Chapter 5 Circuit Theorems

Exercises

Ex 5.3-1 $R = 10\ \Omega$ and $i_s = 1.2$ A.

Ex 5.3-2 $R = 10\ \Omega$ and $i_s = -1.2$ A.

Ex 5.3-3 $R = 8\ \Omega$ and $v_s = 24$ V.

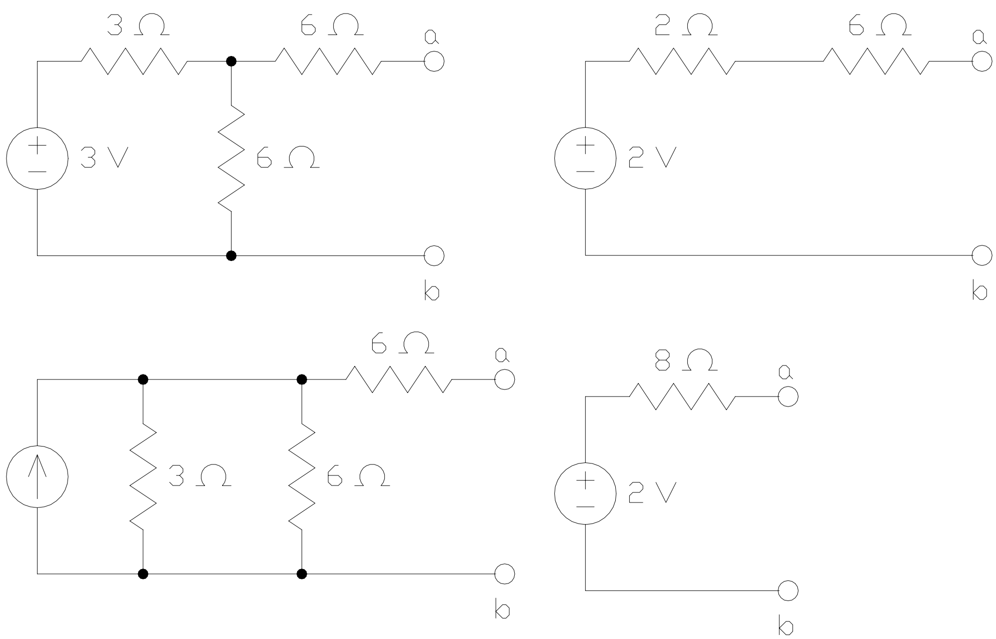
Ex 5.3-4 $R = 8\ \Omega$ and $v_s = -24$ V.

Ex 5.4-1 $v_m = \frac{20}{10+20+20} 15 + 20\left(-\frac{10}{10+(20+20)} 2\right) = 6 + 20\left(-\frac{2}{5}\right) = -2 \text{ V}$

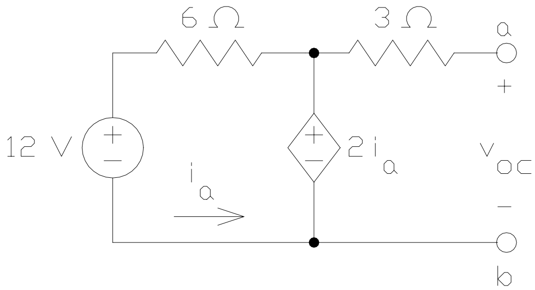
Ex 5.4-2 $i_m = \frac{25}{3+2} - \frac{3}{2+3} 5 = 5 - 3 = 2 \text{ A}$

Ex 5.4-3 $v_m = 3\left(\frac{3}{3+(3+3)} 5\right) - \frac{3}{3+(3+3)} 18 = 5 - 6 = -1 \text{ A}$

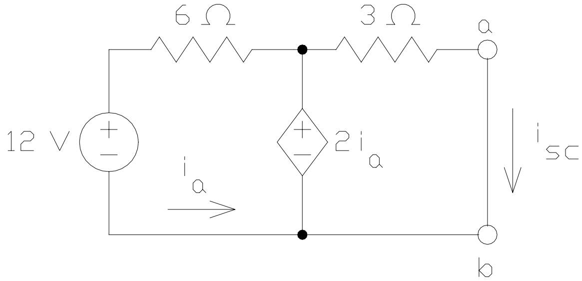
Ex 5.5-1



Ex 5.5-2



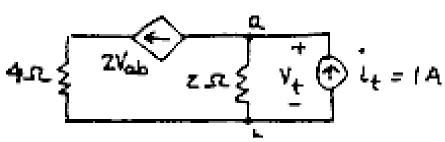
$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3\text{ A}$$
$$v_{oc} = 2\,i_a = -6\text{ V}$$



$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3\text{ A}$$
$$3 i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2\text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

Ex. 5.5–3 No independent sources $\therefore$ $v_{oc} = i_{sc} = 0 \Rightarrow$ apply 1A test source

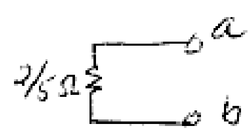


$v_{ab} = v_t$

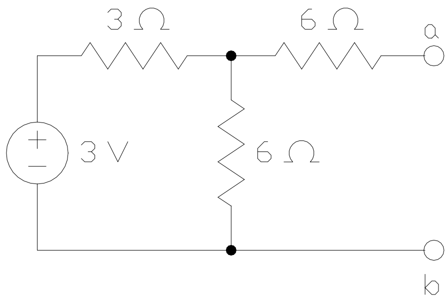
KCL at a: $2v_t + \frac{v_t}{2} - 1 = 0 \Rightarrow v_t = 2/5$ V

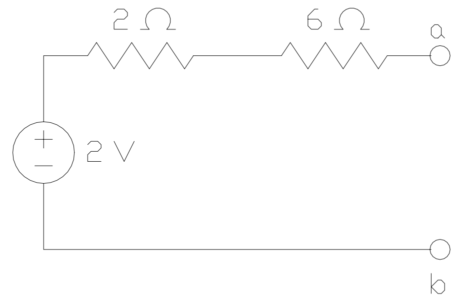
$\therefore R_T = \frac{v_t}{i_t} = 2/5\ \Omega$

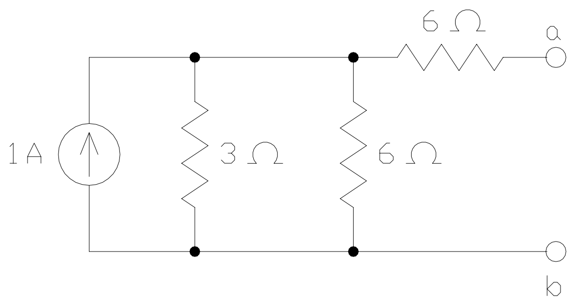
Thev. equiv. ckt

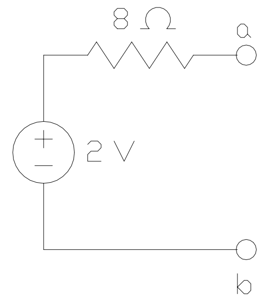


Ex 5.6-1

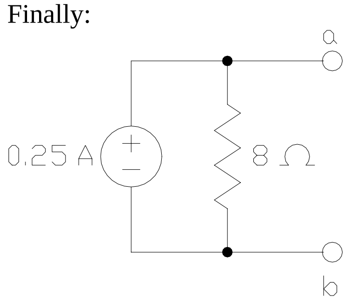




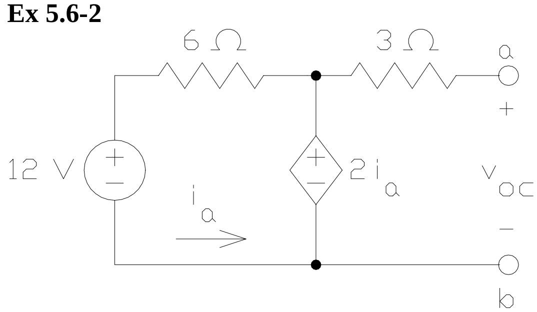




Finally:

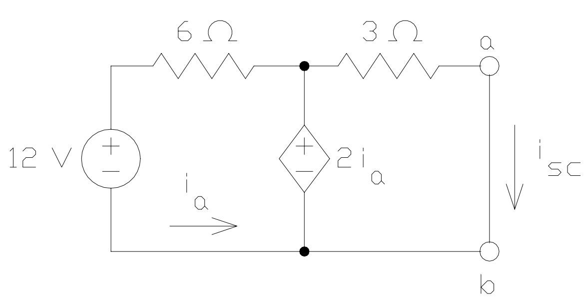


Ex 5.6-2



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$v_{oc} = 2\,i_a = -6 \text{ V}$$

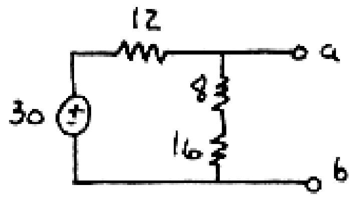


$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$3\,i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2 \text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

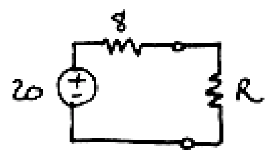
Ex. 5.6-3



$$R_T = \frac{12\times24}{12+24} = \frac{12\times24}{36} = 8\Omega$$

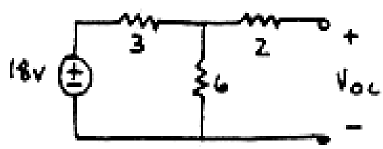
$$v_{oc} = \frac{24}{12+24}\,30 = 20V$$

So we have



$$i = \frac{20}{8+R}\ A$$

Ex. 5.7-1 Find v_{oc}



From voltage divider

$$v_{oc} = 18V\left(\frac{6}{6+3}\right) = 12V$$

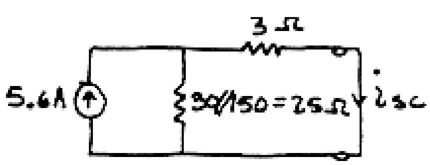
Find R_T (short 18V source)

$= 3\|6 + 2 = 4\Omega$

∴ Thev. equiv ckt ⇒

For max power to R_L ⇒ $R_L = R_T = 4\Omega$ ∴ $P_{\substack{max\\ to\,R_L}} = \frac{(v_{RL})^2}{R_L} = \frac{(6)^2}{4} = \underline{9W}$

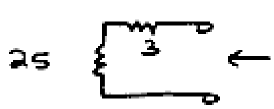
Ex. 5.7-2 Find i_{sc}



From current divider

$$i_{sc} = 5.6A\left(\frac{25}{25+3}\right)$$

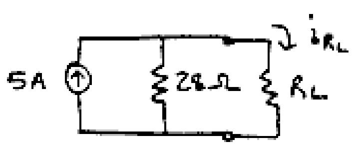
$$\underline{i_{sc} = 5A}$$



Find R_T (open 5.6A source)

$R_T = 25+3 = \underline{28\Omega}$

∴ Norton equiv. ckt ⇒



For max power $R_L = R_T = 28\Omega$ ∴ $P_{L_{max}} = (i_{R_L})^2 R_L = (5/2)^2\ (28) = \underline{175W}$

Ex. 5.7-3

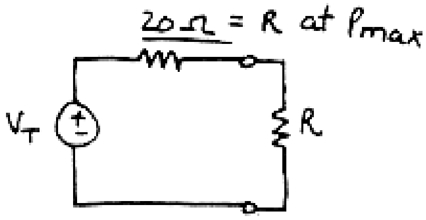
$$P_{L_{max}} = \frac{(v_{L_{max}})^2}{R_L} = \frac{\left[10V\left(\frac{5}{5+R_t}\right)\right]^2}{R_L}$$

Now for V_L to be maximized, R_t must be minimized

$\therefore$ choose $R_t = 1\Omega$

$$\therefore P_{L_{max}} = \frac{\left[10\left(\frac{5}{6}\right)\right]^2}{5} = \underline{13.9W}$$

Ex. 5.7-4



$$P_{max} = 5 = \left(\frac{v_T}{40}\right)^2 20 = \frac{v_T^{\,2}}{80}$$

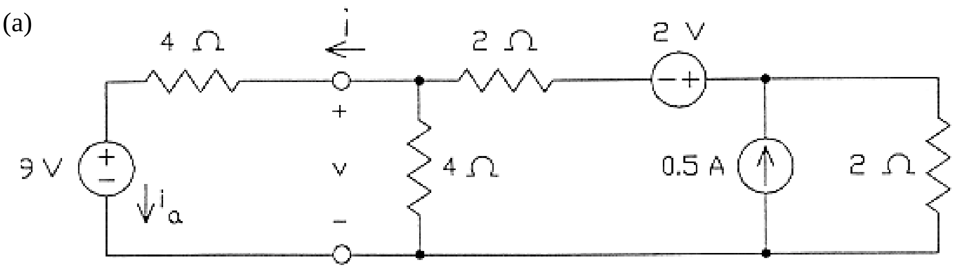
$$v_T = \sqrt{400} = \underline{20V}$$

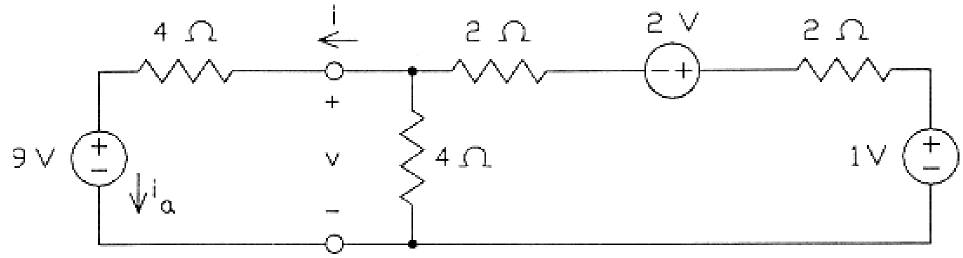
PROBLEMS

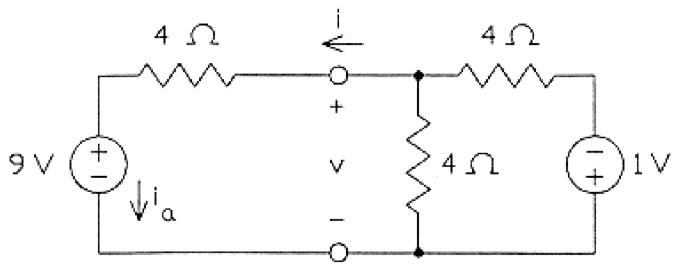
<u>Section 5-3:</u> Source Transformations

