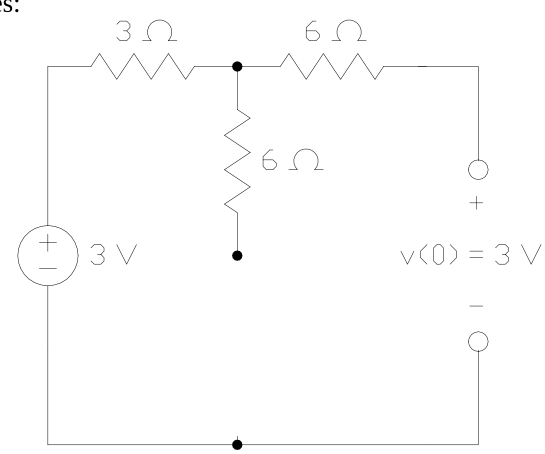
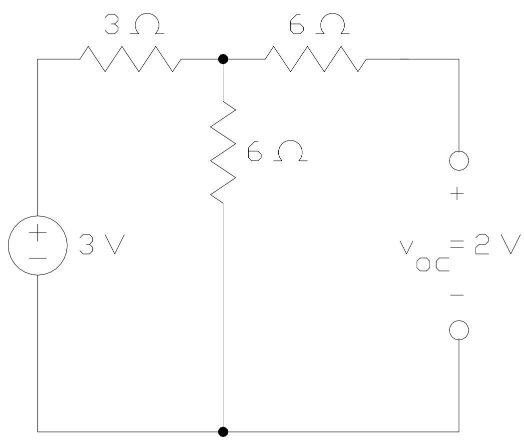
Chapter 8 – The Complete Response of RL and RC Circuits

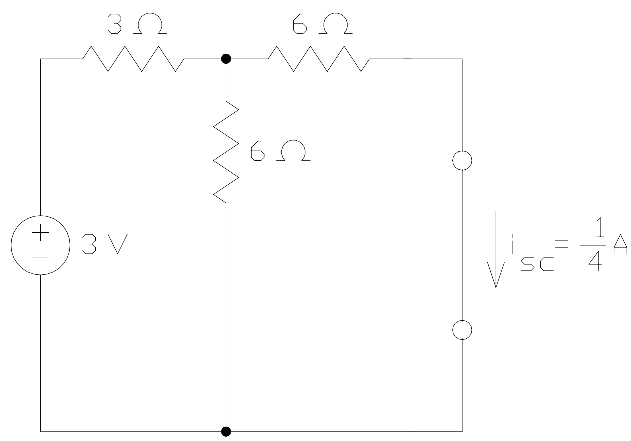
Exercises

Ex 8.3-1 Before the switch closes:



After the switch closes:

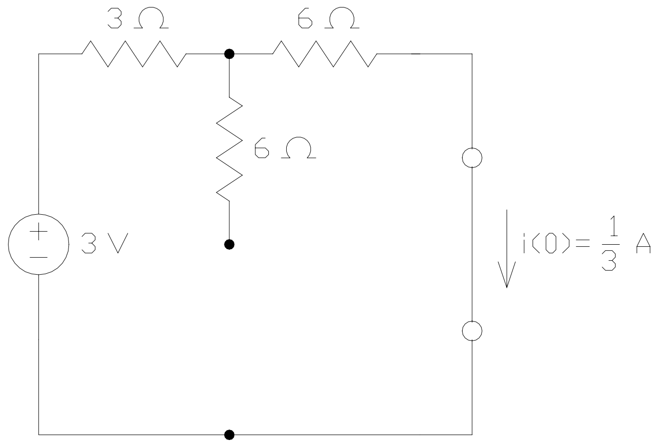




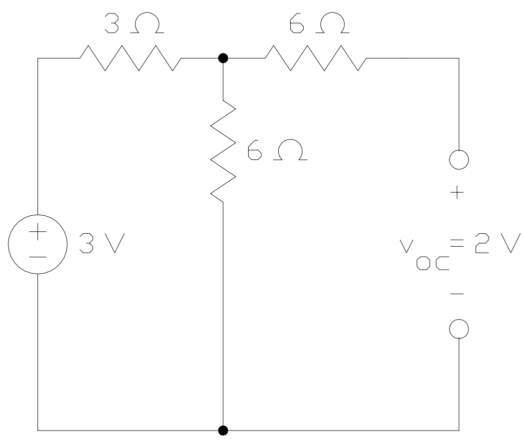
Therefore $R_t = \dfrac{2}{0.25} = 8\,\Omega$ so $\tau = 8(0.05) = 0.4$ s .

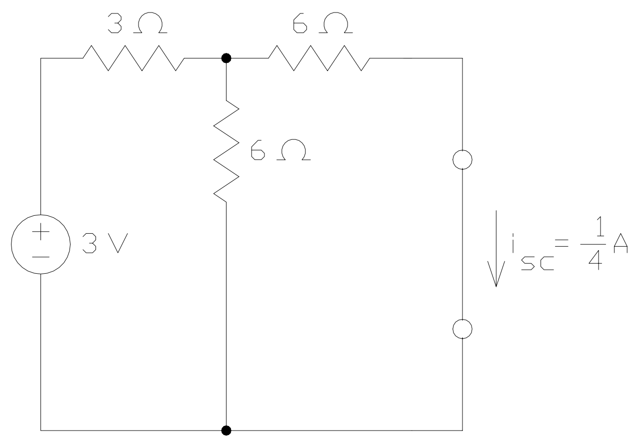
Finally, $v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = 2 + e^{-2.5t}$ V for $t > 0$

Ex 8.3-2 Before the switch closes:



After the switch closes:

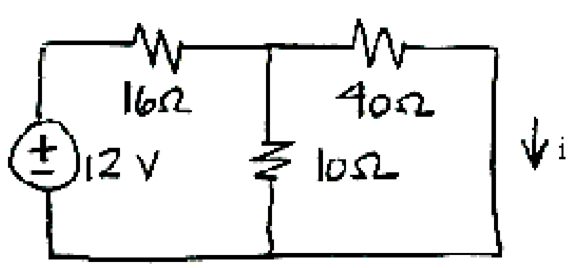




Therefore $R_t = \dfrac{2}{0.25} = 8\,\Omega$ so $\tau = \dfrac{6}{8} = 0.75$ s .

