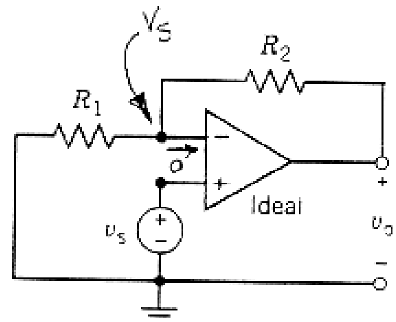
Chapter 6: The Operational Amplifier

Exercises

Ex. 6.4–1

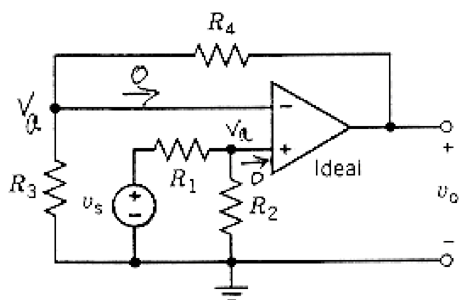
$$\frac{v_s}{R_1}+\frac{v_s-v_0}{R_2}+0 = 0$$

$$\frac{v_0}{v_s} = 1+\frac{R_2}{R_1}$$



Ex. 6.4-2

a)



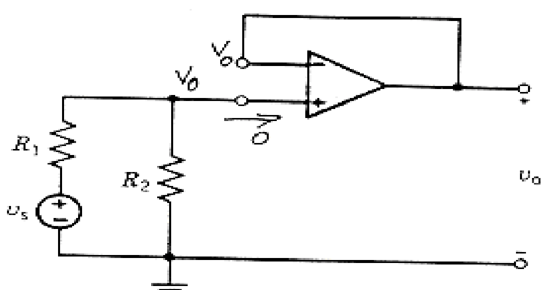
$$v_a = \frac{R_2}{R_1+R_2}v_s$$

$$\frac{v_a}{R_3}+\frac{v_a-v_0}{R_4}+0 = 0$$

$$\frac{v_0}{v_a} = 1+\frac{R_4}{R_3} \quad \Rightarrow \quad \frac{v_0}{v_s} = \left(\frac{R_2}{R_1+R_2}\right)\left(1+\frac{R_4}{R_3}\right)$$

b) When $R_2 >> R_1$ then $\frac{R_2}{R_1+R_2} \simeq \frac{R_2}{R_2} = 1$ and $\frac{v_0}{v_s} \simeq 1+\frac{R_4}{R_3}$

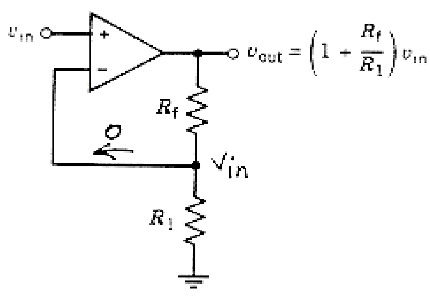
Ex. 6.5-1



$$\frac{v_0}{R_2}+\frac{v_0-v_s}{R_1}+0 = 0$$

$$\frac{v_0}{v_s} = \frac{R_2}{R_1+R_2}$$

Ex. 6.6–1



(b) Noninverting amplifier

$$\frac{v_{in}-v_{out}}{R_f}+\frac{v_{in}}{R_1}+0=0 \quad \Rightarrow \quad v_{out} = \left(1+\frac{R_f}{R_1}\right)v_{in}$$

when $R_f = 100k\Omega$ and $R_1 = 25k\Omega$ then

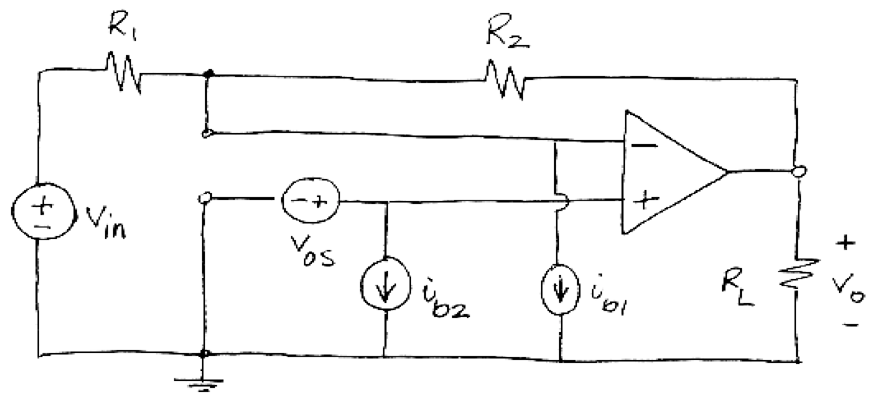
$$\frac{v_{out}}{v_{in}} = \left(1+\frac{100 \cdot 10^3}{25 \cdot 10^3}\right) = 5$$

Ex. 6.7-1 Analysis of the circuit in Section 6.7 showed that output offset voltage = $6\ v_{os} + (50.10^3)i_{b1}$

For a μA741 opamp, $|v_{os}| \le 1mV$ and $|i_{b1}| \le 80nA$ so

$$|\text{output offset voltage}| = \left|6\ v_{os} + (50.10^3)i_{b1}\right| \le 6\ (10^{-3})+(50.10^3)(80.10^{-9}) = 10mV$$

Ex. 6.7-2



$$v_0 = -\frac{R_2}{R_1} v_{in} + \left(1+\frac{R_2}{R_1}\right) v_{os} + R_2 i_{b1}$$

When $R_2 = 10k\Omega$, $R_1 = 2k\Omega$, $|v_{os}| \le 5mV$ and $|i_{bl}| \le 500nA$ then

output offset voltage $\le 6\left(5.10^{-3}\right) + \left(10.10^{3}\right)\left(500.10^{-9}\right) \le 35.10^{-3} = 35mV$

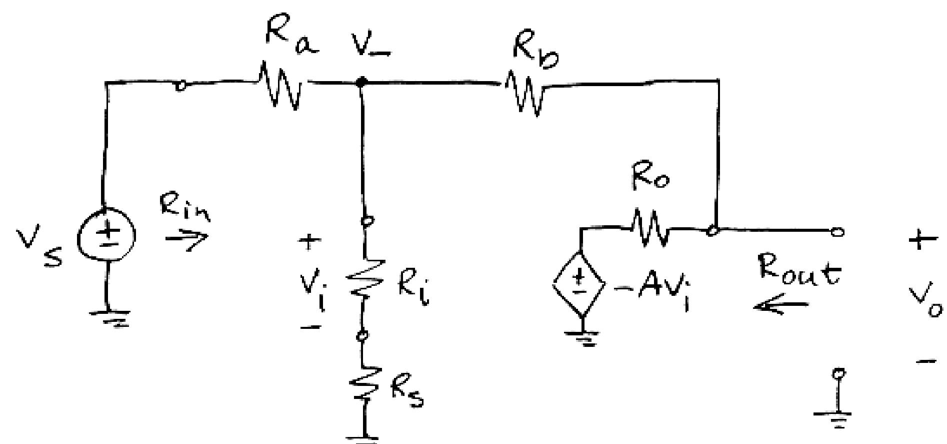
Ex. 6.7-3

Analysis of this circuit in Section 6.7 showed that output offset voltage = $6v_{os} + \left(50\times10^{3}\right) i_{b1}$

For a typical OPA1O1AM, $v_{os} = 0.1mV$ and $i_b = 0.012nA$ so

$$\begin{aligned}|\text{output offset voltage}| &\le 6\left[0.1\times10^{-3}\right]+\left(50\times10^{3}\right)\left[0.012\times10^{-9}\right]\\ &\le 0.6\times10^{-3} + 0.6\times10^{-6} \simeq 0.6\times10^{-3}\\ &\le 0.6mV\end{aligned}$$

Ex. 6.7-4



$$\frac{V_- - V_s}{R_a} + \frac{V_- - V_o}{R_b} + \frac{V_-}{R_i + R_s} = 0$$

$$\frac{V_0 - \left(-A\frac{R_i}{R_i+R_s}V_-\right)}{R_0} + \frac{V_0 - V_-}{R_b} = 0$$

After some algebra

$$A_v = \frac{V_0}{V_s} = \frac{R_0(R_i+R_s)+AR_iR_f}{(R_f+R_0)(R_i+R_s)+R_a(R_f+R_0+R_i+R_s)-AR_iR_a}$$

For the given values, $A_v = -2.00006$

Ex. 6.8-1

```
Spice deck
V1      1 0      200mV
V2      2 0      125mV
V3      3 0      250mV
R1      1 5      50k
R2      2 5      25k
R3      5 4      100k
XOA1    5 0 4    IDEAL_OP_AMP
R4      4 6      25k
R5      3 6      10k
R6      6 7      50k
R7      7 0      100k
XOA2    6 0 7    IDEAL_OP_AMP

.SUBCKT IDEAL_OP_AMP 1 2 3
E 3 0 1 2 -1G
.ENDS IDEAL_OP_AMP
.END

 NODE        VOLTAGE      NODE        VOLTAGE      NODE        VOLTAGE
(    1)         .2000   (    2)          .1250   (    3)           .2500
(    4)        -.9000   (    5)     900.0E-12   (    6)      -550.0E-12
(    7)         .5500
```

Ex. 6.8-2

```
Spice deck
V1    1 0         200mV
V2    2 0         125mV
V3    3 0         250mV
R1    1 5         25k
R2    2 5         50k
R3    5 4         100k
XOA1  5 0 4       TL501_OP_AMP
R4    4 6         25k
R5    3 6         10k
R6    6 7         50k
R7    7 0         100k
XOA2  6 0 7       TL501_OP_AMP


.SUBCKT  TL501_OP_AMP 1 2 5
IB1      1 0     .0175nA
IB2      2 0     .0425nA
VOS      3 2     .59mV
RI       1 3     1MEG
E        4 0 1 3 -105000
RO       4 5     250
.ENDS TL501_OP_AMP
.END

NODE         VOLTAGE      NODE        VOLTAGE      NODE        VOLTAGE
(    1)        .2000    (    2)          .1250   (    3)          .2500
(    4)       -.8958    (    5)      598.6E-06   (    6)      584.8E-06
(    7)        .5463    (XOA1.3)     590.0E-06   (XOA1.4)        -.9070
(XOA2.3)    590.0E-06   (XOA2.4)         .5504
```

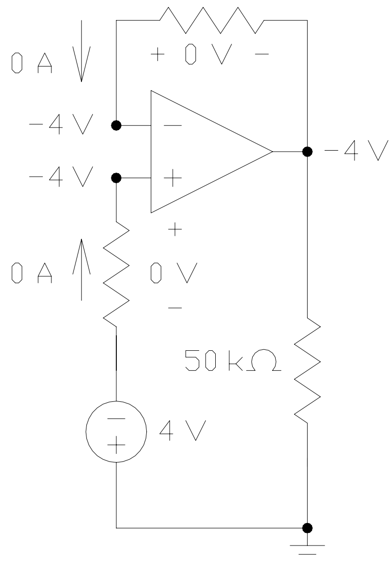
Ex. 6.8-3

```
Spice deck
V1    1 0          200mV
V2    2 0          125mV
V3    3 0          250mV
R1    1 5          5k
R2    2 5          2.5k
R3    5 4          10k
XOA1  5 0 4        UA741_OP_AMP
R4    4 6          2.5k
R5    3 6          1k
R6    6 7          5k
R7    7 0          10k
XOA2  6 0 7        UA741_OP_AMP

SUBCKT UA741_OP_AMP 1 2 5
IB1      1 0      70nA
IB2      2 0      90nA
VOS      3 2      1mV
RI      1 3      2MEG
E      4 0 1 3  -200000
RO       4 5      75
.ENDS  UA741_OP_AMP
.END

NODE        VOLTAGE      NODE        VOLTAGE      NODE       VOLTAGE
(    1)         .2000   (    2)         .1250   (    3)         .2500
(    4)        -.8958   (    5)         .0010   (    6)     997.2E-06
(    7)         .5429   (XOA1.3)         .0010  (XOA1.4)        -.9258
(XOA2.3)         .0010  (XOA2.4)         .5551
```

Problems

Section 6-4: The Ideal Operational Amplifier

P6.4-1



P6.4-2 KVL a1:

$$-12+3000i+0+2000i = 0$$

$$\Rightarrow i = \frac{12}{5000} = 2.4\text{mA}$$

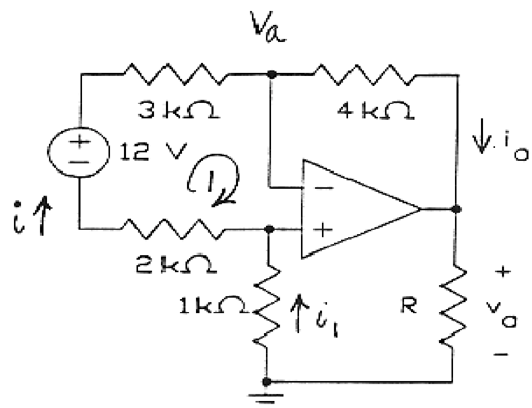
$$i_0 = i = 2.4\text{mA}$$

$$i_1 = i = 2.4\text{mA}$$

$$v_a = -i_1(1000)+0$$

$$= -2.4 \text{ V}$$

$$v_0 = v_a - i_0(4000) = -2.4-\left(2.4\times10^{-3}\right)(4000) = -12 \text{ V}$$