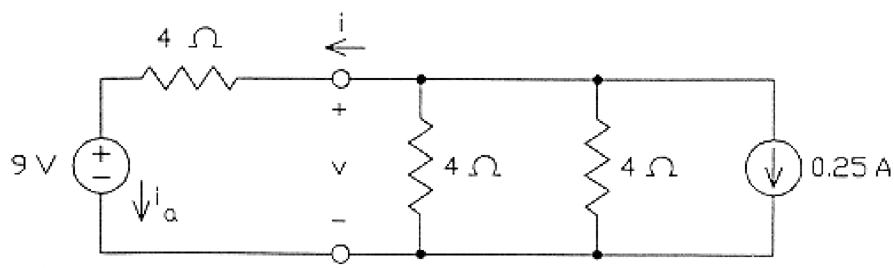
P5.3-1

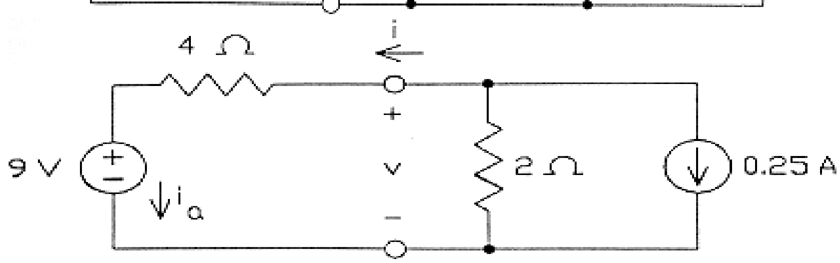
(a)

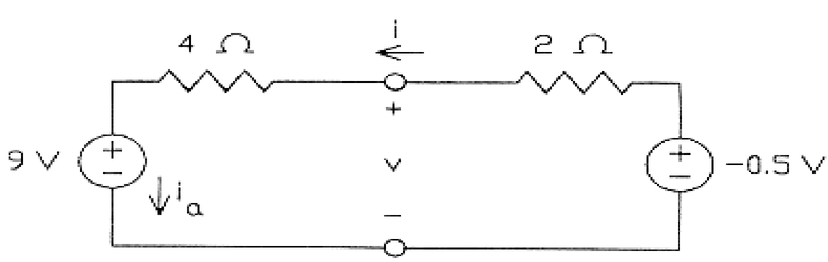












$\therefore\ R_t = 2\Omega$

$v_t = -0.5\text{V}$

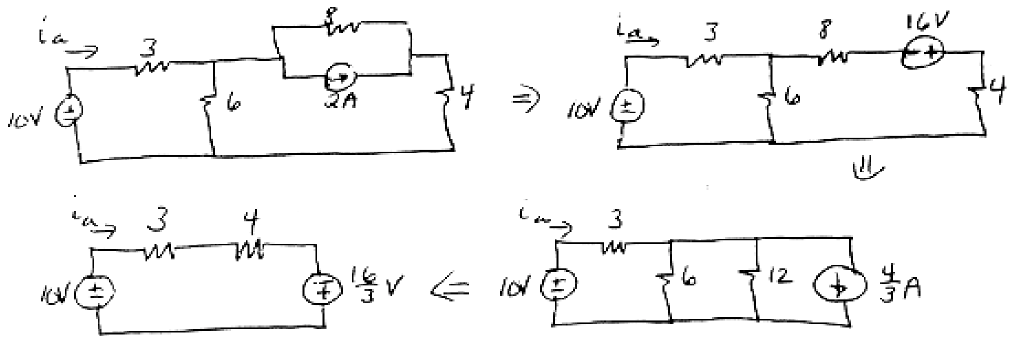
(b) $-9-4i-2i+(-0.5)=0$

$$i = \frac{-9+(-0.5)}{4+2} = -1.58\text{A}$$

$$v = 9+4i = 9+4(-1.58) = 2.67\text{V}$$

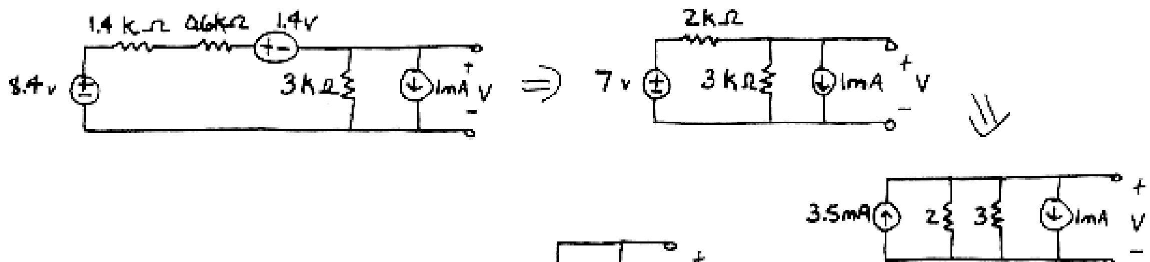
(c) $i_a = i = -1.58\text{A}$

P5.3-2

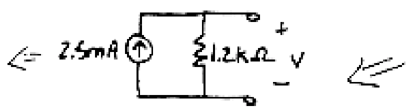


KVL: $-10+3i_a+4i_a-\dfrac{16}{3}=0 \quad \therefore \; \underline{i_a = 2.19\text{A}}$

