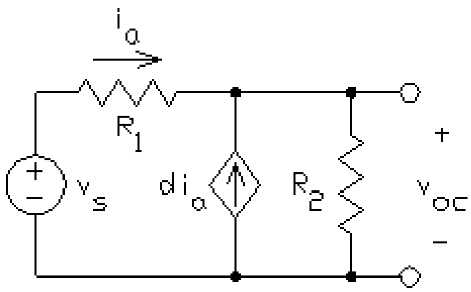
$$\text{KCL}: \frac{1}{100}v_{ab}-9\left[\frac{1}{100}(-v_{ab})\right]+\frac{1}{10}v_{ab}-i = 0$$

$$\Rightarrow i = \frac{1}{5}v_{ab} \quad \therefore R_T = \frac{v_{ab}}{i} = 5\Omega$$

So Thev. equiv.



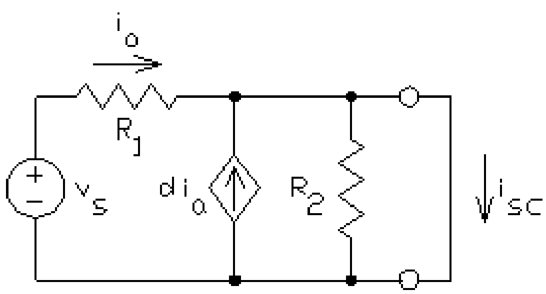
P5.5-7



$$v_s + R_1\, i_a + (d+1) R_2\, i_a = 0$$

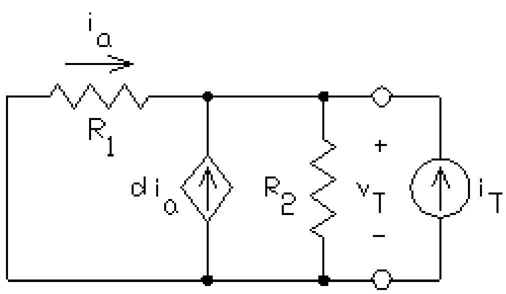
$$i_a = \frac{v_s}{R_1 + (d+1) R_2}$$

$$v_{oc} = \frac{(d+1) R_2 v_s}{R_1 + (d+1) R_2}$$



$$i_a = \frac{v_s}{R_1}$$

$$i_{sc} = (d+1) i_a = \frac{(d+1) v_s}{R_1}$$



$$-i_a - d\, i_a + \frac{v_T}{R_2} - i_T = 0$$

$$R_1\, i_a = -v_T$$

$$i_T = (d+1)\frac{v_T}{R_1} + \frac{v_T}{R_2} = \frac{R_2 (d+1) + R_1}{R_1\, R_2}$$

$$R_t = \frac{v_T}{i_T} = \frac{R_1\, R_2}{R_1 + (d+1) R_2}$$

(b) Let $R_1 = R_2 = 1$ kΩ. Then

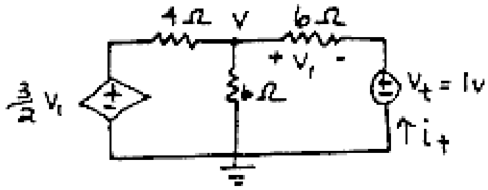
$$625\ \Omega = R_t = \frac{1000}{d+2} \quad \Rightarrow \quad d = \frac{1000}{625} - 2 = -0.4 \text{ A/A}$$

and

$$5 = \frac{(d+1) v_s}{d+2} \quad \Rightarrow \quad v_s = \frac{-0.4+2}{-0.4+1} 5 = 13.33 \text{ V}$$

P5.5-8

Since no independent sources $v_{oc} = i_{sc} = 0 \;\therefore\;$ *apply* test source

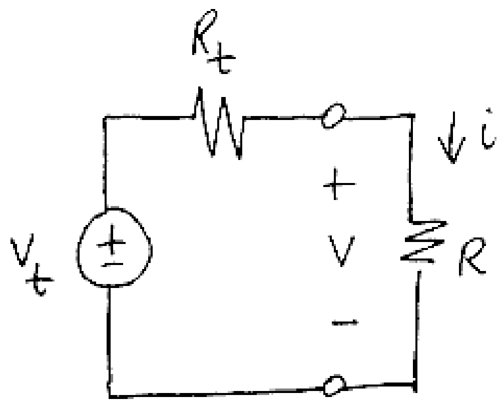


$V = v_1 + v_t$

KCL at V : $\left(V-\tfrac{3}{2}v_1\right)\big/4 + V/6 + v_1/6 = 0$ & with $V = v_1 + 1$ *so* $v_1 = -2A$

now $i_t = -\frac{v_1}{6} = \underline{1/3\,A}$ $\quad\therefore\; R_T = \frac{v_t}{i_t} = \frac{1}{1/3} = \underline{3\Omega}$

P5.5-9



$V = \frac{R}{R+R_t} v_t$

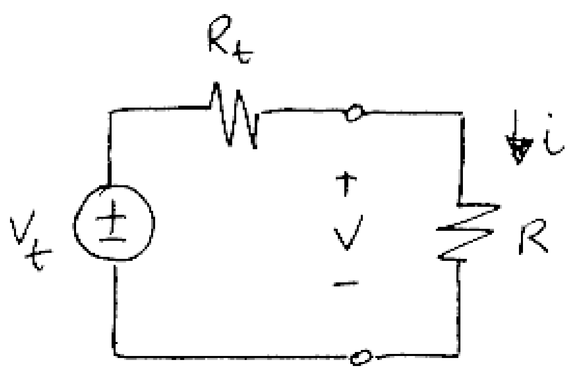
$6 = \frac{2000}{2000+R_t} v_t \qquad$ (line 1)

$2 = \frac{4000}{4000+R_t} v_t \qquad$ (line 2)