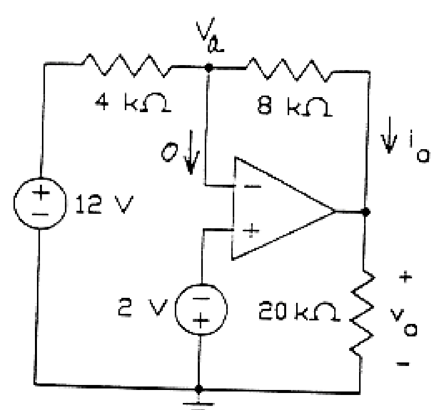
P6.4-3

$$v_a = -2\text{V}$$

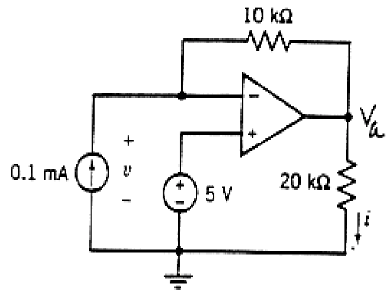
$$\frac{v_0-(-2)}{8000}+\frac{12-(-2)}{4000} = 0$$

$$\Rightarrow v_0 = -30 \text{ V}$$

$$i_0 = \frac{-2-v_0}{8000} = 3.5\text{mA}$$



P6.4-4



$$v = 5\text{ V}$$

$$-\left(\frac{v_a-5}{10000}\right)-0.1\times10^{-3}-0 = 0$$

$$\Rightarrow v_a = 4\text{ V}$$

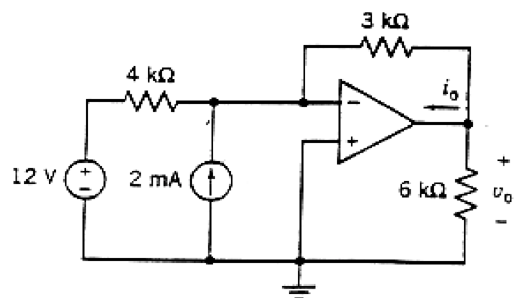
$$i = \frac{v_a}{20000} = \frac{1}{5}\text{mA}$$

P6.4-5

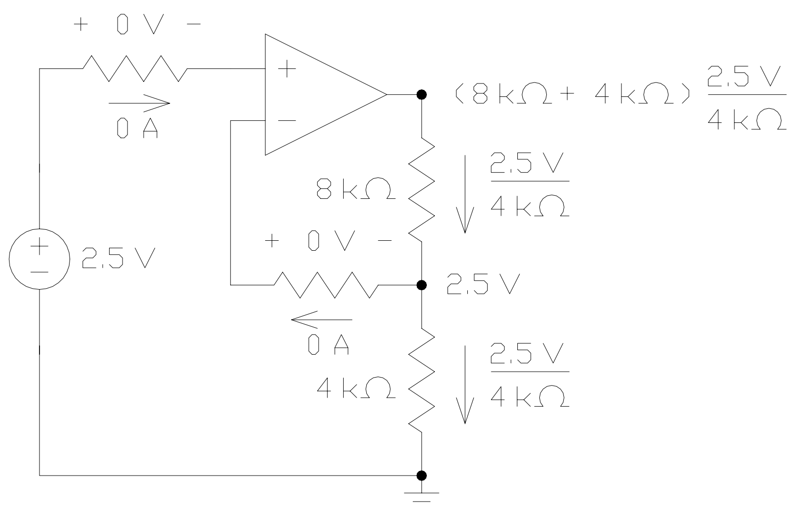
$$-\left(\frac{v_0-0}{3000}\right)-\left(\frac{12-0}{4000}\right)-2\cdot10^{-3}=0$$

$$\Rightarrow v_0 = -15\text{ V}$$

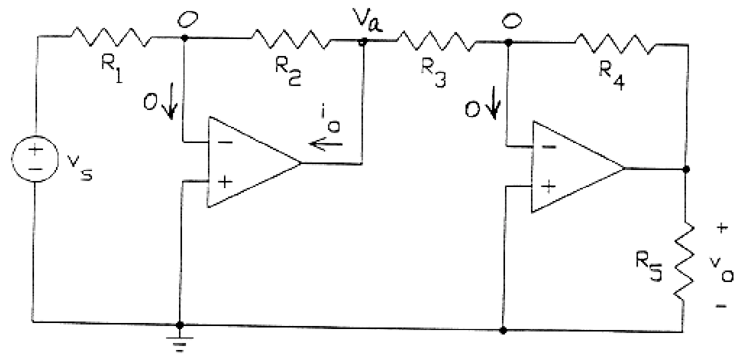
$$i_0+\frac{v_0}{6000}+\frac{v_0}{3000}=0\Rightarrow i_0=7.5\text{mA}$$



P6.4-6



P6.4-7

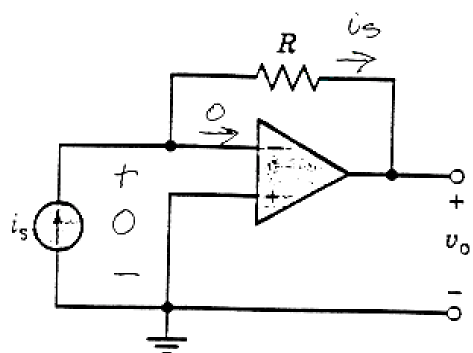


$$-\left(\frac{v_s-0}{R_1}\right)-\left(\frac{v_a-0}{R_2}\right)+0 = 0 \Rightarrow v_a = -\frac{R_2}{R_1}v_s$$

$$i_0 = \frac{0-v_a}{R_2}+\frac{0-v_a}{R_3} = \frac{R_2+R_3}{R_2R_3}v_a = -\left(\frac{R_2+R_3}{R_1R_3}\right)v_s$$

$$-\left(\frac{v_0-0}{R_4}\right)-\left(\frac{v_a-0}{R_3}\right)+0 = 0 \Rightarrow v_0 = -\frac{R_4}{R_3}v_a = \frac{R_2R_4}{R_1R_3}v_s$$

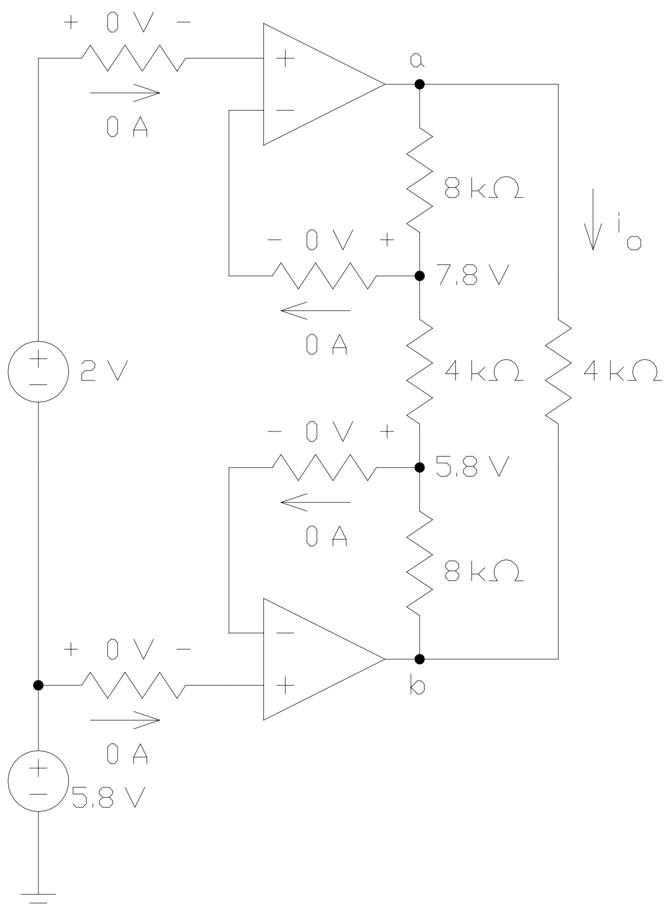
P6.4-8



$$v_0 = 0 - Ri_s$$

$$\therefore \; \frac{v_0}{i_s} = -R$$

P6.4-9



P 6.4-10

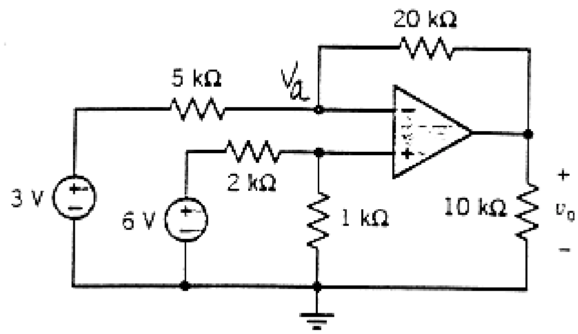
KCL at node a: $\dfrac{v_a-(-18)}{4000}+\dfrac{v_a}{8000}+0=0 \quad\Rightarrow\quad v_a=-12\text{ V}$

The node voltages at the input nodes of ideal op amps are equal so $v_b = v_a$.

Voltage division: $v_o=\dfrac{8000}{4000+8000}v_b=-8\text{ V}$

<u>Section 6-5:</u> Nodal Analysis of Circuits Containing Ideal Operational Amplifiers

P6.5-1



$$v_a = \frac{1000}{2000+1000}6 = 2\text{ V}$$

$$-\left(\frac{v_0-2}{20000}\right)-\left(\frac{3-2}{5000}\right) = 0$$

$$\Rightarrow v_0 = -2\text{ V}$$

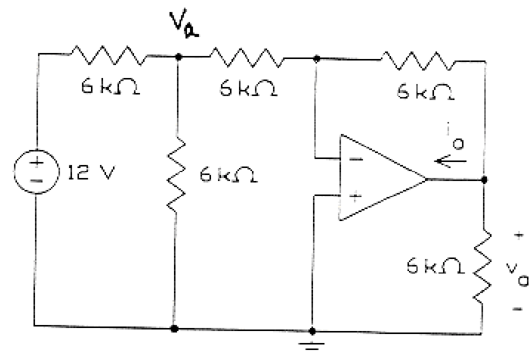
P6.5-2

KCL at node b: $\dfrac{v_b-2}{20e3}+\dfrac{v_b}{40e3}+\dfrac{v_b+5}{40e3}=0 \quad\Rightarrow\quad v_b=-\dfrac{1}{4}\text{ V}$

$v_e = v_b = -\dfrac{1}{4}$ V because the node voltages at the input nodes of an ideal op amp are equal

KCL at node e: $\dfrac{v_e}{1000}+\dfrac{v_d+v_e}{40e3}=0 \quad\Rightarrow\quad v_d = 10\,v_e = -\dfrac{10}{4}\text{ V}$

P6.5-3

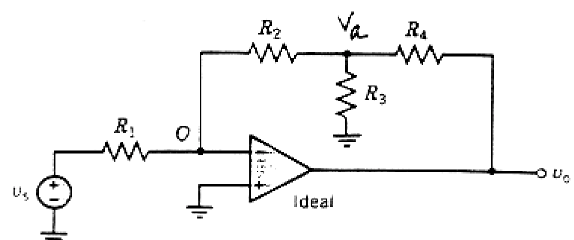


$$0 = \frac{v_a-12}{6000}+\frac{v_a}{6000}+\frac{v_a-0}{6000} \quad\Rightarrow\quad v_a = 4\text{ V}$$

$$-\left(\frac{v_a-0}{6000}\right)+0+\left(\frac{0-v_0}{6000}\right) = 0 \Rightarrow v_0 = -v_a = -4\text{V}$$

$$i_0-\left(\frac{0-v_0}{6000}\right)+\frac{v_0}{6000} = 0 \Rightarrow i_0 = -\frac{v_0}{3000} = 1.33\text{mA}$$

P6.5-4



$$-\left(\frac{v_a-0}{R_2}\right)-\left(\frac{v_s-0}{R_1}\right) = 0$$

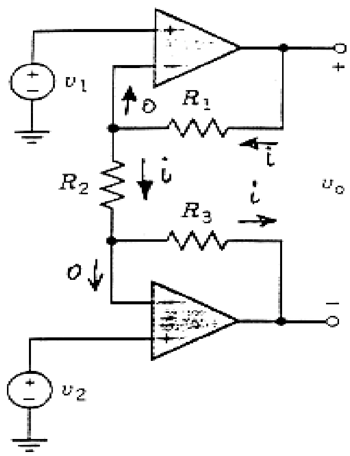
$$\Rightarrow v_a = -\frac{R_2}{R_1}v_s$$

$$\frac{v_a-v_0}{R_4}+\frac{v_a}{R_3}+\frac{v_a-0}{R_2} = 0 \Rightarrow v_0 = R_4\left(\frac{1}{R_4}+\frac{1}{R_3}+\frac{1}{R_2}\right)v_a = \frac{R_2R_3+R_2R_4+R_3R_4}{R_2R_3}v_a$$

$$= -\frac{R_2R_3+R_2R_4+R_3R_4}{R_1R_3}v_s$$

Plug in values $\Rightarrow$ yields $\dfrac{v_0}{v_s} = -200$

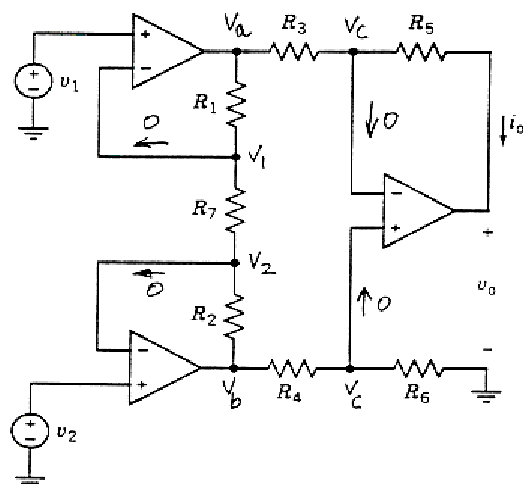
P6.5-5



$$i = \frac{v_1 - v_2}{R_2}$$

$$v_0 = (R_1 + R_2 + R_3)\, i = \frac{R_1 + R_2 + R_3}{R_2}(v_1 - v_2)$$

P6.5-6



$$\frac{v_1 - v_a}{R_1} + \frac{v_1 - v_2}{R_7} + 0 = 0 \;\Rightarrow\; v_a = \left(1 + \frac{R_1}{R_7}\right) v_1 - \frac{R_1}{R_7} v_2$$