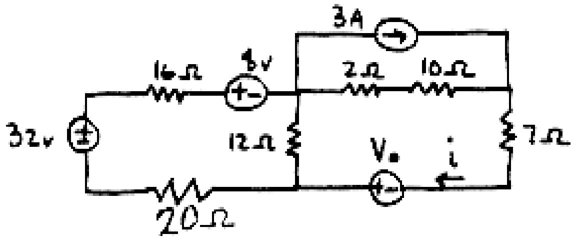
P5.3-3

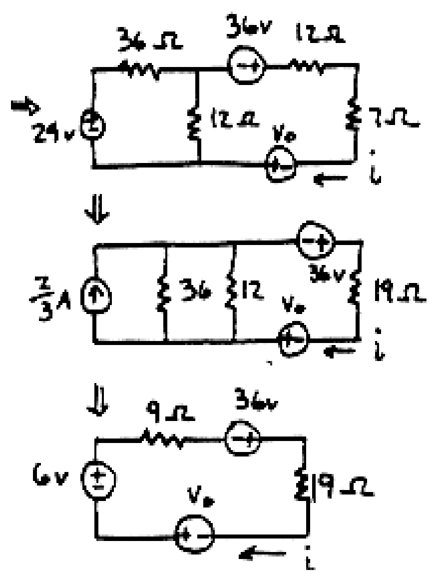


$\underline{V = (1.2\text{k}\Omega)(2.5\text{mA}) = 3\text{V}}$



P5.3-4

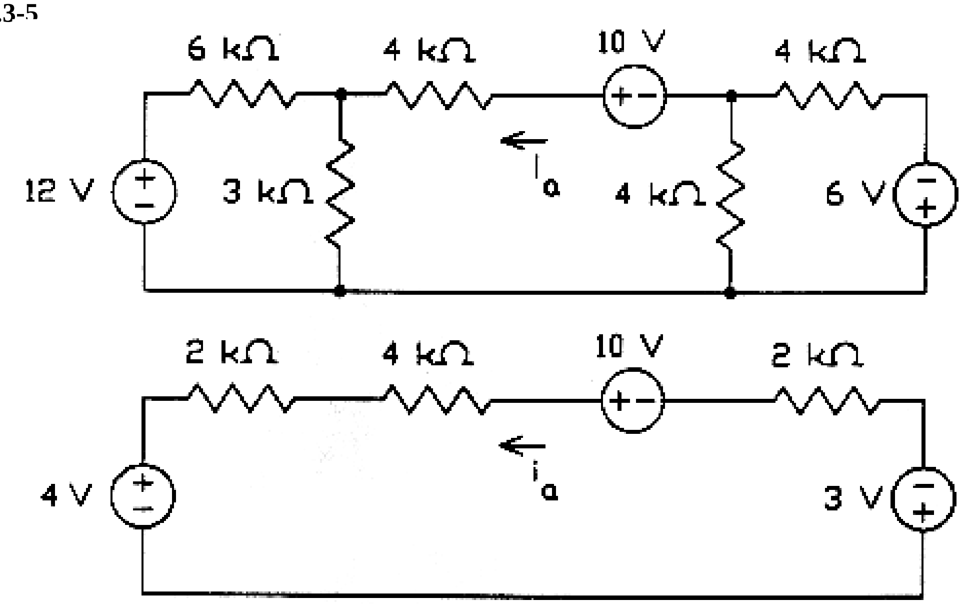




KVL a: $-6 + i(9+19) - 36 - v_0 = 0$

$$\Rightarrow v_o = -42+28\,(5/2) = 28\text{V}$$

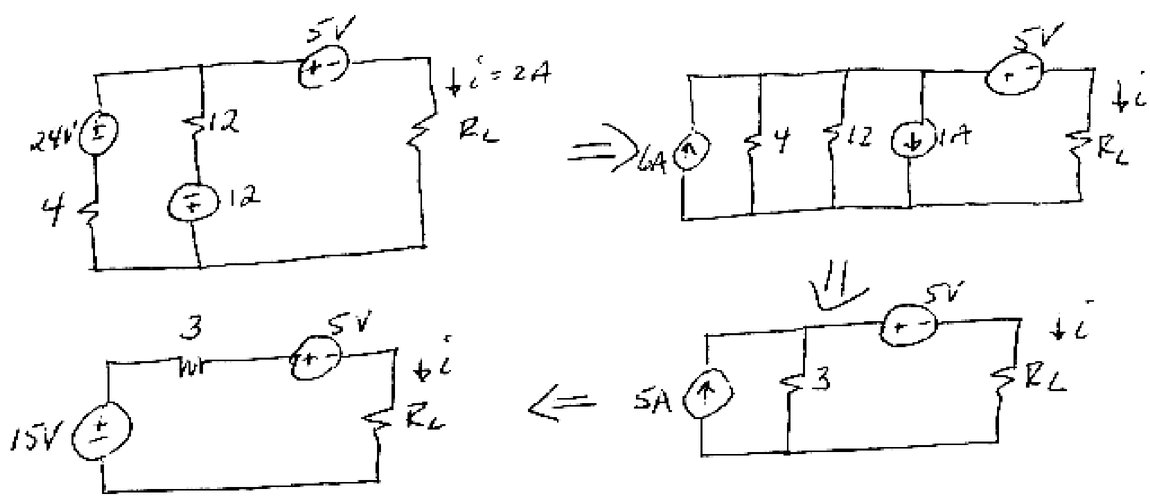
P5.3-5

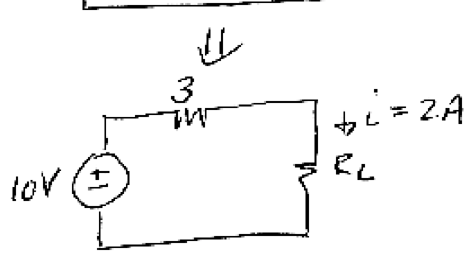


$$-4 - 2000\, i_a - 4000 i_a + 10 - 2000 i_a - 3 = 0$$

$$\therefore\ i_a = 375\ \mu\text{A}$$

P5.3-6





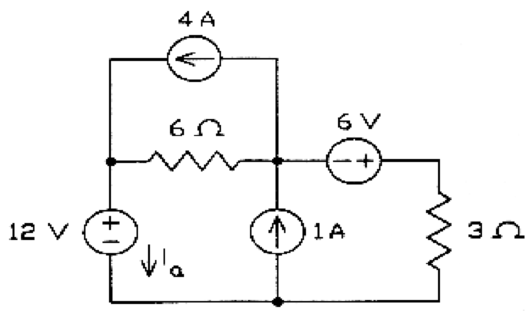
$$0\text{V} = (3 + R_L)\,(2\text{A})$$

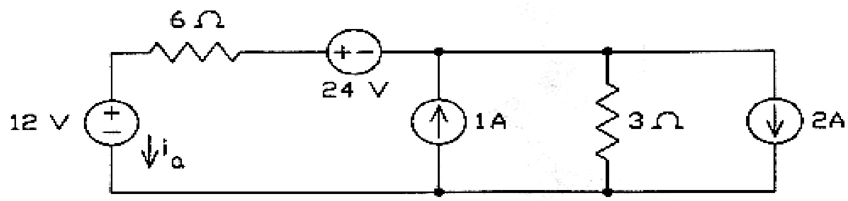
$$5 = 3 + R_L$$

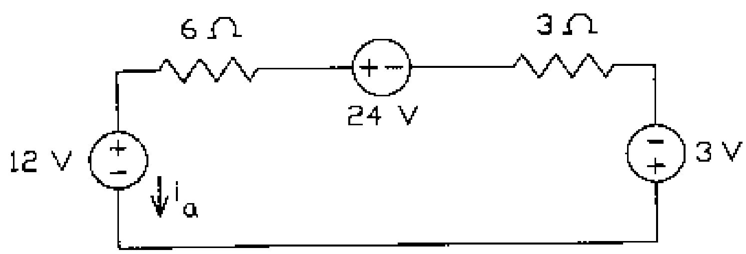
$$\therefore \underline{R_L = 2\Omega}$$

<u>Section 5-4</u> Superposition

P5.4-1







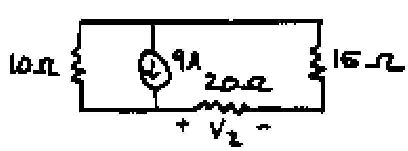
$$-12 - 6i_a + 24 - 3i_a - 3 = 0 \Rightarrow i_a = 1\text{ A}$$

P5.4–2 Consider 6A source only (open 9A source)

From current divider:

$$v_1 / 20 = 6\left[\frac{15}{15 + 30}\right] \Rightarrow \underline{v_1 = 40\text{V}}$$

Consider 9A source only (open 6A source)

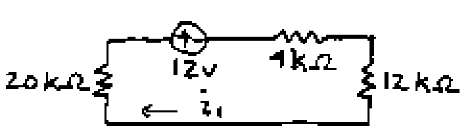


Current divider

$$v_2 / 20 = 9\left[\frac{10}{10 + 35}\right] \Rightarrow \underline{v_2 = 40\text{V}}$$

$$\underline{\therefore v = v_1 + v_2 = 40 + 40 = 80\text{V}}$$

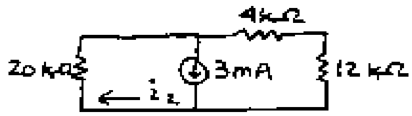
P5.4–3 Consider 12V source only (open both current sources)



KVL a: $20i_1 + 12 + 4i_1 + 12i_1 = 0$

$\Rightarrow \underline{i_1 = -1/3\text{mA}}$

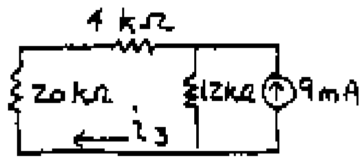
Consider 12mA source only (short 12V and open 6mA sources)



From current divider

$$i_2 = 3\left[\frac{16}{16+20}\right] = \underline{\frac{4}{3}\text{mA}}$$

Consider 9mA source only (short 12V and open 12mA sources)

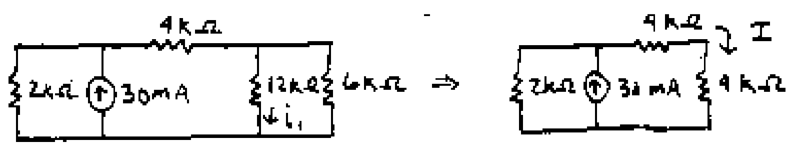


From current divider

$$i_3 = -9\left[\frac{12}{24+12}\right] = \underline{-3\text{mA}}$$

$$\underline{\therefore\ i = i_1 + i_2 + i_3 = -1/3 + 4/3 - 3 = -2\text{mA}}$$

P5.4–4 Consider 30mA source only (open 15mA and short 60V sources)