$\therefore\; \underline{v_t = 1.2V}$ and $\underline{R_t = -1600\Omega}$

When R = 8000, $V = \frac{8000}{8000-1600} 1.2 = \underline{1.5V}$

P5.5-10



$i = \frac{v_t}{R+R_t}$

$0.004 = \frac{v_t}{2000+R_t} \qquad (1)$

$0.003 = \frac{v_t}{4000+R_t} \qquad (2)$

so $\underline{v_t = 24V}$ and $\underline{R_t = 4000\Omega}$

(a) $0.002 = \frac{24}{R+4000} \Rightarrow \underline{R = 8000\Omega}$

(b) when $R = 0$ then $i = \frac{24}{4000} = \underline{6\text{ mA}}$

P5.5-11

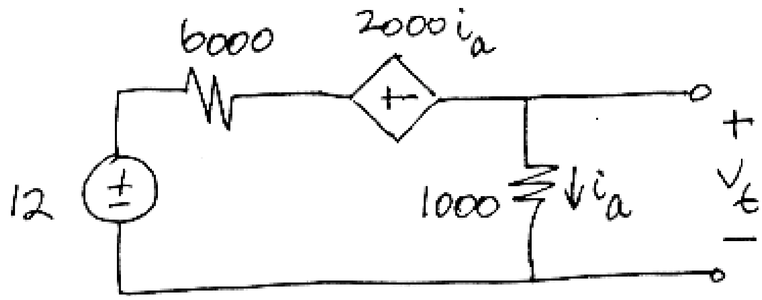
From the graph, when $v_{ab} = v = 0 \Rightarrow i = i_{sc} = 20$ mA

when $i = 0 \quad \Rightarrow v = \underline{v_{oc} = -3\,\text{V}}$

$$\therefore\ R_T = \frac{v_{oc}}{i_{sc}} = \frac{-3\text{V}}{20\,\text{mA}} = -.15\,\text{k}\Omega = -\underline{150\,\Omega}$$

Thev. equiv. ckt $\Rightarrow$

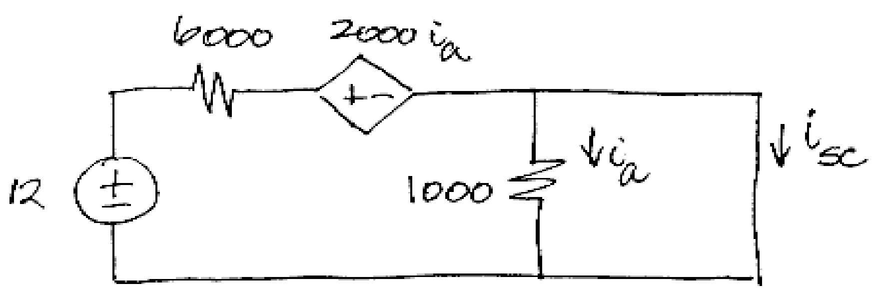
P5.5-12



$$-12 + 6000\,i_a + 2000\,i_a + 1000\,i_a = 0$$

$$i_a = 4/3000 \text{ A}$$

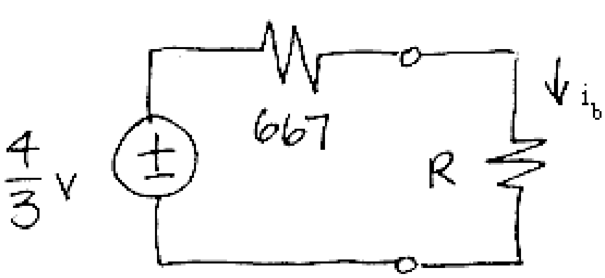
$$v_t = 1000\ i_a = \frac{4}{3}\text{V}$$



$$i_a = \frac{0}{1000} = 0$$

$$-12 + 6000\,i_{sc} = 0 \Rightarrow i_{sc} = 2\,\text{mA}$$

$$R_t = \frac{v_t}{i_{sc}} = \frac{\frac{4}{3}}{.002} = 667\,\Omega$$

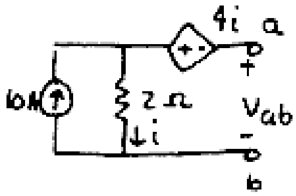


$$i_b = \frac{\frac{4}{3}}{667 + R}$$

$$\therefore\ i_b = 0.002 \text{ requires } R = \frac{\frac{4}{3}}{0.002} - 667 = 0$$

P5.5-13

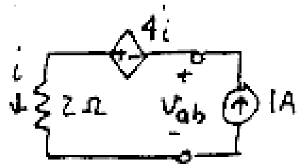
1) disconnect R_L
   open circuit a – b



KVL : $-v_{ab} - 4i + 2i = 0, \ i = 10\,A$

$\Rightarrow v_T = v_{ab} = -2i = \underline{-20\,V}$

2) set independent source = 0 and place 1A source at a – b

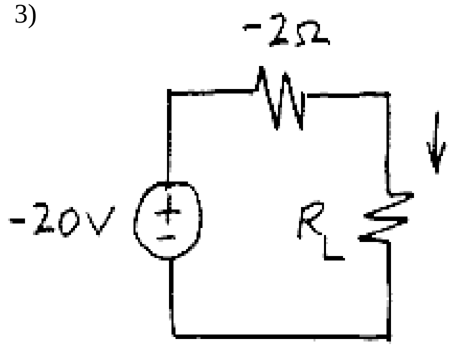


KVL : $-v_{ab} - 4i + 2i = 0, \ i = 1A$

$\Rightarrow v_{ab} = -2A$

$\therefore R_T = v_{ab}/1A = \underline{-2\,\Omega}$

3)



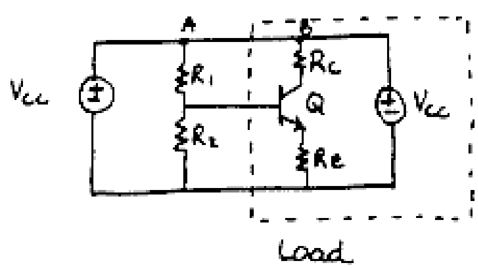
$$-2\,A = i_L = \frac{-20\,V}{R_L - 2\,\Omega} \Rightarrow R_L = 12\,\Omega$$

P5.5-14

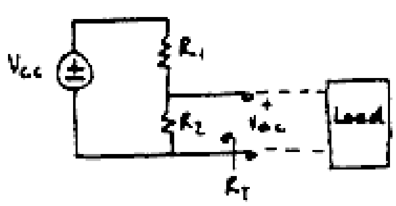
When the terminals of the boxes are open-circuited, no current flows in Box A, but the resistor in Box B dissipates 1 watt. Box B is therefore warmer than Box A. If you short the terminals of each box, the resistor in Box A will draw 1 amp and dissipate 1 watt. The resistor in Box B will be shorted, draw no current, and dissipate no power. Then Box A will warm up and Box B will cool off.