$$\frac{v_2 - v_b}{R_2} - \frac{v_1 - v_2}{R_7} + 0 = 0 \;\Rightarrow\; v_b = \left(1 + \frac{R_2}{R_7}\right) v_2 - \frac{R_2}{R_7} v_1$$

$$-\left(\frac{v_b - v_c}{R_4}\right) + \frac{v_c - 0}{R_6} + 0 = 0 \;\Rightarrow\; v_c = \frac{R_6}{R_4 + R_6} v_b$$

$$-\left(\frac{v_a - v_c}{R_3}\right) + \left(\frac{v_c - v_0}{R_5}\right) + 0 = 0 \;\Rightarrow\; v_0 = -\frac{R_5}{R_3} v_a + \left(1 + \frac{R_5}{R_3}\right) v_c$$

$$v_0 = \left[\frac{R_5 R_1}{R_3 R_7} + \frac{R_6(R_3 + R_5)}{R_3(R_4 + R_6)}\left(1 + \frac{R_2}{R_7}\right)\right] v_2 - \left[\frac{R_5}{R_3}\left(1 + \frac{R_1}{R_7}\right) + \frac{R_6(R_3 + R_5)\, R_2}{R_3\,(R_4 + R_6)\, R_7}\right] v_1$$

$$i_0 = \frac{v_c - v_0}{R_5} = \cdots$$

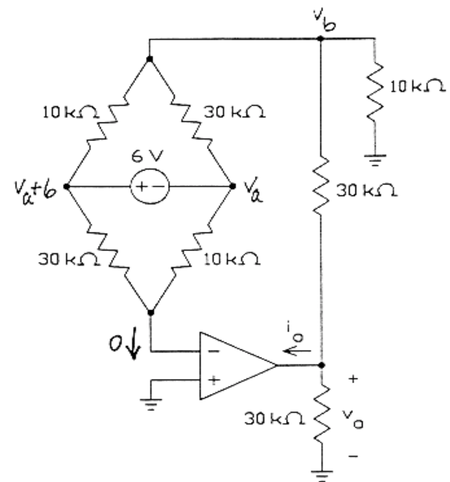
P6.5-7

KCL at node b: $\dfrac{v_a}{20e3} + \dfrac{v_c}{25e3} = 0 \;\Rightarrow\; v_c = -\dfrac{5}{4} v_a$

KCL at node a: $\dfrac{v_a - (-12)}{40e3} + \dfrac{v_a}{10e3} + \dfrac{v_a + 0}{20e3} + \dfrac{v_a - \left(-\frac{5}{4} v_a\right)}{10e3} = 0 \;\Rightarrow\; v_a = -\dfrac{3}{4}$ V

so $v_c = -\dfrac{5}{4} v_a = -\dfrac{15}{16}$.

P6.5-8



$$-\left(\frac{v_a-0}{10000}\right)+0-\left(\frac{(v_a+6)-0}{30000}\right) = 0$$

$$\Rightarrow v_a = -1.5 \text{ V}$$

$$\frac{v_a-0}{10000}+\frac{v_a+6-0}{30000}+\frac{v_a-v_b}{30000}+\frac{(v_a+6)-0}{10000} = 0$$

$$\Rightarrow \quad 3v_a+v_a+6+v_a-v_b+3\left[(v_a+6)-v_b\right] = 0$$

$$\Rightarrow \quad v_b = 2v_a+6 = 3 \text{ V}$$

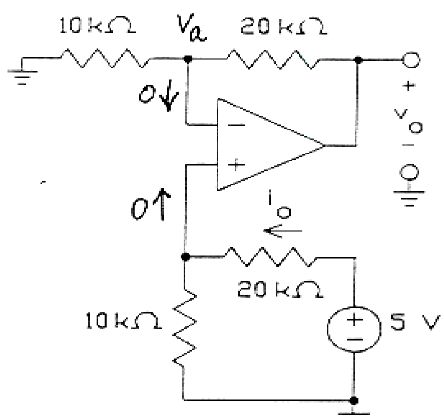
$$\frac{v_b}{10000}+\frac{v_b-v_0}{30000}-\left(\frac{v_a-v_b}{30000}\right)-\left(\frac{(v_a+6)-v_b}{10000}\right) = 0$$

$$\Rightarrow \quad 3v_b+(v_b-v_0)-(v_a-v_b)-3\left[(v_a+6)-v_b\right] = 0$$

$$\Rightarrow \quad v_0 = 8v_b-4v_a-18 = \underline{12 \text{ V}}$$

$$i_0+\frac{v_0}{30000}+\frac{v_0-v_b}{30000} = 0 \Rightarrow i_0 = -0.7\text{mA}$$

P6.5-9



$$-i_0(10000)-i_0(20000)+5 = 0$$

$$i_0 = \frac{1}{6} \text{ mA}$$

$$v_a = 10000 \ i_0 = \frac{10}{6} \text{ V}$$

$$\frac{v_a}{10000}+\frac{v_a-v_0}{20000} = 0$$

$$\Rightarrow v_0 = 3v_a = 5 \text{ V}$$

P6.5-10

KCL at node b: $\frac{v_b+12}{40e3}+\frac{v_b}{20e3}=0 \quad \Rightarrow \quad v_b=-4 \text{ V}$

$v_c = v_b = -4$ V because the node voltages at the input nodes of an ideal op amp are equal.

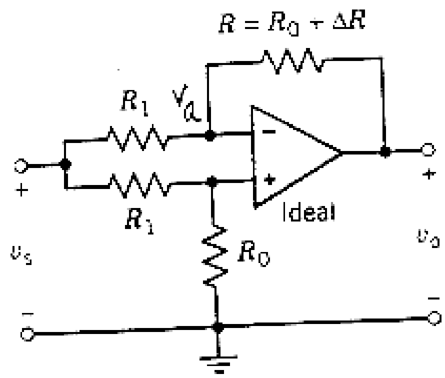
$v_d = v_c + 0\cdot 10e3 = -4$ V because the currents into the inputs of an ideal op amp are zero.

KCL at node g: $-\left(\frac{v_f-v_g}{20e3}\right)+\frac{v_g}{40e3}=0 \quad \Rightarrow \quad v_g=\frac{2}{3}v_f$

KCL at node d: $\frac{v_d-v_f}{20e3}+\frac{v_d-\frac{2}{3}v_f}{20e3}=0 \quad \Rightarrow \quad v_f=\frac{6}{5}v_d=-\frac{24}{5}$ V so $v_g=\frac{2}{3}v_f=-\frac{15}{5}$ V.

$v_c = v_g = -\frac{16}{5}$ V because the node voltages at the input nodes of an ideal op amp are equal

P6.5-11



$$v_a = \frac{R_0}{R_1+R_0} v_s$$

$$\frac{v_a - v_s}{R_1} + \frac{v_a - v_0}{R_0 + \Delta R} = 0$$

$$\frac{R_0 + \Delta R}{R_1}(v_a - v_s) + v_a = v_0$$

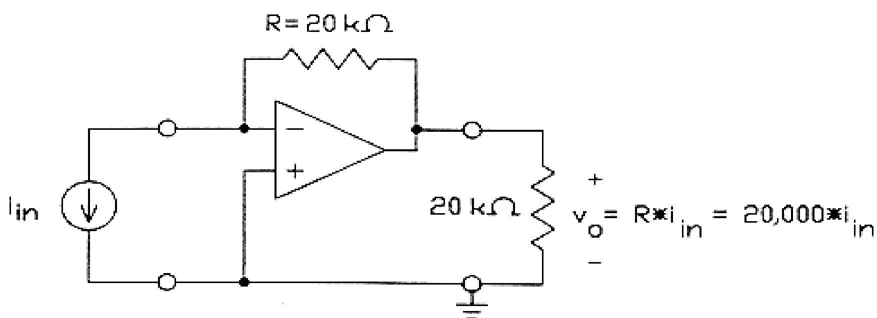
$$v_0 = \left[\left(\frac{R_0 + \Delta R}{R_1} + 1\right)\frac{R_0}{R_1 + R_0} - \frac{R_0 + \Delta R}{R_1}\right] v_s$$

$$= -\frac{\Delta R}{R_1 + R_0} v_s$$

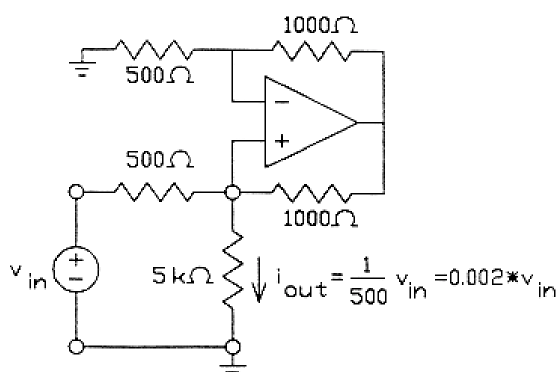
$$= \left(-v_s \frac{R_0}{R_1 + R_0}\right)\frac{\Delta R}{R_0}$$

Section 6-6: Design Using Operational Amplifier

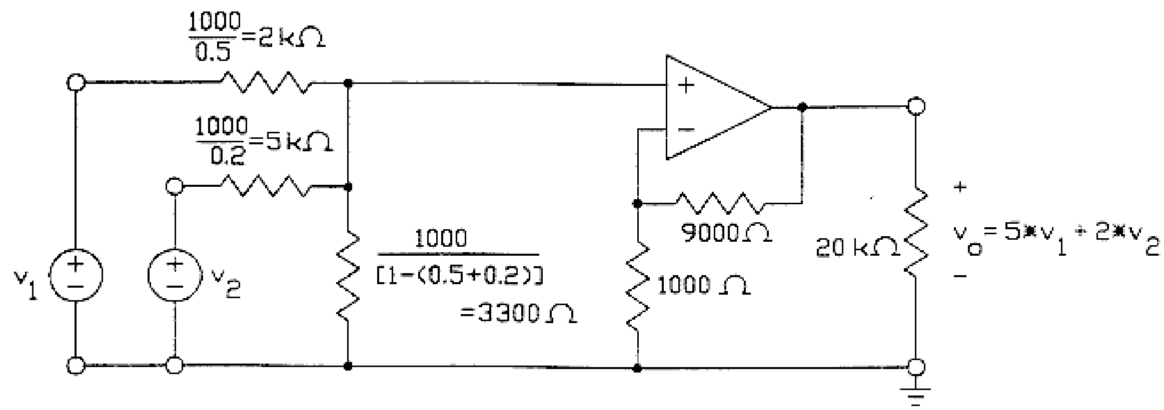
P6.6-1



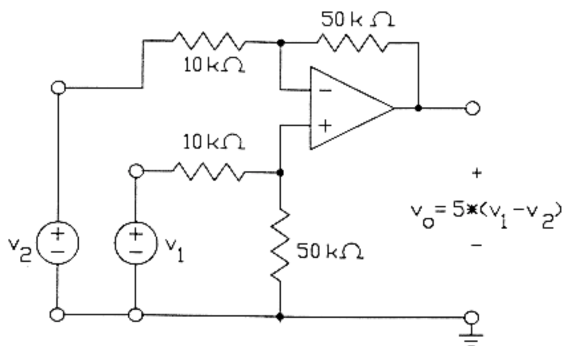
P6.6-2



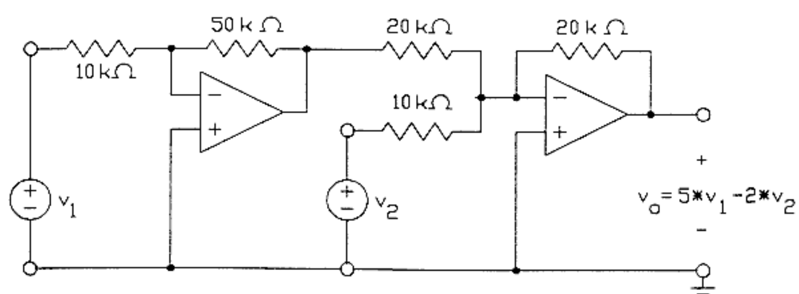
P6.6-3



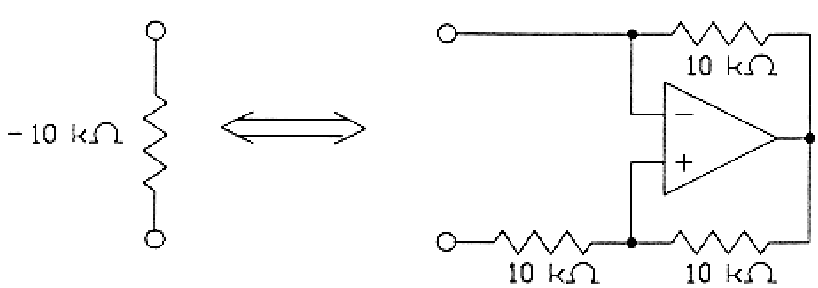
P6.6-4



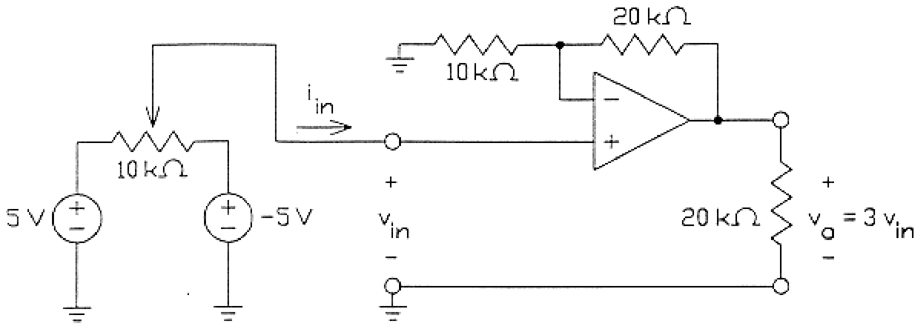
P6.6-5



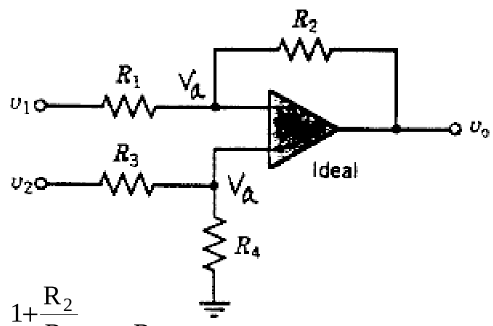
P6.6-6



P6.6-7



P6.6-8



(a) $$v_a = \frac{R_4}{R_3+R_4} v_2$$

$$\frac{v_a - v_1}{R_1} + \frac{v_a - v_0}{R_2} = 0$$

$$\Rightarrow v_0 = \left(\frac{R_2}{R_1}+1\right) v_a - \frac{R_2}{R_1} v_1$$

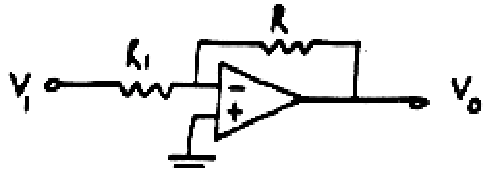
$$= \left(\frac{R_2}{R_1}+1\right)\frac{R_4}{R_3+R_4} v_2 - \frac{R_2}{R_1} v_1 = \frac{1+\frac{R_2}{R_1}}{1+\frac{R_3}{R_4}} v_2 - \frac{R_2}{R_1} v_1$$

