Chapter 7   Energy Storage Elements

Exercises

Ex. 7.3-1

$$i_C(t) = 1\frac{d}{dt}v_s(t) = \begin{cases} 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad i_R(t) = 1\,v_s(t) = \begin{cases} 2t-4 & 2 < t < 4 \\ 8-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{so } i(t) = i_C(t) + i_R(t) = \begin{cases} 2t-2 & 2 < t < 4 \\ 7-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$$

Ex. 7.3-2

$$v(t) = \frac{1}{C}\int_{t_0}^t i_s(\tau)\,d\tau + v(t_0) = \frac{1}{\frac{1}{3}}\int_0^t i_s(\tau)\,d\tau - 12$$

$$v(t) = 3\int_0^t 4\,d\tau - 12 = 12\,t - 12 \quad \text{for} \quad 0 < t < 4 \qquad \text{In particular, } v(4) = 36 \text{ V.}$$

$$v(t) = 3\int_4^t -2\,d\tau + 36 = 60 - 6t \quad \text{for} \quad 4 < t < 10 \qquad \text{In particular, } v(10) = 0 \text{ V.}$$

$$v(t) = 3\int_{10}^t 0\,d\tau + 0 = 0 \quad \text{for} \quad 10 < t$$

Ex. 7.3-3   Break up into intervals
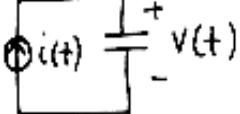
$$i = \begin{cases} 0 & t < 1 \\ 1 & 1 < t < 2 \\ 0 & t > 2 \end{cases} \qquad \text{now } v(t) = v(0) + \frac{1}{c}\int_0^t i\,dt = \frac{1}{10^{-4}}\int_0^t i\,dt$$

$$\therefore \quad 0 < t < 1 \Rightarrow v(t) = 10^4\int_0^t 0\,dt = \underline{0}$$

$$1 < t < 2 \Rightarrow v(t) = v(1) + 10^4\int_1^t 1\,dt = 0 + 10^4\,\underline{t = 10^4\,t\text{ V}}$$

$$t > 2 \quad \Rightarrow v(t) = v(2) + 10^4\int_2^t 0\,dt = 10^4(2) \underline{\underline{= 2\times10^4\text{ V}}}$$

Ex. 7.3-4

because $i(t) = 0$ when $t < 0$

$$v(t) = \frac{1}{c}\int_{-\infty}^t i(t)dt = \frac{1}{5}\int_0^t (2+2\cos 5t)\,dt + v(0) = \underline{\frac{2}{5}t + \frac{2}{25}\sin 5t\,\frac{1}{5}\text{ V}}$$

Ex. 7.4-1 $W = \frac{1}{2}Cv^2 = \frac{1}{2}(2\times10^{-4})(100)^2 = \underline{1\,J}$

$v_c(0^+) = v_c(0^-) = \underline{100\ V}$

Ex. 7.4-2 $W(t) = W(0) + \int_0^t vi\,dt \quad \Rightarrow\ W(0) = 0\ \text{since}\ v(0) = 0$

First find $v(t) = v(0) + \frac{1}{C}\int_0^t i\,dt = 10^4\int_0^t 2\,dt = \underline{2\times10^4 t}$

$\therefore\ W(t) = \int_0^t (2\times10^4)(2)dt = 2\times10^4 t^2$

$W(1s) = 2\times10^4\,J = \underline{20\ kJ}$

b) $W(100s) = 2\times10^4(100)^2 = 2\times10^8\,J = \underline{200\ mJ}$

Ex. 7.4-3

We have $v(0^+) = v(0^-) = 3V$

$V_c(t) = \frac{1}{C}\int_{-\infty}^t i(t)dt = 5\int_0^t 3e^{5t}\ dt + 3\ V = 3(e^{5t}-1)+3 = 3e^{5t}\ V$
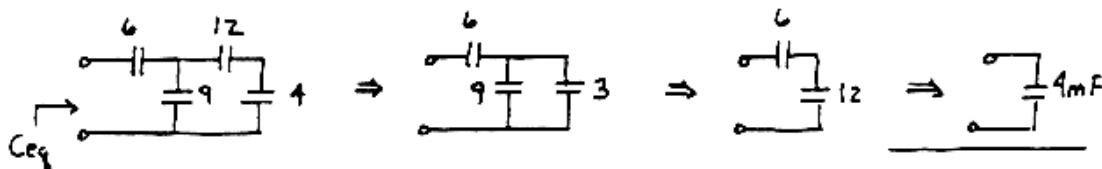
a)   Now $v(t) = v_R(t) + v_c(t) = 5i(t) + v_c(t) = 15e^{5t} + 3e^{5t} = \underline{18e^{5t}\ V},\ 0 \le t < 1$

b) $W_c(t) = \frac{1}{2}Cv_c^2(t) = \frac{1}{2}\times.2(3e^{5t})^2 = 0.9e^{10t}\ J$