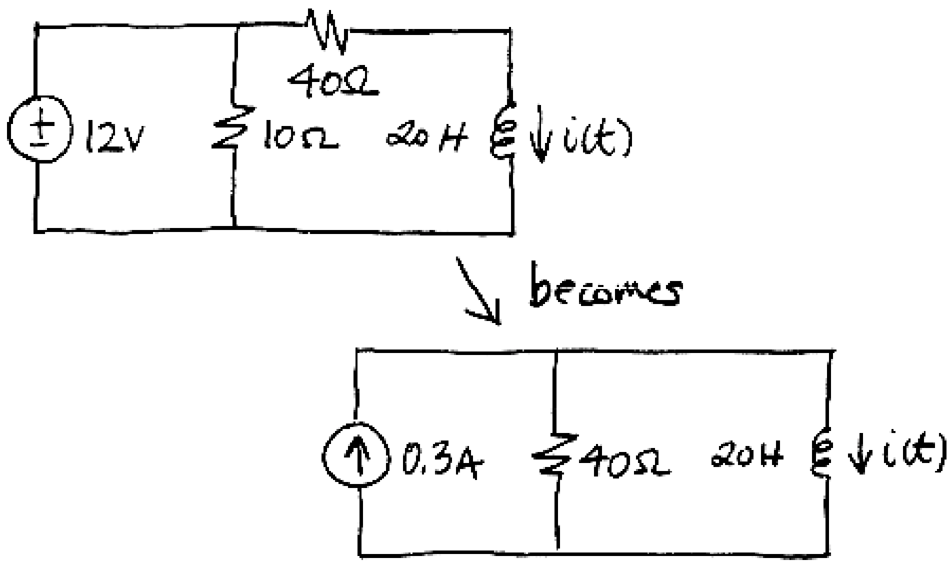
Finally, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = \dfrac{1}{4} + \dfrac{1}{12} e^{-1.33t}$ A for $t > 0$

Ex. 8.3-3 At steady-state before t = 0:



$i = \left(\dfrac{10}{10+40}\right)\left(\dfrac{12}{16+40\|10}\right) = 0.1$ A

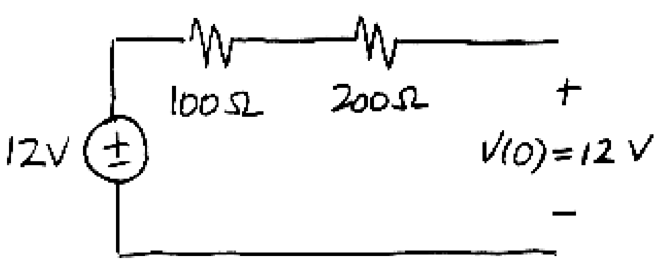
After t = 0, the Norton equivalent of the circuit connected to the inductor is found to be



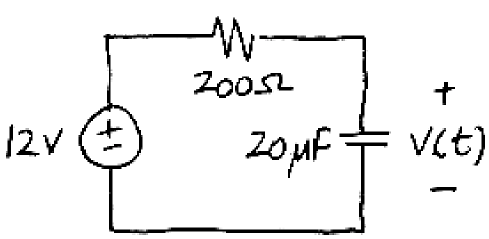
so $I_{sc} = 0.3\text{ A},\ R_{th} = 40\Omega,\ \tau = \frac{L}{R_{th}} = \frac{20}{40} = \frac{1}{2}$

Finally: $i(t) = (0.1 - 0.3)e^{-2t} + 0.3 = 0.3 - 0.2e^{-2t}$ A

Ex. 8.3-4 At steady-state for t < 0



After t = 0, replace the circuit connected to the capacitor by its Thèvenin equivalent

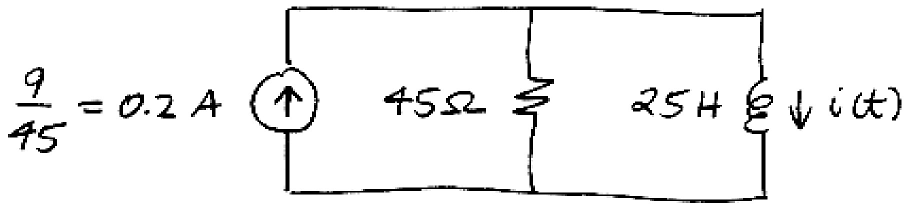


so $V_{oc} = 12\text{V},\ R_{th} = 200\Omega,\ \tau = R_{th}\, C = (200)(20 \cdot 10^{-6}) = 4$ ms

Finally: $v(t) = (12 - 12)e^{-\frac{t}{4}} + 12 = 12$ V

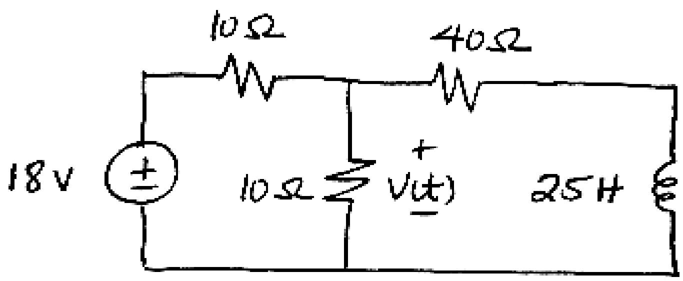
Ex. 8.3-5 Before $t = 0$, $i(t) = 0$ so $I_o = 0$

After $t = 0$, replace the circuit connected to the inductor by its Norton equivalent



$$I_{sc} = 0.2 \text{ A}, \; R_{th} = 45\Omega, \; \tau = \frac{L}{R_{th}} = \frac{25}{45} = \frac{5}{9}$$

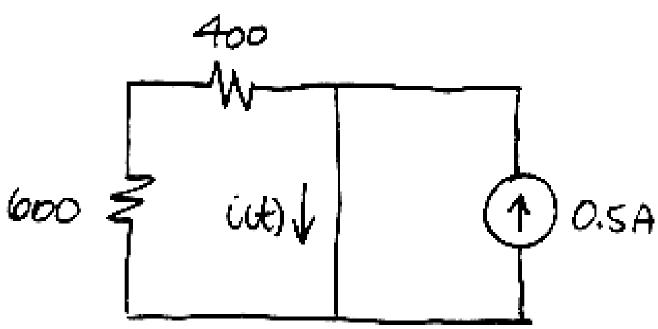
So



$i(t) = 0.2\,(1 - e^{-1.8t})$ A

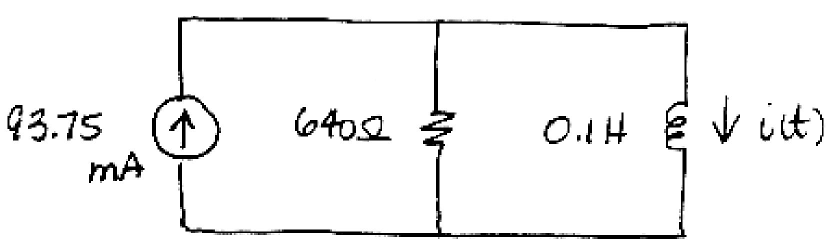
Finally: $v(t) = 40\, i(t) + 25 \frac{d}{dt} i(t) = 8(1 - e^{-1.8t}) + 5(1.8)e^{-1.8t} = 8 + e^{-1.8t}$ V

Ex. 8.3-6 $t < 0$:



$\Rightarrow i(0) = 0.5$ A

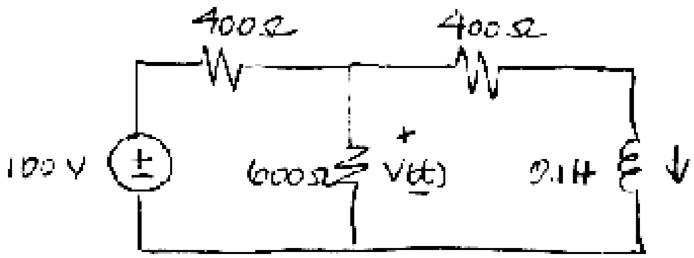
$t > 0$: Replace the circuit connected to the inductor by its Norton equivalent to get