(b) $$11 = \frac{R_2}{R_1} \text{ and } 4 = \frac{1+\frac{R_2}{R_1}}{1+\frac{R_3}{R_4}} = \frac{12}{1+\frac{R_3}{R_4}} \Rightarrow \frac{R_3}{R_4} = 2$$

For example:

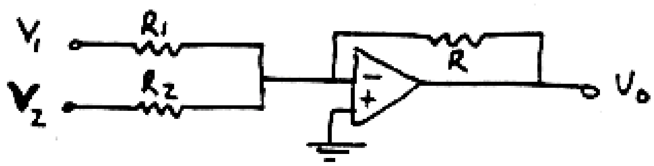
$R_1 = 10k\Omega,\quad R_2 = 110k\Omega,\quad R_3 = 20k\Omega \;\&\; R_4 = 10k\Omega$

P6.6-9



We know this ckt yields $v_0 = -\frac{R}{R_1} v_1$

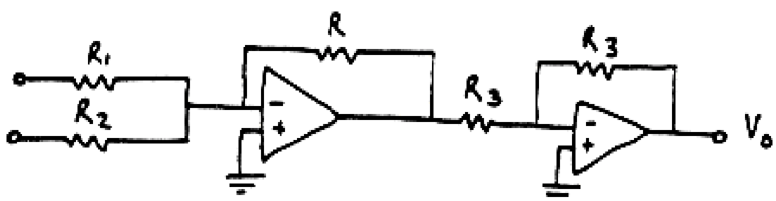
so consider adding another input lead into the negative terminal



with $v_i \simeq 0$, KCL at negative input yields

$$-v_1/R_1 - v_2/R_2 = v_0/R \Rightarrow v_0 = -\frac{R}{R_1} v_1 - \frac{R}{R_2} v_2$$

Since need to invert the answer, add an inverter to the output,

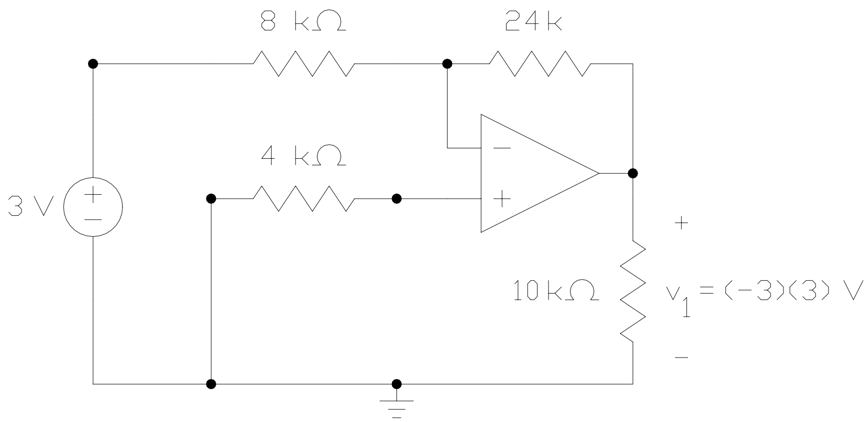


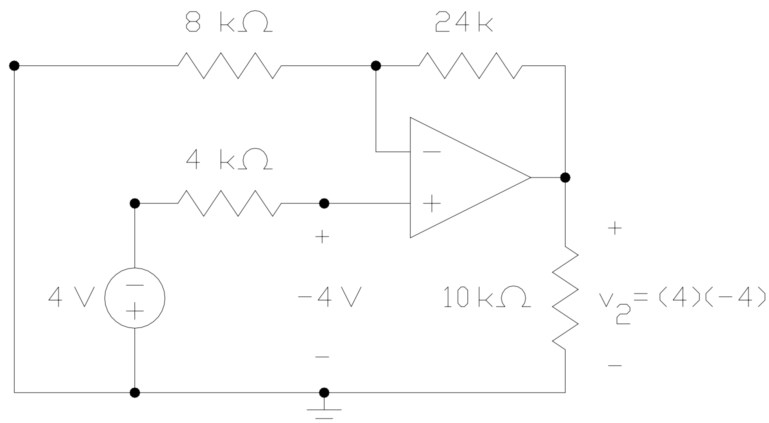
Let $\underline{R_3 = 10\text{k}\Omega}$

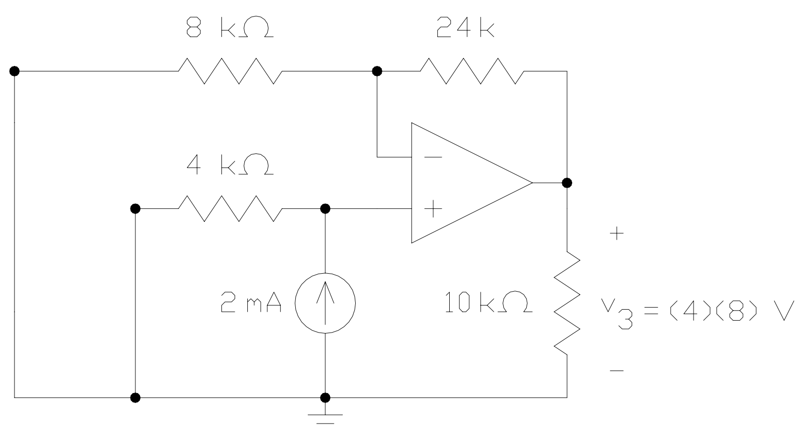
Now $\frac{R}{R_1} = 6$ and $\frac{R}{R_2} = 2$ $\therefore$ let R = 60 kΩ

$R_1 = 10\text{k}\Omega$ and $\underline{R_2 = 30\text{k}\Omega}$

P6.6-10





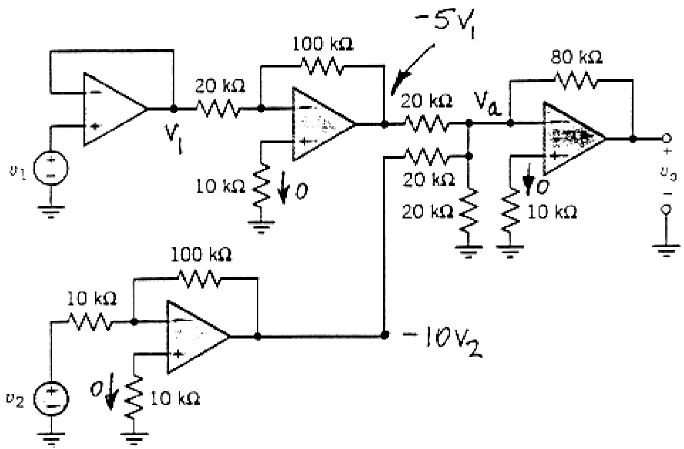


Using superposition, $v_o = v_1 + v_2 + v_3 = -9 - 16 + 32 = 7 \text{ V}$

P6.6-11

R_1	6	12	24	6‖12	6‖24	12‖12	12‖24	6‖12‖12	6‖12‖24	12‖12‖24
R_2	12‖12‖24	6‖12‖24	6‖12‖12	12‖24	12‖12	6‖24	6‖12	24	12	6
$-v_o/v_s$	0.8	0.286	0.125	2	1.25	0.8	0.5	8	3.5	1.25

P6.6-12



$$\frac{v_a-(-5v_1)}{20k\Omega}+\frac{v_a-(-10v_2)}{20k\Omega}+\frac{v_a}{20k\Omega}+\frac{v_a-v_0}{80k\Omega} = 0$$

$$4(v_a+5v_1)+4\,(v_a+10v_2)+4v_a+v_a-v_0 = 0$$

$$v_a = 0(10k\Omega) = 0$$

$$v_0 = 20v_1+40v_2$$

P 6.6-13

$$v_c=-\left(3\,v_a+1\right)$$

$$v_o=-\left(3\,v_b+2\,v_c+2\right)=-\left(3\,v_b-2\left(3\,v_a+1\right)+2\right)=-3\,v_b+6\,v_a$$

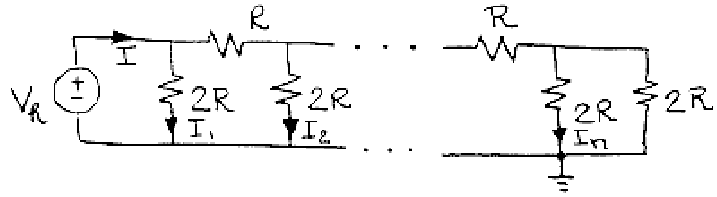
P6.6-14

(a) $2R||2R = R$

R in series with $R = 2R$

$\vdots$

$R_{eq} = R$



$\underline{I = \frac{v_R}{R}}$, $\qquad I_1 = \frac{v_R}{R}\left(\frac{2R}{4R}\right) = \frac{v_R}{2R}$

$$I_2 = \frac{v_R}{2R}\left(\frac{2R}{4R}\right) = \frac{v_R}{2^2 R}$$

$\vdots$

$$I_n = \frac{v_R}{2^n R}$$

(b) $$\frac{V_0}{R_f} = b_1 I_1 + b_2 I_2 + ... + b_n I_n$$

$$V_0 = R_f\left[b_1\frac{V_R}{2R} + b_2\frac{V_R}{2^2R} + ... + b_n\frac{V_R}{2^nR}\right] = \underline{\frac{R_f V_R}{R}\left[b_1 2^{-1} + b_2 2^{-2} + ... + b_n 2^{-n}\right]}$$

(c) $$I^+ + I^- = \frac{V_R}{2R} + \frac{V_R}{2^2 R} + ... + \frac{V_R}{2^n R}$$

$$= \frac{V_R}{R}\left[2^{-1} + 2^{-2} + ... + 2^{-n}\right]$$

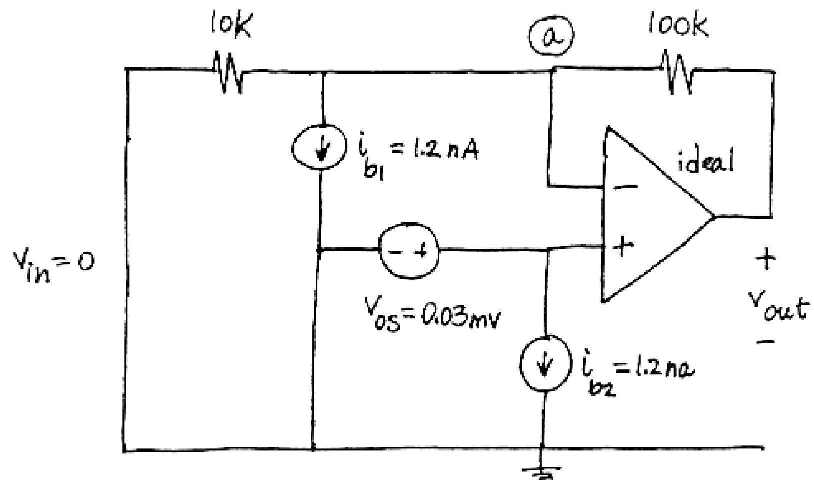
$$= \underline{\frac{V_R}{R}\left[1 - 2^{-n}\right]}$$

(d)

$-V_0$	b1	b2	b3	b4
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	0	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Section 6-7: Characteristics of the Practical Operational Amplifier

P6.7-1



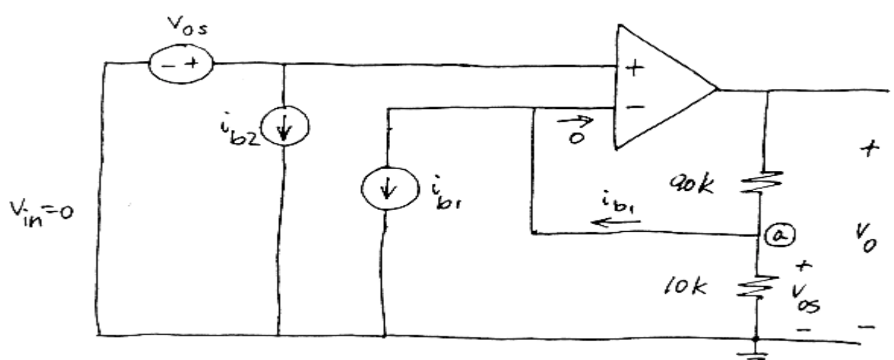
The node equation at node a is $\frac{v_{out}-v_{os}}{100k} = \frac{v_{os}}{10k}+i_{b1}$

so $v_{out} = \left(1+\frac{100k\Omega}{10k\Omega}\right)v_{os}+100k\Omega\cdot i_{b1} = 11v_{os}+100k\Omega\cdot i_{b1}$