Now when $t = 0.2s \Rightarrow W_c(t)\big|_{t=.2s} = \underline{6.65\ J}$

When $t = .8s \Rightarrow W_c(t)\big|_{t=.8s} = \underline{2.68k\ J}$

Ex. 7.5-1

Ex. 7.5-2

$v_1 = v_2 \Rightarrow \dfrac{dv_1}{dt} = \dfrac{dv_2}{dt} \Rightarrow \dfrac{i_1}{C_1} = \dfrac{i_2}{C_2} \Rightarrow \underline{i_1 = \dfrac{C_1}{C_2}i_2}$

from KCL : $i = i_1 + i_2 = \left(\dfrac{C_1}{C_2}+1\right)i_2 \Rightarrow \underline{i_2 = \dfrac{C_2}{C_1+C_2}i}$

Ex. 7.5-3

$\dfrac{1}{C_1} = \dfrac{1}{\frac{1}{3}} + \dfrac{1}{\frac{1}{3}} + \dfrac{1}{\frac{1}{3}} \Rightarrow C_1 = 1/9\,mF$

$C_2 = 1 + C_1 = 1 + 1/9 = 10/9\ mF$

So $\dfrac{1}{C_{eq}} = \dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{10/9}$, $1/C_{eq} = 19/10$

then $\underline{C_{eq} = \dfrac{10}{19}\ mF}$
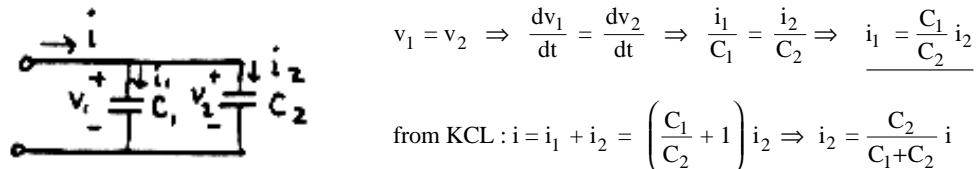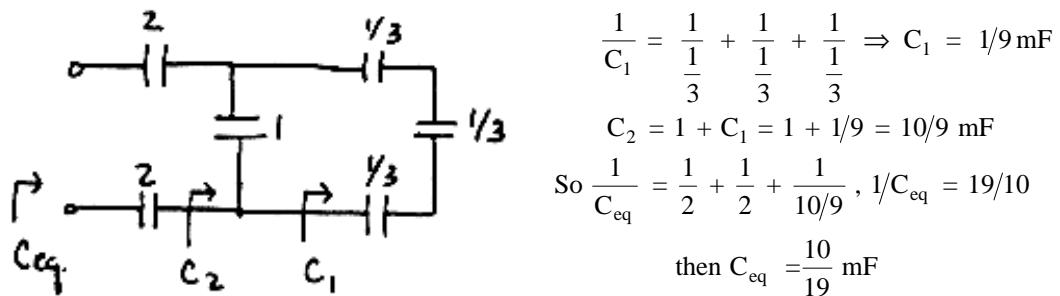
Ex. 7.6-1

$$v_L(t) = 1\frac{d}{dt}i_s(t) = \begin{cases} 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad v_R(t) = 1\,i_s(t) = \begin{cases} 2t-4 & 2 < t < 4 \\ 8-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$$

so $v(t) = v_L(t) + v_R(t) = \begin{cases} 2t-2 & 2 < t < 4 \\ 7-t & 4 < t < 8 \\ 0 & \text{otherwise} \end{cases}$

Ex. 7.6-2

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v_s(\tau)\,d\tau + i(t_0) = \frac{1}{\frac{1}{3}}\int_{0}^{t} v_s(\tau)\,d\tau - 12$$

$i(t) = 3\int_{0}^{t} 4\,d\tau - 12 = 12\,t - 12 \quad \text{for} \quad 0 < t < 4$ In particular, $i(4) = 36$ A.

$i(t) = 3\int_{4}^{t} -2\,d\tau + 36 = 60 - 6t \quad \text{for} \quad 4 < t < 10$ In particular, $i(10) = 0$ A.

$i(t) = 3\int_{10}^{t} 0\,d\tau + 0 = 0 \quad \text{for} \quad 10 < t$

Ex. 7.6-3

$$i(t) = i(0) + \frac{1}{L}\int_{0}^{t} v(t) = 0 + \frac{1}{1}\int_{0}^{t} 9t^2\,dt = \underline{3t^3\ A}$$

Ex. 7.7-1

$$v = L\frac{di}{dt} = \left(\frac{1}{4}\right)\frac{d}{dt}\left(4te^{-t}\right) = \underline{(1-t)e^{-t}\ V}$$

$$P = vi = \left[(1-t)e^{-t}\right]\left(4te^{-t}\right) = \underline{4t(1-t)e^{-2t}\ W}$$

$$W = \frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{4}\right)\left(4te^{-t}\right)^2 = \underline{2t^2\,e^{-2t}\ J}$$