Continued

$$I_{sc} = 93.75\text{mA}, \; R_{th} = 640\Omega, \; t = \frac{L}{R_{th}} = \frac{.1}{640} = \frac{1}{6400}$$

So

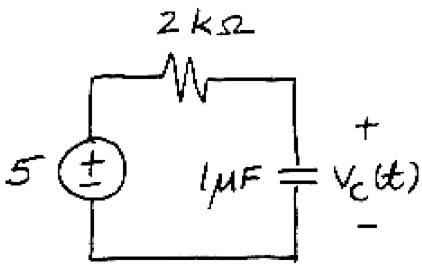


$$i(t) = 406.25e^{-6400t} + 93.75 \text{ mA}$$

Finally

$$v(t) = 400\, i(t) + 0.1 \frac{d}{dt} i(t) = 400\,(.40625e^{-6400t} + .09375) + 0.1\,(-6400)\,(0.40625e^{-6400t}) = 37.5 - 97.5e^{-6400t} \text{ V}$$

Ex. 8.4-1



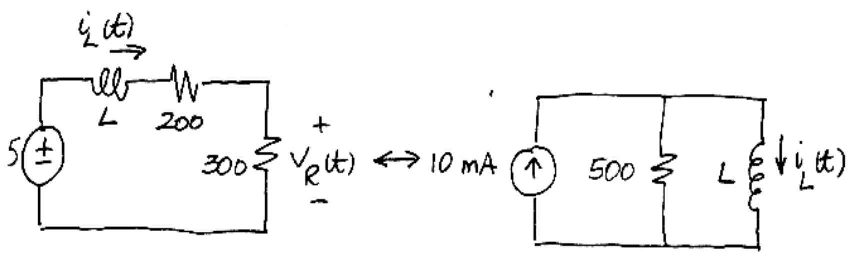
$$\tau = \left(2{\cdot}10^{-3}\right)\left(1{\cdot}10^{-6}\right) = 2 \cdot 10^{-3}$$

$$v_c(t) = 5 + (1.5-5)\,e^{-\frac{t}{2}} \quad \text{where t is in ms}$$

$$v_c(1) = 5 - 3.5e^{-\frac{1}{2}} = 2.88\text{V}$$

So $v_c(t)$ will be equal to v_T at t=1 ms if $v_T = 2.88$ V

Ex. 8.4-2



$$\left.\begin{array}{l} i_L(0) = 1\text{mA},\ I_{sc} = 10\text{mA} \\ R_{th} = 500\Omega,\ \tau = \dfrac{L}{500} \end{array}\right\} \Rightarrow i_L(t) = 10 - 9e^{\frac{-500}{L}t} \text{mA}$$

$$v_R(t) = 300\, i_L(t) = 3 - 2.7e^{-\frac{500}{L}t} \text{ V}$$

We require that $v_R = 1.5$V at t = 10ms = 0.01 s

That is

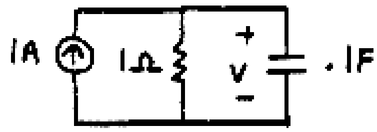
$$1.5 = 3 - 2.7e^{-\frac{500}{L}(0.01)}$$

$$e^{-\frac{5}{L}} = \frac{1.5-3}{-2.7} = 0.555$$

$$-\frac{5}{L} = \ln(0.555) = -.588$$

$$L = \frac{5}{0.588} = 8.5 \text{ H}$$

Ex. 8.6-1 $\underline{0 < t < t_1}$

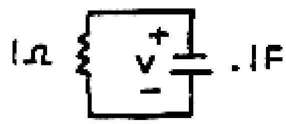


$v(t) = v(\infty) + Ae^{-t/RC}$ where $v(\infty) = (1A)(1\Omega) = 1V$

$v(t) = 1 + Ae^{-t/(1)(.1)} = 1 + Ae^{-10t}$

Now $v(0^-) = v(0^+) = 0 = 1 + A \Rightarrow A = -1$ $\therefore \underline{v(t) = 1 - e^{-10t}\ V}$

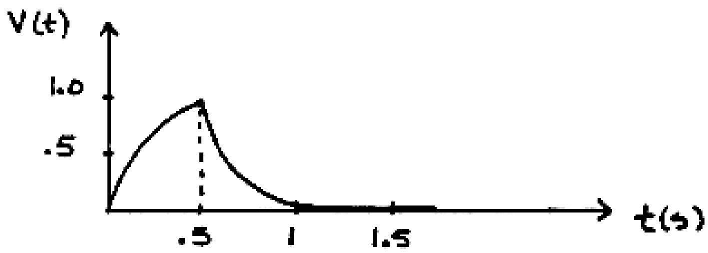
$\underline{t > t_1}$



$t_1 = .5s \qquad v(t) = v(t_1)e^{-\frac{t-.5}{(1)(.1)}} = v(.5)e^{-10(t-.5)}$

Now $v(.5) = 1 - e^{-10(.5)} = .993V$

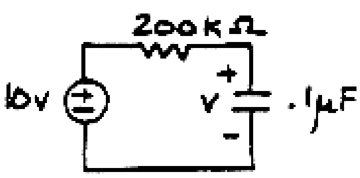
$\therefore \underline{v(t) = .993e^{-10(t-.5)}\ V}$



Ex. 8 6-2

$\underline{t < 0}$ no sources $\therefore v(0^-) = v(0^+) = 0$

$\underline{0 < t < t_1}$



$v(t) = v(\infty) + Ae^{-t/RC} = v(\infty) + Ae^{-t/2\times10^5(10^{-7})}$

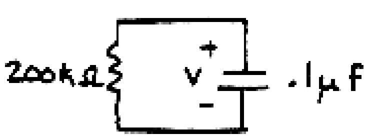
where for $t = \infty$ (steady-state)

$\therefore$ capacitor becomes an open $\Rightarrow v(\infty) = 10V$

$v(t) = 10 + Ae^{-50t}$

Now $v(0) = 0 = 10 + A \Rightarrow A = -10$ $\therefore \underline{v(t) = 10(1 - e^{-50t})\ V}$

$\underline{t > t_1}$, $t_1 = .1s$



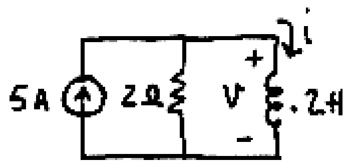
$v(t) = v(.1)e^{-50(t-.1)}$

where $v(.1) = 10(1 - e^{-50(.1)}) = 9.93\ V$

$\therefore \underline{v(t) = 9.93e^{-50(t-.1)}\ V}$

Ex. 8.6–3 <u>for t < 0</u> $i = 0$

<u>0 < t < .2</u>



$$\left.\begin{array}{l}\text{KCL: } -5 + v/2 + i = 0 \\ \text{also: } v = 0.2\,\dfrac{di}{dt}\end{array}\right\} \dfrac{di}{dt} + 10i = 50$$

$\therefore\ i(t) = 5 + Ae^{-10t}$

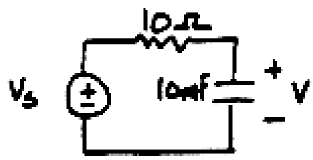
$i(0) = 0 = 5 + A \Rightarrow A = -5$

so have $\underline{i(t) = 5\ (1 - e^{-10t})\ A}$

$t > .2$

$i(.2) = 4.32\ A \quad \therefore\ \underline{i(t) = 4.32e^{-10(t-.2)}\ A}$

Ex. 8.7-1



$v_s = 10\sin 20t\ V$

KVL a: $-10\sin 20t + 10\left(.01\dfrac{dv}{dt}\right) + v = 0 \quad \Rightarrow \quad \dfrac{dv}{dt} + 10v = 100\sin 20t$

