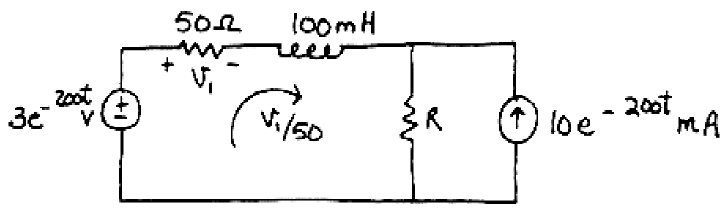
P7.6-7



$v_1(t) = e^{-200t}\,V$

$$\text{KVL a: } -3e^{-200t} + v_1 + .1\frac{d}{dt}\left(\frac{v_1}{50}\right) + R\left[\frac{v_1}{50} + .01e^{-200t}\right] = 0$$

$$-3e^{-200t} + e^{-200t} - .4e^{-200t} + R\left[\frac{1}{50} + .01\right]e^{-200t} = 0$$

$$-2.4 + R(.03) = 0 \Rightarrow \underline{R = \frac{2.4}{.03} = 80\Omega}$$

P 7.6-8

(a) $$v(t) = L\frac{d}{dt}i(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$$

(b) $$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0) = 2\int_0^t v(\tau)\,d\tau$$

For $0 < t < 2$, $v(t) = 0$ V so $i(t) = 2\int_0^t 0\,d\tau + 0 = 0$ A

For $2 < t < 6$, $v(t) = 0.2\,t - 0.4$ V so

$$i(t) = 2\int_0^t (0.2\tau - 0.4)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_1^t = 0.2t^2 - 0.8t + 0.8 \text{ A}$$

$i(6) = 0.2\left(6^2\right) - 0.8(6) + 0.8 = 3.2$ A .

For $6 < t$, $v(t) = 0.8$ V so $i(t) = 2\int_0^t 0.8\,d\tau + 3.2 = 1.6t - 6.4$ A

<u>Section 7-7</u>: Energy Storage in an Inductor

P7.7-1

$$v(t) = 100 \cdot 10^{-3}\frac{d}{dt}i(t) = \begin{cases} 0 & t < 0 \\ 0.4 & 0 < t > 1 \\ 0 & t > 1 \end{cases}$$

$$p(t) = v(t)\,i(t) = \begin{cases} 0 & t < 0 \\ 1.6t & 0 < t < 1 \\ 0 & t > 1 \end{cases}$$

$$W(t) = \int_0^t p(\tau)\,d\tau = \begin{cases} 0 & t < 0 \\ 0.8t^2 & 0 < t < 1 \\ 0.8 & t > 1 \end{cases}$$

P7.7-2

$$p(t)=v(t)\,i(t) = \left[5\frac{d}{dt}(4\sin 2t)\right](\,4\sin 2t\,) = 5\,(8\cos 2t)\,(4\sin 2t)$$

$$= 80\,[2\cos 2t\sin 2t] = 80\,[\sin(2t+2t) + \sin(2t-2t)] = 80\sin 4t \text{ W}$$

$$W(t)=\int_0^t p(\tau)\,d\tau = 80\int_0^t \sin 4\tau\,d\tau = -\frac{80}{4}\,[\cos 4\tau\,|_0^t\,] = 20\,(1-\cos 4t)$$

P7.7-3

$$i(t)= \frac{1}{25\cdot 10^{-3}}\int_0^t 6\cos 100t + 0 = \frac{6}{(25\cdot 10^{-3})(100)}\,[\sin 100\tau\,|_0^t\,] = 2.4\sin 100\,t$$

$$p(t) = v(t)\,i(t) = (6\cos 100t)(2.4\sin 100t) = 7.2\,[\,2(\cos 100t)(\sin 100t)\,] = 7.2\,[\,\sin 200t + \sin 0\,] = 7.2\sin 200t$$

$$W(t) = \int_0^t p(\tau)\,d\tau = 7.2\int_0^t \sin 200\tau\,d\tau = -\frac{7.2}{200}\left[\cos 200\tau|_0^t\right] = 0.036[1-\cos 200t]\text{J} = 36\,[1-\cos 200t\,]\text{ mJ}$$