Ex. 7.7-2

$$v(t) = L \frac{di}{dt} = \frac{1}{2} \frac{di}{dt}$$

$$i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ -2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases} \qquad \therefore v(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t<1 \\ -1 & 1<t<2 \\ 0 & t>2 \end{cases}$$

$$\text{now } p(t) = v(t)i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ 2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases}$$
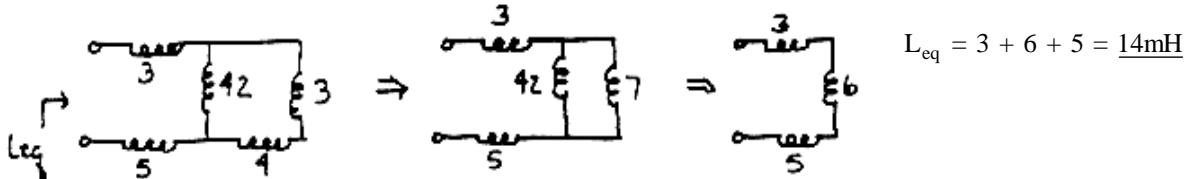
$$W(t) = W(t_0) + \int_{t_0}^{t} p(t)dt \quad \Rightarrow \quad t<0:\ W(t) = 0 \text{ since } p(t) = 0$$

$$0<t<1:\ W(t) = \int_0^t 4t\,dt = 2t^2$$
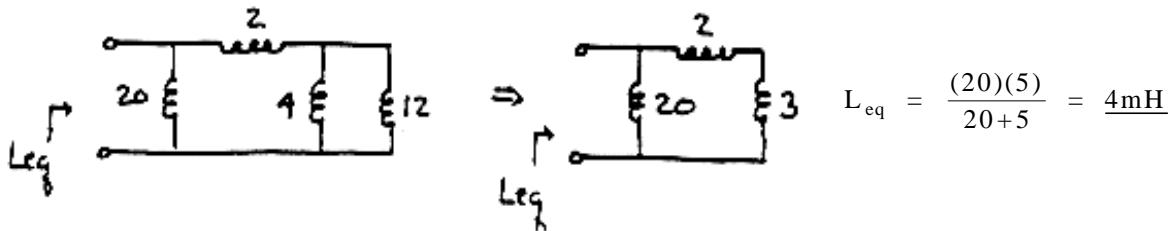
$$1<t<2:\ W(t) = W(1) + \int_1^t 4(t-2)dt = 2t^2 - 8t + 8$$

$$\underline{t>2:\ W(t) = W(2) = 0}$$

Ex. 7.8-1

$$L_{eq} = 3 + 6 + 5 = \underline{14mH}$$

Ex. 7.8-2

$$L_{eq} = \frac{(20)(5)}{20+5} = \underline{4mH}$$

Ex. 7.8-3

$$i = i_1 + i_2$$

$$i_1 = \frac{1}{L_1}\int v\,dt + i_1(0), \quad i_2 = \frac{1}{L_2}\int v\,dt + i_2(0) \qquad \text{but } i_1(0)=0 \text{ and } i_2(0)=0$$
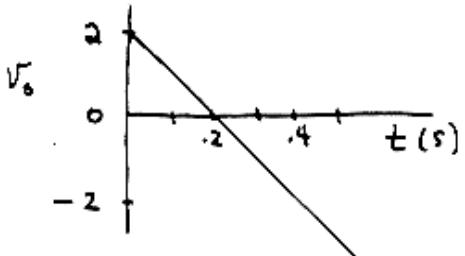
$$\text{so } i = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt, \text{ now also } i = \frac{1}{L_P}\int v\,dt + i(0) \qquad \text{where } i(0)=0$$

$$\therefore \frac{i_1}{i} = \frac{\frac{1}{L_1}\int v\,dt}{\frac{1}{L_P}\int v\,dt} = \frac{\frac{1}{L_1}}{\frac{1}{L_1}+\frac{1}{L_2}} = \frac{L_2}{L_1+L_2}$$

Ex. 7.10-1

$$v_o \;=\; -\frac{1}{RC}\int_0^t v_s\,dt \;+\; v_o(0) \;=\; -2\int_0^t 5\,dt + 2$$

$$\frac{1}{RC} \;=\; \frac{1}{\left(1\times10^3\right)\left(500\times10^{-6}\right)} \;=\; 2 \qquad \underline{v_o = (2-10\,t)\ \text{V}\ ,\ t\geq0}$$

Ex. 7.10-2

$$v_o \;=\; -RC\,\frac{dv_s}{dt} \;=\; -1\,\frac{d\left(0.2\sin 10t\right)}{dt} \;=\; \underline{-2\cos 10t\ \text{V},\ t\geq0}$$

$$RC \;=\; \left(20\times10^3\right)\left(50\times10^{-6}\right) \;=\; 1$$

Problems

Section 7-3:   Capacitors

P7.3-1

$$v \;=\; v(0) + \tfrac{1}{C}\int i\,dt \quad\text{and}\quad q \;=\; Cv$$

$$\therefore Cv = Cv(0) + \int i\,dt = Cv(0) + it \;\leftarrow\; \text{since } i = \text{constant}$$

$$\therefore\quad t \;=\; \frac{q - Cv(0)}{i} \;=\; \frac{150\mu C - \left(15\mu F\right)\left(5V\right)}{25mA} \;=\; \underline{3ms}$$

