

Chapter 7 Energy Storage Elements

Exercises

Ex. 7.3-1

$$i_C(t) = 1 \frac{d}{dt} v_s(t) = \begin{cases} 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad i_R(t) = 1 v_s(t) = \begin{cases} 2t-4 & 2 < t < 4 \\ 8-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{so } i(t) = i_C(t) + i_R(t) = \begin{cases} 2t-2 & 2 < t < 4 \\ 7-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$$

Ex. 7.3-2

$$v(t) = \frac{1}{C} \int_{t_0}^t i_s(\tau) d\tau + v(t_0) = \frac{1}{\frac{1}{3}} \int_0^t i_s(\tau) d\tau - 12$$

$$v(t) = 3 \int_0^t 4 d\tau - 12 = 12t - 12 \quad \text{for } 0 < t < 4 \quad \text{In particular, } v(4) = 36 \text{ V.}$$

$$v(t) = 3 \int_4^t -2 d\tau + 36 = 60 - 6t \quad \text{for } 4 < t < 10 \quad \text{In particular, } v(10) = 0 \text{ V.}$$

$$v(t) = 3 \int_{10}^t 0 d\tau + 0 = 0 \quad \text{for } 10 < t$$

Ex. 7.3-3 Break up into intervals

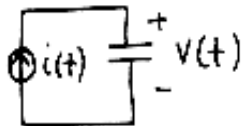
$$i = \begin{cases} 0 & t < 1 \\ 1 & 1 < t < 2 \\ 0 & t > 2 \end{cases} \quad \text{now } v(t) = v(0) + \frac{1}{C} \int_0^t i dt = \frac{1}{10^{-4}} \int_0^t i dt$$

$$\therefore 0 < t < 1 \Rightarrow v(t) = 10^4 \int_0^t 0 dt = \underline{0}$$

$$1 < t < 2 \Rightarrow v(t) = v(1) + 10^4 \int_1^t 1 dt = 0 + 10^4 t = \underline{10^4 t \text{ V}}$$

$$t > 2 \Rightarrow v(t) = v(2) + 10^4 \int_2^t 0 dt = 10^4(2) = \underline{2 \times 10^4 \text{ V}}$$

Ex. 7.3-4



because $i(t) = 0$ when $t < 0$

$$v(t) = \frac{1}{C} \int_{-\infty}^t i(t) dt = \frac{1}{5} \int_0^t (2 + 2 \cos 5t) dt + v(0) = \underline{\underline{\frac{2}{5}t + \frac{2}{25} \sin 5t \frac{1}{5} \text{ V}}}}$$

Ex. 7.4-1 $W = \frac{1}{2}Cv^2 = \frac{1}{2}(2 \times 10^{-4})(100)^2 = \underline{1 \text{ J}}$
 $v_c(0^+) = v_c(0^-) = \underline{100 \text{ V}}$

Ex. 7.4-2 $W(t) = W(0) + \int_0^t v i dt \Rightarrow W(0) = 0$ since $v(0) = 0$

First find $v(t) = v(0) + \frac{1}{C} \int_0^t i dt = 10^4 \int_0^t 2 dt = \underline{2 \times 10^4 t}$

$\therefore W(t) = \int_0^t (2 \times 10^4) (2) dt = 2 \times 10^4 t^2$

$W(1s) = 2 \times 10^4 \text{ J} = \underline{20 \text{ kJ}}$

b) $W(100s) = 2 \times 10^4 (100)^2 = 2 \times 10^8 \text{ J} = \underline{200 \text{ mJ}}$

Ex. 7.4-3 We have $v(0^+) = v(0^-) = 3 \text{ V}$

$V_c(t) = \frac{1}{C} \int_{-\infty}^t i(t) dt = 5 \int_0^t 3e^{-5t} dt + 3 \text{ V} = 3(e^{-5t} - 1) + 3 = 3e^{-5t} \text{ V}$

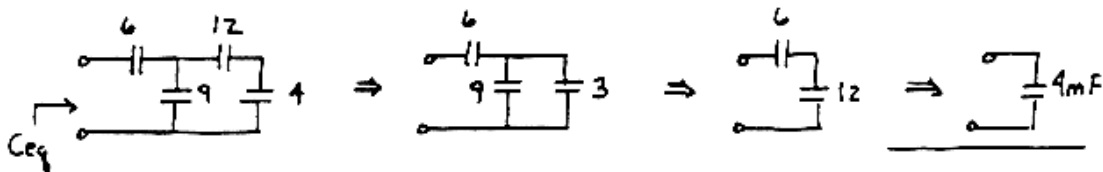
a) Now $v(t) = v_R(t) + v_c(t) = 5i(t) + v_c(t) = 15e^{-5t} + 3e^{-5t} = \underline{18e^{-5t} \text{ V}}, 0 \leq t < 1$

b) $W_c(t) = \frac{1}{2} C v_c^2(t) = \frac{1}{2} \times 2 (3e^{-5t})^2 = 0.9e^{-10t} \text{ J}$

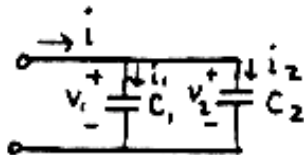
Now when $t = 0.2s \Rightarrow W_c(t)|_{t=0.2s} = \underline{6.65 \text{ J}}$

When $t = .8s \Rightarrow W_c(t)|_{t=.8s} = \underline{2.68 \text{ kJ}}$

Ex. 7.5-1



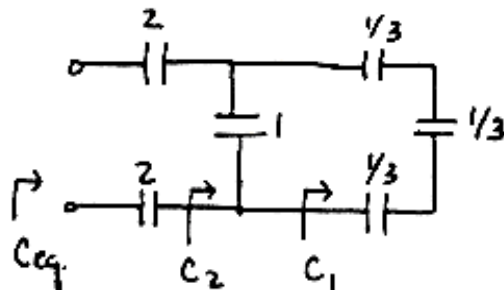
Ex. 7.5-2



$v_1 = v_2 \Rightarrow \frac{dv_1}{dt} = \frac{dv_2}{dt} \Rightarrow \frac{i_1}{C_1} = \frac{i_2}{C_2} \Rightarrow \underline{i_1 = \frac{C_1}{C_2} i_2}$

from KCL : $i = i_1 + i_2 = \left(\frac{C_1}{C_2} + 1 \right) i_2 \Rightarrow \underline{i_2 = \frac{C_2}{C_1 + C_2} i}$

Ex. 7.5-3



$\frac{1}{C_1} = \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}} \Rightarrow C_1 = 1/9 \text{ mF}$

$C_2 = 1 + C_1 = 1 + 1/9 = 10/9 \text{ mF}$

So $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{10/9}, 1/C_{eq} = 19/10$

then $C_{eq} = \underline{\underline{\frac{10}{19} \text{ mF}}}$

Ex. 7.6-1

$$v_L(t) = 1 \frac{d}{dt} i_s(t) = \begin{cases} 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad v_R(t) = 1 i_s(t) = \begin{cases} 2t-4 & 2 < t < 4 \\ 8-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{so } v(t) = v_L(t) + v_R(t) = \begin{cases} 2t-2 & 2 < t < 4 \\ 7-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$$