<u>Section 7-8</u>: Series and Parallel Inductors

P7.8-1

$$6H \parallel 3H = \frac{6H\cdot 3H}{6H+3H} = 2H$$

$$2H + 2H = 4H$$

$$i(t) = \frac{1}{4}\int_0^t 6\cos 100\tau\;d\tau = \frac{6}{4\cdot 100}\left[\sin 100\tau|_0^t\right] = 15\sin 100t \text{ mA}$$

P7.8-2

$$4mH + 4mH = 8mH$$

$$8mH \parallel 8mH = \frac{8mH\cdot 8mH}{8mH+8mH} = 4mH$$

$$4mH + 4mH = 8mH$$

$$v(t) = (8\cdot 10^{-3})\frac{d}{dt}(5+3e^{-250t})(10^{-3}) = (8\cdot 10^{-6})(0+3(-250)e^{-250t}) = -6e^{-250t}\text{ mV}$$

P7.8-3

$$L \parallel L = \frac{L\cdot L}{L+L} = \frac{L}{2}$$

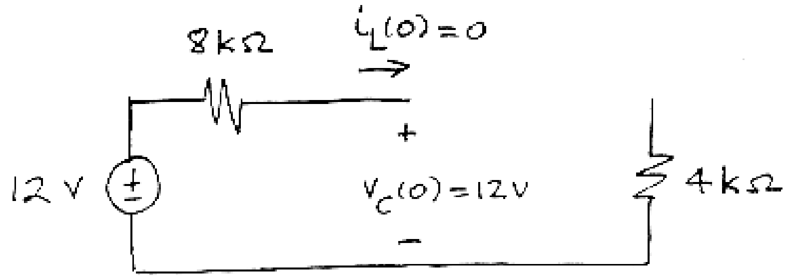
$$L + L + \frac{L}{2} = \frac{5}{2}L$$

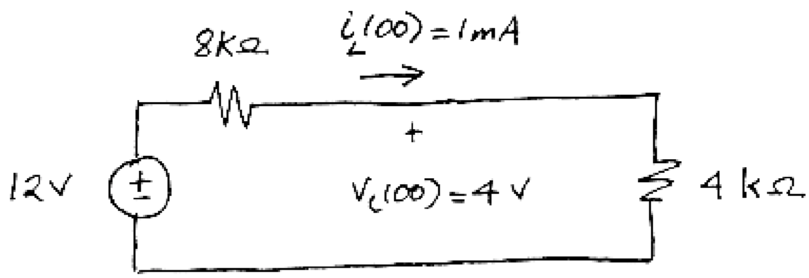
$$25\cos 250t = \frac{5}{2}L\frac{d}{dt}(14\cdot 10^{-3}\sin 250\,t) = \left(\frac{5}{2}L\right)(14\cdot 10^{-3})(250)\cos 250t$$

$$\text{so } L = \frac{25}{\frac{5}{2}(14\cdot 10^{-3})(250)} = 2.86\text{ H}$$

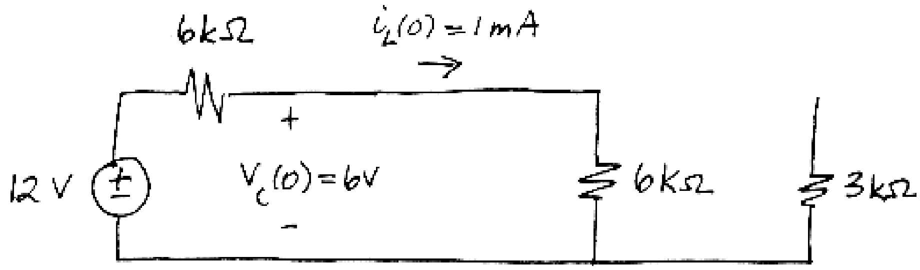
Section 7-9: Initial Conditions of Switched Circuits

