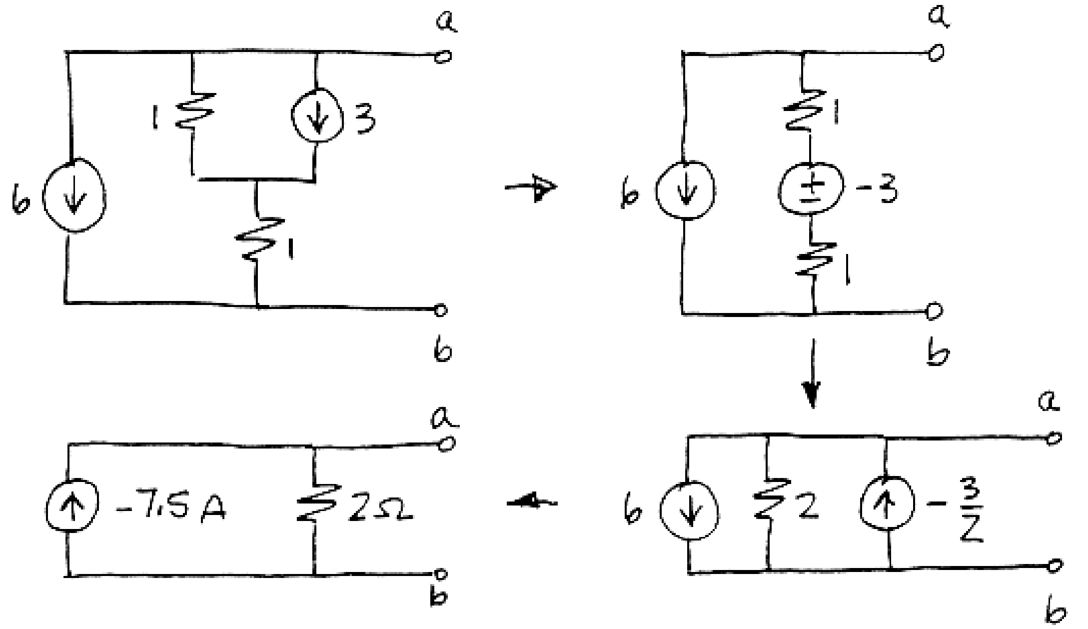
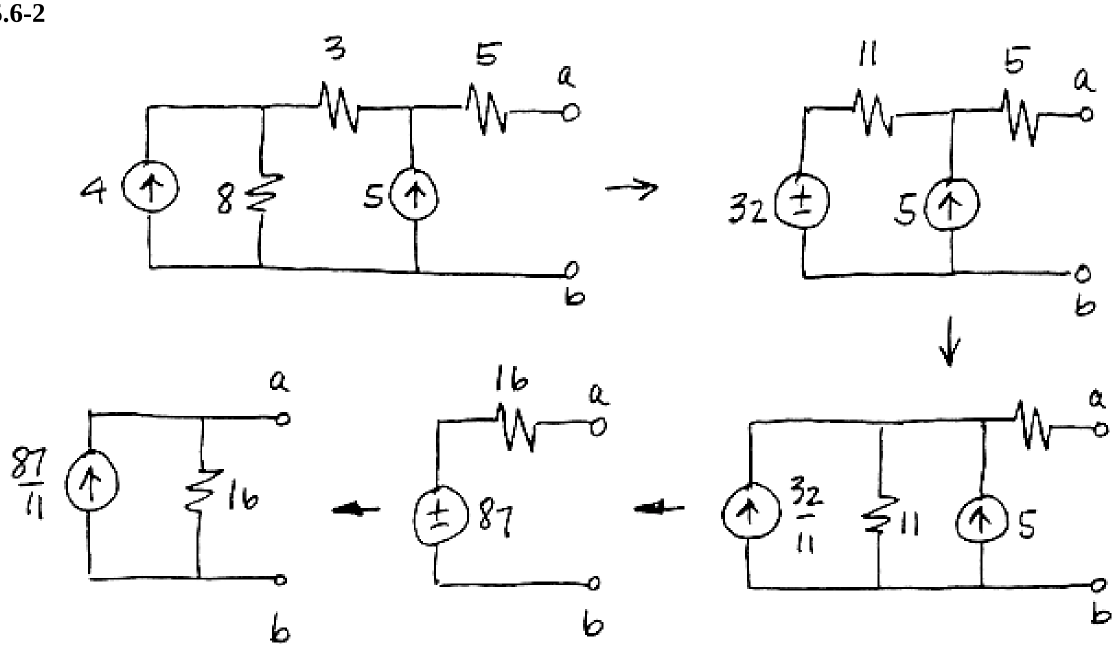
Section 5-6: Norton's Theorem