P5.5-15

Redraw ckt as:



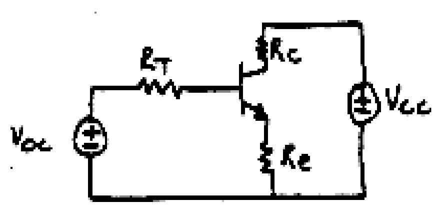
Since points A & B are at same potential, virtually no current exists between A-B $\therefore$ open ckt.



<u>Find R_T</u> : kill v_{cc} source $\Rightarrow R_T = R_1 \| R_2 = R_1R_2/R_1 + R_2$

<u>Find v_{oc}</u> : voltage divider $v_{oc} = v_{cc}\left(\frac{R_2}{R_1+R_2}\right)$

can replace above ckt as :

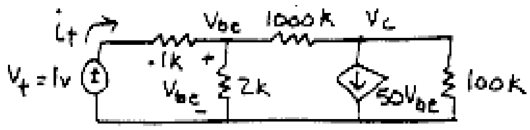


where $\underline{v_{oc} = v_{cc}\left(\frac{R_2}{R_1+R_2}\right)}$

$\underline{R_T = \frac{R_2R_1}{R_2+R_1}}$

P5.5-16

(a) Since there are no independent sources, apply test source



KCL at v_{be}: $-i_t + \frac{1}{2} v_{be} + (v_{be} - v_c)/1000 = 0$

$\downarrow -1000\, i_t + 501\, v_{be} - v_c = 0$ (1)

KCL at v_c: $(v_c - v_{be})/1000 + 50\, v_{be} + v_c/100 = 0$

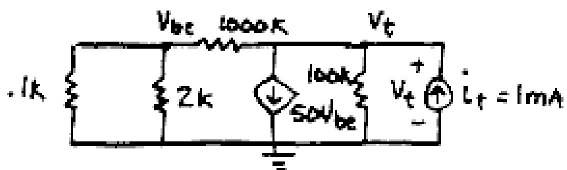
$\downarrow 11 v_c + 50000\, v_{be} = 0$ (2)

also $(1 - v_{be})/.1 = i_t$ (3)

Solving (1), (2), & (3) simultaneously yields $i_t = 3.35\,\text{mA}$

$$\therefore\ R_{IN} = \frac{v_t}{i_t} = \frac{1\,\text{V}}{3.35\,\text{mA}} = .299\,\text{k}\Omega = 299\,\Omega$$

(b) Apply test source



KCL at v_{be}: $v_{be}/.1 + v_{be}/2 + (v_{be} - v_t)/1000 = 0$

$\downarrow 10501 v_{be} = v_t$ (1)

KCL at v_t: $(v_t - v_{be})/1000 + 50\, v_{be} + v_t/100 - 1 = 0$

$\downarrow 11 v_t + 49999\, v_{be} - 1000 = 0$ (2)

Solving (1) & (2) yields: $v_t = 63.5$ V

$\therefore R_{out} = v_t/i_t = 63.5\text{V}/1\text{mA} = 63.5\,\text{k}\Omega$

P5.5-17

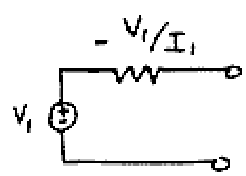
When $0 < V < V_p$, it works as a pure resistor

so $R = V_p/I_p \quad V_{oc} = 0$

When $V_p < V < V_m$, it is linear but shows negative resistance characteristic

$$\Rightarrow V_{oc} = V_{oc}\big|_{I=0} = V_1$$

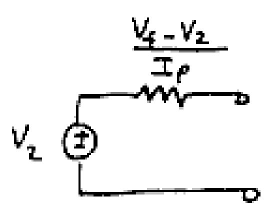
$$R = \frac{V_{oc}}{I_{sc}} = -\frac{V_1}{I_1}$$