$= 11(0.03mV)+100k\Omega(1.2nA)$

$= 0.45mV$

P6.7-2



The node equation at node a is

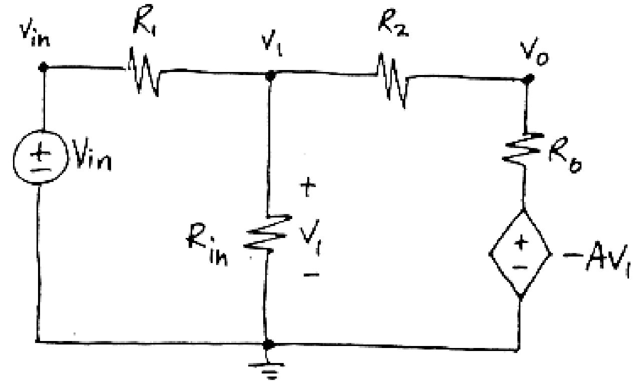
$$\frac{v_{os}}{10k}+i_{b1} = \frac{v_0-v_{os}}{90k\Omega}$$

so $v_0 = \left(1+\frac{90k\Omega}{10k\Omega}\right)v_{os}+90k\Omega\ i_{b1} = 10v_{os}+90k\Omega\ i_{b1}$

$= 10(5mV)+90k\Omega(.05nA)$

$= 50.0045mV \simeq 50mV$

P6.7-3



$$\left.\begin{aligned}\frac{v_1-v_{in}}{R_1}+\frac{v_1}{R_{in}}+\frac{v_1-v_0}{R_2} &= 0\\ \frac{v_0+Av_1}{R_0}+\frac{v_0-v_1}{R_2} &= 0\end{aligned}\right\} \Rightarrow \frac{v_0}{v_{in}} = \frac{R_{in}(R_0-AR_2)}{(R_1+R_{in})(R_0+R_2)+R_1R_{in}(1+A)}$$

P6.7-4

a) $\frac{v_0}{v_{in}} = -\frac{R_2}{R_1} = -\frac{49k\Omega}{5.1k\Omega} = \underline{-9.6078}$

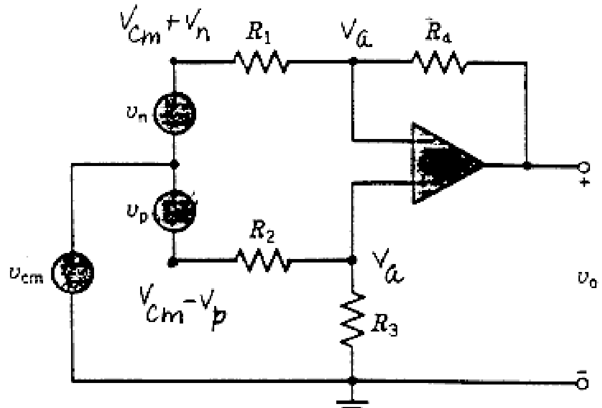
b) $\frac{v_0}{v_{in}} = \frac{2M\Omega\left(75-(200{,}000)(50k\Omega)\right)}{(5k\Omega+2M\Omega)(75+50k\Omega)+(5k\Omega)(2M\Omega)(1+200{,}000)}$

$= \underline{-9.9995}$

c) $\frac{v_0}{v_{in}} = \frac{2M\Omega(75-(200{,}000)(49k\Omega))}{(5.1k\Omega+2M\Omega)(75+49k\Omega)+(5.1k\Omega)(2M\Omega)(1+200{,}000)}$

$= \underline{-9.6073}$

P6.7-5



$$v_a = \frac{R_3}{R_2+R_3}(v_{cm}-v_p)$$

$$\frac{v_a-v_0}{R_4}+\frac{v_a-(v_{cm}+v_n)}{R_1} = 0$$

$$v_0 = -\frac{R_4}{R_1}(v_{cm}+v_n)+\frac{R_4+R_1}{R_1}v_a$$

$$= -\frac{R_4}{R_1}(v_{cm}+v_n)+\frac{(R_4+R_1)R_3}{R_1(R_2+R_3)}(v_{cm}-v_p)$$

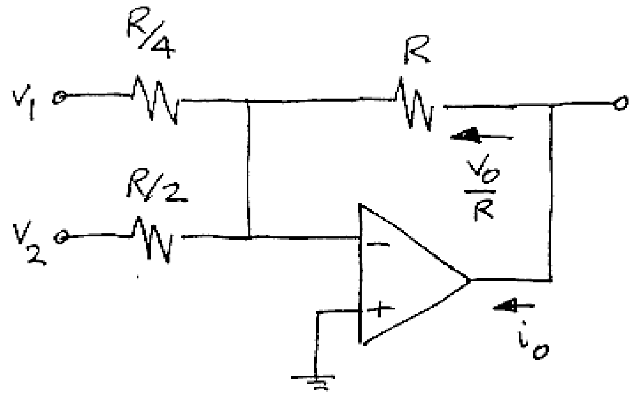
when $\frac{R_4}{R_1} = \frac{R_3}{R_2}$ then $\frac{(R_4+R_1)R_3}{R_1(R_2+R_3)} = \frac{\frac{R_4}{R_1}+1}{\frac{R_3}{R_2}+1}\times\frac{R_3}{R_2} = \frac{R_4}{R_1}$

so

$$v_0 = -\frac{R_4}{R_1}(v_{cm}+v_n)+\frac{R_4}{R_1}(v_{cm}-v_p) = -\frac{R_4}{R_1}(v_n+v_p)$$

P6.7-6

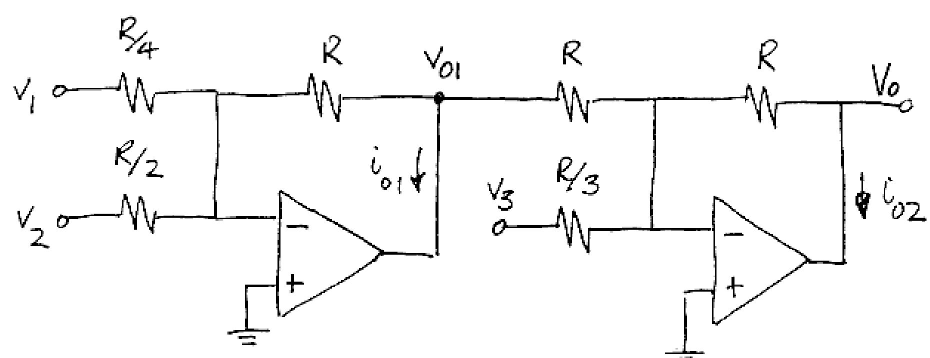
(a)



$v_0 = -(4v_1 + 2v_2)$

$$|i_0| \le \frac{4|v_1| + 2|v_2|}{R} \le \frac{6}{R} < i_{sat} \quad \text{so } R > \frac{6}{i_{sat}} = \frac{6}{2\times 10^{-3}} = 3\ \text{k}\Omega \text{ is required}$$

(b)



$$v_{01} = -(4v_1 + 2v_2); \qquad v_0 = -(v_{01} + 3v_3) = 4v_1 + 2v_2 - 3v_3$$

$$i_{01} = -\frac{v_{01}}{R} - \frac{v_{01}}{R} = -\frac{2v_{01}}{R}$$

$$|i_{01}| \le \frac{2}{R}(4|v_1| + 2|v_2|) \le \frac{12}{R} \le i_{sat}$$

$$\text{so } R \ge \frac{12}{i_{sat}} = \frac{12}{2\cdot 10^{-3}} = 6\text{k}\Omega$$

Also

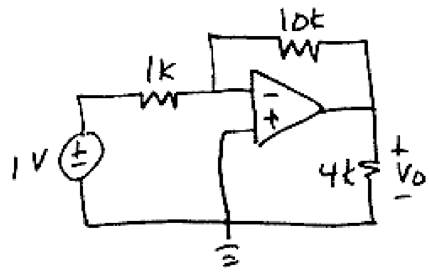
$$i_{02} = -\frac{v_0}{R} = -\frac{4v_1 + 2v_2 - 3v_3}{R}$$

$$|i_{02}| \le \frac{4|v_1| + 2|v_2| + 3|v_3|}{R} \le \frac{9}{R} < i_{sat}$$

$$\text{so } \quad R \ge \frac{9}{i_{sat}} = \frac{9}{2\cdot 10^{-3}} = 4.5\ \text{k}\Omega$$

$\therefore$ $R \ge 6$ kΩ satisfies both constraints.

P6.7-7

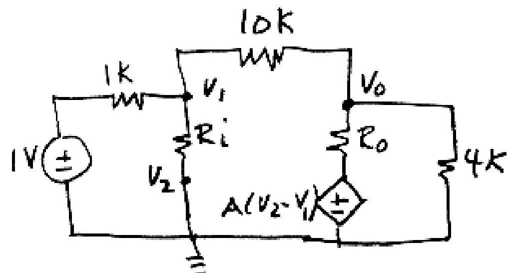


(a) Ideal op amp, KCL

$$\frac{1}{1k\Omega}+\frac{v_0}{10k\Omega}=0$$

$$\underline{v_0=-10\ V}$$

b) Finite gain op amp



$$A=10^4,\ R_i=200k\Omega,\ R_0=5k\Omega$$

$$\text{KCL @ } v_1: \frac{1-v_1}{1k}+\frac{v_0-v_1}{10k\Omega}-\frac{v_1}{R_i}=0$$

$$\text{KCL @ } v_0: \frac{v_1-v_0}{10k\Omega}+\frac{A(v_2-v_1)-v_0}{R_0}-\frac{v_0}{4k\Omega}=0$$