Ex. 7.6-2

$$i(t) = \frac{1}{L} \int_{t_0}^t v_s(\tau) d\tau + i(t_0) = \frac{1}{\frac{1}{3}} \int_0^t v_s(\tau) d\tau - 12$$

$$i(t) = 3 \int_0^t 4 d\tau - 12 = 12t - 12 \quad \text{for } 0 < t < 4 \quad \text{In particular, } i(4) = 36 \text{ A.}$$

$$i(t) = 3 \int_4^t -2 d\tau + 36 = 60 - 6t \quad \text{for } 4 < t < 10 \quad \text{In particular, } i(10) = 0 \text{ A.}$$

$$i(t) = 3 \int_{10}^t 0 d\tau + 0 = 0 \quad \text{for } 10 < t$$

Ex. 7.6-3

$$i(t) = i(0) + \frac{1}{L} \int_0^t v(t) dt = 0 + \frac{1}{1} \int_0^t 9t^2 dt = \underline{3t^3 \text{ A}}$$

Ex. 7.7-1

$$v = L \frac{di}{dt} = \left(\frac{1}{4}\right) \frac{d}{dt} (4te^{-t}) = \underline{(1-t)e^{-t} \text{ V}}$$

$$P = vi = \left[(1-t)e^{-t} \right] (4te^{-t}) = \underline{4t(1-t)e^{-2t} \text{ W}}$$

$$W = \frac{1}{2} Li^2 = \frac{1}{2} \left(\frac{1}{4}\right) (4te^{-t})^2 = \underline{2t^2 e^{-2t} \text{ J}}$$

Ex. 7.7-2

$$v(t) = L \frac{di}{dt} = \frac{1}{2} \frac{di}{dt}$$

$$i(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 < t < 1 \\ -2(t-2) & 1 < t < 2 \\ 0 & t > 2 \end{cases} \quad \therefore v(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$

$$\text{now } p(t) = v(t)i(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 < t < 1 \\ 2(t-2) & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$

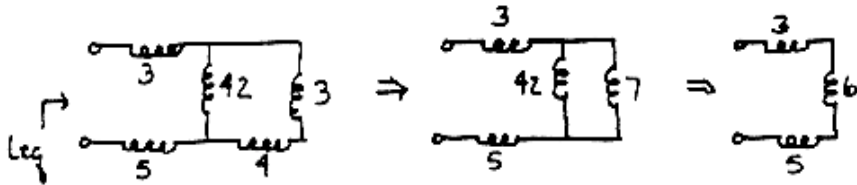
$$W(t) = W(t_0) + \int_{t_0}^t p(t) dt \Rightarrow t < 0: W(t) = 0 \text{ since } p(t) = 0$$

$$0 < t < 1: W(t) = \int_0^t 4t dt = 2t^2$$

$$1 < t < 2: W(t) = W(1) + \int_1^t 4(t-2) dt = 2t^2 - 8t + 8$$

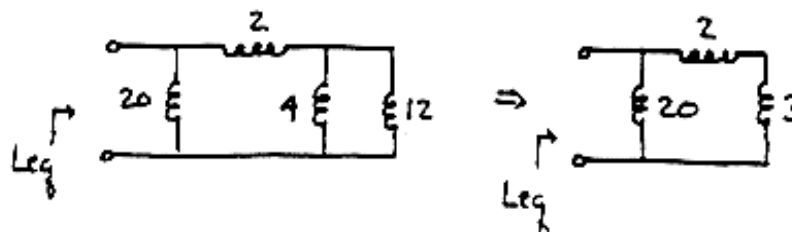
$$t > 2: W(t) = W(2) = 0$$

Ex. 7.8-1



$$L_{eq} = 3 + 6 + 5 = \underline{14\text{mH}}$$

Ex. 7.8-2



$$L_{eq} = \frac{(20)(5)}{20+5} = \underline{4\text{mH}}$$

Ex. 7.8-3

$$i = i_1 + i_2$$

$$i_1 = \frac{1}{L_1} \int v dt + i_1(0), \quad i_2 = \frac{1}{L_2} \int v dt + i_2(0) \quad \text{but } i_1(0)=0 \text{ and } i_2(0)=0$$

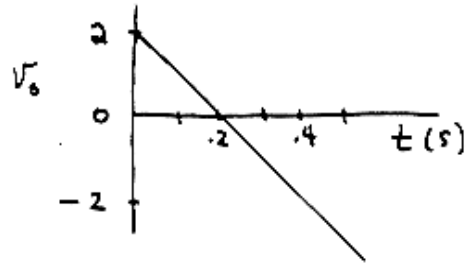
$$\text{so } i = \frac{1}{L_1} \int v dt + \frac{1}{L_2} \int v dt, \text{ now also } i = \frac{1}{L_P} \int v dt + i(0) \quad \text{where } i(0)=0$$

$$\therefore \frac{i_1}{i} = \frac{\frac{1}{L_1} \int v dt}{\frac{1}{L_P} \int v dt} = \frac{\frac{1}{L_1}}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_2}{L_1 + L_2}$$

Ex. 7.10-1

$$v_o = -\frac{1}{RC} \int_0^t v_s dt + v_o(0) = -2 \int_0^t 5 dt + 2$$

$$\frac{1}{RC} = \frac{1}{(1 \times 10^3)(500 \times 10^{-6})} = 2 \quad \underline{v_o = (2-10t) \text{ V}, t \geq 0}$$

**Ex. 7.10-2**

$$v_o = -RC \frac{dv_s}{dt} = -1 \frac{d(0.2 \sin 10t)}{dt} = \underline{-2 \cos 10t \text{ V}, t \geq 0}$$

$$RC = (20 \times 10^3)(50 \times 10^{-6}) = 1$$

ProblemsSection 7-3: Capacitors**P7.3-1**

$$v = v(0) + \frac{1}{C} \int i dt \quad \text{and} \quad q = Cv$$

$$\therefore Cv = Cv(0) + \int i dt = Cv(0) + it \quad \leftarrow \text{since } i = \text{constant}$$

$$\therefore t = \frac{q - Cv(0)}{i} = \frac{150 \mu\text{C} - (15 \mu\text{F})(5\text{V})}{25\text{mA}} = \underline{3\text{ms}}$$

P7.3-2

$$i(t) = C \frac{d}{dt} v(t) = \frac{1}{8} \frac{d}{dt} 12 \cos(2t + 30^\circ) = \frac{1}{8} (12)(2) \cos(2t + 30^\circ + 90^\circ) = 3 \cos(2t + 120^\circ) \text{ A}$$

P7.3-3

$$(3 \cdot 10^{-3}) \cos(500t + 45^\circ) = C \frac{d}{dt} 12 \cos(500t - 45^\circ) = C (12)(500) \cos(500t - 45^\circ + 90^\circ) = C (6000) \cos(500t + 45^\circ)$$

$$\text{so } C = \frac{3 \cdot 10^{-3}}{6 \cdot 10^3} = \frac{1}{2} \cdot 10^{-6} = \underline{\frac{1}{2} \mu\text{F}}$$

P7.3-4

$$v(t) = -1 \text{ mV} + \frac{1}{2 \text{ pF}} \int_0^t i_s(\tau) d\tau$$

for $0 \leq t < 2 \text{ ns}$ $i_s(t) = 0$ So $v(t) = -1 \text{ mV}$.

for $2 \text{ ns} \leq t < 3 \text{ ns}$ $i_s(t) = 4 \mu\text{A}$

$$v(t) = -1 \text{ mV} + \frac{1}{2 \text{ pF}} \int_{2 \text{ ns}}^t 4 \mu\text{s} d\tau = -5 \text{ mV} + \left(2 \frac{\mu\text{A}}{\text{pF}} \right) t$$

$v(2 \text{ ns}) \text{-----} \uparrow$

$$v(3 \text{ ns}) = -5 \text{ mV} + 2 \frac{\mu\text{A}}{\text{pF}} \cdot 3 \text{ ns} = 1 \text{ mV}$$

for $3 \text{ ns} \leq t < 5 \text{ ns}$ $i_s(t) = -2 \mu\text{A}$

$$v(t) = 1 \text{ mV} + \frac{1}{2 \text{ pF}} \int_{3 \text{ ns}}^t -2 \mu\text{A} d\tau = 4 \text{ mV} - \left(\frac{\mu\text{A}}{\text{pF}} \right) t$$

$$v(5 \text{ ns}) = 4 \text{ mV} - \left(\frac{\mu\text{A}}{\text{pF}} \right) 5 \text{ ns} = -1 \text{ mV}$$

for $5 \text{ ns} < t$ $i_s(t) = 0$ so $v(t)$ remains -1 mV .