When $V_m < V < V_f$, it is linear

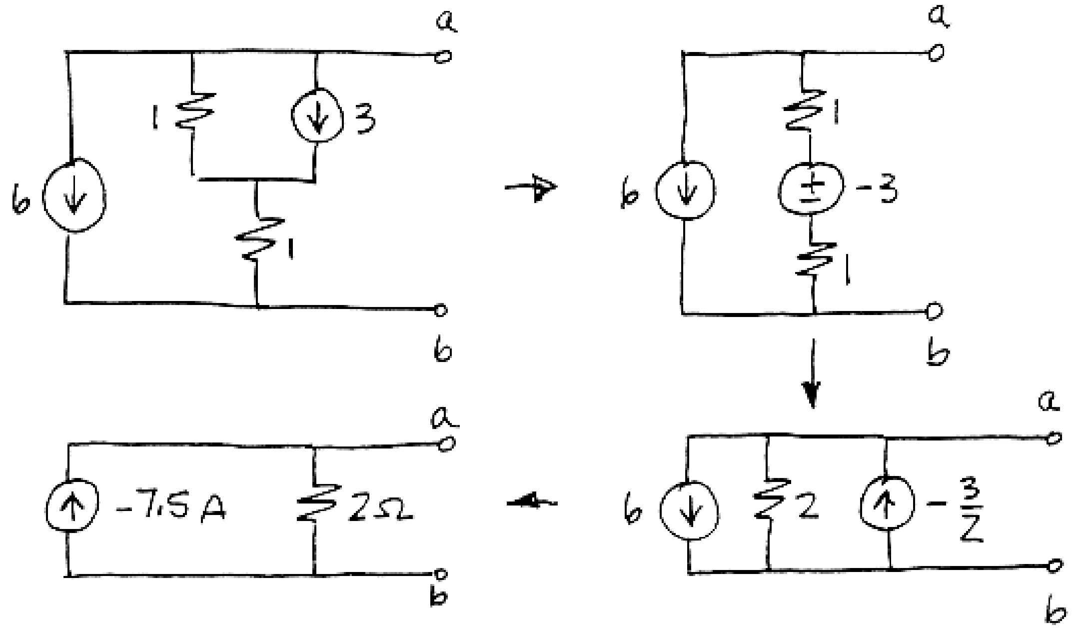
so $V_{oc} = V\big|_{I=0} = V_2$

$$R = \frac{V_{oc}}{I_{sc}} = \frac{V_f - V_2}{I_p}$$

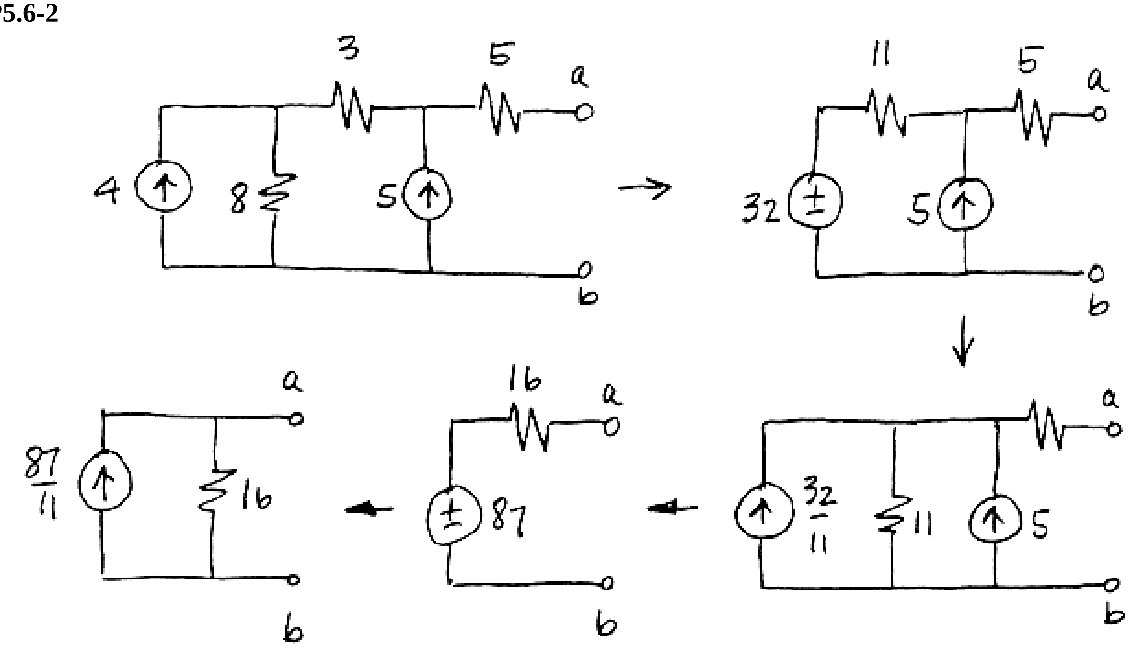


Section 5-6: Norton's Theorem

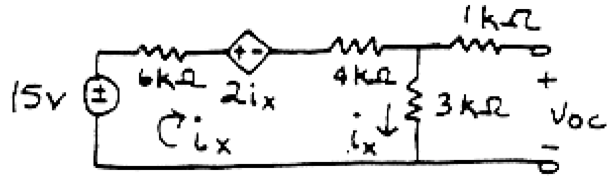
P5.6-1



P5.6-2



P5.6-3 Find v_{oc}

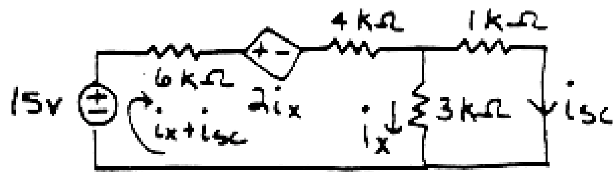


KVL a i_x: $-15+i_x(6+4)+2i_x+3i_x=0$

$\Rightarrow i_x = 1\text{mA}$

$\therefore \underline{v_{oc}} = 3i_x = 3\text{V}$

Find i_{sc}



KVL a i_x+i_{sc}: $-15+2i_x+10(i_x+i_{sc})+3i_x=0$

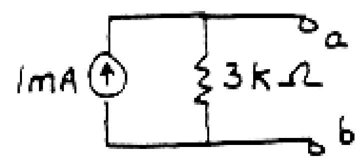
$\Rightarrow \underline{-15+15i_x+10i_{sc}=0}$ (1)

KVL a i_{sc}: $\underline{-3i_x+i_{sc}=0}$ (2)

Solving (1) & (2) simultaneously yields : $\underline{i_{sc}=1\text{mA}}$

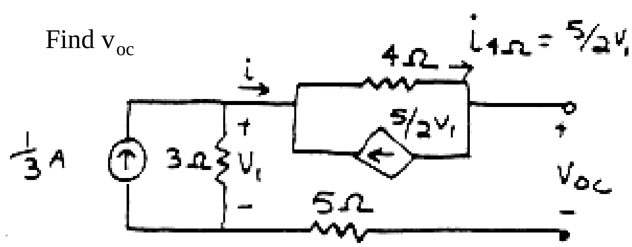
$\therefore R_T = \dfrac{v_{oc}}{i_{sc}} = \dfrac{3}{1} = 3\text{k}\Omega$

Norton equiv. ckt.