Solving yields $\underline{v_0=-10.03\ V}$

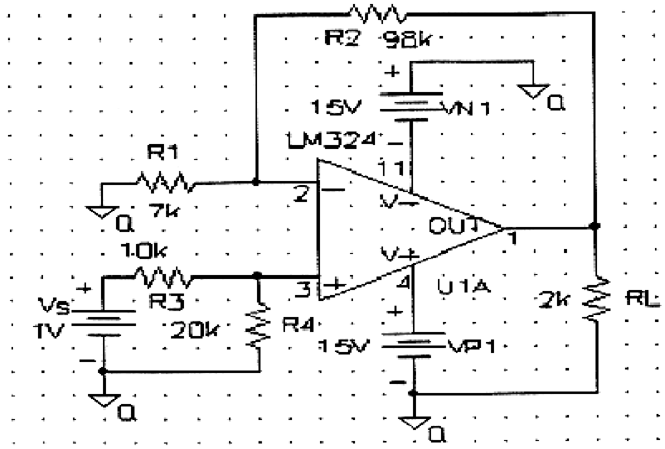
PSpice Problems

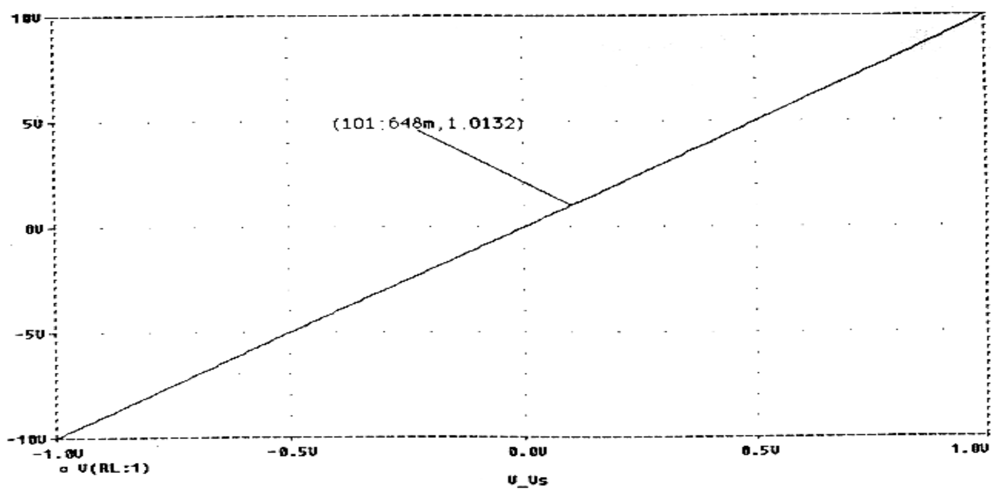
SP 6-1

From the PROBE plot shown below

$v_0 \simeq 1\text{ V}$ when $v_s \simeq 0.1\text{ V}$

Then $i_0 = \dfrac{v_0}{2000} \simeq 0.5\text{ mA}$



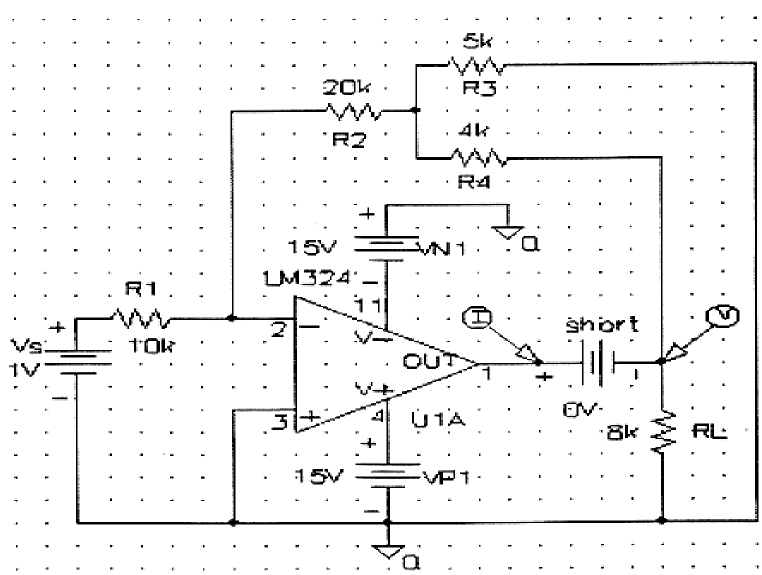


SP 6-2

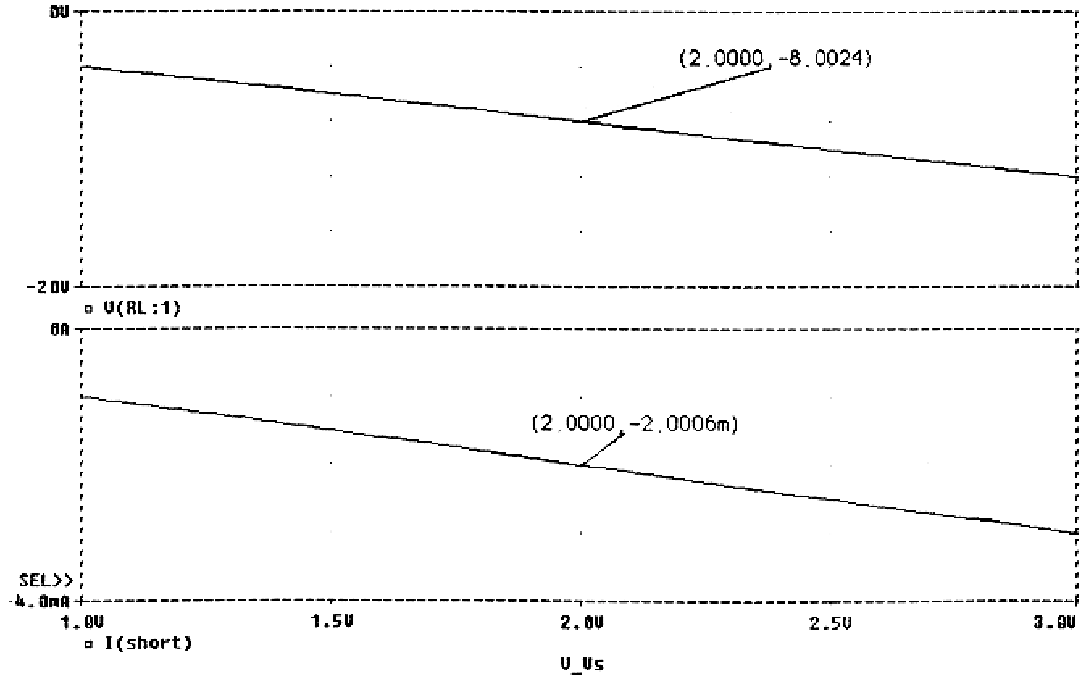
These PROBE plots indicates that

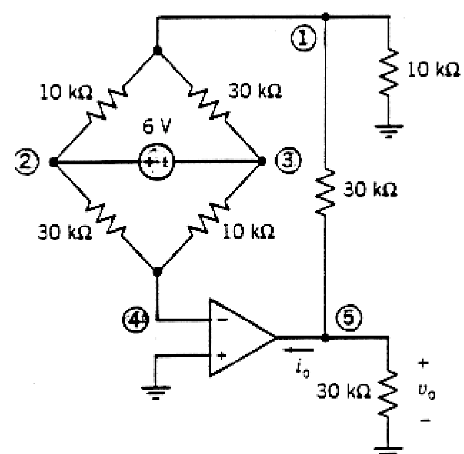
$$v_0 = -8.0024\text{V and}$$
$$i_0 = -2.0006\text{mA}$$

Notice that a model of the LM324 op amp was used instead of an ideal op amp



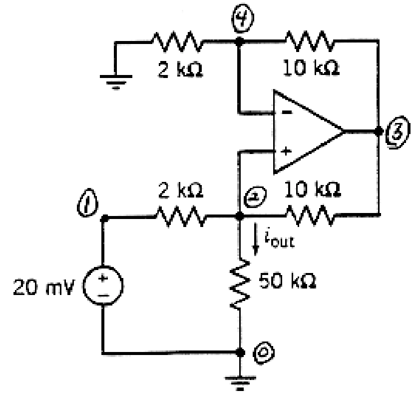
SP 6-2 (continued)






```
SP 6-3 Spice deck corresponding to SP 6-3
R1 1  0 10K
R2 1  5 30K
R3 1  2 10K
R4 1  3 30K
R5 2  4 30K
R6 3  4 10K
VS 2  3 DC   6
RI 4  0 10MEG
X0A1  4 0  5  IDEAL
R7 5  0 30K

.SUBCKT IDEAL 1 2 3
E 3 0 1 2 -1G
.ENDS IDEAL
.END

 NODE    VOLTAGE       NODE    VOLTAGE       NODE    VOLTAGE
(    1)     3.0000    (    2)      4.5000   (    3)   -1.5000
(    4)-12.00E-09    (    5)     12.0000

VOLTAGE SOURCE CURRENTS
NAME         CURRENTS
VS          -3.000E-04
```

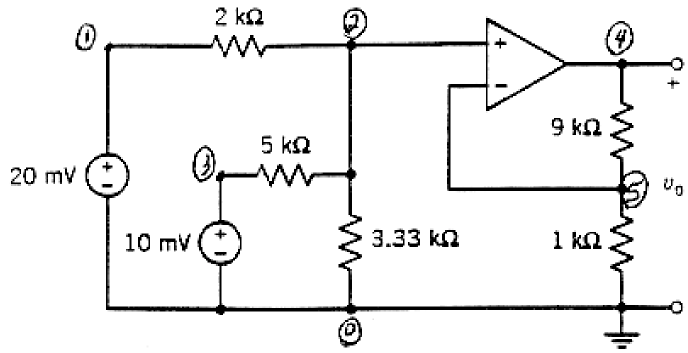
```
SP 6-4 Spice deck corresponding to Problem SP 6-4(a)
V1    1 0   DC .02
R1    1 2   2K
R2    2 0   50K
R3    2 3   10K
R4    3 4   10K
R5    4 0   2K
X0A   1 4   2 3  IDEAL

.SUBCKT IDEAL 1 2 3
E 3 0 1 2 -1G
.ENDS IDEAL
.END
```




```
NODE     VOLTAGE       NODE     VOLTAGE       NODE   VOLTAGE
(    1)    .0200     (    2)      .5000    (    3)   3.0000
(    4)    .5000

VOLTAGE SOURCE CURRENTS
NAME          CURRENTS
VS            2.400E-04
```

```
SP 6-4b Spice deck corresponding to Problem SP 6-4(b)
VS    1 0   DC .02
R1    1 2   2K
R2    2 0   50K
R3    2 3   10K
R4    3 4   10K
R5    4 0   2K
X0A   1 4   2  3  UA741

.SUBCKT UA741 1 2 5
IB1 1 0 70nA
Ib2 2 0 90nA
VOS 3 2 1mV
RI 1 3 2MEG
E 4 0 1 3 -200000
RO 4 5 75
.ENDS UA741
.END

 NODE   VOLTAGE      NODE     VOLTAGE     NODE    VOLTAGE
(    1)    .0200    (    2)      .5285   (    3)    3.1777
(    4)    .5295    (X0A1.3)      .5295  (X0A1.4)    3.2174

VOLTAGE SOURCE CURRENTS
NAME         CURRENT
VS           2.543E-04
X0A1.VOS     -8.044E-12
```

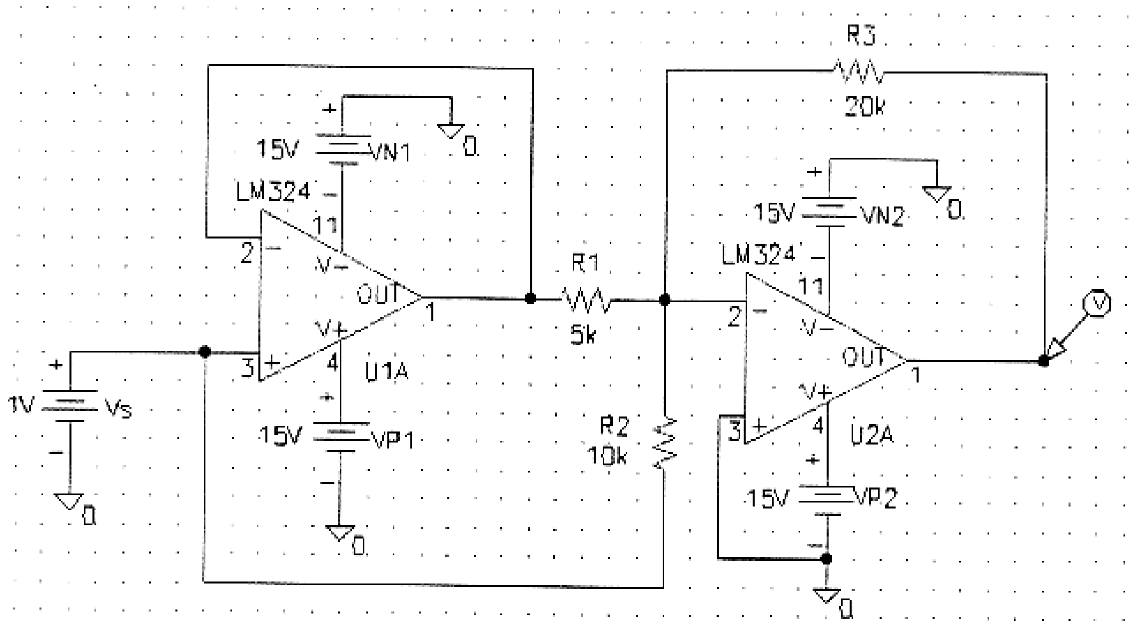
```
SP 6-5 Spice deck corresponding to Problem SP 6-5(a)
V1    1 0   DC .02
R1    1 2   2K
R2    2 3   5K
R3    2 0   3.33K
V2    3 0   DC .01
X0A   1 5   2  4 IDEAL
R4    4 5   9K
R5    5 0   1K

.SUBCKT IDEAL 1 2 3
E 3 0 1 2 -1G
.ENDS IDEAL
.END
```

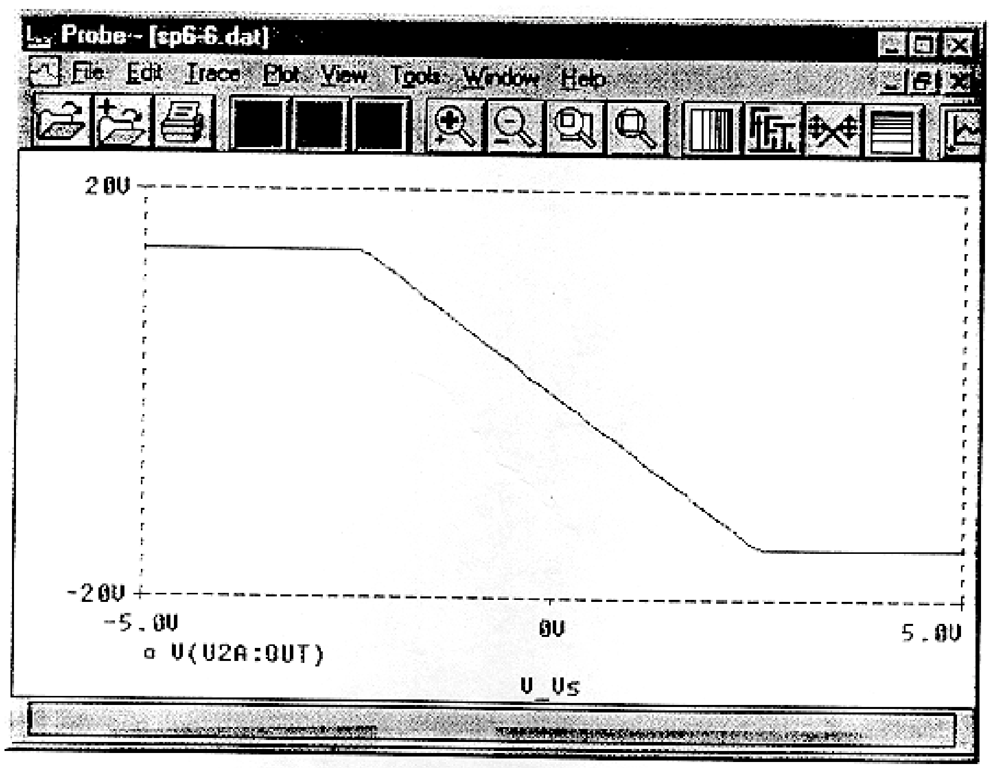



```
NODE     VOLTAGE        NODE     VOLTAGE        NODE     VOLTAGE
(    1)     .0200     (     2)       .0120   (     3)       .0100
(    4)     .1200     (     5)       .0120

VOLTAGE SOURCE CURRENTS
NAME          CURRENTS
V1           -4.002E-06
V2            3.993E-07
```