P 7.3-5

$$(a) i(t) = C \frac{d}{dt} v(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \\ -4 & 2 < t < 3 \\ 0 & 3 < t \end{cases}$$

$$(b) v(t) = \frac{1}{C} \int_0^t i(\tau) d\tau + v(0) = \int_0^t i(\tau) d\tau$$

$$\text{For } 0 < t < 1, i(t) = 0 \text{ A so } v(t) = \int_0^t 0 d\tau + 0 = 0 \text{ V}$$

$$\text{For } 1 < t < 2, i(t) = 4t - 4 \text{ A so } v(t) = \int_0^t (4\tau - 4) d\tau + 0 = (2\tau^2 - 4\tau) \Big|_1^t = 2t^2 - 4t + 2 \text{ V}$$

$$v(2) = 4(2^2) - 4(2) + 2 = 2 \text{ V}. \text{ For } 2 < t < 3, i(t) = -4t + 12 \text{ A so}$$

$$v(t) = \int_0^t (-4\tau + 12) d\tau + 2 = (-2\tau^2 + 12\tau) \Big|_1^t + 2 = -2t^2 + 12t - 14 \text{ V}$$

$$v(3) = -4(3^2) + 12(3) - 14 = 4 \text{ V}$$

$$\text{For } 0 < t < 1, i(t) = 0 \text{ A so } v(t) = \int_0^t 0 d\tau + 4 = 4 \text{ V}$$

P 7.3-6

$$(a) i(t) = C \frac{d}{dt} v(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$$

$$(b) v(t) = \frac{1}{C} \int_0^t i(\tau) d\tau + v(0) = 2 \int_0^t i(\tau) d\tau$$

For $0 < t < 2$, $i(t) = 0$ A so $v(t) = 2 \int_0^t 0 d\tau + 0 = 0$ V

For $2 < t < 6$, $i(t) = 0.2t - 0.4$ V so

$$v(t) = 2 \int_0^t (0.2\tau - 0.4) d\tau + 0 = (0.2\tau^2 - 0.8\tau) \Big|_0^t = 0.2t^2 - 0.8t + 0.8$$

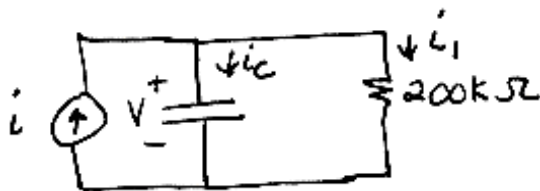
$v(6) = 0.2(6^2) - 0.8(6) + 0.8 = 3.2$ V. For $6 < t$, $i(t) = 0.8$ A so $v(t) = 2 \int_0^t 0.8 d\tau + 3.2 = 1.6t - 6.4$ V

P7.3-7

$$v(t) = v(0) + \frac{1}{C} \int_0^t i dt = 25 + 2.5 \times 10^4 \int_0^t (6 \times 10^{-3}) e^{-6t} dt = 25 + 150 \int_0^t e^{-6t} dt = 25 + 150 \left[-\left(\frac{1}{6}\right) e^{-6t} \right]_0^t$$

$$v(t) = \underline{50 - 25e^{-6t}} \text{ V}$$

P7.3-8



$$i_1 = \frac{v}{200} = \frac{1}{40} (1 - 2e^{-2t}) \text{ mA} = 25(1 - 2e^{-2t}) \mu\text{A}$$

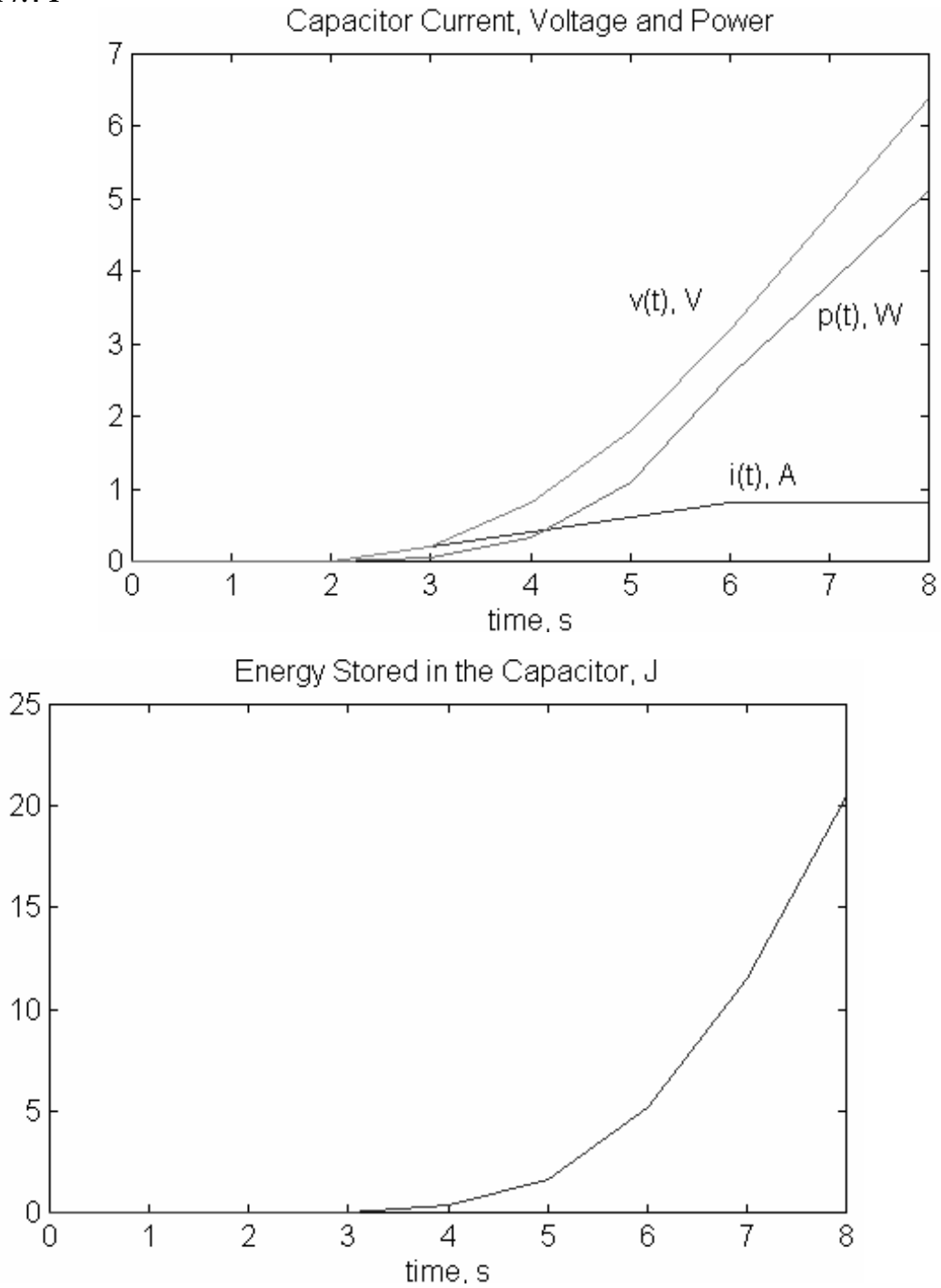
$$i_c = C \frac{dv}{dt} = 10(-2)(-10e^{-2t}) = 200e^{-2t} \mu\text{A}$$