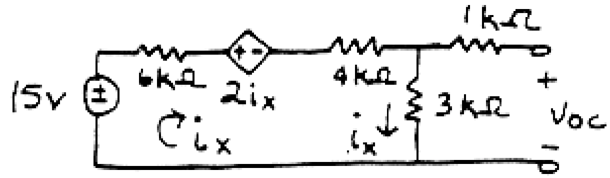
P5.6-1



P5.6-2



P5.6-3 Find v_{oc}

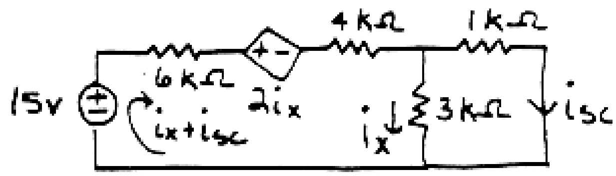


KVL a i_x: $-15+i_x(6+4)+2i_x+3i_x=0$

$\Rightarrow i_x = 1\text{mA}$

$\therefore \underline{v_{oc}} = 3i_x = 3\text{V}$

Find i_{sc}



KVL a i_x+i_{sc}: $-15+2i_x+10(i_x+i_{sc})+3i_x=0$

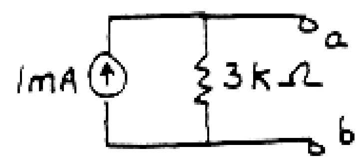
$\Rightarrow \underline{-15+15i_x+10i_{sc}=0}$ (1)

KVL a i_{sc}: $\underline{-3i_x+i_{sc}=0}$ (2)

Solving (1) & (2) simultaneously yields : $\underline{i_{sc}=1\text{mA}}$

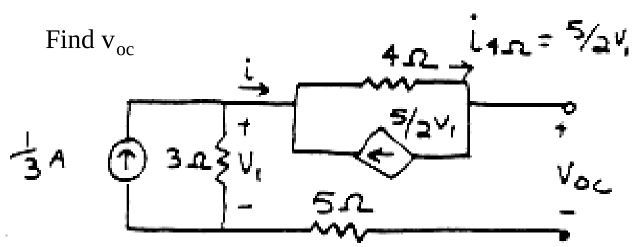
$\therefore R_T = \dfrac{v_{oc}}{i_{sc}} = \dfrac{3}{1} = 3\text{k}\Omega$

Norton equiv. ckt.



P5.6-4

Find v_{oc}



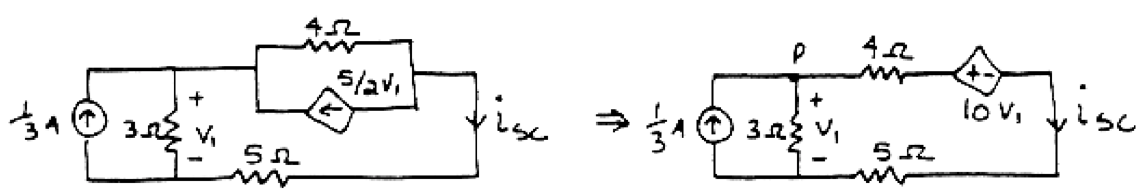
by inspection $i = 0$

from left mesh : $v_1 = 3(1/3) = 1\text{V}$

from KVL a: $-v_1+4i_{4\Omega}+v_{oc}=0$

$\Rightarrow v_{oc} = v_1 - 4(5/2\, v_1) = -9\text{V}$

Find i_{sc}



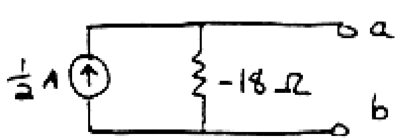
from KVL a: $-v_1+4i_{sc}+10v_1+5i_{sc}=0$