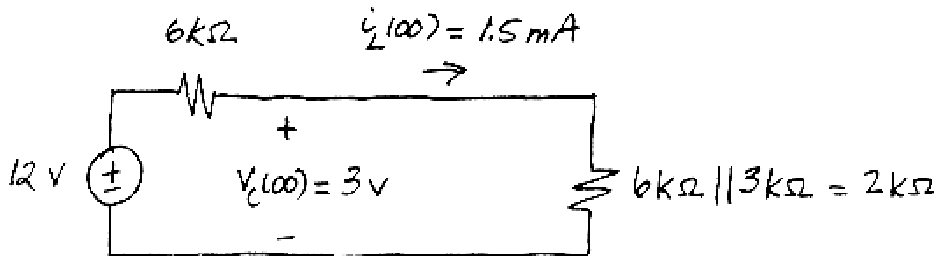
P7.9-1



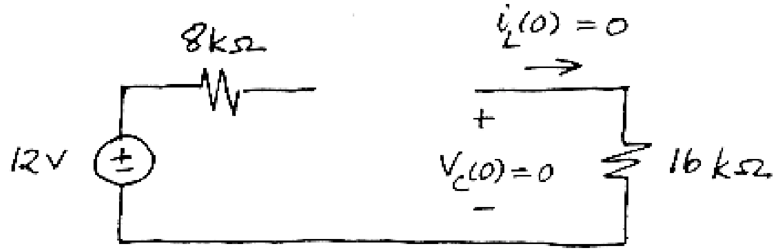


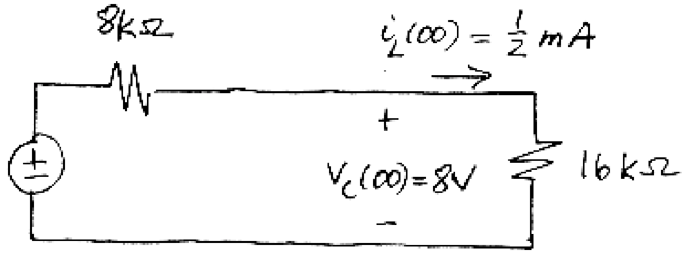
P7.9-2



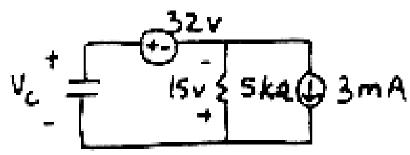


P7.9-3





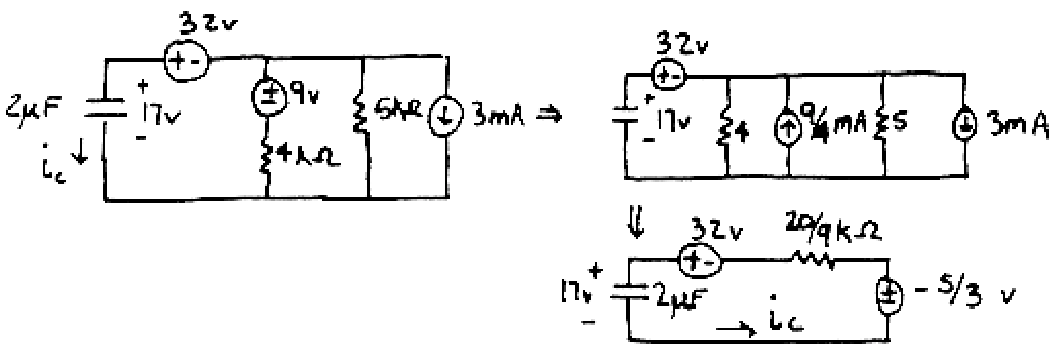
P7.9-4 at $t=0^-$



KVL: $-v_c(0^-)+32-15=0$

$\Rightarrow \underline{v_c(0^-)=v_c(0^+)=17\text{ V}}$

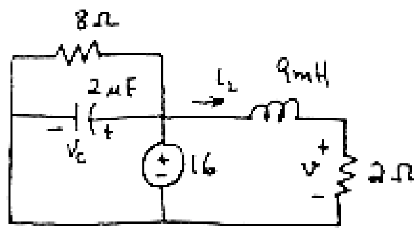
at $t=0^+$



KVL: $\frac{5}{3}+\left(\frac{20}{9}\right)i_c-32+17=0 \;\Rightarrow \underline{i_c(0^+)\;=\;6\text{mA}}$

Now $i_c(0^+)\;=\;C\dfrac{dv_c(0^+)}{dt} \;\Rightarrow\; \dfrac{dv_c(0^+)}{dt}\;=\;\dfrac{6\text{mA}}{2\mu\text{F}}\;=\;\underline{3\,\text{V}/\text{ms}}$