Natural response: $s + 10 = 0 \Rightarrow s = -10 \quad \therefore\ v_n(t) = Ae^{-10t}$

Forced response: try $v_f(t) = B_1\cos 20t + B_2\sin 20t$

plugging $v_f(t)$ into the differential equation and equating like terms

yields: $B_1 = -40$ & $B_2 = 20$

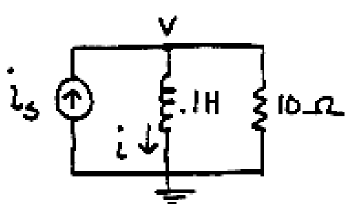
Complete response: $v(t) = v_n(t) + v_f(t)$

$$v(t) = Ae^{-10t} - 40\cos 20t + 20\sin 20t$$

Now $v(0^-) = v(0^+) = 0 = A - 40 \quad \therefore A = 40$

$\therefore\ \underline{v(t) = 40e^{-10t} - 40\cos 20t + 20\sin 20t\ V}$

Ex. 8.7-2



$i_s = 10e^{-5t}$

KCL at top node: $-10e^{-5t} + i + v/10 = 0$

Now $v = .1\dfrac{di}{dt} \quad \Rightarrow \quad \underline{\dfrac{di}{dt} + 100i = 1000e^{-5t}}$

Natural response: $s + 100 = 0 \Rightarrow s = -100 \ \therefore\ i_n(t) = Ae^{-100t}$

Forced response: try $i_f(t) = Be^{-5t}$ & plug into D.E.

$\Rightarrow -5Be^{-5t} + 100Be^{-5t} = 1000e^{-5t}$

$\Rightarrow B = 10.53$

Complete response: $i(t) = Ae^{-100t} + 10.53e^{-5t}$

Now $i(0^-) = i(0^+) = 0 = A + 10.53 \Rightarrow A = -10.53$

$\therefore\ \underline{i(t) = 10.53\,(e^{-5t} - e^{-100t})\ A}$

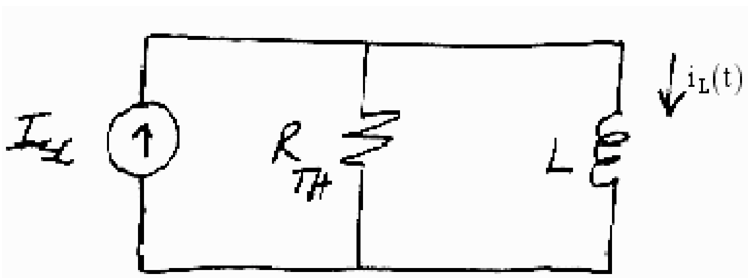
Ex. 8.7-3

A current $i_L = v_s / 1$ flows in the inductor with the switch closed. When the switch opens, i_L cannot change instantaneously. Thus, the energy stored in the inductor dissipated in the spark. Add a resistor (say 1 kΩ across the switch terminals.)

Problems

Section 8.3: The Response of a First Order Circuit to a Constant Input

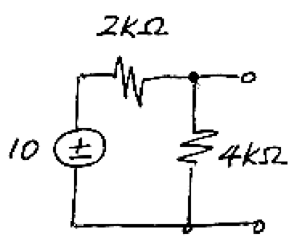
P8.3-1 We know that



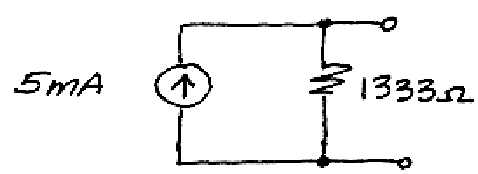
$$i_L(t) = (I_o - I_{SC})e^{-\left(\frac{t-t_o}{\tau}\right)} + I_{SC}$$

where $I_o = i_L(t_o)$ and $\tau = \dfrac{L}{R_{TH}}$. In this problem $t_o = 0$ and $I_o = i_L(0) = 3\text{mA}$.

The Norton equivalent of



is

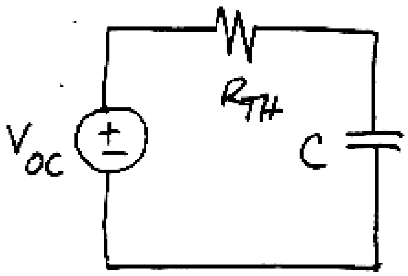


So $R_{th} = 1333\Omega$ and $I_{sc} = 5\text{mA}$.

L= 5 H so $\tau = \dfrac{L}{R_{th}} = \dfrac{5}{1333} = 3.75$ ms

Finally $\quad i_L(t) = -2e^{-\frac{t}{3.75}} + 5 \text{ mA} \quad t > 0 \quad$ where t has units of ms.

P8.3-2 We know that

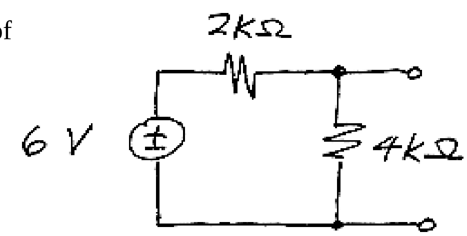


$$v_c(t) = (V_0 - V_{oc})e^{-\left(\frac{t-t_0}{\tau}\right)} + V_{oc}$$

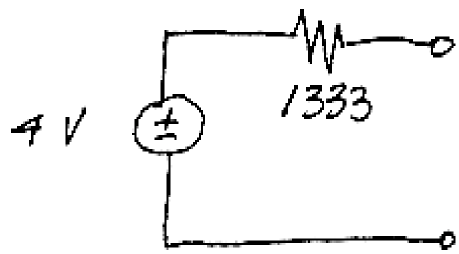
where $v_0 = v_C(t_0)$ and $\tau = R_{th}C$,

In this problem, $t_0 = 0$ and $v_0 = v_c(0) = 8V$.

The Thèvenin equivalent of



is

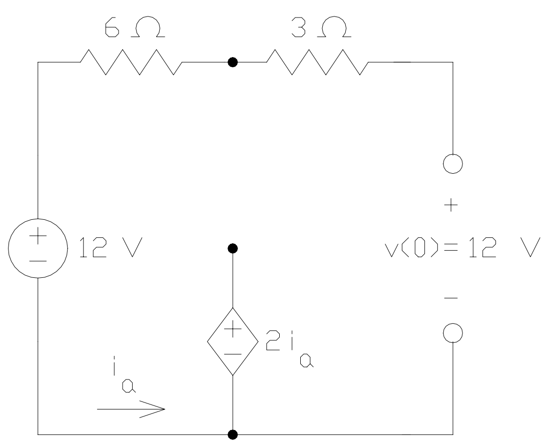


so $R_{th} = 1333$ and $V_{oc} = 4$ V.