P5.6-4

Find v_{oc}



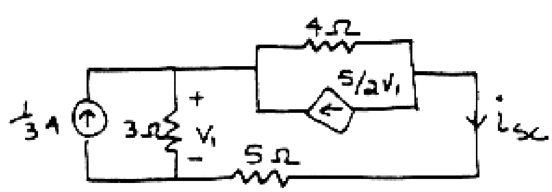
by inspection $i = 0$

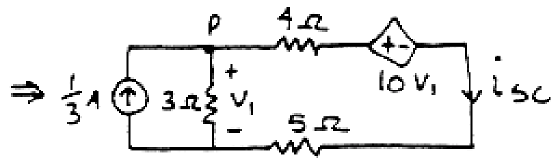
from left mesh : $v_1 = 3(1/3) = 1\text{V}$

from KVL a: $-v_1+4i_{4\Omega}+v_{oc}=0$

$\Rightarrow v_{oc} = v_1 - 4(5/2\,v_1) = -9\text{V}$

Find i_{sc}





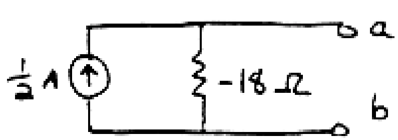
from KVL a: $-v_1+4i_{sc}+10v_1+5i_{sc}=0$

$\Rightarrow 9v_1+9i_{sc}=0$ (1)

from KCL at P : $\underline{-\frac{1}{3}+\frac{v_1}{3}+i_{sc}=0}$ (2) (1) & (2) yields

$\therefore R_T = \dfrac{v_{oc}}{i_{sc}} = \dfrac{-9}{1/2} = \underline{-18\Omega}$

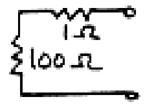
Norton equiv. ckt : $\underline{i_{sc} = \frac{1}{2}\text{A}}$



Section 5-7: Maximum Power Transfer

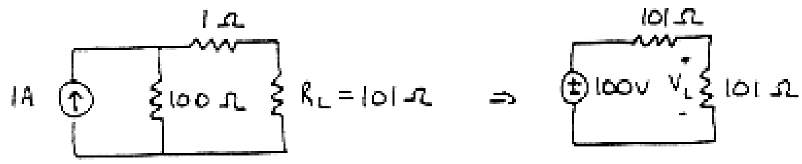
P5.7-1

$\therefore$



$\leftarrow \underline{R_T = 101\Omega = R_L}$

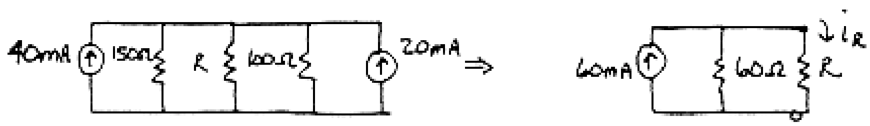
(b)



$$\underline{P_{max} = v_L^2/101 = \frac{(50)^2}{101} = 24.75\ \text{W}}$$

P5.7-2

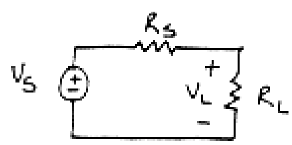
(a) Use source transformations to reduce ckt.



Norton equiv. where $R_T = 60\Omega$ $\quad \underline{\therefore \text{want } R_L = 60\Omega}$

(b) $\quad P_{max} = i_R^2(R) = (30)^2(60) \ \underline{= 54{,}000\mu\ \text{W} = 54\text{mW}}$

P5.7-3

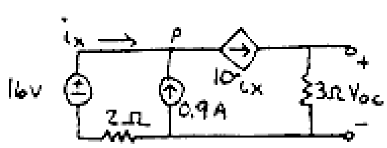


$$V_L = V_S\left[\frac{R_L}{R_S + R_L}\right]$$

$$\therefore P_L = \frac{V_L^2}{R_L} = \frac{V_S^2 R_L}{(R_S + R_L)^2}$$

By inspection, P_L is max when you vary R_s to get the smallest denominator. $\underline{\therefore \text{ set } R_s = 0}$

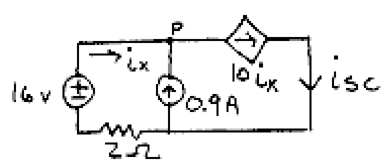
P5.7-4 Find R_T using $R_T = v_{oc}/i_{sc}$. First find v_{oc} :



KCL at P: $-i_x - .9 + 10i_x = 0 \quad \Rightarrow \ i_x = 0.1\text{A}$

$\therefore \ \underline{v_{oc} = 3\ (10 i_x) = 3\text{V}}$

Find i_{SC}

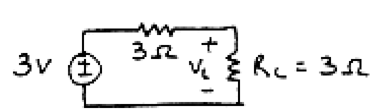


KCL at P: $-i_x - .9 + 10 i_x = 0 \quad \Rightarrow \ i_x = 0.1\text{A}$

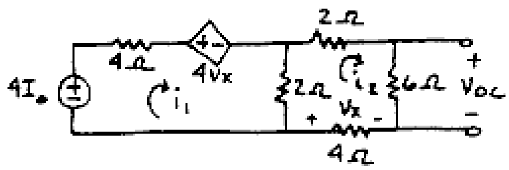
$\therefore \underline{i_{sc} = 10 i_x = 1\ \text{A}}$

$$p_{L_{max}} = \frac{{V_L}^2}{R_L} = \frac{(1.5)^2}{3} = \underline{0.75\ \text{W}}$$

$\therefore R_T = v_{oc}/i_{sc} = 3\Omega = R_L \qquad$ for max power



P5.7-5 (a) For max power $R_L = R_T$. First find v_{oc} :



KVL $a\,i_1$: $-4I_0 + 4i_1 - 4v_x + 2(i_1 - i_2) = 0$

$\Rightarrow \underline{6i_1 - 2i_2 + 4v_x - 4I_0 = 0}$ (1)

KVL $a\,i_2$: $2i_2 + 6i_2 - v_x + 2(i_2 - i_1) = 0$

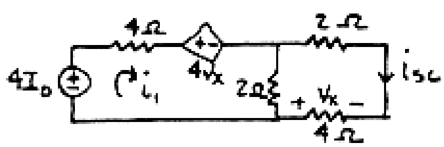
$\Rightarrow \underline{-2i_1 + 10i_2 - v_x = 0}$ (2)

also $\underline{v_x = 4i_2}$ (3)

and $\underline{v_{oc} = 6i_2}$ (4)

Solving (1), (2), (3), & (4) yields $\underline{v_{oc} = I_0}$

Find i_{sc}



KVL $a\,i_1$: $-4I_0 + 4i_1 + 4v_x + 2(i_1 - i_{sc}) = 0$

$\Rightarrow \underline{6i_1 - 2i_{sc} + 4v_x - 4I_0 = 0}$ (1)