P7.3-2

$$i(t) \;=\; C\frac{d}{dt}v(t) \;=\; \frac{1}{8}\frac{d}{dt}12\cos\left(2t+30^\circ\right) \;=\; \frac{1}{8}(12)(2)\ \cos\left(2t+30^\circ+90^\circ\right) \;=\; 3\ \cos\left(2t+120^\circ\right)\ \text{A}$$

P7.3-3

$$\left(3{\cdot}10^{-3}\right)\ \cos\left(500t+45^\circ\right) \;=\; C\,\frac{d}{dt}\ 12\ \cos\left(500t-45^\circ\right) \;=\; C\,(12)(500)\ \cos\left(500t-45+90\right) \;=\; C\,(6000)\ \cos\left(500t+45^\circ\right)$$

$$\text{so}\quad C = \frac{3{\cdot}10^{-3}}{6{\cdot}10^3} \;=\; \frac{1}{2}{\cdot}10^{-6} \;=\; \frac{1}{2}\ \mu F$$

P7.3-4

$$v(t) = -1\ \text{mV} + \frac{1}{2\,\text{pF}} \int_0^t i_s(\tau)\, d\tau$$

for $0 \le t < 2\,\text{ns}$ $i_s(t) = 0$ So $v(t) = -1\ \text{mV}$.

for $2\,\text{ns} \le t < 3\,\text{ns}$ $i_s(t) = 4\,\mu A$

$$v(t) = -1\ \text{mV} + \frac{1}{2\ \text{pF}} \int_{2\text{ns}}^t 4\mu s\, d\tau = -5\,\text{mV} + \left(2\,\frac{\mu A}{\text{pF}} \right) t$$

$v(2\,\text{ns})\text{------}\uparrow$

$$v(3\,\text{ns}) = -5\,\text{mV} + 2\,\frac{\mu A}{\text{pF}} \cdot 3\,\text{ns} = 1\,\text{mV}$$

for $3\ \text{ns} \le t < 5\,\text{ns}$ $i_s(t) = -2\,\mu A$

$$v(t) = 1\ \text{mV} + \frac{1}{2\ \text{pF}} \int_{3\text{ns}}^t -2\mu A\ d\tau = 4\ \text{mV} - \left(\frac{\mu A}{\text{pF}} \right) t$$

$$v(5\ \text{ns}) = 4\,\text{mV} - \left(\frac{\mu A}{\text{pF}} \right) 5\,\text{ns} = -1\ \text{mV}$$

for $5\,\text{ns} < t$ $i_s(t) = 0$ so $v(t)$ remains $-1\,\text{mV}$.

P 7.3-5

(a) $i(t) = C \dfrac{d}{dt} v(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \\ -4 & 2 < t < 3 \\ 0 & 3 < t \end{cases}$

(b) $v(t) = \dfrac{1}{C} \int_0^t i(\tau)\, d\tau + v(0) = \int_0^t i(\tau)\, d\tau$

For $0 < t < 1$, $i(t) = 0$ A so $v(t) = \int_0^t 0\, d\tau + 0 = 0$ V

For $1 < t < 2$, $i(t) = 4t - 4$ A so $v(t) = \int_0^t (4\tau - 4)\, d\tau + 0 = \left(2\tau^2 - 4\tau \right) \Big|_1^t = 2t^2 - 4t + 2$ V

$v(2) = 4\left(2^2 \right) - 4(2) + 2 = 2$ V . For $2 < t < 3$, $i(t) = -4t + 12$ A so

$v(t) = \int_0^t (-4\tau + 12)\, d\tau + 2 = \left(-2\tau^2 + 12\tau \right) \Big|_1^t + 2 = -2t^2 + 12t - 14$ V

$v(3) = -4\left(3^2 \right) + 12(3) - 14 = 4$ V

For $0 < t < 1$, $i(t) = 0$ A   so $v(t) = \int_0^t 0\, d\tau + 4 = 4$ V

P 7.3-6

(a) $i(t) = C \dfrac{d}{dt} v(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$

(b) $v(t) = \dfrac{1}{C} \displaystyle\int_0^t i(\tau)\, d\tau + v(0) = 2 \int_0^t i(\tau)\, d\tau$

For $0 < t < 2$, $i(t) = 0$ A so $v(t) = 2 \displaystyle\int_0^t 0\, d\tau + 0 = 0$ V

For $2 < t < 6$, $i(t) = 0.2\, t$ - 0.4 V so
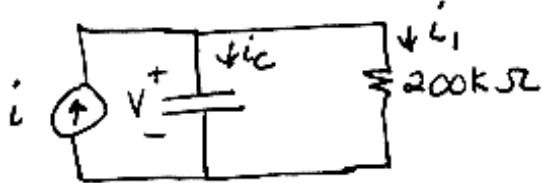
$v(t) = 2 \displaystyle\int_0^t (0.2\tau - 0.4)\, d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_1^t = 0.2t^2 - 0.8t + 0.8$ V