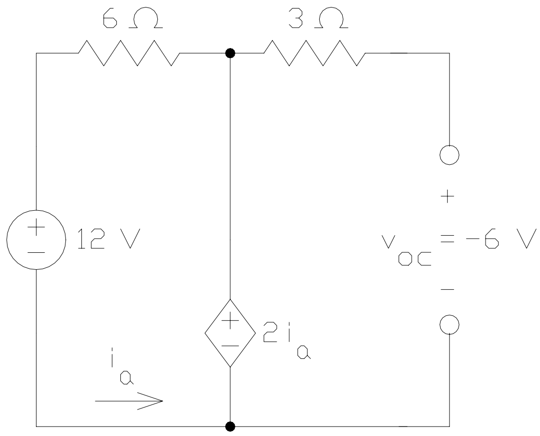
Next, $C = 0.5\mu F$ so $\tau = \left(0.5 \cdot 10^{-6}\right) 1333 = 0.67ms$

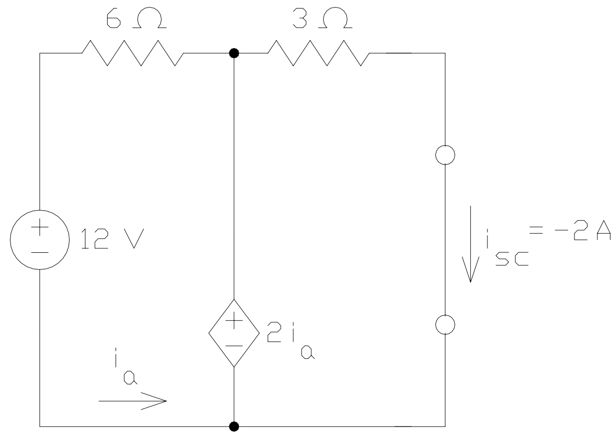
Finally $v_c(t) = 4e^{-\frac{t}{.67}} + 4$ V where t has units of ms.

P 8.3-3 Before the switch closes:



After the switch closes:

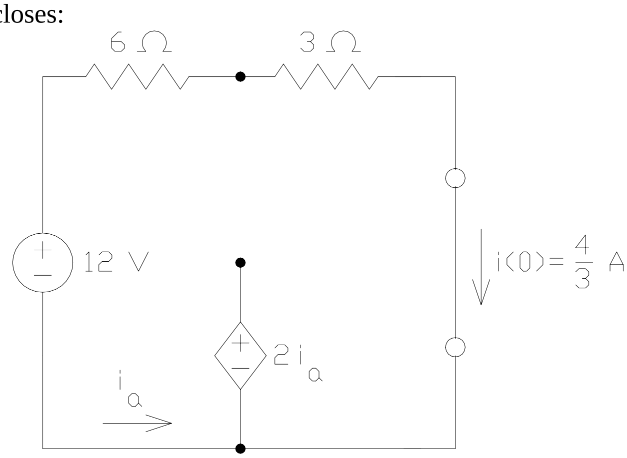




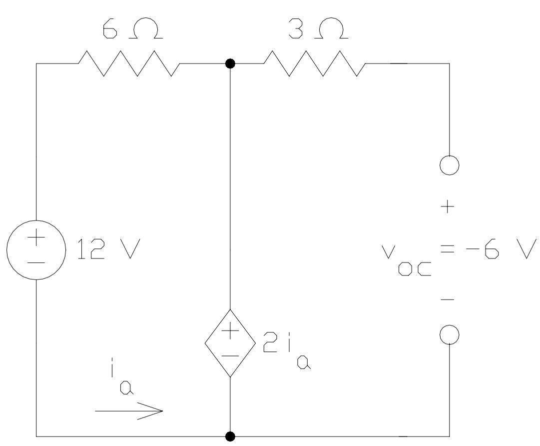
Therefore $R_t = \dfrac{-6}{-2} = 3\,\Omega$ so $\tau = 3(0.05) = 0.15$ s.

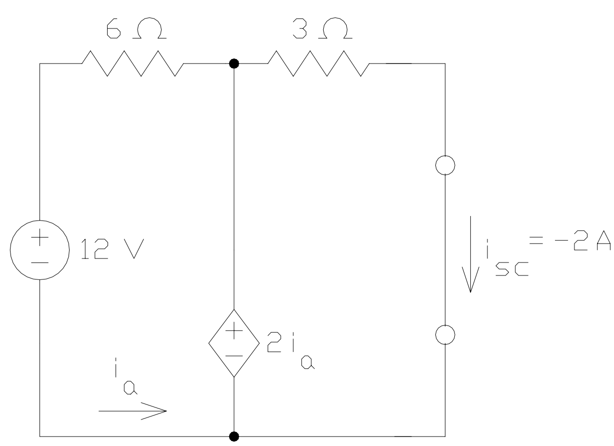
Finally, $v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = -6 + 18\, e^{-6.67t}$ V for $t > 0$

P 8.3-4 Before the switch closes:



After the switch closes:



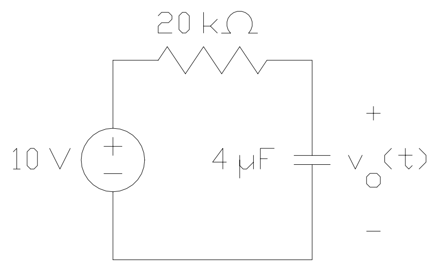


Therefore $R_t = \frac{-6}{-2} = 3\,\Omega$ so $\tau = \frac{6}{3} = 2$ s .

Finally, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = -2 + \frac{10}{3}\, e^{-0.5t}$ A for $t > 0$

P8.3-5

Before the switch opens, $v_o(t) = 0$ V $\Rightarrow v_o(0) = 0$ V . After the switch opens the part of the circuit connected to the capacitor can be replaced by it's Thevenin equivalent circuit to get:



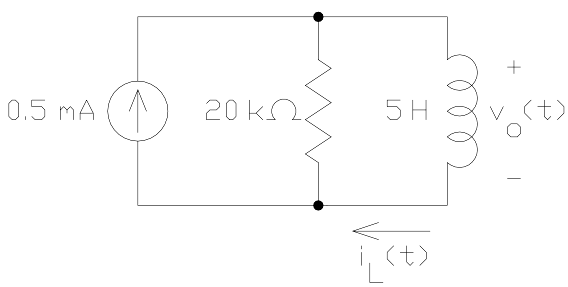
Therefore $\tau = \left(20\times10^{3}\right)\left(4\times10^{-6}\right) = 0.08$ s .

Next, $v_C(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = 10 - 10\, e^{-12.5t}$ V for $t > 0$

Finally, $v_0(t) = v_C(t) = 10 - 10\, e^{-12.5t}$ V for $t > 0$

P8.3-6

Before the switch opens, $v_o(t) = 0$ V $\Rightarrow v_o(0) = 0$ V . After the switch opens the part of the circuit connected to the capacitor can be replaced by it's Norton equivalent circuit to get:



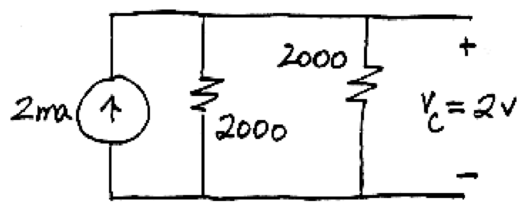
Therefore $\tau = \frac{5}{20\times10^{3}} = 0.25$ ms .