$$\therefore i = i_1 + i_c = 200e^{-2t} + 25 - 50e^{-2t}$$

$$= \underline{25 + 150e^{-2t} \mu\text{A}}$$

Section 7-4: Energy Storage in a Capacitor

P7.4-1



These plots were produced using three MATLAB scripts:

```

capvol.m          function v = CapVol(t)
                  if t<2
                    v = 0;
                  elseif t<6
                    v = 0.2*t*t - .8*t + .8;
                  else
                    v = 1.6*t - 6.4;
                  end

capcur.m          function i = CapCur(t)
                  if t<2
                    i=0;
                  elseif t<6
                    i=.2*t - .4;
                  else
                    i =.8;
                  end

c7s4p1.m         t=0:1:8;
                  for k=1:1:length(t)
                    i(k)=CapCur(k-1);
                    v(k)=CapVol(k-1);
                    p(k)=i(k)*v(k);
                    w(k)=0.5*v(k)*v(k);
                  end

                  plot(t,i,t,v,t,p)
                  text(5,3.6,'v(t), V')
                  text(6,1.2,'i(t), A')
                  text(6.9,3.4,'p(t), W')
                  title('Capacitor Current, Voltage and Power')
                  xlabel('time, s')

                  % plot(t,w)
                  % title('Energy Stored in the Capacitor, J')
                  % xlabel('time, s')

```

P7.4-2

$$i_c = C \frac{dv}{dt} = (10 \times 10^{-6}) (-5) (-4000) e^{-4000t} = \underline{0.2 e^{-4000t} \text{ A}}$$

$$\underline{i_c(0) = 0.2 \text{ A}} \quad \underline{i_c(10\text{ms}) = 8.5 \times 10^{-19} \text{ A}}$$

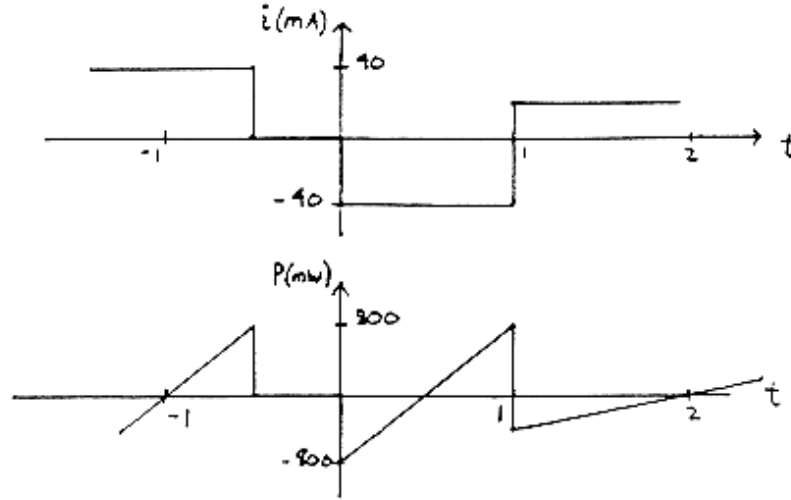
$$W(t) = \frac{1}{2} C v^2(t) \quad v(0) = 5 - 5e^0 = 0 \quad \underline{W(0) = 0}$$

$$v(10\text{ms}) = 5 - 5e^{-40} = 5 - 4.24 \times 10^{-18} \cong 5 \quad \underline{W(10) = 1.25 \times 10^{-4} \text{ J}}$$

P7.4-3

$i(t) = C dv_c / dt$ so read off slope of $v_c(t)$ to get $i(t)$

$P(t) = v_c(t)i(t)$ so multiply $v_c(t)$ & $i(t)$ curves to get $p(t)$



P7.4-4 $v_c(t) = v_c(0) + \frac{1}{C} \int_0^t i dt = v_c(0) + \frac{1}{2} \int_0^t 50 \cos(10t + \pi/6) dt = [v_c(0) - \frac{5}{2} \sin \pi/6] + \frac{5}{2} \sin(10t + \pi/6)$

Now since $v_c(t)_{ave} = 0 \Rightarrow v_c(0) = \frac{5}{2} \sin \pi/6$

$\therefore v_c(t) = \frac{5}{2} \sin(10t + \pi/6) \text{ V}$

$\therefore W_{max} = \frac{1}{2} C v_{c,max}^2 = \frac{1}{2} (2) (2.5)^2 = \underline{6.25 \mu \text{ J}}$

First non-negative t for max energy occurs: $10t + \pi/6 = \pi/2 \Rightarrow t = \frac{\pi/30} = \underline{0.1047 \text{ s}}$

P7.4-5

$W = \frac{1}{2} C v^2 \Rightarrow v = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(4e^{-10t})}{.001}} = 40 \sqrt{5} e^{-5t}$

$\therefore v(0.1) = 40 \sqrt{5} e^{-5(0.1)} = \underline{54.25 \text{ V}}$

$i(t) = C \frac{dv_c}{dt} = (10^{-3}) (-5) (40 \sqrt{5} e^{-5t}) = \frac{1}{\sqrt{5}} e^{-5t}$

$\therefore i(0.1) = \frac{1}{\sqrt{5}} e^{-5(0.1)} = \underline{-0.2712 \text{ A}}$

P7.4-6

Max. charge on capacitor = $Cv = (10 \mu\text{F})(6\text{V}) = 60 \mu\text{C}$

$\Delta t = \frac{\Delta q}{i} = \frac{60 \mu\text{C}}{10 \mu\text{A}} = \underline{6 \text{ sec to charge}}$

stored energy = $W = \frac{1}{2} C v^2 = \frac{1}{2} (10 \mu\text{F})(6\text{V})^2 = \underline{180 \mu \text{ J}}$

P7.4-7

$$P(\Delta t) = (1.5 \times 10^3 \text{ W}) (1 \text{ hr}) \left(\frac{60 \text{ min}}{1 \text{ hr}} \right) \left(\frac{60 \text{ sec}}{1 \text{ min}} \right) = 5.4 \times 10^6 \text{ J}$$

$$W = \frac{1}{2} C v^2 \Rightarrow C = \frac{2W}{v^2}$$

$$C = \frac{2(5.4 \times 10^6)}{(500)^2} = 43.2 \text{ F}$$

$$\text{size of capacitor} = (43.2 \text{ F}) \left(\frac{10^6 \mu\text{F}}{\text{F}} \right) \left(\frac{1 \text{ cm}^3}{100 \mu\text{F}} \right)$$

$$\text{size} = 4.32 \times 10^5 \text{ cm}^3 \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = \underline{0.432 \text{ m}^3}$$

Capacitor may be practical, it depends on size of car! Would be 76 cm on a side- pretty large !