$\Rightarrow 9v_1+9i_{sc}=0$ (1)

from KCL at P : $\underline{-\frac{1}{3}+\frac{v_1}{3}+i_{sc}=0}$ (2) (1) & (2) yields

$\therefore R_T = \dfrac{v_{oc}}{i_{sc}} = \dfrac{-9}{1/2} = \underline{-18\Omega}$

Norton equiv. ckt : $\underline{i_{SC} = \frac{1}{2}\text{A}}$



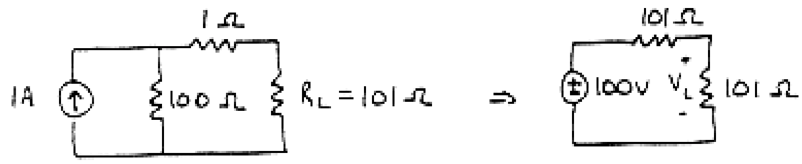
Section 5-7: Maximum Power Transfer

P5.7-1

$\therefore$

$\leftarrow$ $\underline{R_T = 101\Omega = R_L}$

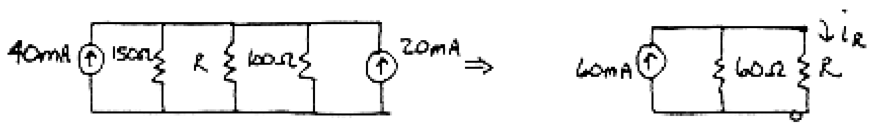
(b)



$$\underline{P_{max} = v_L^2/101 = (50)^2/101 = 24.75\ \text{W}}$$

P5.7-2

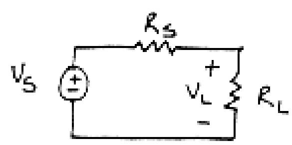
(a) Use source transformations to reduce ckt.



Norton equiv. where $R_T = 60\Omega$ $\quad\therefore$ $\underline{\text{want } R_L = 60\Omega}$

(b) $P_{max} = i_R^2(R) = (30)^2(60) = \underline{54{,}000\mu\ \text{W} = 54\text{mW}}$

P5.7-3

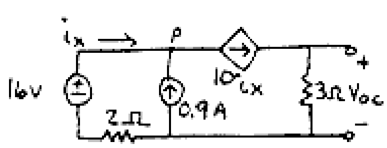


$$V_L = V_S\left[\frac{R_L}{R_S + R_L}\right]$$

$$\therefore P_L = \frac{V_L^2}{R_L} = \frac{V_S^2 R_L}{(R_S + R_L)^2}$$

By inspection, P_L is max when you vary R_s to get the smallest denominator. $\underline{\therefore \text{ set } R_s = 0}$

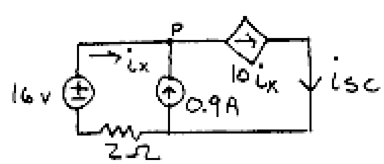
P5.7-4 Find R_T using $R_T = v_{oc}/i_{sc}$. First find v_{oc} :



KCL at P: $-i_x - .9 + 10i_x = 0 \quad \Rightarrow \quad i_x = 0.1\text{A}$

$\therefore$ $\underline{v_{oc} = 3\ (10 i_x) = 3\text{V}}$

Find i_{SC}



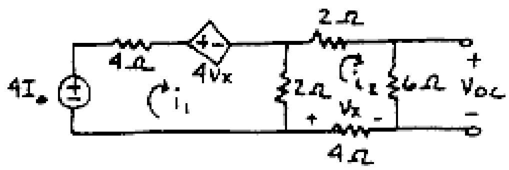
KCL at P: $-i_x - .9 + 10i_x = 0 \quad \Rightarrow i_x = 0.1\text{A}$