Next, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = 0.5\times10^{-3}\left(1 - e^{-4000t}\right)$ A for $t > 0$

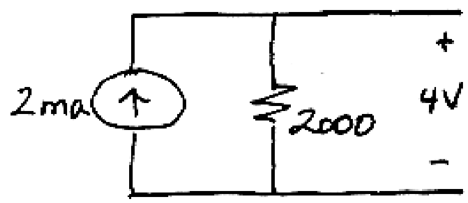
Finally, $v_o(t) = 5\frac{d}{dt} i_L(t) = 10\, e^{-4000t}$ V for $t > 0$

P8.3-7 Since the input to this circuit is constant, the capacitor will act like an open circuit when the circuit is at steady-state:

t < 0

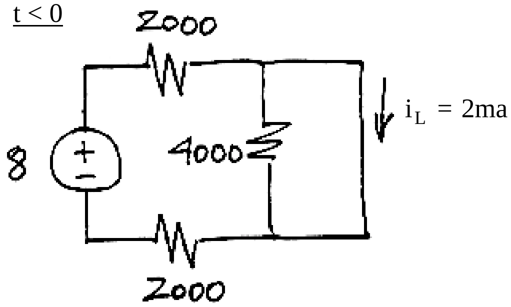


t > 0

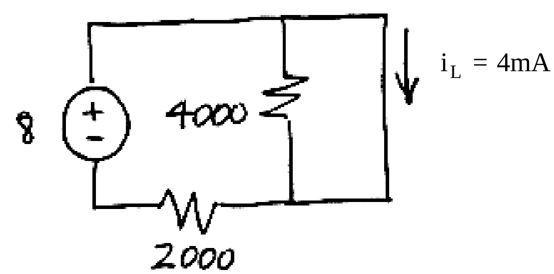


P8.3-8 Since the input to this circuit is constant, the inductor will act like a short circuit when the circuit is at steady-state:

t < 0

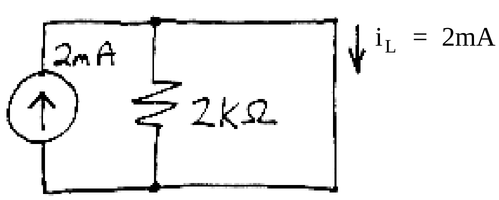


t > 0

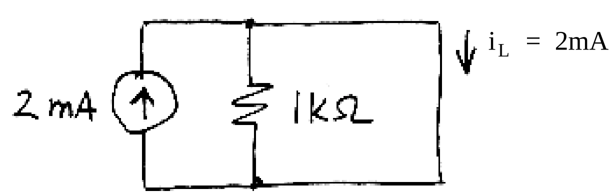


P8.3-9 Since the input to this circuit is constant, the inductor will act like a short circuit when the circuit is at steady-state:

t < 0

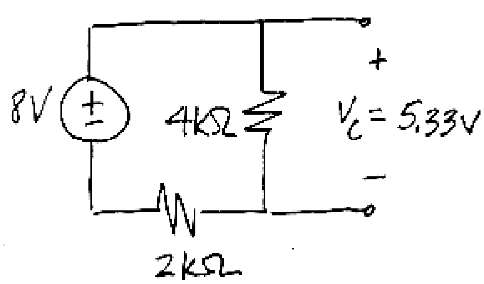


t > 0

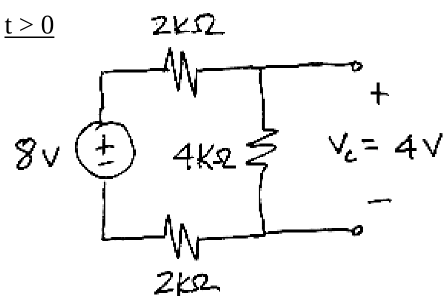


P8.3-10 Since the input to this circuit is constant, the capacitor will act like an open circuit when the circuit is at steady-state:

t < 0

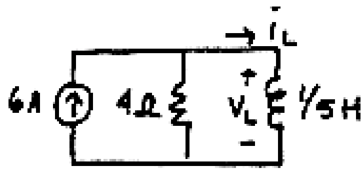


t > 0



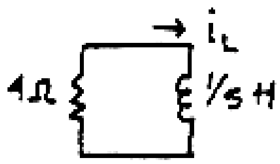
P8.3-11

at $t = 0^-$ (steady-state)



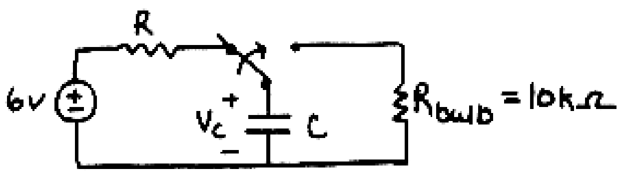
$v_L(0^-) = 0 \quad \therefore\ i_L(0^-) = 6\text{A} = i_L(0^+)$

for $t > 0$



$i_L(t) = i_L(0)e^{-(R/L)\,t} = 6e^{-20t}\text{A}$

P8.3-12



Assume the capacitor is charged at $t = 5\tau$, i.e., $\dfrac{V_c}{V_{c_{max}}} = 1 - e^{-5} = .993$

Similarly, assume the capacitor is discharged when $\dfrac{V_c}{V_{c_{max}}} = e^{-5} = 6.74\times10^{-3}$.

Now determine C from discharging condition

$$v_c(t) = v_c(t_o)e^{-(t-t_o)/C\,R_{bulb}} \Rightarrow 6.74\times10^{-3} = e^{-0.5/CR_{bulb}} \Rightarrow \underline{C = 10^{-5}\text{F} = 10\mu F}$$

Now determine a condition for R from charging circuit at the instant

$v_c = 0 \Rightarrow \dfrac{6\text{V}}{R} < 100\times10^{-6}\text{A} \Rightarrow R > 60\ \text{k}\Omega$

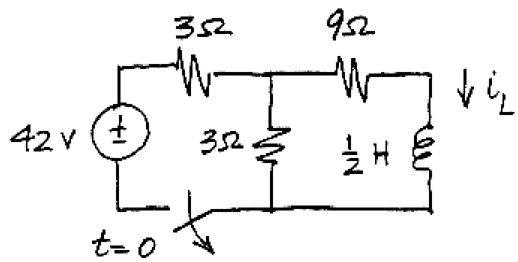
then for the charging ckt.

$$.993 = 1 - e^{-5/RC}$$

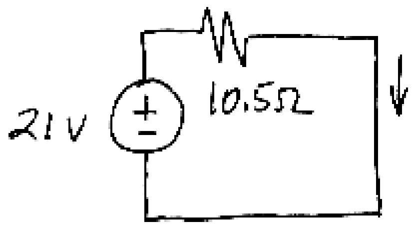
$$-4.96 = -5/RC \Rightarrow \underline{R = \frac{5}{(4.96)(10^{-5})} = 100.8\ \text{k}\Omega}$$

and see that $R \simeq 100\text{k}\Omega > 60\text{k}\Omega$.