Section 7-5: Series and Parallel capacitors

P7.5-1

$$2 \mu\text{F} \parallel 4 \mu\text{F} = 6 \mu\text{F}$$

$$6 \mu\text{F} \text{ in series with } 3 \mu\text{F} = \frac{6 \mu\text{F} \cdot 3 \mu\text{F}}{6 \mu\text{F} + 3 \mu\text{F}} = 2 \mu\text{F}$$

$$i(t) = 2 \mu\text{F} \frac{d}{dt} (6 \cos 100t) = (2 \cdot 10^{-6}) (6) (100) (-\sin 100t) = \underline{-1.2 \sin 100t \text{ mA}}$$

P7.5-2

$$4 \mu\text{F} \text{ in series with } 4 \mu\text{F} = \frac{4 \mu\text{F} \cdot 4 \mu\text{F}}{4 \mu\text{F} + 4 \mu\text{F}} = 2 \mu\text{F}$$

$$2 \mu\text{F} \parallel 2 \mu\text{F} = 4 \mu\text{F}$$

$$4 \mu\text{F} \text{ in series with } 4 \mu\text{F} = 2 \mu\text{F}$$

$$i(t) = 2 \mu\text{F} \frac{d}{dt} (3 + 3e^{-250t}) = (2 \cdot 10^{-6}) (0 + 3(-250)e^{-250t}) = -1.5 \cdot 10^{-3} e^{-250t} \text{ A} = \underline{-1.5 e^{-250t} \text{ mA}}$$

P7.5-3 C in series with C = $\frac{C \cdot C}{C + C} = \frac{C}{2}$

$$C \parallel C \parallel \frac{C}{2} = \frac{5}{2} C$$

$$C \text{ in series with } \frac{5}{2} C = \frac{C \cdot \frac{5}{2} C}{C + \frac{5}{2} C} = \frac{5}{7} C$$

$$(25 \cdot 10^{-3}) \cos 250t = \frac{5}{7} C \frac{d}{dt} (14 \sin 250t) = \left(\frac{5}{7} C \right) (14) (250) \cos 250t$$

$$\text{so } 25 \cdot 10^{-3} = 2500C \Rightarrow C = 10 \cdot 10^{-6} = 10 \mu\text{F}$$

Section 7-6: Inductors

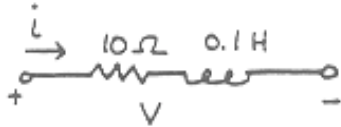
P7.6-1 Find max. voltage across coil:

$$v(t) = L \frac{di}{dt} = 200 [100(400)\cos 400t] \text{ V}$$

$$\therefore v_{\max} = 8 \times 10^6 \text{ V}$$

thus have a field of $8 \times 10^6 \text{ V}/2\text{m} = 4 \times 10^6 \text{ V}/\text{m}$ which exceeds dielectric strength in air of $3 \times 10^6 \text{ V}/\text{m}$ \therefore will get a discharge as the air is ionized.

P7.6-2



$$\begin{aligned} v &= L \frac{di}{dt} + 10i \\ &= (.1)(4e^{-t} - 4te^{-t}) + 10(4te^{-t}) \\ &= \underline{\underline{0.4e^{-t} + 39.6te^{-t} \text{ V}}} \end{aligned}$$

P 7.6-3

$$(a) v(t) = L \frac{d}{dt} i(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \\ -4 & 2 < t < 3 \\ 0 & 3 < t \end{cases}$$

$$(b) i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau + i(0) = \int_0^t v(\tau) d\tau$$

$$\text{For } 0 < t < 1, v(t) = 0 \text{ V so } i(t) = \int_0^t 0 d\tau + 0 = 0 \text{ A}$$

$$\text{For } 1 < t < 2, v(t) = 4t - 4 \text{ V so } i(t) = \int_0^t (4\tau - 4) d\tau + 0 = (2\tau^2 - 4\tau) \Big|_1^t = 2t^2 - 4t + 2 \text{ A}$$

$$i(2) = 4(2^2) - 4(2) + 2 = 2 \text{ A}$$

For $2 < t < 3$, $v(t) = -4t + 12 \text{ V}$ so

$$i(t) = \int_0^t (-4\tau + 12) d\tau + 2 = (-2\tau^2 + 12\tau) \Big|_1^t + 2 = -2t^2 + 12t - 14 \text{ A}$$

$$i(3) = -4(3^2) + 12(3) - 14 = 4 \text{ A}$$

$$\text{For } 0 < t < 1, v(t) = 0 \text{ V so } i(t) = \int_0^t 0 d\tau + 4 = 4 \text{ A}$$

P7.6-4

$$v(t) = (250 \cdot 10^{-3}) \frac{d}{dt} (120 \cdot 10^{-3}) \sin(500t - 30^\circ) = (.25)(.12)(500) \sin(500t - 30^\circ + 90^\circ) = 15 \sin(500t + 60^\circ)$$

P7.6-5

$$i_L(t) = \frac{1}{5\text{mH}} \int_0^t v_s(\tau) d\tau - 2\mu\text{A}$$

for $0 \leq t < 1 \mu\text{s}$ $v_s(t) = 4 \text{mV}$

$$i_L(t) = \frac{1}{5\text{mH}} \int_0^t 4 \text{mV} d\tau - 2\mu\text{A} = \left(\frac{4}{5} \frac{\text{mV}}{\text{mH}} \right) t - 2\mu\text{A}$$

$$i_L(1\mu\text{s}) = \left(\frac{4}{5} \frac{\text{mV}}{\text{mH}} \cdot 1\mu\text{s} \right) - 2\mu\text{A} = -\frac{6}{5} \mu\text{A}$$

for $1\mu\text{s} \leq t < 3\mu\text{s}$ $v_s(t) = -1 \text{mV}$

$$i_L(t) = \frac{1}{5\text{mH}} \int_{1\mu\text{s}}^t -1 \text{mV} d\tau - \frac{6}{5} \mu\text{A} = -\frac{1}{5} \frac{\text{mV}}{\text{mH}} (t - 1\mu\text{s}) - \frac{6}{5} \mu\text{A} = \left(-\frac{1}{5} \frac{\text{mV}}{\text{mH}} \right) t - 1\mu\text{A}$$

$$i_L(3\mu\text{s}) = \left(-\frac{1}{5} \frac{\text{mV}}{\text{mH}} \cdot 3\mu\text{s} \right) - 1\mu\text{A} = -\frac{8}{5} \mu\text{A}$$

for $3\mu\text{s} \leq t$ $v_s(t) = 0$ so $i_L(t)$ remains $-\frac{8}{5} \mu\text{A}$

P7.6-6

$$v(t) = 2\text{k}\Omega \cdot i_s(t) + 4\text{mH} \cdot \frac{d}{dt} i_s(t) \text{ (in general)}$$

$$\text{for } 0 < t < 1 \mu\text{s} \quad i_s(t) = 1 \frac{\text{mA}}{\mu\text{s}} \cdot t \Rightarrow \frac{d}{dt} i_s(t) = 1 \frac{\text{mA}}{\mu\text{s}}$$

$$\text{so } v(t) = (2\text{k}\Omega) \left(1 \frac{\text{mA}}{\mu\text{s}} \right) t + 4 \text{mH} \left(1 \frac{\text{mA}}{\mu\text{s}} \right) = \left(\frac{2\text{V}}{\mu\text{s}} \right) t + 4 \text{V}$$

$$\text{for } 1\mu\text{s} < t < 3\mu\text{s} \quad i_s(t) = 1 \text{mA} \Rightarrow \frac{d}{dt} i_s(t) = 0$$

$$\text{so } v(t) = 2\text{k}\Omega \cdot 1\text{mA} + 4\text{mH} \cdot 0 = 2 \text{V}$$

$$\text{for } 3\mu\text{s} < t < 5\mu\text{s} \quad i_s(t) = 4\text{mA} - \left(1 \frac{\text{mA}}{\mu\text{s}} \right) t$$

$$\text{so } \frac{d}{dt} i_s(t) = -1 \frac{\text{mA}}{\mu\text{s}} \quad \text{and} \quad v(t) = 4 \text{V} - \left(\frac{2\text{V}}{\mu\text{s}} \right) t$$

$$\text{when } 5\mu\text{s} < t < 7\mu\text{s} \quad i_s(t) = -1 \text{mA} \quad \text{and} \quad \frac{d}{dt} i_s(t) = 0$$

$$\text{so } v(t) = -2 \text{V}$$

$$\text{when } 7\mu\text{s} < t < 8\mu\text{s} \quad i_s(t) = 1 \frac{\text{mA}}{\mu\text{s}} \cdot t - 8\text{mA}$$