$\therefore \underline{i_{sc} = 10 i_x = 1\text{ A}}$

$$p_{L_{max}} = \frac{{V_L}^2}{R_L} = \frac{(1.5)^2}{3} = \underline{0.75\ \text{W}}$$

$\therefore R_T = v_{oc}/i_{sc} = 3\Omega = R_L$ for max power

P5.7-5 (a) For max power $R_L = R_T$. First find v_{oc} :



KVL $\text{a}\,i_1$: $-4I_0 + 4i_1 - 4v_x + 2(i_1 - i_2) = 0$

$\Rightarrow \underline{6i_1 - 2i_2 + 4v_x - 4I_0 = 0}$ (1)

KVL $\text{a}\,i_2$: $2i_2 + 6i_2 - v_x + 2(i_2 - i_1) = 0$

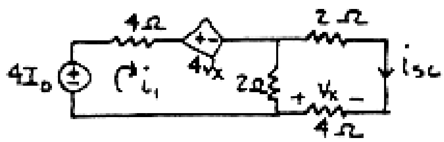
$\Rightarrow \underline{-2i_1 + 10i_2 - v_x = 0}$ (2)

also $\underline{v_x = 4i_2}$ (3)

and $\underline{v_{oc} = 6i_2}$ (4)

Solving (1), (2), (3), & (4) yields $\underline{v_{oc} = I_0}$

Find i_{sc}



KVL $\text{a}\,i_1$: $-4I_0 + 4i_1 + 4v_x + 2(i_1 - i_{sc}) = 0$

$\Rightarrow \underline{6i_1 - 2i_{sc} + 4v_x - 4I_0 = 0}$ (1)

KVL $\text{a}\,i_x$: $2i_{sc} + 4i_{sc} + 2(i_{sc} - i_1) = 0$

$\Rightarrow \underline{-2i_1 + 8i_{sc} = 0}$ (2)

also $\underline{v_x = -4i_{sc}}$ (3)

Solving (1), (2), & (3) yields $\underline{i_{sc} = \frac{2}{3} I_0}$ $\quad \underline{\therefore R_T = \frac{v_{oc}}{i_{sc}} = \frac{3}{2}\Omega = R_L}$

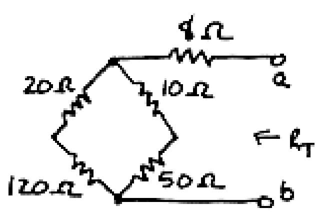
(b) $P_{L_{max}} = 54\text{ W} = \dfrac{(v_{oc}/2)^2}{R_L} = I_0^2/6$

$\Rightarrow \underline{I_0 = 18\text{A}}$

P5.7-6

$P_{max} = v_T^2/4R_T$

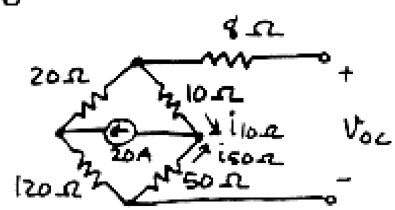
Find $R_T \Rightarrow$ kill i source



$R_T = 8 + (20+120)\|(10+50)$

$= 50\Omega$

find v_{oc}:



$i_{10\Omega} = \dfrac{120 + 50}{120+50+20+10}\, 20\text{A}$

$= 17\text{A}$

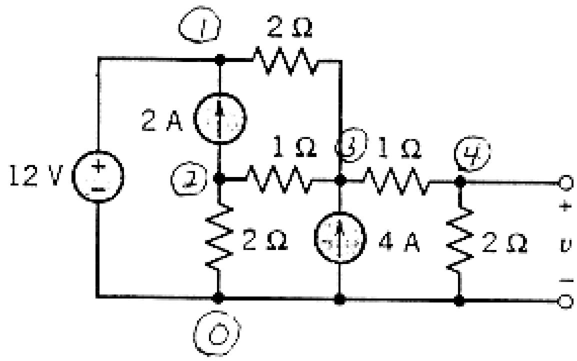
$\therefore v_{10\Omega} = 10(17) = 170\text{V}$