P8.3-13 First, use source transformations to obtain the equivalent circuit

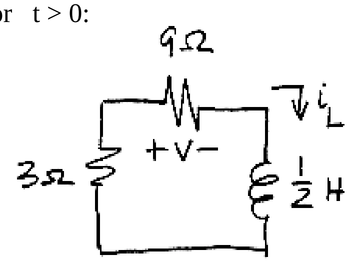


for $t < 0$:



$i_L = 2A$

for $t > 0$:

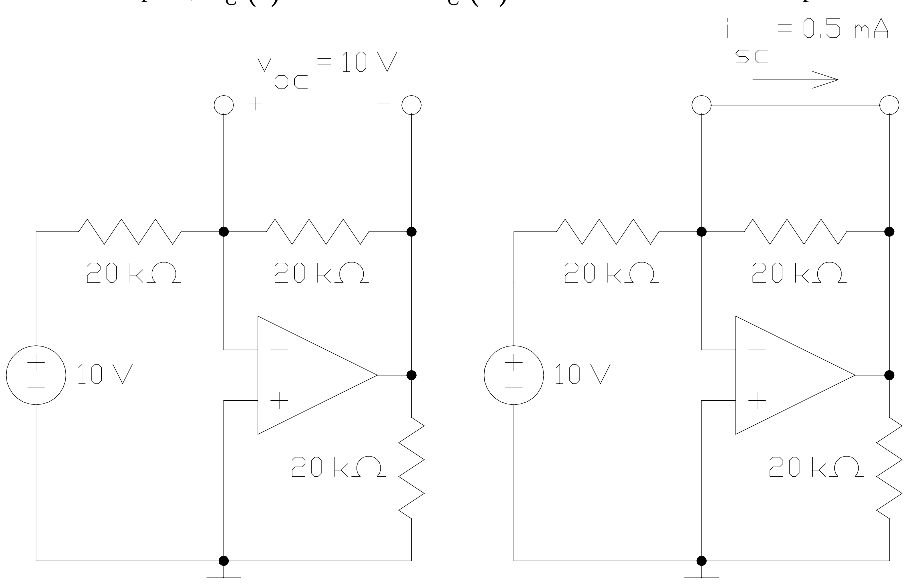


So $I_0 = 2A$, $I_{sc} = 0$, $R_{th} = 3\Omega + 9\Omega = 12\Omega$, $\tau = \frac{L}{R_{th}} = \frac{\frac{1}{2}}{12} = \frac{1}{24}$

and $i_L(t) = 2e^{-24t} \quad t > 0$

Finally $v(t) = 9\, i_L(t) = 18\, e^{-24t} \quad t>0$

P 8.3-14 Before the switch opens, $v_C(t) = 0 \text{ V} \Rightarrow v_C(0) = 0 \text{ V}$. After the switch opens:



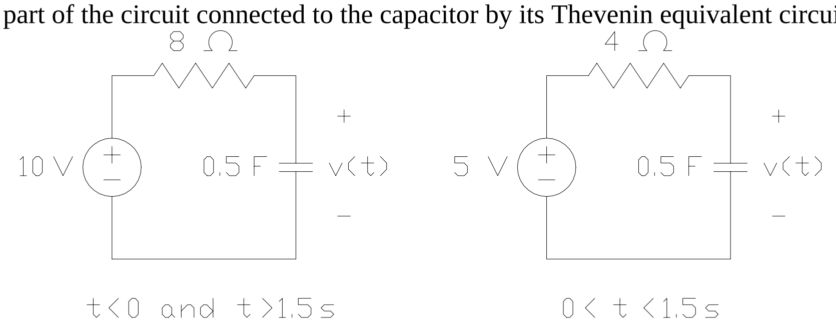
Therefore $R_t = \frac{10}{0.5\times10^{-3}} = 20 \text{ k}\Omega$ so $\tau = \left(20\times10^3\right)\left(4\times10^{-6}\right) = 0.08 \text{ s}$.

Next, $v_C(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = 10 - 10\, e^{-12.5t} \text{ V} \quad \text{for } t > 0$

Finally, $v_0(t) = -v_C = -10 + 10\, e^{-12.5t} \text{ V} \quad \text{for } t > 0$

Section 8-4: Sequential Switching

P 8.4-1 Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:



Before the switch closes at t = 0 the circuit is at steady state so $v(0) = 10$ V. For $0 < t < 1.5$s, $v_{oc} = 5$ V and $R_t = 4\ \Omega$ so $\tau = 4 \times 0.5 = 2$ s. Therefore

$$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = 5 + 5e^{-0.5t}\ \text{V} \quad \text{for } 0 < t < 1.5\ \text{s}$$

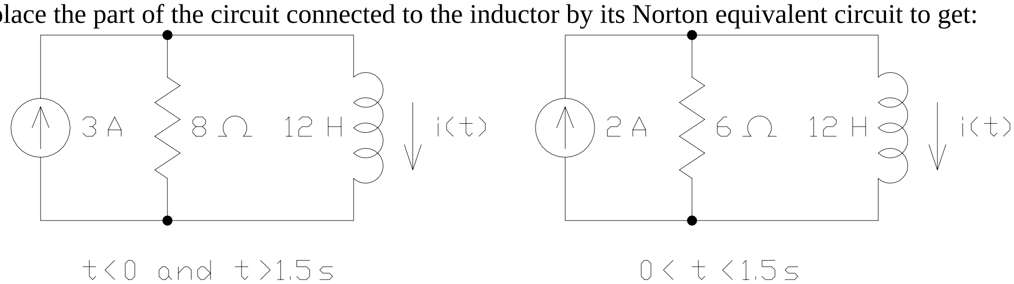
At t =1.5 s, $v(1.5) = 5 + 5e^{-0.5(1.5)} = 7.36\ \text{V}$. For 1.5s < t, $v_{oc} = 10$ V and $R_t = 8\ \Omega$ so $\tau = 8 \times 0.5 = 4$ s. Therefore

$$v(t) = v_{oc} + (v(1.5) - v_{oc})\, e^{-\frac{t-1.5}{\tau}} = 10 - 2.34\, e^{-0.25(t-1.5)}\ \text{V} \quad \text{for } 1.5\ \text{s} < t$$

Finally

$$v(t) = \begin{cases} 5 + 5\, e^{-0.5t}\ \text{V} & \text{for } 0 < t < 1.5\ \text{s} \\ 10 - 2.34\, e^{-0.25(t-1.5)}\ \text{V} & \text{for } 1.5\ \text{s} < t \end{cases}$$

P 8.4-2 Replace the part of the circuit connected to the inductor by its Norton equivalent circuit to get:



Before the switch closes at t = 0 the circuit is at steady state so $i(0) = 3$ A. For $0 < t < 1.5$s, $i_{sc} = 2$ A and $R_t = 6\ \Omega$ so $\tau = \frac{12}{6} = 2$ s. Therefore

$$i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = 2 + e^{-0.5t}\ \text{A} \quad \text{for } 0 < t < 1.5\ \text{s}$$

At t =1.5 s, $i(1.5) = 2 + e^{-0.5(1.5)} = 2.47\ \text{A}$. For 1.5 s < t, $i_{sc} =$ 3 A and $R_t = 8\ \Omega$ so $\tau = \frac{12}{8} = 1.5$ s. Therefore

$$i(t) = i_{sc} + (i(1.5) - i_{sc})\, e^{-\frac{t-1.5}{\tau}} = 3 - 0.64\, e^{-0.667(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

$$i(t) = i_{sc} + (i(1.5) - i_{sc})\, e^{-\frac{t-1.5}{\tau}} = 3 - 0.64\, e^{-0.667(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