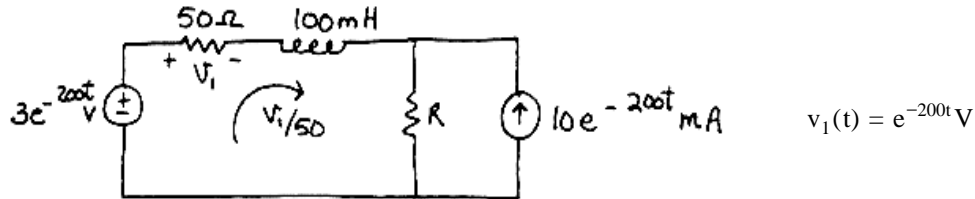
$$\text{so } \frac{d}{dt} i_s(t) = 1 \frac{\text{mA}}{\mu\text{s}}$$

$$\text{and } v(t) = -12 \text{V} + \left(\frac{2 \text{V}}{\mu\text{s}} \right) t$$

$$\text{when } 8\mu\text{s} < t, \text{ then } i_s(t) = 0 \Rightarrow \frac{d}{dt} i_s(t) = 0$$

$$\text{so } v(t) = 0$$

P7.6-7



$$\begin{aligned} \text{KVL a: } -3e^{-200t} + v_1 + .1 \frac{d}{dt} \left(\frac{v_1}{50} \right) + R \left[\frac{v_1}{50} + .01e^{-200t} \right] &= 0 \\ -3e^{-200t} + e^{-200t} - .4e^{-200t} + R \left[\frac{1}{50} + .01 \right] e^{-200t} &= 0 \\ -2.4 + R(.03) = 0 \Rightarrow R = \frac{2.4}{.03} = 80\Omega \end{aligned}$$

P 7.6-8

$$(a) v(t) = L \frac{d}{dt} i(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$$

$$(b) i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau + i(0) = 2 \int_0^t v(\tau) d\tau$$

$$\text{For } 0 < t < 2, v(t) = 0 \text{ V so } i(t) = 2 \int_0^t 0 d\tau + 0 = 0 \text{ A}$$

For $2 < t < 6$, $v(t) = 0.2t - 0.4$ V so

$$i(t) = 2 \int_0^t (0.2\tau - 0.4) d\tau + 0 = (0.2\tau^2 - 0.8\tau) \Big|_0^t = 0.2t^2 - 0.8t + 0.8 \text{ A}$$

$$i(6) = 0.2(6^2) - 0.8(6) + 0.8 = 3.2 \text{ A.}$$

$$\text{For } 6 < t, v(t) = 0.8 \text{ V so } i(t) = 2 \int_0^t 0.8 d\tau + 3.2 = 1.6t - 6.4 \text{ A}$$

Section 7-7: Energy Storage in an Inductor

P7.7-1

$$v(t) = 100 \cdot 10^{-3} \frac{d}{dt} i(t) = \begin{cases} 0 & t < 0 \\ 0.4 & 0 < t < 1 \\ 0 & t > 1 \end{cases}$$

$$p(t) = v(t) i(t) = \begin{cases} 0 & t < 0 \\ 1.6t & 0 < t < 1 \\ 0 & t > 1 \end{cases}$$

$$W(t) = \int_0^t p(\tau) d\tau = \begin{cases} 0 & t < 0 \\ 0.8t^2 & 0 < t < 1 \\ 0.8 & t > 1 \end{cases}$$

P7.7-2

$$p(t) = v(t) i(t) = \left[5 \frac{d}{dt} (4 \sin 2t) \right] (4 \sin 2t) = 5 (8 \cos 2t) (4 \sin 2t)$$

$$= 80 [2 \cos 2t \sin 2t] = 80 [\sin(2t+2t) + \sin(2t-2t)] = 80 \sin 4t \text{ W}$$

$$w(t) = \int_0^t p(\tau) d\tau = 80 \int_0^t \sin 4\tau d\tau = -\frac{80}{4} [\cos 4\tau]_0^t = 20 (1 - \cos 4t)$$

P7.7-3

$$i(t) = \frac{1}{25 \cdot 10^{-3}} \int_0^t 6 \cos 100\tau d\tau + 0 = \frac{6}{(25 \cdot 10^{-3})(100)} [\sin 100\tau]_0^t = 2.4 \sin 100t$$

$$p(t) = v(t) i(t) = (6 \cos 100t)(2.4 \sin 100t) = 7.2 [2(\cos 100t)(\sin 100t)] = 7.2 [\sin 200t + \sin 0] = 7.2 \sin 200t$$

$$W(t) = \int_0^t p(\tau) d\tau = 7.2 \int_0^t \sin 200\tau d\tau = -\frac{7.2}{200} [\cos 200\tau]_0^t = 0.036 [1 - \cos 200t] \text{ J} = 36 [1 - \cos 200t] \text{ mJ}$$

Section 7-8: Series and Parallel Inductors**P7.8-1**

$$6\text{H} \parallel 3\text{H} = \frac{6\text{H} \cdot 3\text{H}}{6\text{H} + 3\text{H}} = 2\text{H}$$

$$2\text{H} + 2\text{H} = 4\text{H}$$

$$i(t) = \frac{1}{4} \int_0^t 6 \cos 100\tau d\tau = \frac{6}{4 \cdot 100} [\sin 100\tau]_0^t = 15 \sin 100t \text{ mA}$$

P7.8-2

$$4\text{mH} + 4\text{mH} = 8\text{mH}$$

$$8\text{mH} \parallel 8\text{mH} = \frac{8\text{mH} \cdot 8\text{mH}}{8\text{mH} + 8\text{mH}} = 4\text{mH}$$

$$4\text{mH} + 4\text{mH} = 8\text{mH}$$

$$v(t) = (8 \cdot 10^{-3}) \frac{d}{dt} (5 + 3e^{-250t})(10^{-3}) = (8 \cdot 10^{-6})(0 + 3(-250)e^{-250t}) = -6e^{-250t} \text{ mV}$$

P7.8-3

$$L \parallel L = \frac{L \cdot L}{L + L} = \frac{L}{2}$$

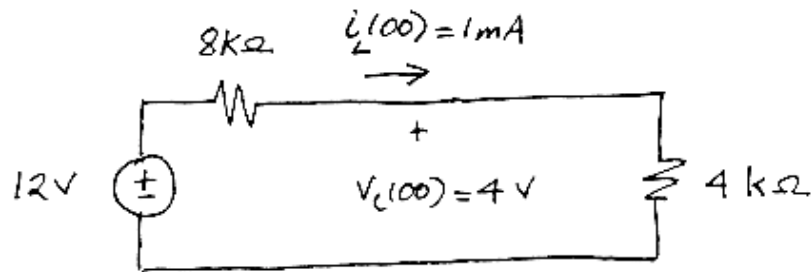
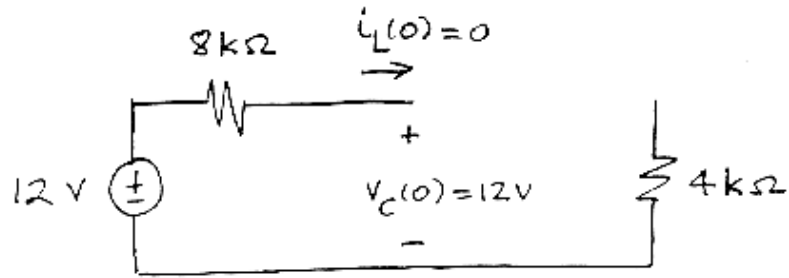
$$L + L + \frac{L}{2} = \frac{5}{2} L$$

$$25 \cos 250t = \frac{5}{2} L \frac{d}{dt} (14 \cdot 10^{-3} \sin 250t) = \left(\frac{5}{2} L \right) (14 \cdot 10^{-3})(250) \cos 250t$$