$v(6) = 0.2\left(6^2\right) - 0.8(6) + 0.8 = 3.2$ V . For $6 < t$, $i(t) = 0.8$ A so $v(t) = 2\displaystyle\int_0^t 0.8\, d\tau + 3.2 = 1.6t - 6.4$ V

P7.3-7

$v(t) \;=\; v(0) \;+\; \dfrac{1}{C}\displaystyle\int_0^t i\, dt \;=\; 25 \;+\; 2.5 \times 10^4 \int_0^t \left(6 \times 10^{-3}\right) e^{-6t}\, dt \;=\; 25 + 150 \int_0^t e^{-6t}\, dt = 25 + 150\left[-\left(\tfrac{1}{6}\right) e^{-6t} \right]_0^t$

$v(t) \;=\; \underline{50 - 25 e^{-6t}}$ V

P7.3-8
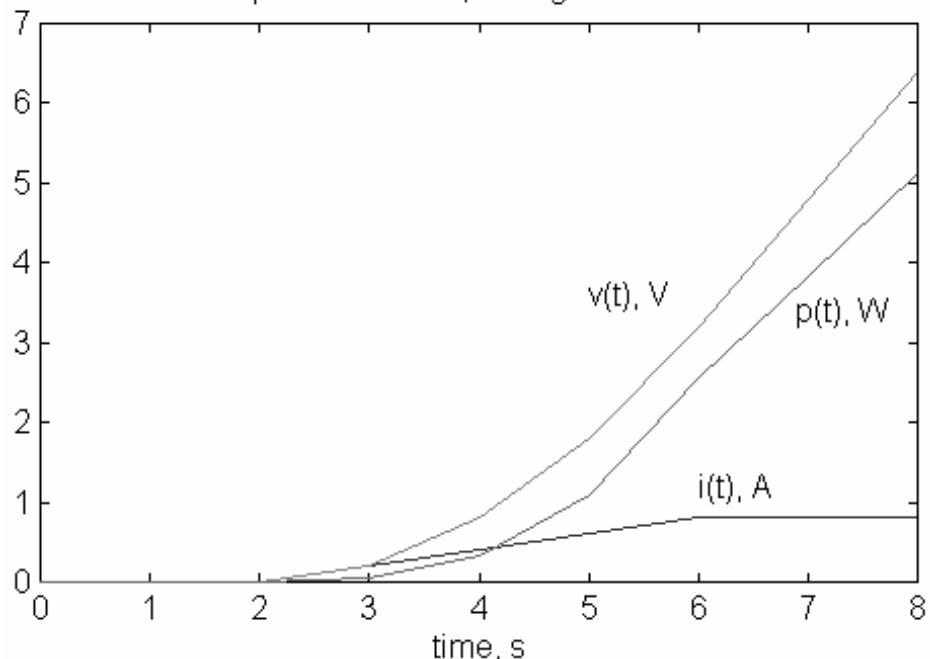
$i_1 \;=\; \dfrac{v}{200} \;=\; \dfrac{1}{40}\left(1 - 2e^{-2t}\right) \text{mA} \;=\; 25\left(1 - 2e^{-2t}\right) \mu A$

$i_C \;=\; C\dfrac{dv}{dt} \;=\; 10(-2)\left(-10 e^{-2t}\right) \;=\; 200 e^{-2t} \mu A$

$\therefore i \;=\; i_1 + i_c \;=\; 200 e^{-2t} + 25 - 50 e^{-2t}$

$\;=\; \underline{25 + 150 e^{-2t} \mu A}$
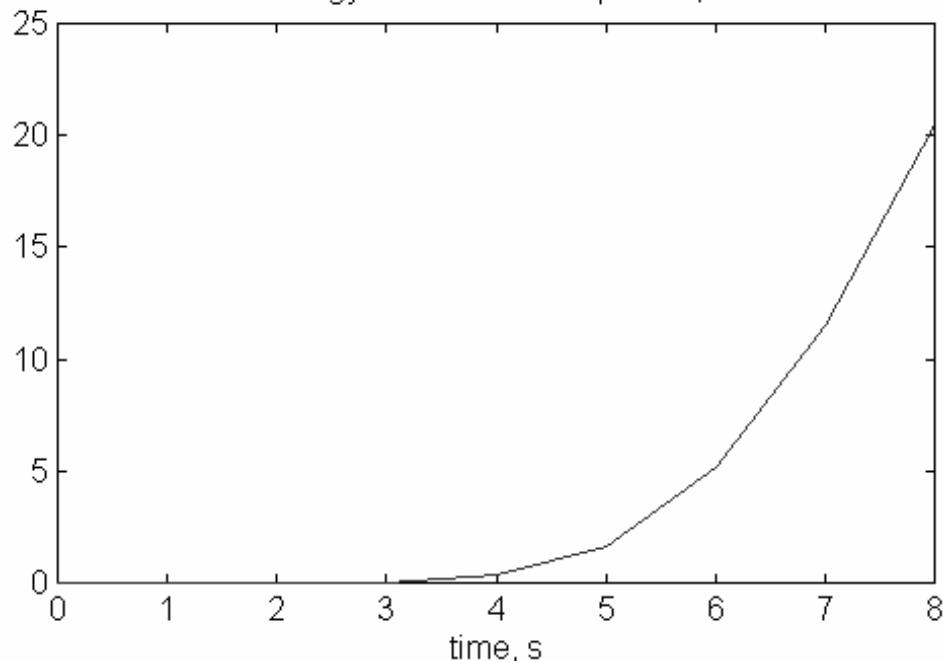
P7.4-1