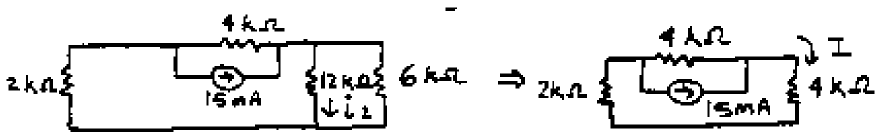


$$\text{Current divider} \Rightarrow I = 30\left(\frac{2}{2+8}\right) = 6\text{mA}$$

$$\therefore\ i_1 = I\left(\frac{6}{6+12}\right) = \underline{2\text{mA}}$$

Consider 15mA source only (open 30mA source and short 60V source)

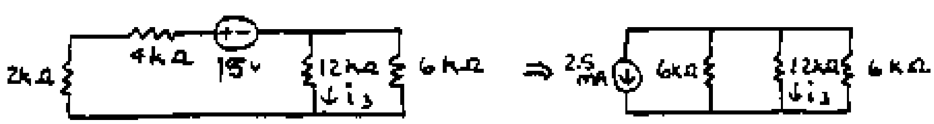
Continued



$$\text{Current divider} \Rightarrow I = 15\left(\frac{4}{4+6}\right) = 6\text{mA}$$

$$\therefore\ i_2 = I\left(\frac{6}{6+12}\right) = \underline{2\text{mA}}$$

Consider 15V source only (open both current sources)

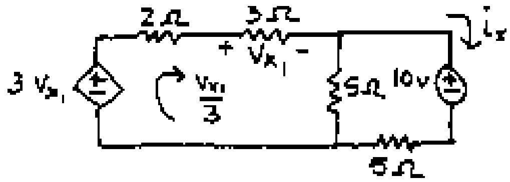


From current divider

$$i_3 = -2.5\left(\frac{6//6}{6//6+12}\right) = -10\left(\frac{3}{3+12}\right) = \underline{-.5\text{mA}}$$

$$\therefore\ \underline{i = i_1 + i_2 + i_3 = 2 + 2 - .5 = 3.5\text{mA}}$$

P5.4-5 Consider 10V source only (open 4A source)



KVL 1st mesh a:

$$-3v_{x_1}+5\left(\frac{v_{x_1}}{3}\right)+5\left(\frac{v_{x_1}}{3}-i_x\right)=0$$

$$\Rightarrow \underline{v_{x_1}=15i_x} \quad (1)$$

KVL 2nd mesh a: $5\,(i_x - v_{x_1}/3) + 10 + 5i_x = 0$ (2)

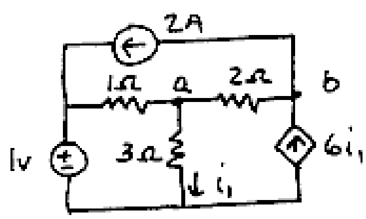
Solving (1) and (2) simultaneously $\Rightarrow$ $\underline{v_{x_1} = 10\text{V}}$

Consider 4A source only (short 10V source)

Using current divider: $\frac{v_{x_2}}{3}=\left(3/2v_{x_2}-4\right)\left(\frac{2}{2+3+5/2}\right) \Rightarrow \underline{v_{x_2}=16\text{V}}$

$$\therefore \underline{v_x = v_{x_1}+v_{x_2} = 10+16 = 26\text{V}}$$

P5.4-6



KCL at b : $i+6i_1-2 = 0$

$\Rightarrow i_1 = 1/3-1/6\,i$ (1)

KVL around left lower mesh:

$1(i_1+i)+3i_1-1 = 0$ (2)

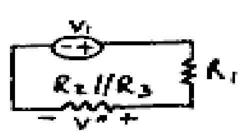
Plugging (1)into (2) $\Rightarrow$ $\underline{i = -1\text{A}}$

P5.4-7 Consider v_2 source only

Voltage divider : $v' = -v_2\left[\frac{R_2}{R_2+R_1\|R_3}\right]$

$$v' = -v_2\left[\frac{R_2(R_1+R_3)}{R_1R_2+R_1R_3+R_2R_3}\right]$$

Consider v_1 source only



Voltage divider $v'' = v_1\left[\frac{R_2\|R_3}{R_2\|R_3+R_1}\right]$

$$v'' = v_1\left[\frac{R_2\,R_3}{R_1R_2+R_1R_3+R_2R_3}\right]$$

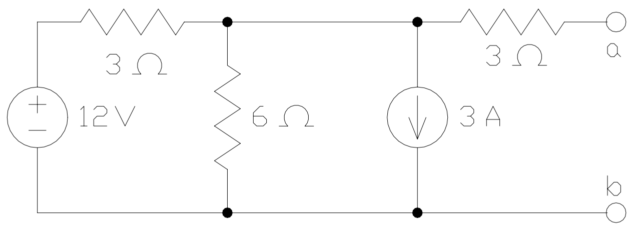
Consider i_1 source only

$v''' = 0$ since no current flows through R_2, R_3 and R_1

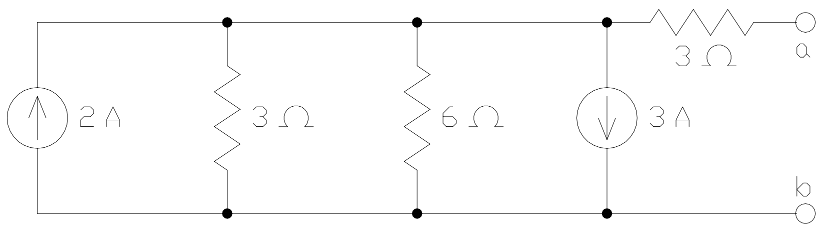
$$\therefore \underline{v = v'+v''+v''' = \frac{v_1R_2R_3-v_2(R_2(R_1+R_3))}{R_1R_2+R_1R_3+R_2R_3}}$$

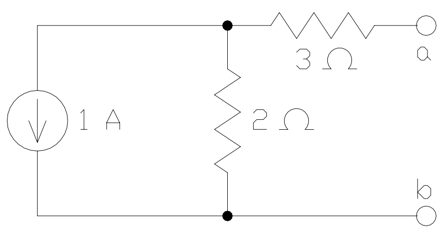
Section 5-5: Thèvenin’s Theorem

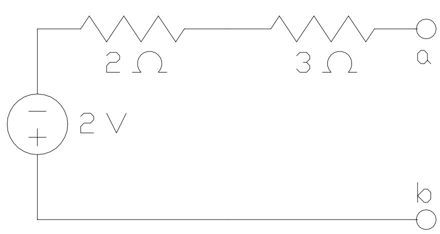
Ex 5.5-1

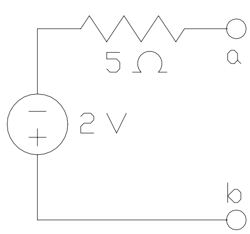


(a)