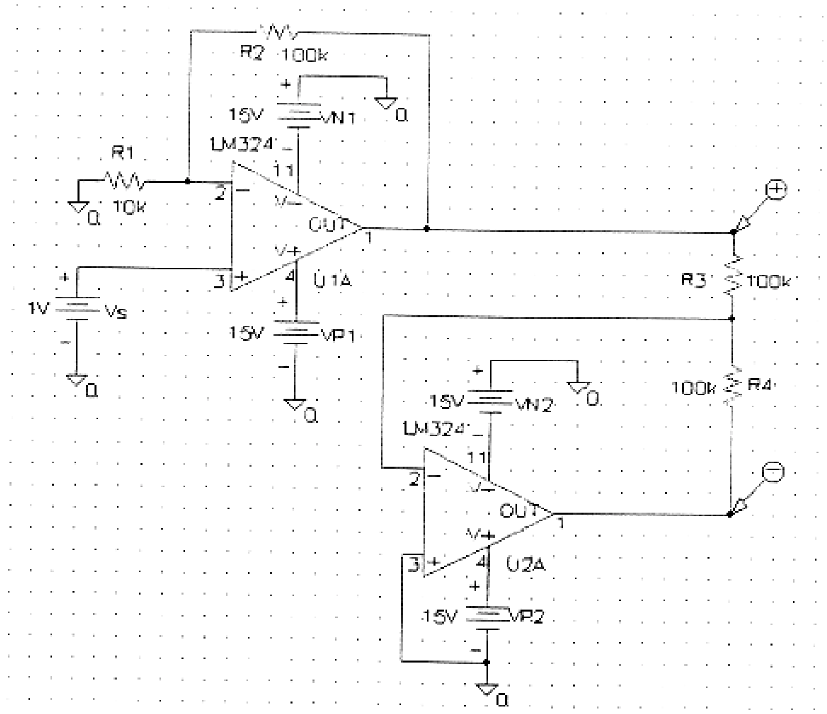
```
SP 6-5 Spice deck corresponding to Problem SP 6-5(b)
V1    1 0   DC .02
R1    1 2   2K
R2    2 3   5K
R3    2 0   3.33K
V2    3 0   DC .01
X0A   1 5   2  4 UA741
R4    4 5   9K
R5    5 0   1K

.SUBCKT UA741 1 2 5
IB1 1 0 70nA
IB2 2 0 90nA
VOS 3 2 1mV
RI 1 3 2MEG
E 4 0 1 3 -200000
RO 4 5 75
.ENDS UA741
.END

 NODE   VOLTAGE       NODE     VOLTAGE      NODE    VOLTAGE
(    1)     .0200    (     2)      .0119   (     3)     .0100
(    4)     .1297    (     5)      .0129   (X0A1.3)     .0129
(X0A1.4)     .1307

VOLTAGE SOURCE CURRENTS
NAME          CURRENT
V1           -4.047E-06
V2            3.813E-07
X0A1.VOS     -3.267E-13
```

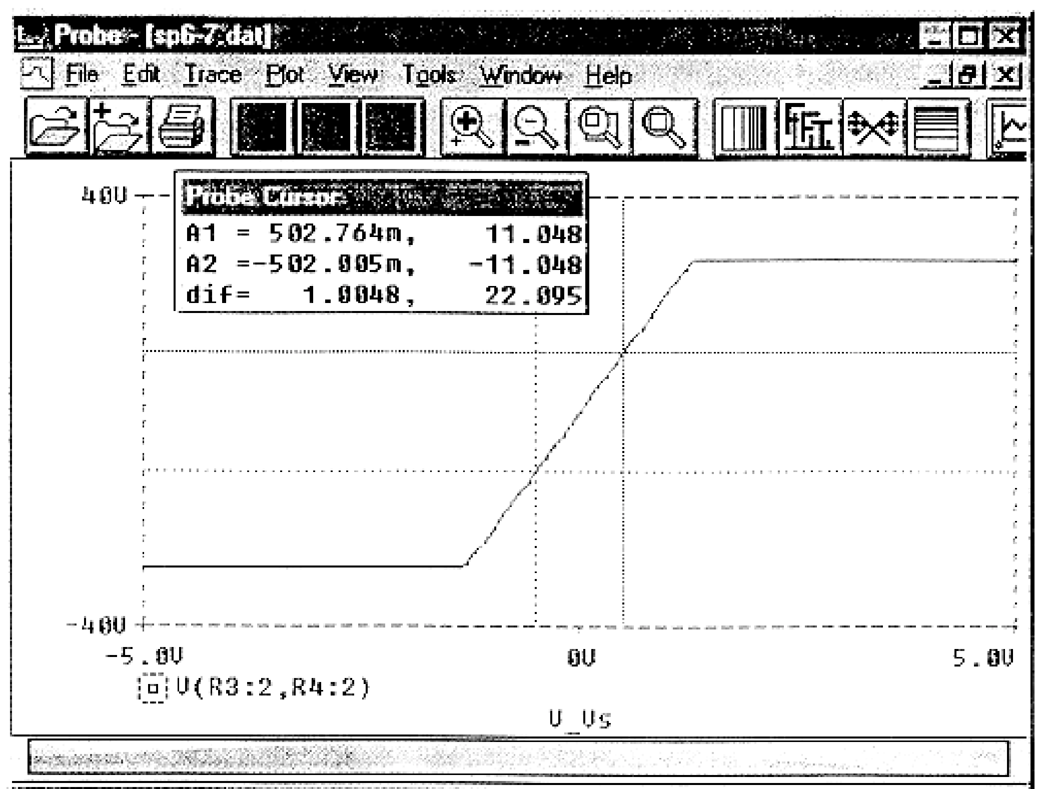
SP 6-6


R3
20k
15V
VN1
0
LM324
11
2
V-
OUT
1
V+
3
4
U1A
1V
Vs
15V
VP1
0
R1
5k
R2
10k
15V
VN2
0
LM324
11
2
V-
OUT
1
V+
3
4
U2A
15V
VP2
0
V

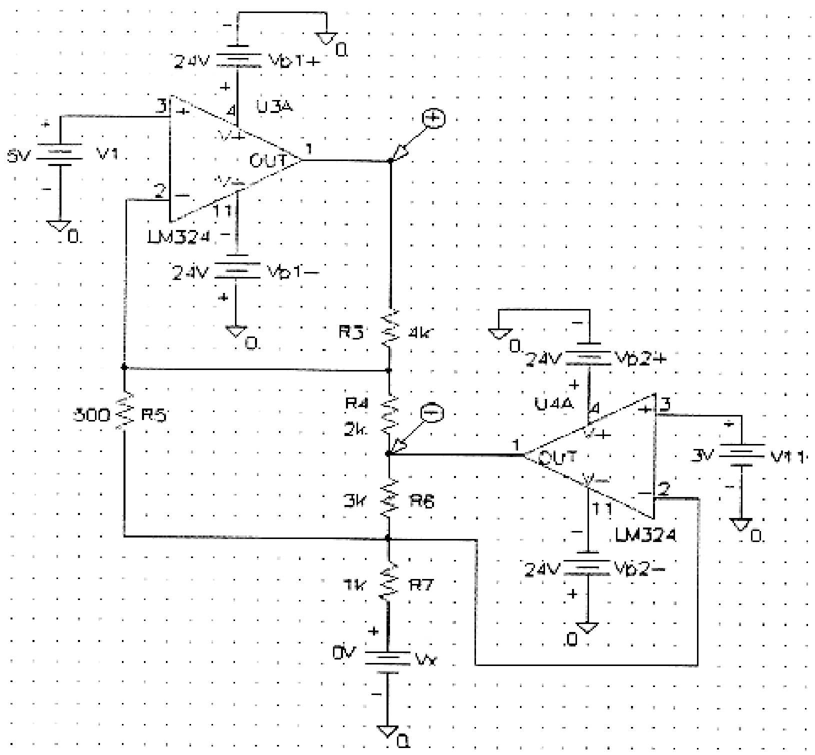

Probe - [sp6-6.dat]
File Edit Trace Plot View Tools Window Help
20V
-20V
-5.0V
0V
5.0V
V(U2A:OUT)
V_Vs

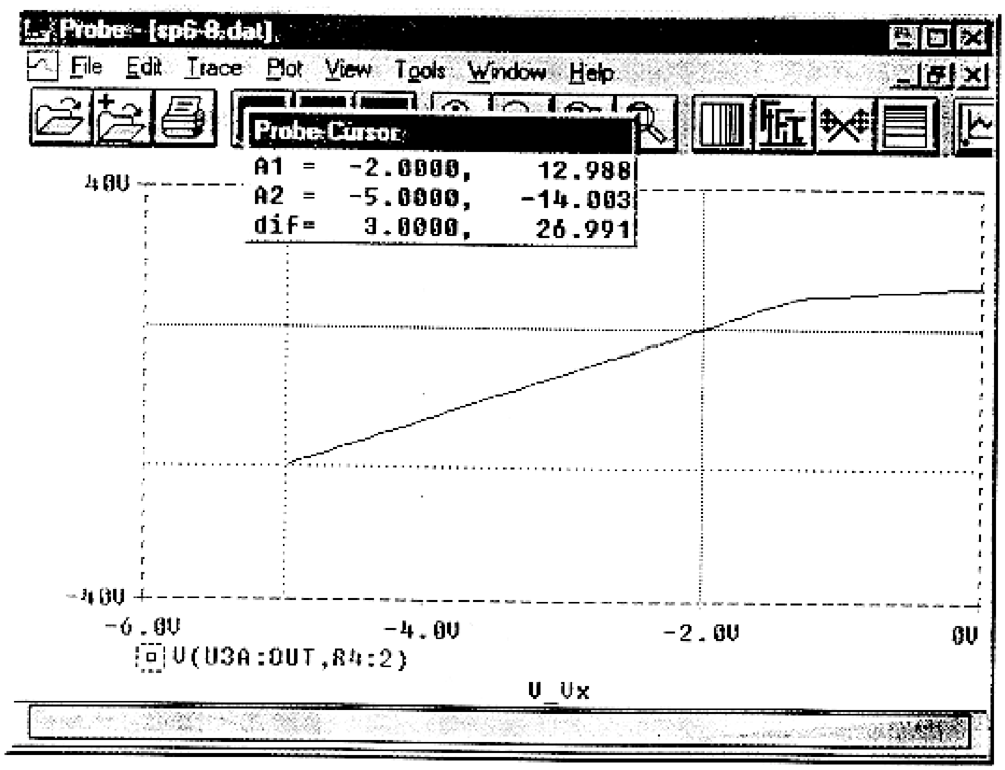
SP 6-7


R2 100k
15V
VN1
0
LM324
R1
10k
11
2
V-
OUT
1
V+
3
4
U1A
1V
Vs
15V
VP1
0
R3
100k
100k
R4
15V
VN2
0
LM324
11
2
OUT
1
V+
3
4
U2A
15V
VP2
0


Probe - [sp6-7.dat]
File Edit Trace Plot View Tools Window Help
Probe Cursor
A1 = 502.764m, 11.048
A2 =-502.005m, -11.048
dif= 1.0048, 22.095
40V
-40V
-5.0V
0V
5.0V
V(R3:2,R4:2)
V_Vs

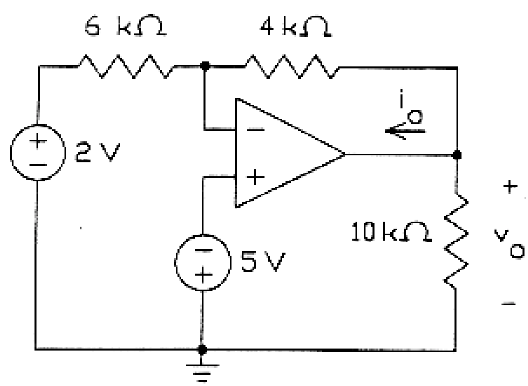
SP 6-8


24V
Vb1+
U3A
LM324
5V
V1
OUT
24V
Vb1-
R3
4k
R4
2k
300
R5
3k
R6
1k
R7
0V
Vx
U4A
24V
Vb2+
3V
V11
LM324
24V
Vb2-


Probe - [sp6-8.dat]
File Edit Trace Plot View Tools Window Help
Probe Cursor
A1 = -2.0000, 12.988
A2 = -5.0000, -14.003
dif= 3.0000, 26.991
40V
-40V
-6.0V
-4.0V
-2.0V
0V
V(U3A:OUT,R4:2)
V_Vx

Verification Problems

VP 6-1



Apply KCL at the ouput node of the op amp to get

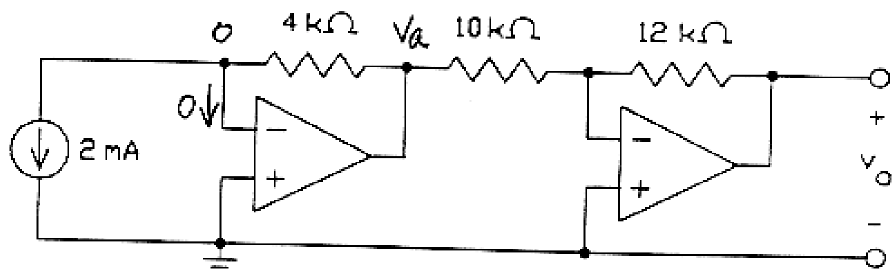
$$i_0+\frac{v_0}{10{,}000}+\frac{v_0-(-5)}{4000}=0$$

or, when $v_o = 7$ V and $i_o = -1$mA,

$$-10^{-3}+\frac{7}{10{,}000}+\frac{12}{4000}=2.7\times10^{-3}\neq 0$$

KCL is not satisfied, so the analysis cannot be correct.

VP 6-2



$$v_a=\left(4\times10^{3}\right)\left(2\times10^{-3}\right)=8\text{ V}$$

$$v_0=-\frac{12\times10^{3}}{10\times10^{3}}v_a=-1.2\,(8)=-9.6\text{ V}$$

So $v_0 = -9.6$ V instead of 9.6 V.