KVL $a\,i_x$: $2i_{sc} + 4i_{sc} + 2(i_{sc} - i_1) = 0$

$\Rightarrow \underline{-2i_1 + 8i_{sc} = 0}$ (2)

also $\underline{v_x = -4i_{sc}}$ (3)

Solving (1), (2), & (3) yields $\underline{i_{sc} = \tfrac{2}{3} I_0}$ $\quad \underline{\therefore R_T = \frac{v_{oc}}{i_{sc}} = \tfrac{3}{2}\Omega = R_L}$

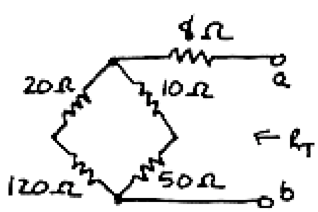
(b) $P_{L_{max}} = 54\text{ W} = \dfrac{(v_{oc}/2)^2}{R_L} = I_0^2/6$

$\Rightarrow \underline{I_0 = 18A}$

P5.7-6

$P_{max} = v_T^2/4R_T$

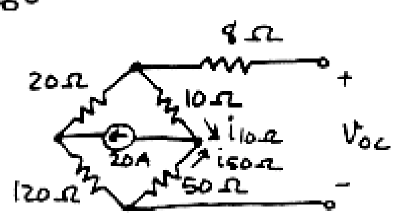
Find $R_T \Rightarrow$ kill i source



$R_T = 8 + (20+120)\|(10+50)$

$= 50\Omega$

find v_{oc}:



$i_{10\Omega} = \dfrac{120 + 50}{120+50+20+10}\, 20A$

$= 17A$

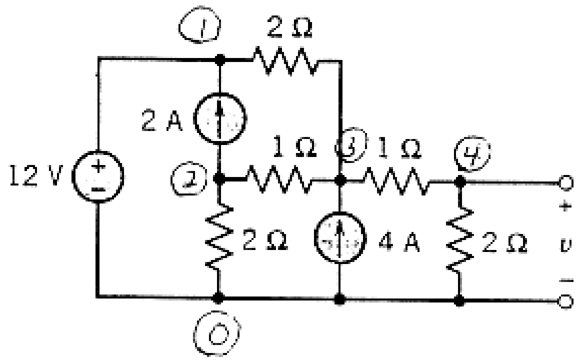
$\therefore v_{10\Omega} = 10(17) = 170V$

$v_{50\Omega} = 50(17-20) = -150V \quad \Rightarrow \quad v_{oc} = v_{10\Omega} + v_{50\Omega} = 170 - 150 = \underline{20V}$

$\therefore P_{max} = \underline{20^2} = 2W$

PSpice Problems

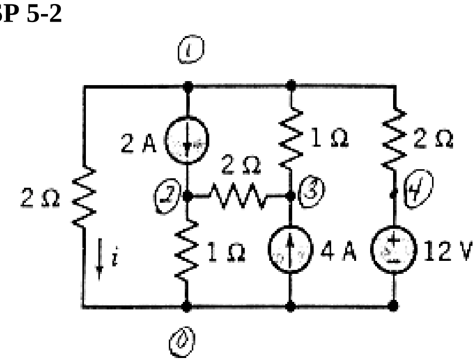
SP 5-1




```
Input file
 V1     1     0     dc     12
 I2     2     1     dc      2
 R3     2     0             2
 R4     2     3             1
 R5     1     3             2
 R6     3     4             1
 I7     0     3     dc      4
 R8     4     0             2
.dc  V1 12  12 1
.print  dc  V (4)
.END
```

result V(4)= v =4.952E+00V

SP 5-2

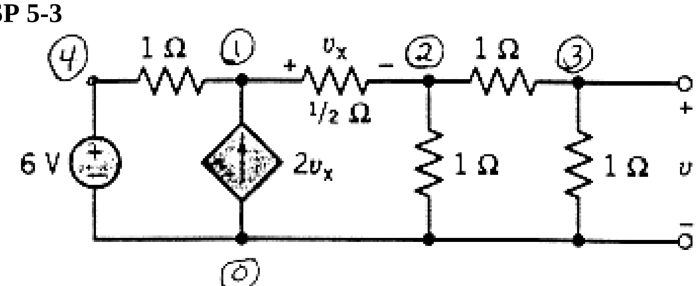



```
Input file
R1      1     0           2
I2      1     2    dc     2
R3      2     0           1
R4      2     3           2
R5      1     3           1
I6      0     3    dc     4
R7      1     4           2
V8      4     0    dc     12
.dc  V8  12  12  1
.print   dc  I(R1)
.END
```

result

i = I(R1)=3.000E+00A

SP 5-3

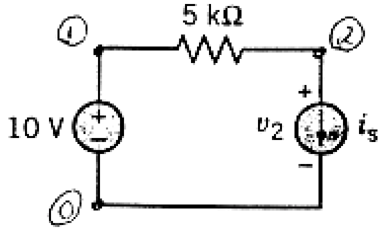



```
Input file
V1     4      0      dc     6
G2     0      1      1      2      2
R3     1      2             500m
R4     1      4             1
R5     2      0             1
R6     2      3             1
R7     3      0             1
.dc  V1  6  6 1
.print   dc  V(3)
.END
```

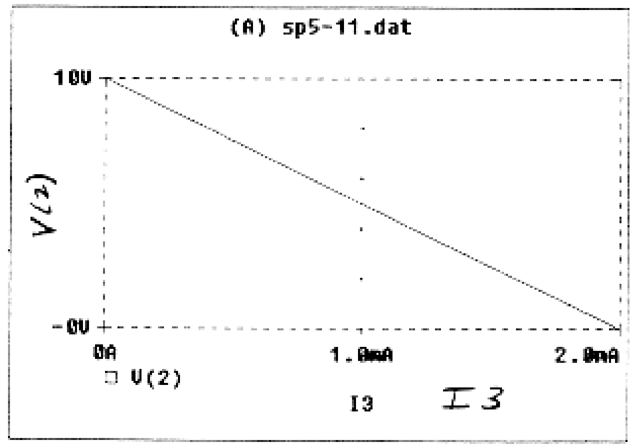
result

v = v(3) = 1.714E+00V

SP 5.4

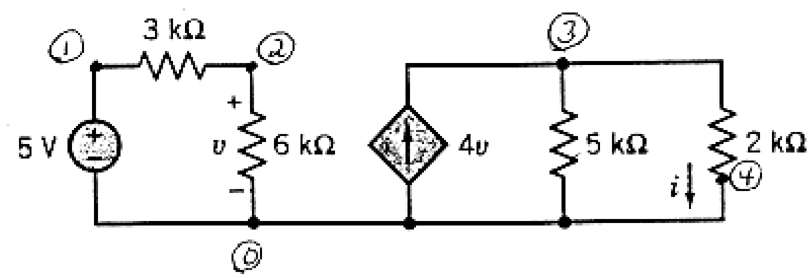


Probe result




```
Input file
V1    1     0     dc    10
R2    1     2           5k
I3    2     0     dc    1m
.dc   I3 0  2e-3   0.2e-3
.probe V(2)
.end
```