P7.9-5 at $t=0^-$

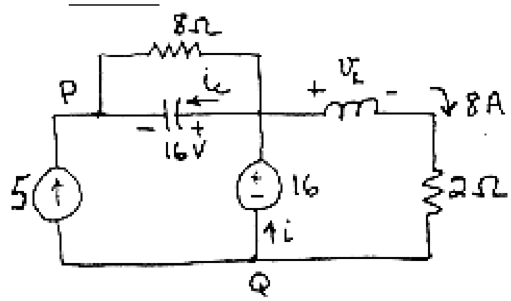


at steady-state

$$v_c(0^-) = v_c(0^+) = 16\text{V}$$

and $$i_L(0^-) = i_L(0^+) = \frac{16}{2} = 8\text{A}$$

at $t = 0^+$



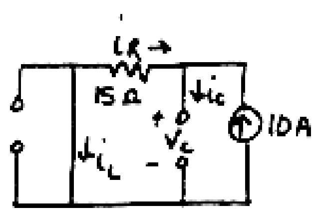
KVL: $-16 + v_L + 8(2) = 0 \Rightarrow v_L(0^+) = 0$

KCL at P: $-5 - i_c - \frac{16}{8} = 0 \Rightarrow i_c(0^+) = -7\text{A}$

KCL at Q: $5 + i - 8 = 0 \Rightarrow i(0^+) = 3\text{A}$

Then $$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = 0 \text{ and } \frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{-7}{2\mu\text{F}} = -3.5\,\text{A}/\mu\text{s}$$

P7.9-6 $t=0^-$

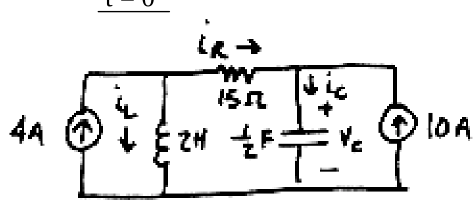


$v_c(0^-) = (15)10 = \underline{150\text{V}}$ $\quad$ $i_c(0^-) = \underline{0}$

$v_L(0^-) = \underline{0\text{V}}$ $\quad$ $i_L(0^-) = \underline{10\text{A}}$

$v_R(0^-) = 15(-10) = -\underline{150\text{V}}$ $\quad$ $i_R(0^-) = -\underline{10\text{A}}$

$t = 0^+$



$v_c(0^+) = v_c(0^-) = \underline{150\text{V}}$ $\quad$ $i_L(0^+) = i_L(0^-) = \underline{10\text{A}}$

$v_R(0^+) = -6(15) = -\underline{90\text{V}}$ $\quad$ $i_R(0^+) = 4\text{A} - 10\text{A} = \underline{-6\text{A}}$

$v_L(0^+) = v_R(0^+) + v_c(0^+) = \underline{60\text{V}}$ $\quad$ $i_c(0^+) = i_R(0^+) + 10\text{A} = 4\text{A}$

Section 7-10: The Operational Amplifier and RC Circuits

P7.10-1

$$v_0(t) = -\frac{1}{RC}\int_0^t v_s(\tau)d\tau + v_0(0) = \frac{-1}{(20 \cdot 10^3)(2 \cdot 10^{-6})}\int_0^t 12\cos 100\tau \, d\tau + 0$$

$$= -25\left(\frac{12 \sin 100\tau}{100}\right)\Bigg|_0^t = -3 \sin 100t$$

P7.10-2

$$v_0(t) = -\frac{1}{RC}\int_0^t v_s(\tau)\,d\tau + v_0(0) = \frac{-1}{(2\cdot10^3)(10^{-6})}\int_0^t -4d\tau + 0 = 2000\,t \qquad 0<t<3ms$$

$$v_0(3ms) = (2\cdot10^3)(3\cdot10^{-3}) = 6$$

$$v_0(t) = -\frac{1}{RC}\int_3^t 0d\tau + 6 = 6 \qquad t>3ms$$

P7.10-3

$$250t = -\frac{1}{RC}\int_0^t -5\,dt\text{ , when }0<t<20ms = \frac{5}{RC}\,t$$

$$\text{so}\quad 250 = \frac{5}{RC} \Rightarrow RC = \frac{5}{250} = \frac{1}{50}$$

$$\text{Let } C = 1\mu F\text{, then } R = \frac{1}{50(10^{-6})} = 20\cdot10^3 = 20k\Omega$$

Verification Problems

VP 7-1

$$\text{at } t=1 \qquad 0.025 \overset{?}{=} -\frac{1}{2} + 0.065$$

$$\text{at } t=3 \qquad -\frac{3}{25} + 0.065 \overset{?}{=} \frac{3}{50} - 0.115$$

$$-0.55 \neq -0.485$$

The equation for the inductor current indicates that this current changes instantaneously at t = 3s. This equation cannot be correct.