VP 6-3

When $v_0 = -12$ V, the current in the 4 kΩ resistor is

$$i=\frac{-12-2}{4000}=-3.5\text{ mA}$$

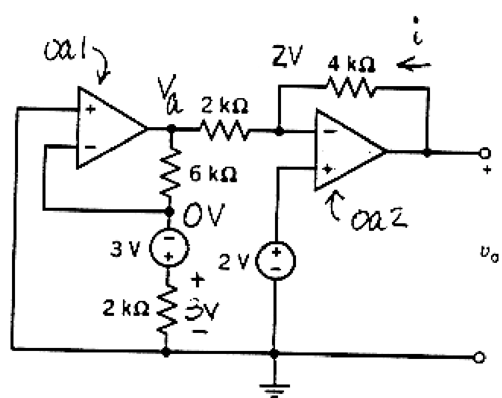
Then

$$i=\frac{2-v_a}{2000}$$

or

$$2-v_a=2000\left(-3.5\cdot10^{-3}\right)$$

$$v_a=2+2(3.5)=9\text{V}$$



Finally, apply KCL at the inverting node of oa1 to get

$$-\frac{9}{6000}+\frac{3}{2000}=0$$

Since KCL is satisfied, this analysis appears to be correct.

VP 6-4

First notice that $v_e = v_f = v_e$ is required by the ideal op amp. (There is zero current into the input lead of an ideal op amp so there is zero current in the 10 kΩ connected between nodes e and f, hence zero volts across this resistor. Also, the node voltages at the input nodes of an ideal op amp are equal.)

The node equations at nodes b, c and d are all satisfied by the given voltages:

node b:
$$\frac{0-(-5)}{10000}+\frac{0}{40000}+\frac{0-2}{4000}=0$$

node c:
$$\frac{0-2}{4000}=\frac{2-5}{6000}+0$$

node d:
$$\frac{2-5}{6000}=\frac{5}{5000}+\frac{5-11}{4000}$$

Therefore, the analysis is correct.

VP 6-5

The node equations at nodes b and e are satisfied by the given voltages:

node b:
$$\frac{-.25-2}{20000}+\frac{-.25}{40000}+\frac{-.25-(-5)}{40000}=0$$

node e:
$$\frac{-2.5-(-0.25)}{9000}=\frac{-0.25}{1000}+0$$

Therefore, the analysis is correct.

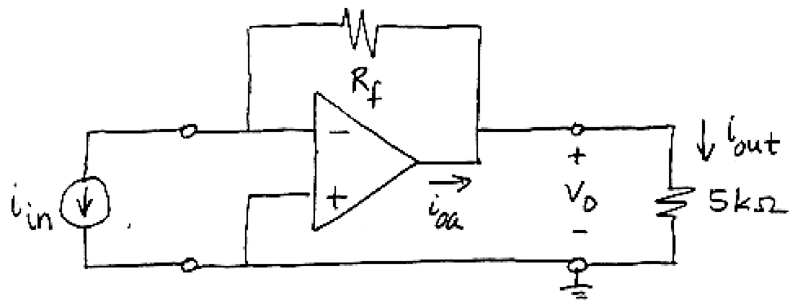
Also, the circuit is an noninverting summer with R_a = 10 kΩ and R_b = 1 kΩ, K_1 =1/ 2, K_2 = 1/ 4 and K_4 = 9. The given node voltages satisfy the equation

$$-2.5=v_d=K_4\left(K_1 v_a+K_2 v_c\right)=10\left(\frac{1}{2}(2)+\frac{1}{4}(-5)\right)$$

Again, we see that the analysis is correct.

Design Problems

DP 6-1 From Figure 6.6-1(g), this circuit



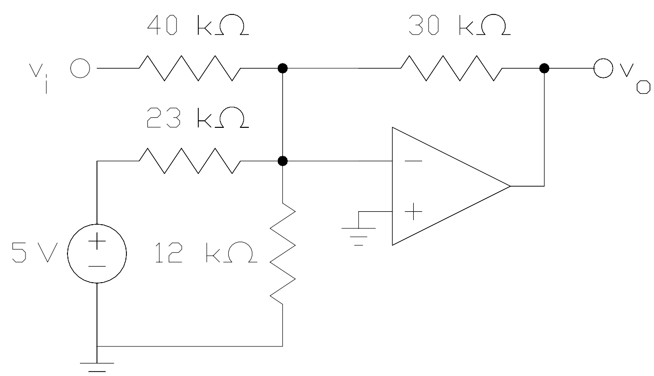
is described by $v_0 = R_f i_{in}$. Since $i_{out} = \frac{v_0}{5k\Omega}$

we require $\frac{1}{4} = \frac{i_{out}}{i_{in}} = \frac{R_f}{5k\Omega}$

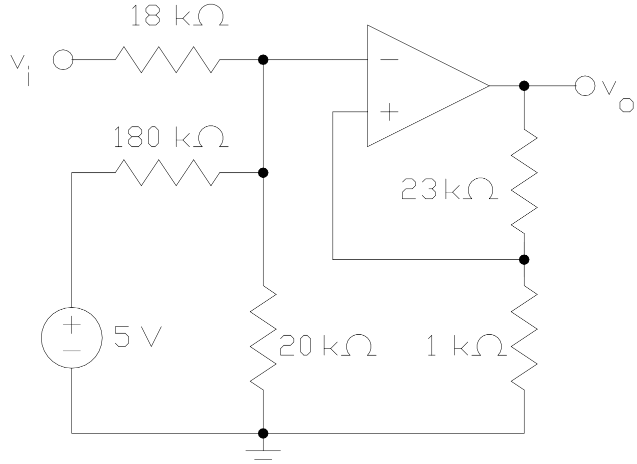
or $R_f = 1250\ \Omega$

Notice that $i_{oa} = i_{in} + \frac{1250 \cdot i_{in}}{5000} = \frac{5}{4} i_{in}$. To avoid current saturation of the op amp $\frac{5}{4} i_{in} < i_{sat}$ or $i_{in} < \frac{4}{5} i_{sat}$. For example, if $i_{sat} = 2\text{mA}$, then $i_{in} < 1.6\text{mA}$ to avoid current saturation.

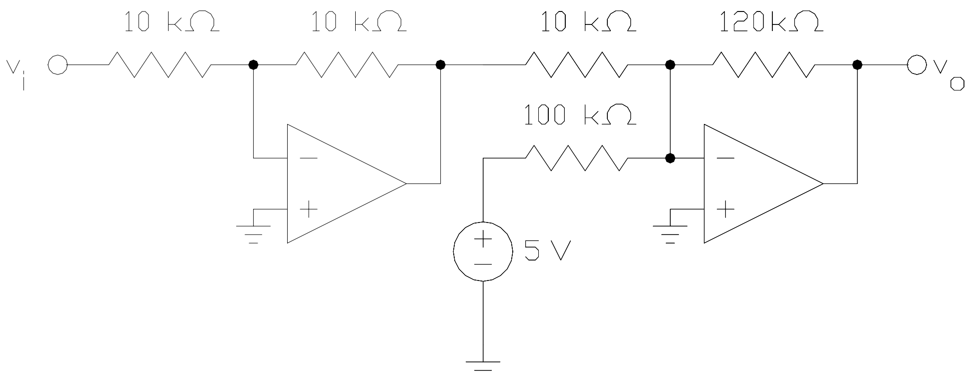
DP 6-2 $v_o = -\frac{3}{4} v_i + 3 = -\frac{3}{4} v_i + \left(1 + \frac{3}{4}\right)\left(\frac{12}{35}\right) 5$



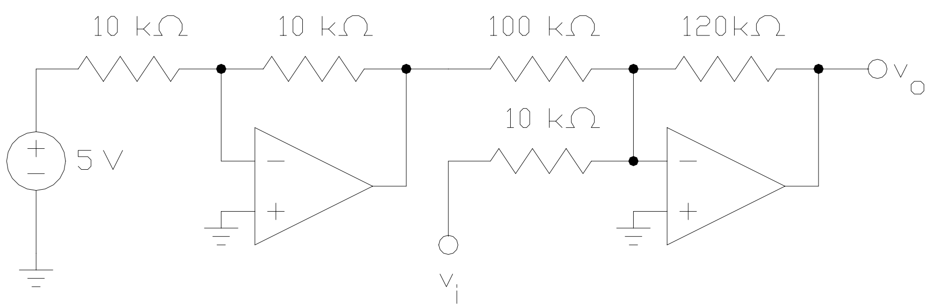
DP 6-3 (a) $12v_i + 6 = 24\left(\frac{1}{2}v_i + \frac{1}{20}(5)\right) \Rightarrow K_4 = 24,\ K_1 = \frac{1}{2}, \text{ and } K_2 = \frac{1}{20}.$ Take R_a = 18 kΩ and R_b = 1 kΩ to get



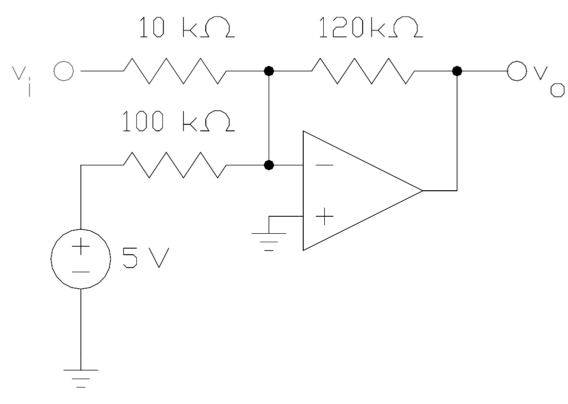
(b)



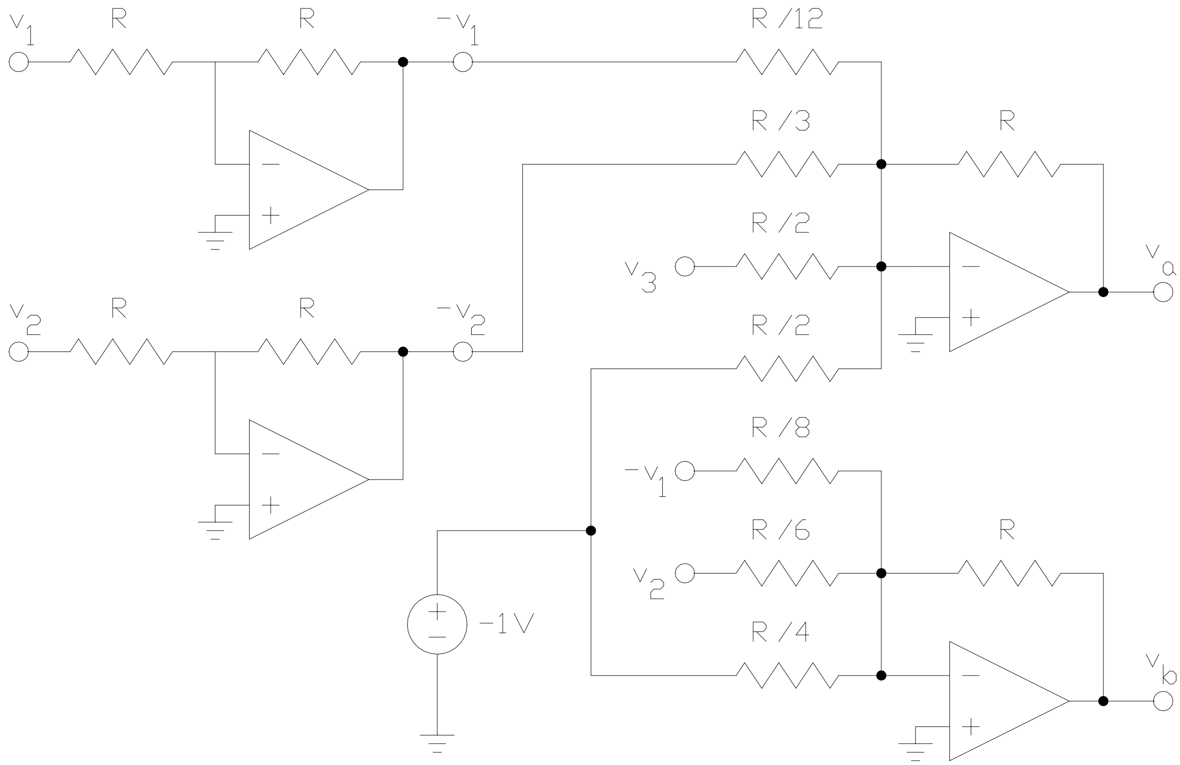
(c)



(d)



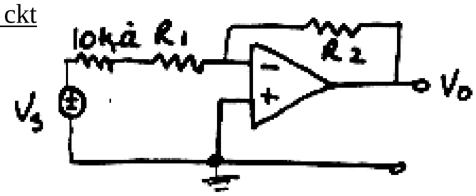
DP6-4



DP 6-5

Need gain $v_o/v_s = \dfrac{4}{20\times10^{-3}} = 200$

<u>Inverting ckt</u>

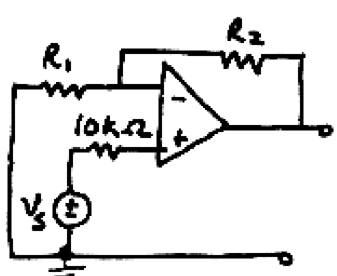


$$\frac{v_0}{v_s} = -\frac{R_2}{R_1+R_s}$$

Use $R_2 = 2\ \text{M}\Omega,\ R_1 = 0$

$R_s = 10\text{k}\Omega$

<u>Noninverting ckt</u>



$$\frac{v_0}{v_s} = \left(1+\frac{R_2}{R_1}\right)v_s$$

R_{IN} = <u>higher</u> for noninverting compared to inverting

Choose $\left(1+{}^{R_2}\!/_{R_1}\right) = 200$ use $R_1 = 1\text{k}\Omega,\ R_2 = 199\text{k}\Omega$

So use noninverting for higher R_{IN} for microphone.