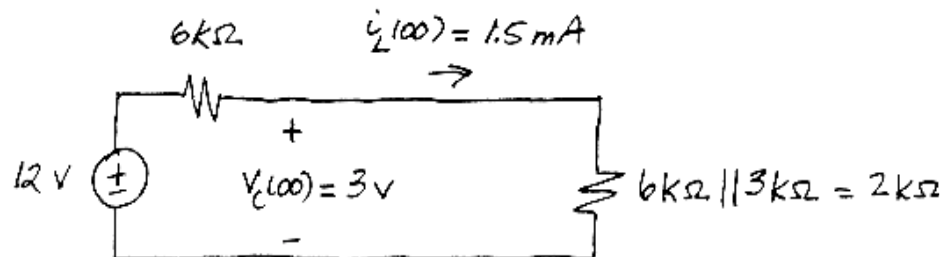
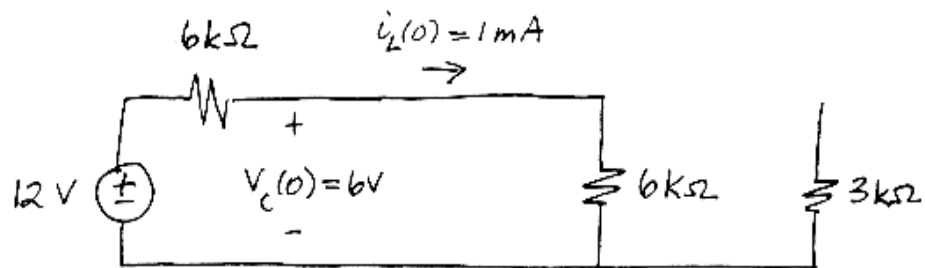
$$\text{so } L = \frac{25}{\frac{5}{2}(14 \cdot 10^{-3})(250)} = 2.86 \text{ H}$$

Section 7-9: Initial Conditions of Switched Circuits

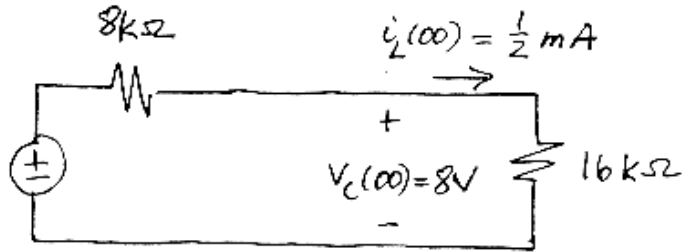
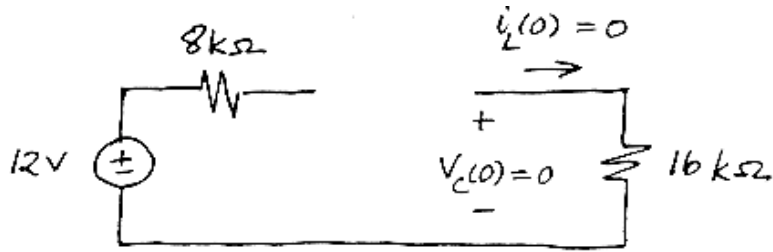
P7.9-1



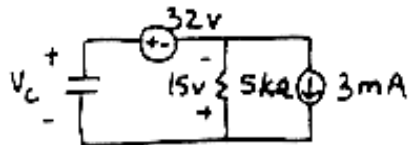
P7.9-2



P7.9-3



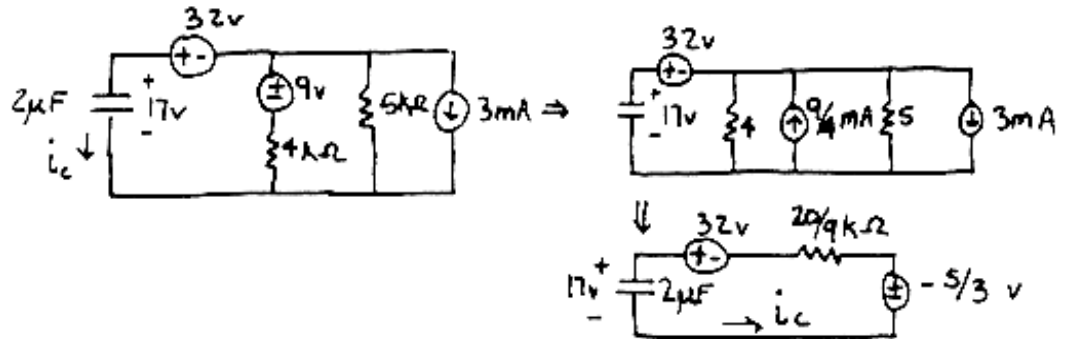
P7.9-4 at $t=0^-$



$$\text{KVL: } -v_c(0^-) + 32 - 15 = 0$$

$$\Rightarrow v_c(0^-) = v_c(0^+) = 17\text{ V}$$

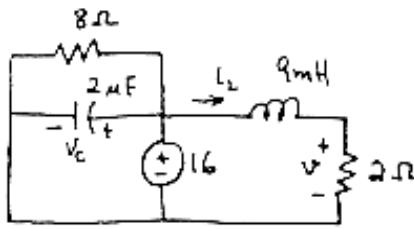
at $t=0^+$



$$\text{KVL: } \frac{5}{3} + \left(\frac{20}{9}\right) i_c - 32 + 17 = 0 \Rightarrow i_c(0^+) = 6\text{mA}$$

$$\text{Now } i_c(0^+) = C \frac{dv_c(0^+)}{dt} \Rightarrow \frac{dv_c(0^+)}{dt} = \frac{6\text{mA}}{2\mu\text{F}} = 3\text{V/ms}$$

P7.9-5 at $t=0^-$

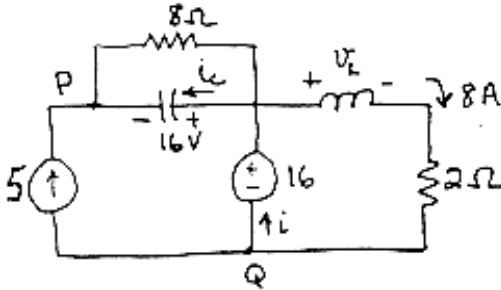


at steady - state

$$v_c(0^-) = v_c(0^+) = 16V$$

$$\text{and } i_L(0^-) = i_L(0^+) = \frac{16}{2} = 8A$$

at $t=0^+$



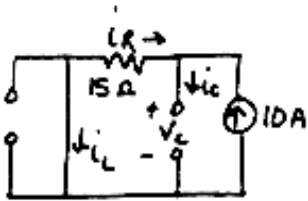
$$\text{KVL: } -16 + v_L + 8(2) = 0 \Rightarrow v_L(0^+) = 0$$

$$\text{KCL at P: } -5 - i_c - \frac{16}{8} = 0 \Rightarrow i_c(0^+) = -7A$$

$$\text{KCL at Q: } 5 + i - 8 = 0 \Rightarrow i(0^+) = 3A$$

$$\text{Then } \frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = 0 \text{ and } \frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{-7}{2\mu F} = -3.5A/\mu s$$