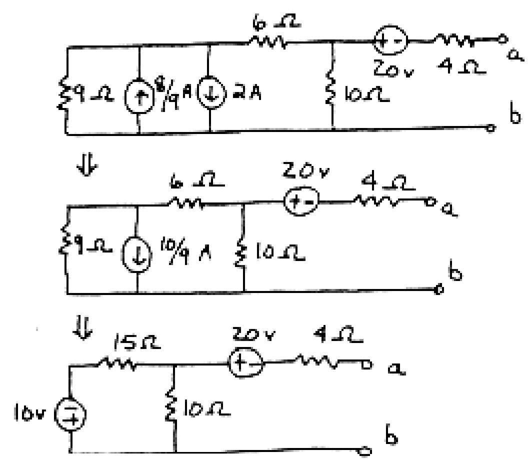


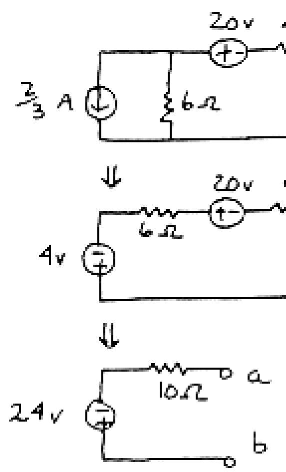






P5.5-2 Use source transformations

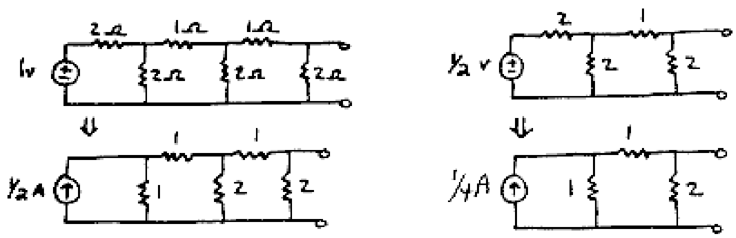


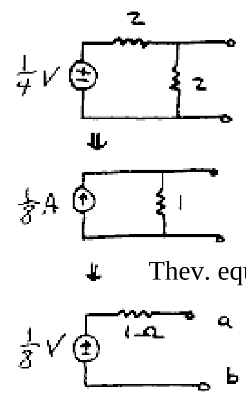


$R_T = 10\Omega$

$v_{oc} = -24V$

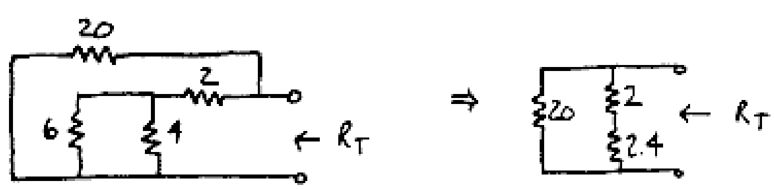
P5.5-3 Use source transformations





Thev. equiv.ckt

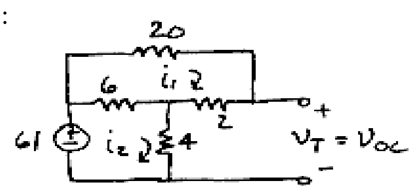
P5.5-4 Find R_T:



$$R_T = \frac{20(2+2.4)}{20+2+2.4} = \underline{3.61\Omega}$$

Continued

Find v_T:



$v_T = 2i_1 + 4i_2$

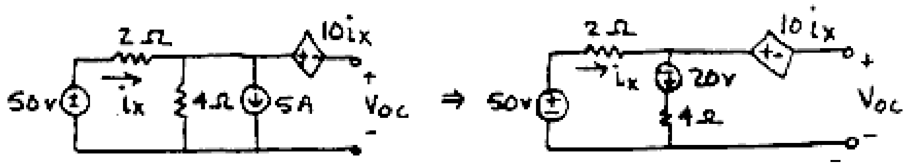
mesh i_1 : $28i_1 - 6i_2 = 0$ (1)

mesh i_2 : $-6i_1 + 10i_2 - 61 = 0$ (2)

Solving (1) & (2) yields: $i_1 = 1.5A$, $i_2 = 7A$

$\therefore$ $\underline{v_T = 3+28 = 31V}$

P5.5-5 Find v_{oc}

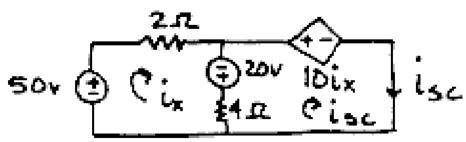


KVL around 1st mesh a: $-50+2i_x-20+4i_x = 0 \Rightarrow i_x = 70/6\ \text{A}$

KVL around 2nd mesh a: $-4i_x+20+10i_x+v_{oc} = 0$

$\Rightarrow \underline{v_{oc} = -90\text{V}}$

Find i_{sc}



KVL i_x mesh a: $-50+2i_x-20+4(i_x-i_{sc}) = 0$

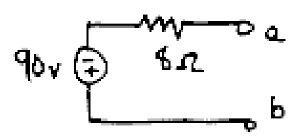
$\underline{6i_x-4i_{sc}-70 = 0}$ (1)

KVL i_{sc} mesh a: $4(i_{sc}-i_x)+20+10i_x = 0$

$\underline{6i_x+4i_{sc}+20 = 0}$ (2)

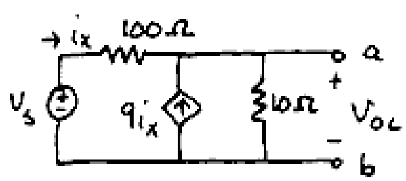
Solving (1) and (2) simultaneously $\Rightarrow \underline{i_{sc} = -45/4\ \text{A}}$

$\therefore R_T = \dfrac{v_{oc}}{i_{sc}} = \underline{8\Omega}$ Thev. equiv.ckt :



P5.5-6

For v_{oc}:



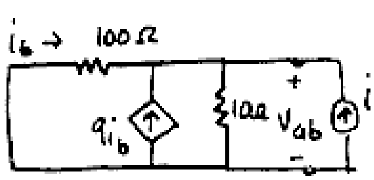
$i_x = \dfrac{v_s-v_{oc}}{100}$

KCL at terminal a:

$$\frac{1}{100}(v_{oc}-v_s)-9\left[\frac{1}{100}(v_s-v_{oc})\right]+\frac{1}{10}v_{oc} = 0$$

$$\Rightarrow \underline{v_{oc} = \frac{1}{2}v_s}$$

Use current source at a-b to find R_T:

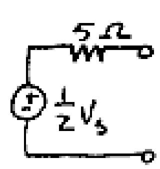


$i_b = -\dfrac{v_{ab}}{100}$

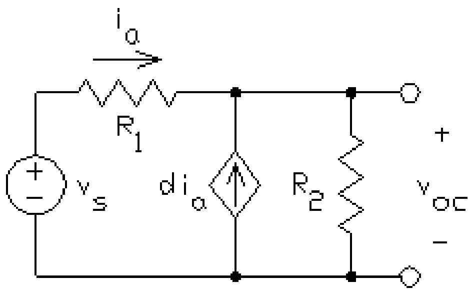
$$\text{KCL}: \frac{1}{100}v_{ab}-9\left[\frac{1}{100}(-v_{ab})\right]+\frac{1}{10}v_{ab}-i = 0$$

$$\Rightarrow i = \frac{1}{5}v_{ab} \quad \therefore R_T = \frac{v_{ab}}{i} = 5\Omega$$

So Thev. equiv.