$v_{50\Omega} = 50(17-20) = -150\text{V} \quad \Rightarrow \quad v_{oc} = v_{10\Omega} + v_{50\Omega} = 170 - 150 = \underline{20\text{V}}$

$\therefore P_{max} = \underline{20^2} = 2\text{W}$

PSpice Problems

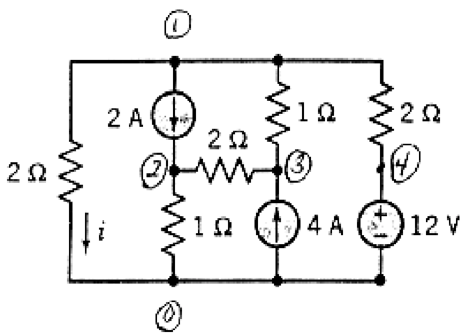
SP 5-1




```
Input file
 V1    1    0    dc    12
 I2    2    1    dc     2
 R3    2    0           2
 R4    2    3           1
 R5    1    3           2
 R6    3    4           1
 I7    0    3    dc     4
 R8    4    0           2
.dc  V1 12  12 1
.print  dc  V (4)
.END
```

result V(4)= v =4.952E+00V

SP 5-2

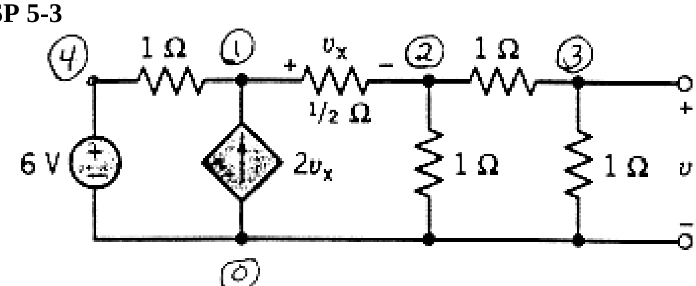



```
Input file
R1      1     0          2
I2      1     2    dc    2
R3      2     0          1
R4      2     3          2
R5      1     3          1
I6      0     3    dc    4
R7      1     4          2
V8      4     0    dc    12
.dc  V8  12  12  1
.print   dc  I(R1)
.END
```

result

i = I(R1)=3.000E+00A

SP 5-3

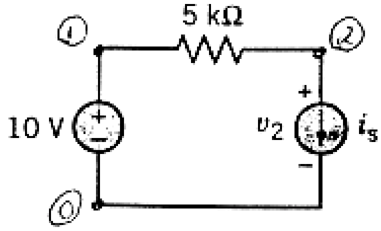


result

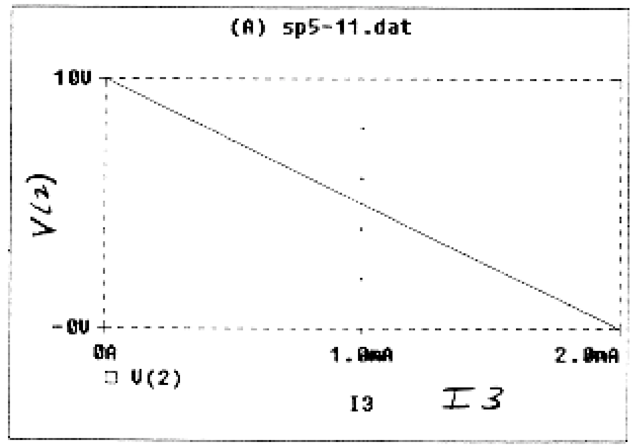
v = v(3) = 1.714E+00V

```
Input file
V1    4     0     dc    6
G2    0     1     1     2     2
R3    1     2           500m
R4    1     4           1
R5    2     0           1
R6    2     3           1
R7    3     0           1
.dc  V1  6  6 1
.print   dc  V(3)
.END
```