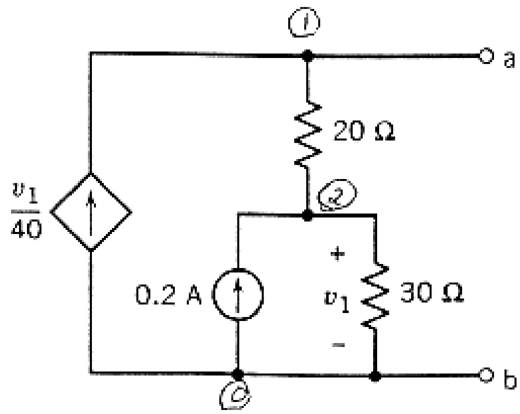
SP 5-5




```
Input file
V1    1     0     dc    5
R1    1     2           3k
R2    2     0           6k
G4    0     3     2  0  4
R3    3     0           5k
R4    3     0           2k
.dc  V1  5  5 1
.print  dc  I(R4)
.end
```

result i = I(R4)= 9.524E+00A

SP 5-6




```
Input file
G1    0     1     2     0     25m
R2    1     2           20
I3    0     2     dc    0.2
R4    2     0           30
.tf   V(1)  I3
.end
```

result

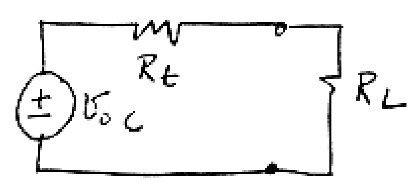
answer: $V_1 = V_{oc} = V_T =$ 36V

NODE	VOLTAGE	NODE	VOLTAGE
(1)	36.0000	(2)	24.0000

R_{TH} = OUTPUT RESISTANCE AT V(1)=2.000E+02Ω

Verification Problems

VP 5-1 Evaluating data

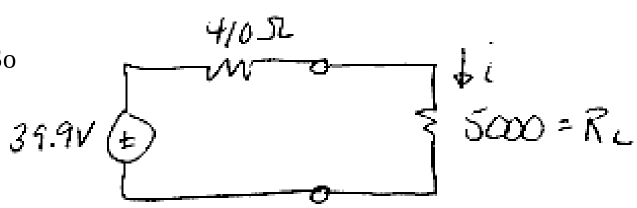


Case 1: $R_L = 0\Omega$; $i = I_{sc} = 97.2\text{mA} = \dfrac{v_{oc}}{R_t}$ (1)

Case 2: $R_L = 500\Omega$; $i = 43.8\text{mA} = \dfrac{v_{oc}}{R_t + 500}$ (2)

Solving 1+2 yields $R_t = 410\Omega$, $v_{oc} = 39.9$ V

So



When $R_L = 5000\Omega$

$$i = \frac{v_{oc}}{R_t + R_L} = 7.37\text{mA}$$

not 16.5mA as recorded ∴ the data is inconsistent.

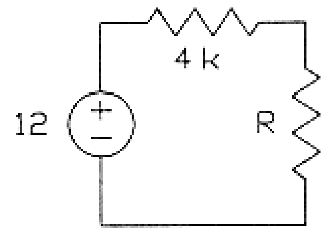
VP 5-2

$V_{oc} = 12\text{ V}$ (line 1 of the table)

$I_{sc} = 3\text{mA}$ (line 3 of the table)

so

$R_{TH} = V_{oc} / I_{sc} = 4\text{K}\Omega$



$$\downarrow i = \frac{12}{R+4K}$$

Hence the circuit can be simplified as shown above right. (Check:

$$\frac{12}{10\text{K}\Omega+4\text{K}\Omega} = 0.857\text{ mA}$$

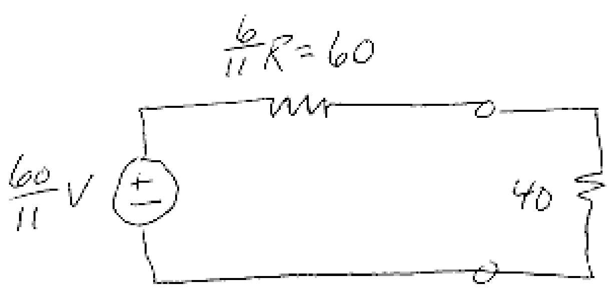
as shown in line 2 of the table.)

When i = 1 mA is required

$$1\text{mA} = \frac{12}{R+4\text{k}\Omega} \Rightarrow R = \frac{12}{1\text{mA}} - 4\text{k}\Omega = 8\text{k}\Omega$$

I agree with my lab partner's claim that R = 8000 causes i = 1 mA.

VP 5-3



$$\downarrow \quad i = \frac{60/11}{60+40} = 54.5\text{mA}$$

The measurement is consistent with the prelab calculations.

Design Problems

DP 5-1 The equation of representing the straight line in Figure DP 5-1b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\frac{0-5}{0.008-0} = 625\ \Omega$ and v_{oc} = 5 V.

Try $R_1 = R_2 = 1\ \text{k}\Omega$. ($R_1 \parallel R_2$ must be smaller than R_t = 625 Ω.) Then

$$5 = \frac{R_2}{R_1 + R_2} v_s = \frac{1}{2} v_s \quad \Rightarrow v_s = 10\ \text{V}$$

and

$$625 = R_3 + \frac{R_1 R_2}{R_1 + R_2} = R_3 + 500 \quad \Rightarrow \quad R_3 = 125\ \Omega$$

Now v_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-2 The equation of representing the straight line in Figure DP 5-2b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\frac{0-(-3)}{-0.006-0} = 500\ \Omega$ and v_{oc} = -3 V.