Capacitor Current, Voltage and Power

Energy Stored in the Capacitor, J

These plots were produced using three MATLAB scripts:

capvol.m	```
	function v = CapVol(t)
	  if t<2
	    v = 0;
	  elseif t<6
	    v = 0.2*t*t - .8*t +.8;
	  else
	    v = 1.6*t - 6.4;
	  end
	``

capcur.m	```
	function i = CapCur(t)
	  if t<2
	    i=0;
	  elseif t<6
	    i=.2*t - .4;
	  else
	    i =.8;
	  end
	``

c7s4p1.m	```
	t=0:1:8;
	for k=1:1:length(t)
	 i(k)=CapCur(k-1);
	 v(k)=CapVol(k-1);
	 p(k)=i(k)*v(k);
	 w(k)=0.5*v(k)*v(k);
	end
	
	 plot(t,i,t,v,t,p)
	 text(5,3.6,'v(t), V')
	 text(6,1.2,'i(t), A')
	 text(6.9,3.4,'p(t), W')
	 title('Capacitor Current, Voltage and Power')
	 xlabel('time, s')
	
	 % plot(t,w)
	 % title('Energy Stored in the Capacitor, J')
	 % xlabel('time, s')
	``
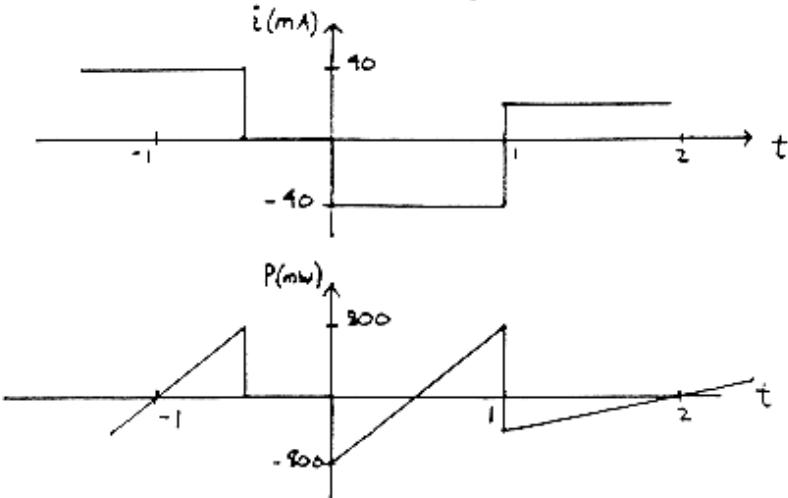
P7.4-2

$$i_c = C\frac{dv}{dt} = \left(10\times10^{-6}\right)\left(-5\right)\left(-4000\right)e^{-4000t} = \underline{0.2e^{-4000t}\ A}$$

$$\underline{i_c(0) = 0.2\ A} \qquad \underline{i_c(10ms) = 8.5\times10^{-19}A}$$

$$W(t) = \frac{1}{2}Cv^2(t) \qquad v(0) = 5-5e^0 = 0 \qquad \underline{W(0) = 0}$$

$$v(10ms) = 5-5e^{-40} = 5 - 4.24\times10^{-18} \cong 5 \qquad \underline{W(10) = 1.25\times10^{-4}J}$$

P7.4–3 $i(t) = C \, dv_c / dt$ so read off slope of $v_c(t)$ to get $i(t)$

$P(t) = v_c(t)i(t)$ so multiply $v_c(t)$ & $i(t)$ curves to get $p(t)$

P7.4-4 $v_c(t) = v_c(0) + \frac{1}{C}\int_0^t i \, dt = v_c(0) + \frac{1}{2}\int_0^t 50 \cos\left(10t + \frac{\pi}{6}\right) dt = \left[v_c(0) - \frac{5}{2}\sin\frac{\pi}{6}\right] + \frac{5}{2}\sin\left(10t + \frac{\pi}{6}\right)$

Now since $v_c(t)_{ave} = 0 \Rightarrow v_c(0) = \frac{5}{2}\sin\frac{\pi}{6}$

$\therefore v_c(t) = \frac{5}{2}\sin\left(10t + \frac{\pi}{6}\right)$ V

$\therefore W_{max} = \frac{1}{2}Cv_{c_{max}}^2 = \frac{1}{2}(2)(2.5)^2 = \underline{6.25\mu \, J}$

First non-negative t for max energy occurs : $10t + \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow \underline{t = \frac{\pi}{30} = 0.1047 \text{ s}}$

P7.4-5

$W = \frac{1}{2}Cv^2 \Rightarrow v = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2\left(4e^{-10t}\right)}{.001}} = 40\sqrt{5}\,e^{-5t}$