P7.9-6 $t=0^-$



$$v_c(0^-) = (15)(10) = \underline{150V}$$

$$i_c(0^-) = \underline{0}$$

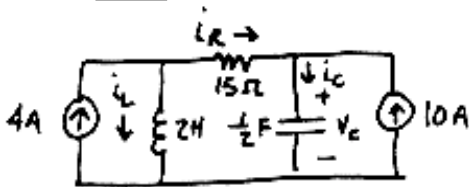
$$v_L(0^-) = \underline{0V}$$

$$i_L(0^-) = \underline{10A}$$

$$v_R(0^-) = 15(-10) = \underline{-150V}$$

$$i_R(0^-) = \underline{-10A}$$

$t=0^+$



$$v_c(0^+) = v_c(0^-) = \underline{150V}$$

$$i_L(0^+) = i_L(0^-) = \underline{10A}$$

$$v_R(0^+) = -6(15) = \underline{-90V}$$

$$i_R(0^+) = 4A - 10A = \underline{-6A}$$

$$v_L(0^+) = v_R(0^+) + v_c(0^+) = \underline{60V}$$

$$i_c(0^+) = i_R(0^+) + 10A$$

$$= 4A$$

Section 7-10: The Operational Amplifier and RC Circuits

P7.10-1

$$\begin{aligned} v_0(t) &= -\frac{1}{RC} \int_0^t v_s(\tau) d\tau + v_0(0) = \frac{-1}{(20 \cdot 10^3)(2 \cdot 10^{-6})} \int_0^t 12 \cos 100\tau d\tau + 0 \\ &= -25 \left(\frac{12 \sin 100\tau}{100} \right)_0^t = -3 \sin 100t \end{aligned}$$

P7.10-2

$$v_0(t) = -\frac{1}{RC} \int_0^t v_s(\tau) d\tau + v_0(0) = \frac{-1}{(2 \cdot 10^3)(10^{-6})} \int_0^t -4 d\tau + 0 = 2000 t \quad 0 < t < 3 \text{ms}$$

$$v_0(3 \text{ms}) = (2 \cdot 10^3)(3 \cdot 10^{-3}) = 6$$

$$v_0(t) = -\frac{1}{RC} \int_3^t 0 d\tau + 6 = 6 \quad t > 3 \text{ms}$$

P7.10-3

$$250t = -\frac{1}{RC} \int_0^t -5 dt, \text{ when } 0 < t < 20 \text{ms} = \frac{5}{RC} t$$

$$\text{so } 250 = \frac{5}{RC} \Rightarrow RC = \frac{5}{250} = \frac{1}{50}$$

$$\text{Let } C = 1 \mu\text{F, then } R = \frac{1}{50(10^{-6})} = 20 \cdot 10^3 = 20 \text{k}\Omega$$

Verification Problems**VP 7-1**

$$\text{at } t = 1 \quad 0.025 \stackrel{?}{=} -\frac{1}{2} + 0.065$$

$$\text{at } t = 3 \quad -\frac{3}{25} + 0.065 \stackrel{?}{=} \frac{3}{50} - 0.115$$

$$-0.55 \neq -0.485$$

The equation for the inductor current indicates that this current changes instantaneously at $t = 3$ s. This equation cannot be correct.

VP 7-2

We need to check the values of the inductor current at the ends of the intervals.

$$\text{at } t = 1 \quad -\frac{1}{200} + 0.025 \stackrel{?}{=} -\frac{1}{100} + 0.03 \quad \text{Yes}$$

$$\text{at } t = 4 \quad -\frac{4}{100} + 0.03 \stackrel{?}{=} \frac{4}{100} - 0.03 \quad \text{No}$$

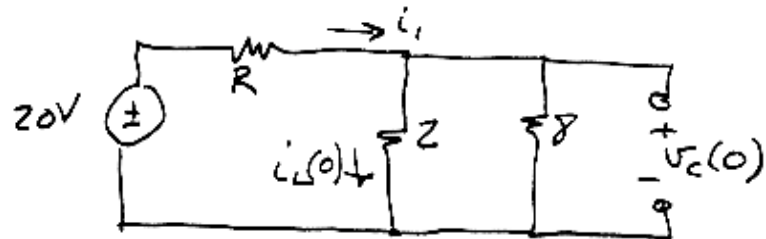
The equation for the inductor current indicates that this current changes instantaneously at $t = 4$ s. This equation cannot be correct.

Design Problems

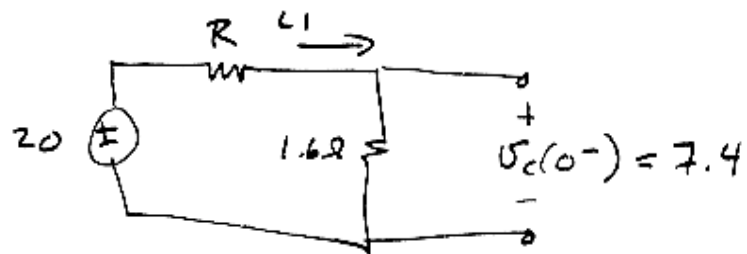
DP 7-1 We have $i(0) = \frac{30}{2+R} = 5$ & $v(0) = \frac{R}{2+R} 30 = 20$

Both relations above are satisfied for $R = 4\Omega$

DP 7-2 at $t = 0^-$



which becomes



by voltage division:

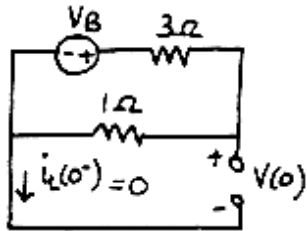
$$v_C(0^-) = \left(\frac{1.6}{1.6+R} \right) 20 = 7.4 \Rightarrow R = 2.7\Omega$$

$$\text{Then } i_1 = \left(\frac{20}{2.7+1.6} \right) = 4.6 \text{ A}$$

Check with current division

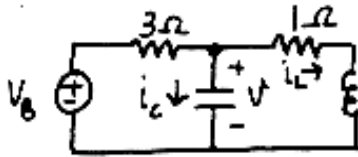
$$i_L(0^-) = \left(\frac{8}{2+8} \right) i_1 = 3.7 \text{ A} \Rightarrow i_1 = 4.6 \text{ A} \quad \underline{\text{OK}}$$

DP 7-3 at $t = 0^-$



$$v(0) = \frac{1}{4}v_B$$

at $t = 0^+$



Now $\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C}$ and $i_L(0^-) = i_L(0^+) = 0$

at top node: $\frac{v-v_B}{3} + i_L + i_c = 0$

$$i_c(0^+) = \frac{v_B - v(0^+)}{3} = \frac{v_B - 3}{3}$$

$$\frac{dv_c}{dt} = 24 = \frac{1}{\left(\frac{1}{8}\right)} \frac{(v_B - 3)}{3} \Rightarrow \underline{v_B = 12V}$$

DP 7-4

$$\frac{1}{2}Li_L^2 = \frac{1}{2}Cv_c^2 \quad (1) \quad \Leftrightarrow \text{in steady-state}$$

now in dc $i_L = v_c/R$ so (1) becomes $L\left(\frac{v_c}{R}\right)^2 = Cv_c^2 \Rightarrow C = \frac{L}{R^2}$

then $R = \sqrt{L/C} = \sqrt{\frac{10^{-2}}{10^{-6}}} = \sqrt{10^4} = 10^2$

So $R = 100 \Omega$