SP 5.4



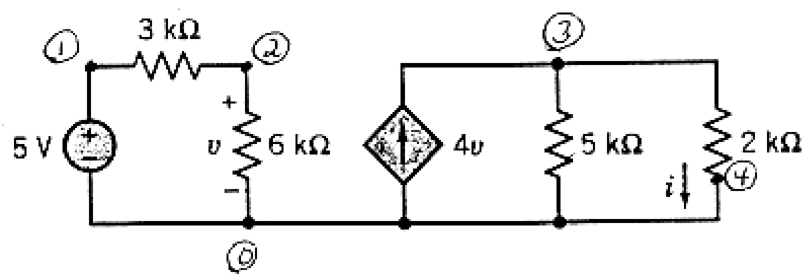
Probe result



<u>Input file</u>

```
V1    1     0     dc     10
R2    1     2            5k
I3    2     0     dc     1m
.dc   I3 0  2e-3   0.2e-3
.probe V(2)
.end
```

SP 5-5

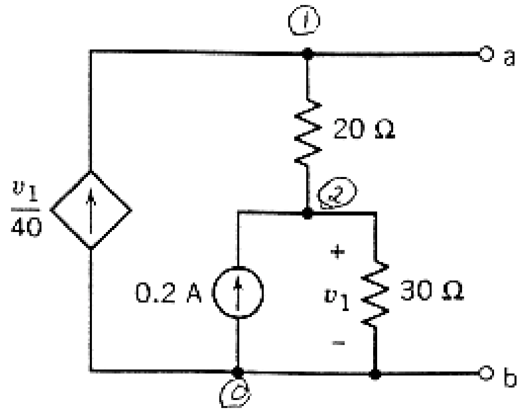


<u>Input file</u>

```
V1    1     0     dc     5
R1    1     2            3k
R2    2     0            6k
G4    0     3     2  0   4
R3    3     0            5k
R4    3     0            2k
.dc  V1  5  5 1
.print  dc  I(R4)
.end
```

<u>result</u> i = I(R4)= 9.524E+00A

SP 5-6




```
Input file
G1    0     1     2     0     25m
R2    1     2           20
I3    0     2     dc    0.2
R4    2     0           30
.tf   V(1)  I3
.end
```

result

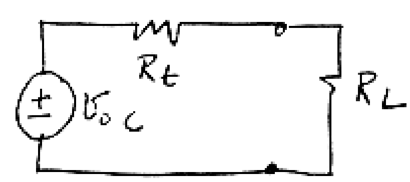
answer: $V_1 = V_{oc} = V_T =$ 36V

NODE	VOLTAGE	NODE	VOLTAGE
(1)	36.0000	(2)	24.0000

R_{TH} = OUTPUT RESISTANCE AT V(1)=2.000E+02Ω

Verification Problems

VP 5-1 Evaluating data

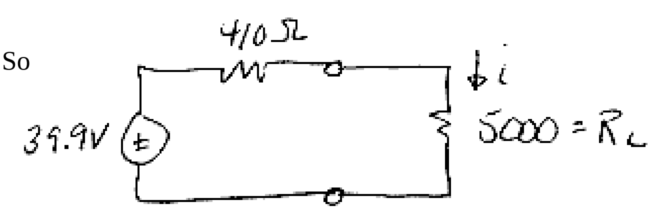


Case 1: $R_L = 0\Omega\,; \quad i = I_{sc} = 97.2\text{mA} = \dfrac{v_{oc}}{R_t}$ (1)

Case 2: $R_L = 500\Omega\,;\ i = 43.8\text{mA} = \dfrac{v_{oc}}{R_t + 500}$ (2)

Solving 1+2 yields $R_t = 410\Omega\,, v_{oc} = 39.9$ V

So



When $R_L = 5000\Omega$

$$i = \frac{v_{oc}}{R_t + R_L} = 7.37\text{mA}$$

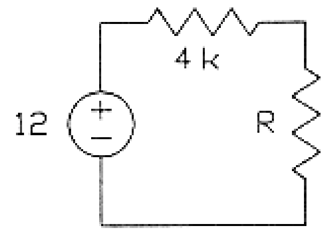
not 16.5mA as recorded ∴ the data is inconsistent.