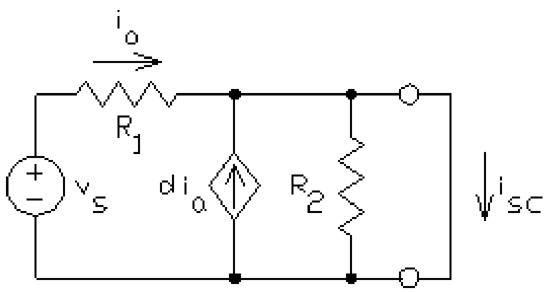
P5.5-7



$$v_s + R_1\, i_a + \left(d+1\right) R_2\, i_a = 0$$

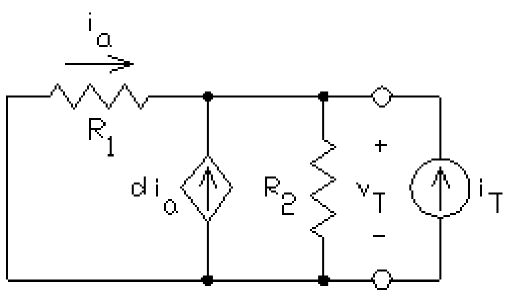
$$i_a = \frac{v_s}{R_1 + \left(d+1\right) R_2}$$

$$v_{oc} = \frac{\left(d+1\right) R_2 v_s}{R_1 + \left(d+1\right) R_2}$$



$$i_a = \frac{v_s}{R_1}$$

$$i_{sc} = \left(d+1\right) i_a = \frac{\left(d+1\right) v_s}{R_1}$$



$$-i_a - d\, i_a + \frac{v_T}{R_2} - i_T = 0$$

$$R_1\, i_a = -v_T$$

$$i_T = \left(d+1\right)\frac{v_T}{R_1} + \frac{v_T}{R_2} = \frac{R_2\left(d+1\right) + R_1}{R_1\, R_2}$$

$$R_t = \frac{v_T}{i_T} = \frac{R_1\, R_2}{R_1 + \left(d+1\right) R_2}$$

(b) Let $R_1 = R_2 = 1$ kΩ. Then

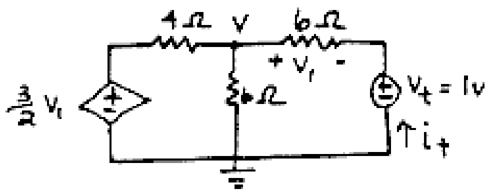
$$625\ \Omega = R_t = \frac{1000}{d+2} \quad \Rightarrow \quad d = \frac{1000}{625} - 2 = -0.4\ \text{A/A}$$

and

$$5 = \frac{\left(d+1\right) v_s}{d+2} \quad \Rightarrow \quad v_s = \frac{-0.4+2}{-0.4+1} 5 = 13.33\ \text{V}$$

P5.5-8

Since no independent sources $v_{oc} = i_{sc} = 0 \;\therefore\;$ *apply* test source

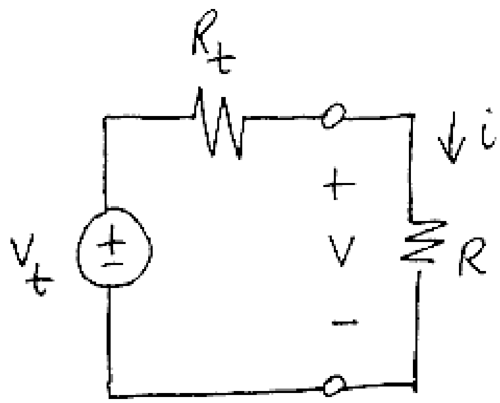


$V = v_1 + v_t$

KCL at V : $\dfrac{\left(V-\frac{3}{2}v_1\right)}{4} + \dfrac{V}{6} + \dfrac{v_1}{6} = 0$ & with $V = v_1 + 1$ *so* $v_1 = -2A$

now $i_t = -\dfrac{v_1}{6} = \underline{\tfrac{1}{3}A}$ $\therefore\; R_T = \dfrac{v_t}{i_t} = \dfrac{1}{\frac{1}{3}} = \underline{3\Omega}$

P5.5-9



$V = \dfrac{R}{R+R_t}v_t$

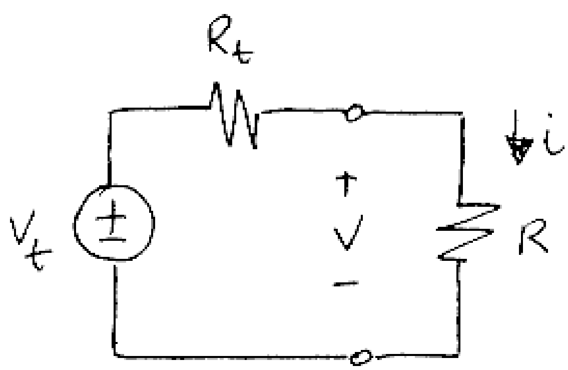
$6 = \dfrac{2000}{2000+R_t}v_t$ (line 1)

$2 = \dfrac{4000}{4000+R_t}v_t$ (line 2)

$\therefore\; \underline{v_t = 1.2\text{V}}$ and $\underline{R_t = -1600\Omega}$

When R = 8000, $V = \dfrac{8000}{8000-1600}1.2 = \underline{1.5\text{V}}$

P5.5-10



$i = \dfrac{v_t}{R+R_t}$

$0.004 = \dfrac{v_t}{2000+R_t}$ (1)

$0.003 = \dfrac{v_t}{4000+R_t}$ (2)

so $\underline{v_t = 24\text{V}}$ and $\underline{R_t = 4000\Omega}$

(a) $0.002 = \dfrac{24}{R+4000} \Rightarrow \underline{R = 8000\Omega}$

(b) when $R = 0$ then $i = \dfrac{24}{4000} = \underline{6\text{ mA}}$

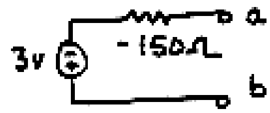
P5.5-11

From the graph, when $v_{ab} = v = 0 \Rightarrow i = i_{sc} = 20$ mA

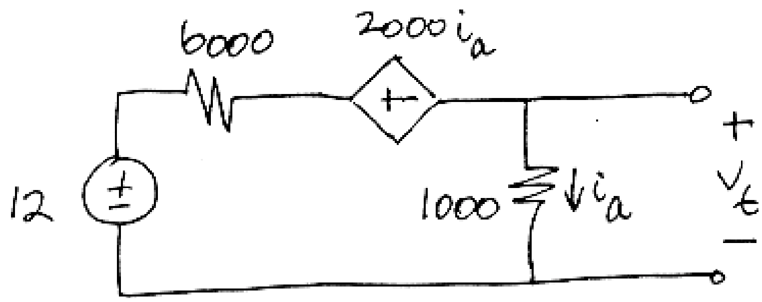
when $i = 0 \quad \Rightarrow v = \underline{v_{oc} = -3\,V}$

$$\therefore\ R_T = \frac{v_{oc}}{i_{sc}} = \frac{-3V}{20\,mA} = -.15\,k\Omega = -\underline{150\,\Omega}$$

Thev. equiv. ckt ⇒



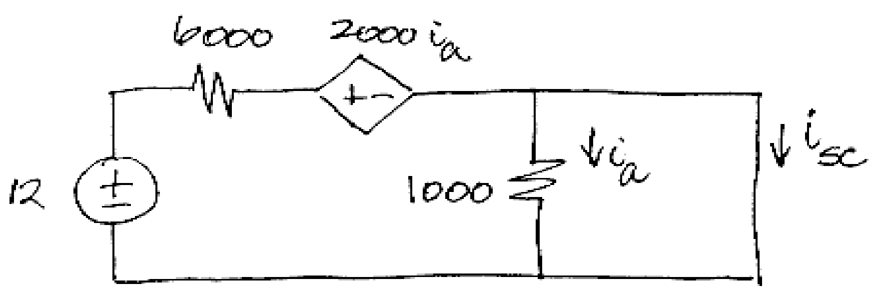
P5.5-12



$$-12 + 6000\,i_a + 2000\,i_a + 1000\,i_a = 0$$

$$i_a = 4/3000 \text{ A}$$

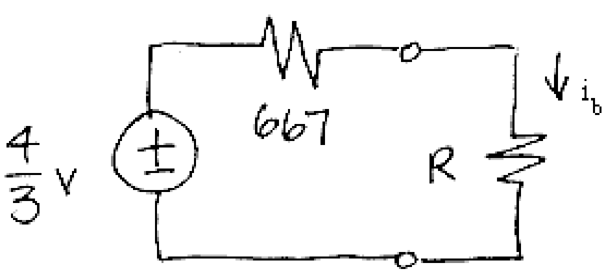
$$v_t = 1000\ i_a = \frac{4}{3}\text{V}$$



$$i_a = \frac{0}{1000} = 0$$

$$-12 + 6000\,i_{sc} = 0 \Rightarrow i_{sc} = 2\,\text{mA}$$

$$R_t = \frac{v_t}{i_{sc}} = \frac{\frac{4}{3}}{.002} = 667\,\Omega$$

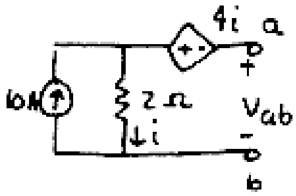


$$i_b = \frac{\frac{4}{3}}{667 + R}$$

$$\therefore\ i_b = 0.002 \text{ requires } R = \frac{\frac{4}{3}}{0.002} - 667 = 0$$