$\therefore \underline{v(0.1) = 40\sqrt{5}\,e^{-5(.1)} = 54.25 \text{ V}}$

$i(t) = C\,\frac{dv_c}{dt} = \left(10^{-3}\right)(-5)\left(40\sqrt{5}\,e^{-5t}\right) = \frac{1}{\sqrt{5}}e^{-5t}$

$\therefore \underline{i(0.1) = \frac{1}{\sqrt{5}}e^{-5(.1)} = -0.2712 \text{ A}}$

P7.4-6   Max. charge on capacitor $= Cv = (10\mu F)(6V) = 60\mu C$

$\Delta t = \frac{\Delta q}{i} = \frac{60\mu C}{10\mu A} = \underline{6 \text{ sec}}$ to charge

stored energy $= W = \frac{1}{2}Cv^2 = \frac{1}{2}(10\mu F)(6V)^2 = \underline{180\mu \, J}$

P7.4-7

$$P(\Delta t) = \left(1.5 \times 10^3\,W\right)(1hr)\left(\frac{60min}{1hr}\right)\left(\frac{60sec}{1min}\right) = 5.4 \times 10^6\,J$$

$$W = \frac{1}{2}Cv^2 \Rightarrow C = \frac{2W}{v^2}$$

$$C = \frac{2\left(5.4 \times 10^6\right)}{(500)^2} = 43.2\ F$$

$$\text{size of capacitor} = (43.2F)\left(\frac{10^6\,\mu F}{F}\right)\left(\frac{1cm^3}{100\mu F}\right)$$

$$\text{size} = 4.32 \times 10^5\ cm^3\left(\frac{1m}{100cm}\right)^3 = \underline{0.432\ m^3}$$

Capacitor may be practical, it depends on size of car! Would be 76 cm on a side- pretty large !

Section 7-5: Series and Parallel capacitors

P7.5-1

$$2\mu F \parallel 4\mu F = 6\mu F$$

$$6\mu F \text{ in series with } 3\mu F = \frac{6\mu F \cdot 3\mu F}{6\mu F + 3\mu F} = 2\mu F$$

$$i(t) = 2\mu F\,\frac{d}{dt}\,(6\cos 100t) = (2 \cdot 10^{-6})\,(6)\,(100)\,(-\sin 100t) = \underline{-1.2\ \sin 100t\ mA}$$

P7.5-2

$$4\mu F \text{ in series with } 4\mu F = \frac{4\mu F \cdot 4\mu F}{4\mu F + 4\mu F} = 2\mu F$$

$$2\ \mu F \parallel 2\mu F = 4\mu F$$

$$4\mu F \text{ in series with } 4\mu F = 2\mu F$$

$$i(t) = 2\mu F\,\frac{d}{dt}(3 + 3e^{-250t}) = (2 \cdot 10^{-6})\,(0 + 3(-250)e^{-250t}) = -1.5 \cdot 10^{-3}\ e^{-250t}\ A = \underline{-1.5\ e^{-250t}\ mA}$$

P7.5-3 C in series with $C = \dfrac{C \cdot C}{C + C} = \dfrac{C}{2}$

$$C \parallel C \parallel \frac{C}{2} = \frac{5}{2}C$$

$$C \text{ in series with } \frac{5}{2}C = \frac{C \cdot \frac{5}{2}C}{C + \frac{5}{2}C} = \frac{5}{7}C$$

$$(25 \cdot 10^{-3})\cos 250t = \frac{5}{7}C\,\frac{d}{dt}(14\ \sin 250t\ ) = \left(\frac{5}{7}C\right)(14)\,(250)\cos 250t$$

$$\text{so } 25 \cdot 10^{-3} = 2500C \Rightarrow C = 10 \cdot 10^{-6} = 10\mu F$$

Section 7-6: Inductors
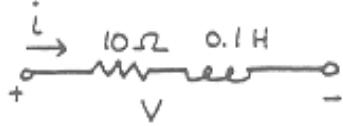
P7.6-1   Find max. voltage across coil:

$$v(t) = L\,\frac{di}{dt} = 200\left[100(400)\cos 400t\right]\text{ V}$$

$$\therefore\ v_{max} = 8\times10^6\text{V}$$

thus have a field of $8\times10^6\,\text{V}/{2m} = 4\times10^6\ \text{V}/m$ which exceeds dielectric strength in air of

3×10^6 V/m $\therefore$ will get a discharge as the air is ionized.