VP 5-2

$V_{oc} = 12$ V (line 1 of the table)
$I_{sc} = 3$mA (line 3 of the table)

so

$R_{TH} = V_{oc} / I_{sc} = 4K\Omega$



$$\downarrow i = \frac{12}{R+4K}$$

Hence the circuit can be simplified as shown above right. (Check:

$$\frac{12}{10K\Omega+4K\Omega} = 0.857 \text{ mA}$$

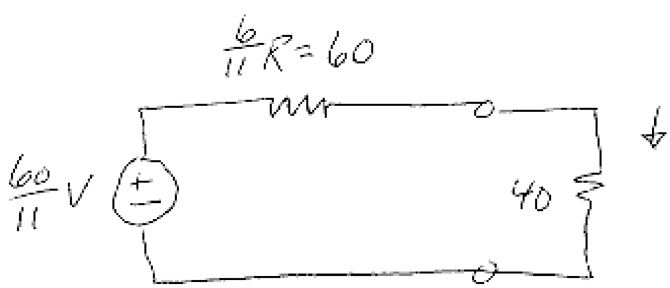
as shown in line 2 of the table.)

When i = 1 mA is required

$$1\text{mA} = \frac{12}{R+4k\Omega} \Rightarrow R = \frac{12}{1\text{mA}} - 4k\Omega = 8k\Omega$$

I agree with my lab partner's claim that R = 8000 causes i = 1 mA.

VP 5-3



$$i = \frac{60/11}{60+40} = 54.5\text{mA}$$

The measurement is consistent with the prelab calculations.

Design Problems

DP 5-1 The equation of representing the straight line in Figure DP 5-1b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{0-5}{0.008-0} = 625\ \Omega$ and v_{oc} = 5 V.

Try $R_1 = R_2 = 1\ \text{k}\Omega$. ($R_1 \parallel R_2$ must be smaller than R_t = 625 Ω.) Then

$$5 = \frac{R_2}{R_1+R_2} v_s = \frac{1}{2} v_s \quad \Rightarrow v_s = 10\ \text{V}$$

and

$$625 = R_3 + \frac{R_1 R_2}{R_1+R_2} = R_3 + 500 \quad \Rightarrow \quad R_3 = 125\ \Omega$$

Now v_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-2 The equation of representing the straight line in Figure DP 5-2b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{0-(-3)}{-0.006-0} = 500\ \Omega$ and v_{oc} = -3 V.

From the circuit we calculate

$$R_t = \frac{R_3\left(R_1+R_2\right)}{R_1+R_2+R_3} \text{ and } v_{oc} = -\frac{R_1 R_3}{R_1+R_2+R_3} i_s$$

so

$$500\ \Omega = \frac{R_3\left(R_1+R_2\right)}{R_1+R_2+R_3} \text{ and } 3\ \text{V} = -\frac{R_1 R_3}{R_1+R_2+R_3} i_s$$

Try $R_3 = 1\text{k}\Omega$ and $R_1 + R_2 = 1\text{k}\Omega$. Then $R_t = 500\ \Omega$ and

$$-3 = -\frac{1000 R_1}{2000} i_s = \frac{R_1}{2} i_s \quad \Rightarrow 6 = R_1\, i_s$$

This equation can be satisfied by taking R_1 = 600 Ω and i_s = 10 mA. Finally, R_2 = 1 kΩ - 400 Ω = 600 Ω.
Now i_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-3 The slope of the graph is positive so the Thevenin resistance is negative. This would require $R_3 + \dfrac{R_1 R_2}{R_1+R_2} < 0$, which is not possible since R_1, R_2 and R_3 will all be non-negative.

Is it not possible to specify values of v_s, R_1, R_2 and R_3 that cause the current i and the voltage v in Figure DP 5-3*a* to satisfy the relationship described by the graph in Figure DP 5-3*b*.

DP 5-4 The equation of representing the straight line in Figure DP 5-4b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to the Thevenin impedance and the "*v* - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{-5-0}{0-0.008} = -625\ \Omega$ and v_{oc} = -5 V.