From the circuit we calculate

$$R_t = \frac{R_3\left(R_1 + R_2\right)}{R_1 + R_2 + R_3} \text{ and } v_{oc} = -\frac{R_1 R_3}{R_1 + R_2 + R_3} i_s$$

so

$$500\ \Omega = \frac{R_3\left(R_1 + R_2\right)}{R_1 + R_2 + R_3} \text{ and } 3\ \text{V} = -\frac{R_1 R_3}{R_1 + R_2 + R_3} i_s$$

Try $R_3 = 1\text{k}\Omega$ and $R_1 + R_2 = 1\text{k}\Omega$. Then $R_t = 500\ \Omega$ and

$$-3 = -\frac{1000 R_1}{2000} i_s = \frac{R_1}{2} i_s \quad \Rightarrow 6 = R_1\, i_s$$

This equation can be satisfied by taking R_1 = 600 Ω and i_s = 10 mA. Finally, R_2 = 1 kΩ - 400 Ω = 600 Ω. Now i_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-3 The slope of the graph is positive so the Thevenin resistance is negative. This would require $R_3 + \frac{R_1 R_2}{R_1 + R_2} < 0$, which is not possible since R_1, R_2 and R_3 will all be non-negative.

Is it not possible to specify values of v_s, R_1, R_2 and R_3 that cause the current i and the voltage v in Figure DP 5-3*a* to satisfy the relationship described by the graph in Figure DP 5-3*b*.

DP 5-4 The equation of representing the straight line in Figure DP 5-4b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to the Thevenin impedance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{-5-0}{0-0.008} = -625\ \Omega$ and v_{oc} = -5 V.

The open circuit voltage, v_{oc}, the short circuit current, i_{sc}, and the Thevenin resistance, R_t, of this circuit are given by

$$v_{oc} = \frac{R_2\left(d+1\right)}{R_1 + \left(d+1\right)R_2} v_s\,,$$

$$i_{sc} = \frac{\left(d+1\right)}{R_1} v_s$$

and

$$R_t = \frac{R_1 R_2}{R_1 + \left(d+1\right)R_2}$$

Let $R_1 = R_2 = 1$ kΩ. Then

$$-625\ \Omega = R_t = \frac{1000}{d+2} \quad \Rightarrow \quad d = \frac{1000}{-625} - 2 = -3.6 \text{ A/A}$$

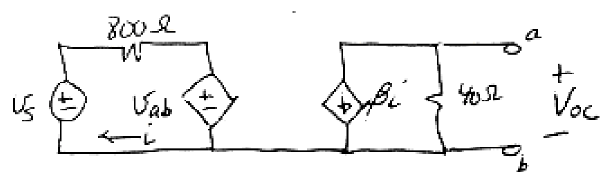
and

$$-5 = \frac{\left(d+1\right)v_s}{d+2} \quad \Rightarrow \quad v_s = \frac{-3.6+2}{-3.6+1}\left(-5\right) = -3.077 \text{ V}$$

Now v_s, R_1, R_2 and d have all been specified so the design is complete.

DP 5-5 a) Find Thev. equiv.

v_{oc}:



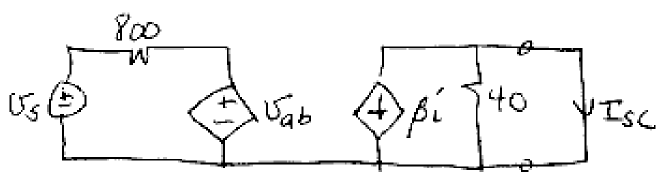
$-v_s + 800\, i + v_{ab} = 0 \qquad (1)$

$v_{ab} = v_{oc} \qquad (2)$

$v_{oc} = -(\beta\, i)(40) \qquad (3)$

Solving eqs. (1) - (3) yields $\underline{v_{oc} = v_s \Big/ \left(1 - \dfrac{20}{\beta}\right)}$

I_{sc}:



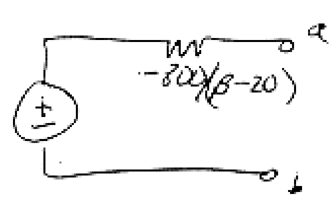
$I_{sc} = -\beta\, i \qquad (4)$

$v_{ab} = 0 \qquad (5)$

$v_s = 800 i \qquad (6)$

Solving eqs. (4) - (6) yields $\underline{I_{sc} = -\beta\, v_s / 800}$

so $\underline{R_t = \dfrac{v_{oc}}{I_{sc}} = \dfrac{-800}{\beta - 20}}$ and Thev. equiv. $\dfrac{v_S}{\left(1 - \dfrac{20}{\beta}\right)}$



b) $R_t = R_L = 400 = \frac{-800}{\beta-20} \Rightarrow \underline{\beta = 18}$

c) Max power to R_L, largest v_{OC}, largest v_S, smallest R_t

$$P_L = \frac{(V_L)^2}{R_L} \qquad (7)$$

$$\text{and } V_L = \frac{400}{R_{total}} v_{oc} \qquad (8)$$

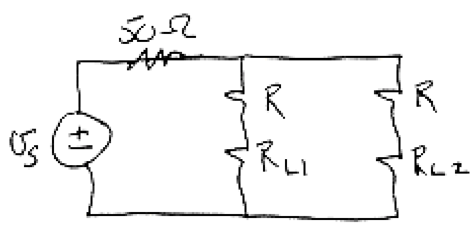
with

$R_{total} = \frac{-800}{\beta-20} + 400$ (a) yields $\underline{\beta = \pm 18}$

d) delivering large amounts of power could melt antenna.

DP 5-6 Max power to load : $R_L = R_t = 50\Omega$

But split power equally ($R_{L1} = R_{L2} = 50\Omega$)



$$\frac{(R+50)(R+50)}{R+50+R+50} = 50 \quad \Rightarrow \text{ yields } \underline{R = 50\Omega}$$