P7.6-2

$$v = L\frac{di}{dt}+10\,i$$

$$= (.1)(4e^{-t}-4te^{-t}) + 10(4te^{-t})$$

$$= 0.4\,e^{-t} + 39.6t\,e^{-t}\text{ V}$$

P 7.6-3

(a) $v(t) = L\dfrac{d}{dt}i(t) = \begin{cases} 0 & 0<t<1 \\ 4 & 1<t<2 \\ -4 & 2<t<3 \\ 0 & 3<t \end{cases}$

(b) $i(t) = \dfrac{1}{L}\displaystyle\int_0^t v(\tau)\,d\tau + i(0) = \int_0^t v(\tau)\,d\tau$

For $0<t<1$, $v(t) = 0$ V so $i(t) = \displaystyle\int_0^t 0\,d\tau + 0 = 0$ A

For $1<t<2$, $v(t) = 4t$ - 4 V so $i(t) = \displaystyle\int_0^t (4\tau - 4)\,d\tau + 0 = \left(2\tau^2 - 4\tau\right)\Big|_1^t = 2t^2 - 4t + 2$ A

$i(2) = 4\left(2^2\right) - 4(2) + 2 = 2$ A

For $2<t<3$, $v(t) = -4t + 12$ V so

$i(t) = \displaystyle\int_0^t (-4\tau + 12)\,d\tau + 2 = \left(-2\tau^2 + 12\tau\right)\Big|_1^t + 2 = -2t^2 + 12t - 14$ A

$i(3) = -4\left(3^2\right) + 12(3) - 14 = 4$ A

For $0<t<1$, $v(t) = 0$ V so $i(t) = \displaystyle\int_0^t 0\,d\tau + 4 = 4$ A

P7.6-4

$$v(t) = (250 \cdot 10^{-3}) \ \frac{d}{dt} \ (120 \cdot 10^{-3}) \ \sin(500t - 30^\circ) = (.25)(.12)(500) \ \sin(500t - 30^\circ + 90^\circ) = 15 \sin(500t + 60^\circ)$$

P7.6-5 $i_L(t) = \dfrac{1}{5\,\text{mH}} \ \displaystyle\int_0^t v_s(\tau)\,d\tau \ - \ 2\mu A$

for $0 \leq t < 1\ \mu s$ $v_s(t) = 4\,\text{mV}$

$$i_L(t) = \frac{1}{5\text{mH}} \ \int_0^t 4\,\text{mV}\,d\tau \ - \ 2\mu A = \left(\frac{4}{5} \ \frac{\text{mV}}{\text{mH}}\right) t - 2\mu A$$

$$i_L\ (1\mu s) = \left(\frac{4}{5} \ \frac{\text{mV}}{\text{mH}} \cdot 1\mu s\right) - 2\mu A = \ - \ \frac{6}{5} \mu A$$

for $1\mu s \leq t < 3\mu s$ $v_s(t) = -1\ \text{mV}$

$$i_L\ (t) = \frac{1}{5\text{mH}} \ \int_{1\mu s}^t -1\text{mV}\,d\tau \ - \ \frac{6}{5}\mu A = -\frac{1}{5}\frac{\text{mV}}{\text{mH}}\ (t-1\mu s) \ - \ \frac{6}{5}\mu A = \left(-\frac{1\,\text{mV}}{5\,\text{mH}}\right) t - 1\mu A$$

$$i_L\ (3\mu s) = \left(-\frac{1\,\text{mV}}{5\,\text{mH}} \cdot 3\mu s\right) - 1\mu A = -\frac{8}{5}\mu A$$

for $3\mu s \ \leq \ t \ \ v_s(t) = 0$ so $i_L(t)$ remains $-\dfrac{8}{5}\ \mu A$

P7.6-6 $v(t) = 2k\Omega \cdot i_s\ (t) + 4\text{mH} \cdot \dfrac{d}{dt}\ i_s\ (t)$ (in general)

for $0 < t < 1\ \mu s$ $i_s(t) = 1\dfrac{\text{mA}}{\mu s} \cdot t \Rightarrow \dfrac{d}{dt} i_s(t) = 1\dfrac{\text{mA}}{\mu s}$

so $v(t) = (2k\Omega)\left(1\dfrac{\text{mA}}{\mu s}\right) t + 4\ \text{mH}\left(1\ \dfrac{\text{mA}}{\mu s}\right) = \left(\dfrac{2V}{\mu s}\right) t + 4\ V$

for $1\mu s < t < 3\mu s$ $i_s(t) = 1\text{mA} \Rightarrow \dfrac{d}{dt}\ i_s(t) = 0$

so $v(t) = 2k\Omega \cdot 1\text{mA} + 4\text{mH} \cdot 0 = 2\ V$

for $3\mu s < t < 5\mu s$ $i_s\ (t) = 4\text{mA} - \left(1\dfrac{\text{mA}}{\mu s}\right) t$

so $\dfrac{d}{dt}i_s(t) = -1\ \dfrac{\text{mA}}{\mu s}$ and $v(t) = 4\,V - \left(\dfrac{2V}{\mu s}\right) t$

when $5\mu s < t < 7\mu s$ $i_s(t) = -1\text{mA}$ and $\dfrac{d}{dt}i_s(t) = 0$

so $v(t) = -2\ V$

when $7\mu s < t < 8\mu s$ $i_s(t) = 1\ \dfrac{\text{mA}}{\mu s} \cdot t - 8\text{mA}$

so $\dfrac{d}{dt}i_s\ (t) = 1\ \dfrac{\text{mA}}{\mu s}$

and $v(t) = -12\ V + \left(\dfrac{2\ V}{\mu s}\right) t$

when $8\mu s < t$, then $i_s(t) = 0 \Rightarrow \dfrac{d}{dt}i_s(t) = 0$

so $v(t) = 0$