The open circuit voltage, v_{oc}, the short circuit current, i_{sc}, and the Thevenin resistance, R_t, of this circuit are given by

$$v_{oc} = \frac{R_2\left(d+1\right)}{R_1+\left(d+1\right)R_2} v_s\ ,$$

$$i_{sc} = \frac{\left(d+1\right)}{R_1} v_s$$

and

$$R_t = \frac{R_1 R_2}{R_1+\left(d+1\right)R_2}$$

Let $R_1 = R_2 = 1\ \text{k}\Omega$. Then

$$-625\ \Omega = R_t = \frac{1000}{d+2} \quad \Rightarrow \quad d = \frac{1000}{-625} - 2 = -3.6\ \text{A/A}$$

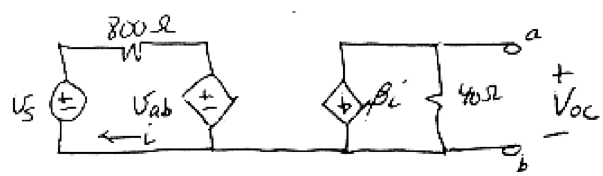
and

$$-5 = \frac{\left(d+1\right)v_s}{d+2} \quad \Rightarrow \quad v_s = \frac{-3.6+2}{-3.6+1}\left(-5\right) = -3.077\ \text{V}$$

Now v_s, R_1, R_2 and d have all been specified so the design is complete.

DP 5-5 a) Find Thev. equiv.

v_{oc}:



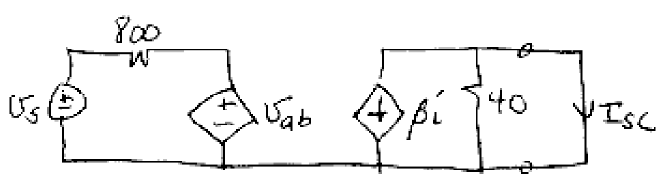
$$-v_s + 800\, i + v_{ab} = 0 \qquad (1)$$

$$v_{ab} = v_{oc} \qquad (2)$$

$$v_{oc} = -(\beta\, i)\,(40) \qquad (3)$$

Solving eqs. (1) - (3) yields $\underline{v_{oc} = v_s \Big/ \left(1 - \dfrac{20}{\beta}\right)}$

I_{sc}:



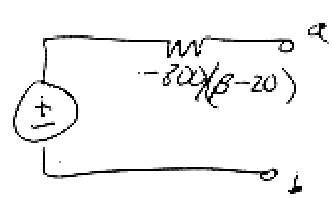
$$I_{sc} = -\beta\, i \qquad (4)$$

$$v_{ab} = 0 \qquad (5)$$

$$v_s = 800 i \qquad (6)$$

Solving eqs. (4) - (6) yields $\underline{I_{sc} = -\beta\, v_s / 800}$

so $\underline{R_t = \dfrac{v_{oc}}{I_{sc}} = \dfrac{-800}{\beta - 20}}$ and Thev. equiv. $\dfrac{v_S}{\left(1 - \dfrac{20}{\beta}\right)}$



b) $R_t = R_L = 400 = \dfrac{-800}{\beta - 20} \Rightarrow \underline{\beta = 18}$

c) Max power to R_L, largest v_{OC}, largest v_S, smallest R_t

$$P_L = \frac{(V_L)^2}{R_L} \qquad (7)$$

$$\text{and } V_L = \frac{400}{R_{total}} v_{oc} \qquad (8)$$

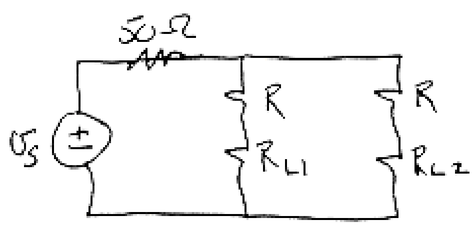
with

$R_{total} = \dfrac{-800}{\beta - 20} + 400$ (a) yields $\underline{\beta = \pm 18}$

d) delivering large amounts of power could melt antenna.

DP 5-6 Max power to load : $R_L = R_t = 50\Omega$

But split power equally ($R_{L1} = R_{L2} = 50\Omega$)



$\dfrac{(R+50)(R+50)}{R+50+R+50} = 50 \quad \Rightarrow$ yields $\underline{R = 50\Omega}$