VP 7-2 We need to check the values of the inductor current at the ends of the intervals.

$$\text{at } t = 1 \qquad -\frac{1}{200} + 0.025 \overset{?}{=} -\frac{1}{100} + 0.03 \qquad \text{Yes}$$

$$\text{at } t = 4 \qquad -\frac{4}{100} + 0.03 \overset{?}{=} \frac{4}{100} - 0.03 \qquad \text{No}$$

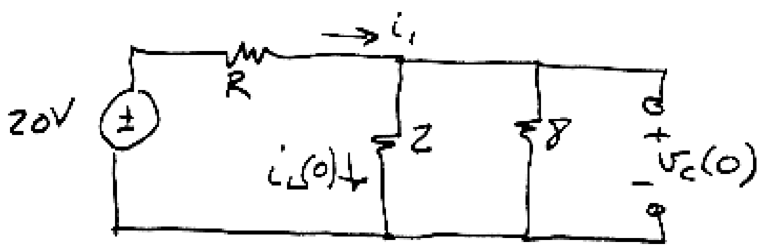
The equation for the inductor current indicates that this current changes instantaneously at t = 4s. This equation cannot be correct.

Design Problems

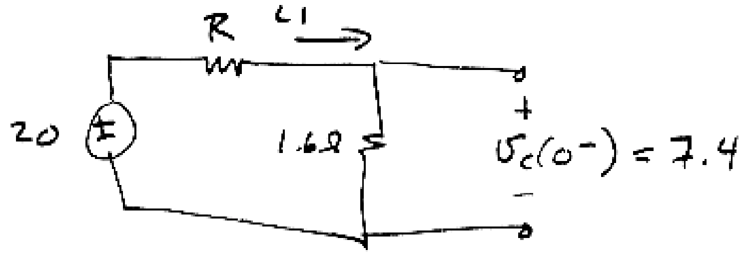
DP 7-1 We have $i(0) = \frac{30}{2+R} = 5$ & $v(0) = \frac{R}{2+R} 30 = 20$

Both relations above are satisfied for <u>R = 4Ω</u>

DP 7-2 <u>at t = 0⁻</u>



which becomes



by voltage division:

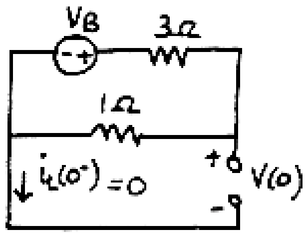
$$v_c(0^-) = \left(\frac{1.6}{1.6+R}\right) 20 = 7.4 \Rightarrow \underline{R = 2.7\Omega}$$

$$\text{Then } i_1 = \left(\frac{20}{2.7+1.6}\right) = 4.6 \text{ A}$$

Check with current division

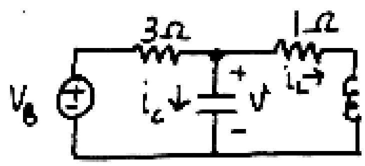
$$i_L(0^-) = \left(\frac{8}{2+8}\right) i_1 = 3.7 \text{ A} \Rightarrow i_1 = 4.6 \text{ A} \qquad \underline{\text{OK}}$$

DP 7-3 at $t = 0^-$



$v(0) = \frac{1}{4}v_B$

at $t = 0^+$



Now $\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C}$ and $i_L(0^-) = i_L(0^+) = 0$

at top node : $\frac{v - v_B}{3} + i_L + i_c = 0$

$$i_c(0^+) = \frac{v_B - v(0^+)}{3} = \frac{v_B - 3}{3}$$

$$\frac{dv_c}{dt} = 24 = \frac{1}{\left(1/8\right)}\frac{(V_B - 3)}{3} \Rightarrow \underline{v_B = 12\text{V}}$$

DP 7-4

$\frac{1}{2}Li_L^2 = \frac{1}{2}Cv_c^2$ (1) $\Leftarrow$ in steady-state

now in dc $i_L = v_c/R$ so (1) becomes $L\left(\frac{v_c}{R}\right)^2 = Cv_c^2 \Rightarrow C = \frac{L}{R^2}$

then $R = \sqrt{L/C} = \sqrt{\frac{10^{-2}}{10^{-6}}} = \sqrt{10^4} = 10^2$

So R = 100 Ω