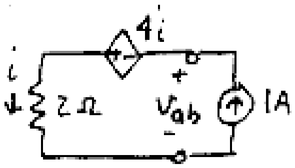
P5.5-13

1) disconnect R_L
open circuit a − b



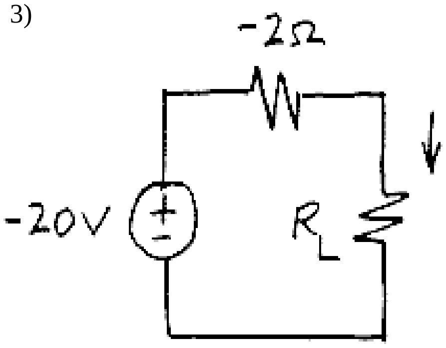
$$\text{KVL}: \; -v_{ab} - 4i + 2i = 0, \; i = 10\,\text{A}$$
$$\Rightarrow \; v_T = v_{ab} = -2i = \underline{-20\,\text{V}}$$

2) set independent source = 0 and place 1A source at a − b



$$\text{KVL}: \; -v_{ab} - 4i + 2i = 0, \; i = 1\text{A}$$
$$\Rightarrow v_{ab} = -2\text{A}$$
$$\therefore R_T = v_{ab}/1\text{A} = \underline{-2\,\Omega}$$

3)



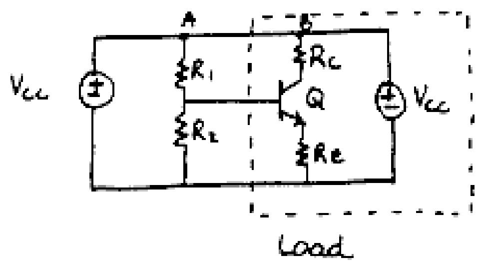
$$-2\,\text{A} = i_L = \frac{-20\,\text{V}}{R_L - 2\,\Omega} \; \Rightarrow \; R_L = 12\,\Omega$$

P5.5-14

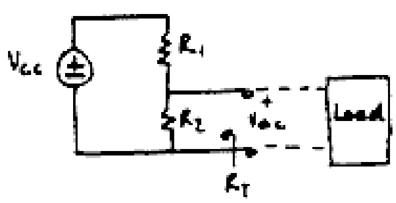
When the terminals of the boxes are open-circuited, no current flows in Box A, but the resistor in Box B dissipates 1 watt. Box B is therefore warmer than Box A. If you short the terminals of each box, the resistor in Box A will draw 1 amp and dissipate 1 watt. The resistor in Box B will be shorted, draw no current, and dissipate no power. Then Box A will warm up and Box B will cool off.

P5.5-15

Redraw ckt as:



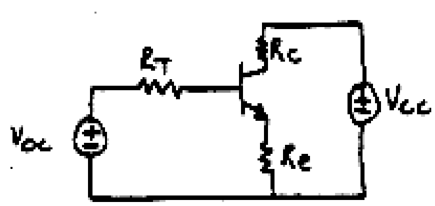
Since points A & B are at same potential, virtually no current exists between A-B ∴ open ckt.



$\underline{\text{Find } R_T}$: kill v_{cc} source $\Rightarrow R_T = R_1 \| R_2 = R_1R_2/R_1 + R_2$

$\underline{\text{Find } v_{oc}}$: voltage divider $\quad v_{oc} = v_{cc}\left(\dfrac{R_2}{R_1+R_2}\right)$

can replace above ckt as :

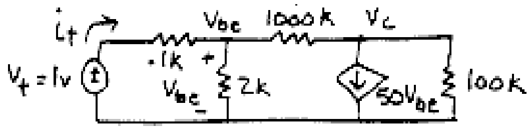


where $v_{oc} = v_{cc}\left(\dfrac{R_2}{R_1+R_2}\right)$

$$R_T = \frac{R_2R_1}{R_2+R_1}$$

P5.5-16

(a) Since there are no independent sources, apply test source



KCL at v_{be} : $-i_t + \frac{1}{2} v_{be} + (v_{be} - v_c)/1000 = 0$

$\downarrow \underline{-1000\, i_t + 501\, v_{be} - v_c = 0}$ (1)

KCL at v_c : $(v_c - v_{be})/1000 + 50\, v_{be} + v_c/100 = 0$

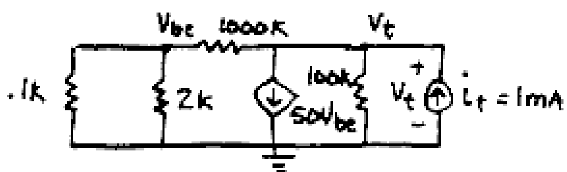
$\downarrow \underline{11 v_c + 50000\, v_{be} = 0}$ (2)

also $\underline{(1 - v_{be})/.1 = i_t}$ (3)

Solving (1), (2), & (3) simultaneously yields $\underline{i_t = 3.35\,\text{mA}}$

$$\therefore\ R_{IN} = \frac{v_t}{i_t} = \frac{1\,\text{V}}{3.35\,\text{mA}} = .299\,\text{k}\Omega = 299\,\Omega$$

(b) Apply test source



KCL at v_{be} : $v_{be}/.1 + v_{be}/2 + (v_{be} - v_t)/1000 = 0$

$\downarrow \underline{10501 v_{be} = v_t}$ (1)

KCL at v_t : $(v_t - v_{be})/1000 + 50\, v_{be} + v_t/100 - 1 = 0$

$\downarrow \underline{11 v_t + 49999\, v_{be} - 1000 = 0}$ (2)

Solving (1) & (2) yields : $\underline{v_t = 63.5\ \text{V}}$

$\underline{\therefore R_{out} = v_t/i_t = 63.5\text{V}/1\,\text{mA} = 63.5\,\text{k}\Omega}$

P5.5-17

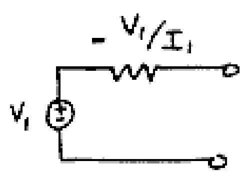
When $\underline{0 < V < V_p}$, it works as a pure resistor

so $R = V_p/I_p \quad V_{oc} = 0$

When $\underline{V_p < V < V_m}$, it is linear but shows negative resistance characteristic

$$\Rightarrow V_{oc} = V_{oc}\big|_{I=0} = V_1$$

$$R = \frac{V_{oc}}{I_{sc}} = -\frac{V_1}{I_1}$$



When $\underline{V_m < V < V_f}$, it is linear

so $V_{oc} = V\big|_{I=0} = V_2$

$$R = \frac{V_{oc}}{I_{sc}} = \frac{V_f - V_2}{I_p}$$