Finally

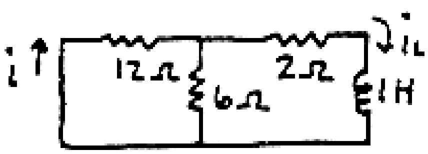
$$i(t) = \begin{cases} 2 + e^{-0.5\,t} \text{ A} & \text{for } 0 < t < 1.5 \text{ s} \\ 3 - 0.64\, e^{-0.667(t-1.5)} \text{ A} & \text{for } 1.5 \text{ s} < t \end{cases}$$

P8.4-3 $\underline{t = 0^-}$ (steady-state)

KVL : $-52 + 18i + (12/8)i = 0 \quad \Rightarrow \quad i(0^-) = 104/39$ A

$\therefore\ i_L = i\left(\frac{6}{6+2}\right) = \underline{2\text{A} = i_L\ (0^+)}$

<u>0 < t < 51ms</u>

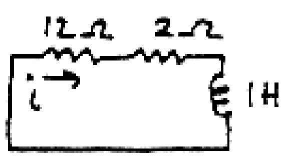


$i_L\ (t) = i_L\ (o)\ e^{-(R/L)t} \qquad R = 6\|12 + 2 = 6\Omega$

$i_L\ (t) = 2e^{-6t}$A

$\underline{\therefore\ i(t) = i_L(t)\left(\frac{6}{6+12}\right) = 2/3\ e^{-6t}\text{A}}$

<u>t > 51ms</u>



$i_L\ (t) = i_L(51\text{ms})\ e^{-(R/L)(t-.051)}$

$i_L\ (51\text{ms}) = 2e^{-6(.051)} = 1.473$

$\underline{i_L\ (t) = 1.473\ e^{-14(t-.051)}\ \text{A}}$

P8.4-4 $\underline{t = 0^-}$

Assume V_1 = voltage across 10μF capacitor = 3V

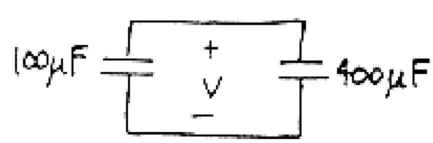
<u>0 < t < 10mS</u>

With R negligibly small, we may assume a static steady-state situation is obtained in the circuit nearly instantaneously ($t = 0^+$). Thus with both capacitors in parallel, the common voltage is obtained by considering charge conservation.

at $t = 0^-$, $q_{100\mu F} = CV = (100\mu F)(3V) = 300\ \mu C$

$q_{400\mu F} = CV = (400\mu F)(0) = 0$

$q_{ToT} = q_{100} + q_{400} = 300\mu C$

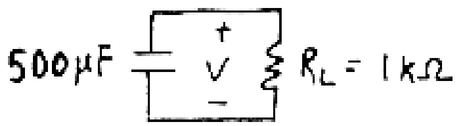


at $t = 0^+$, $q_{100} + q_{400} = 300\mu C$

Now using q = CV $\Rightarrow$ (100μF) (V) + (400μF) (V) = 300μC $\Rightarrow$ <u>V = 0.6 V</u>

10ms < t < ls

Combine 100μF & 400μF in parallel to obtain



with V(10ms) = 0.6V

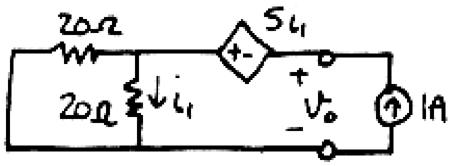
$$v(t) = V(10ms)\, e^{-(t-.01)/RC} = 0.6e^{-(t-.01)/(10^3)\,(5x10^{-4})}$$

$$\underline{v(t) = 0.6\, e^{-2(t-.01)}\ V}$$

P8.4-5

$$V_{oc} = V_T = (40 \times \frac{20}{20+20}) - 5i_1 = 20 - 5 = \left(\frac{40}{40}\right) = \underline{15\ V}$$

for R_T, kill source with V = 0

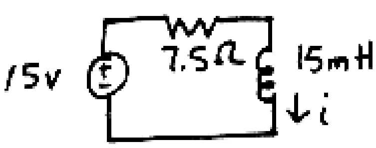


Note $i_1 = \frac{1}{2}$ A

$$R_T = \frac{v_0}{1} = 1(10\Omega) - 5(\frac{1}{2}A) = \underline{7.5\Omega}$$

$$\underline{R_{eq} = 7.5\Omega}$$

Forced response



$i = 2A$

$$\tau = L/R = \frac{15\times10^{-3}}{7.5} = 2ms$$

natural: $i = Be^{-t/\tau} = Be^{-500t}$

total : $i = Be^{-500t} + 2$ now $i(0) = 0 \Rightarrow B = -2$

$$\underline{i(t) = 2(1-e^{-500t})A}$$

time to 99%:

for $e^{-500t} = .01$ or $500t = 4.605 \Rightarrow \underline{t = 9.2ms}$

P8.4-6 $\left.\begin{array}{l}\tau = RC = 10^5\times10^{-6} = .1s \\ v_c(0) = 5\ V\end{array}\right\} \Rightarrow v_c(t) = 5e^{-t/\tau}$

Now $5/2 = v_c(t_1) = 5e^{-t_1/\tau}$

$$e^{-t_1/\tau} = .5 \qquad \Rightarrow \quad \underline{t_1 = .0693s}$$

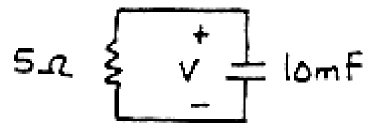
$$i(t_1) = \frac{v(t_1)}{100\ k\Omega} = \frac{5/2}{10^5} = \underline{25\mu\ A}$$

P8.4-7

t = 0^- (steady-state)

$v(0^-) = v(0^+) = (2A)(5\Omega) = 10V$

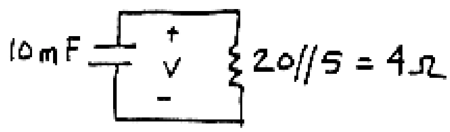
0 < t < 100ms



$v(t) = v(0)e^{-t/RC}$

$\underline{v(t) = 10e^{-t/(5)(.01)} = 10e^{-20t}\ V}$

t > 100ms



$v(t) = v(100ms)\ e^{-\frac{(t-1)}{(4)(.01)}}$

$v(100ms) = 10e^{-20(.1)} = 1.35\ V$

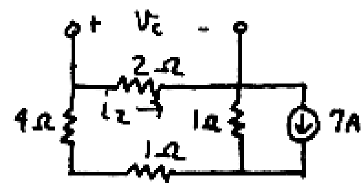
$\therefore\ \underline{v(t) = 1.35e^{-25(t-.1)}\ V}$

P8.4-8

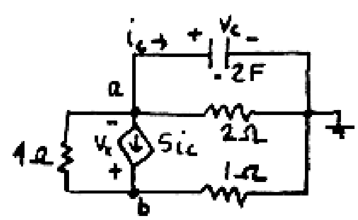
t = 0^- (steady-state)

$i_2 = \dfrac{1}{1+4+2+1}\ 7 = \dfrac{7}{8}A$

$v_c(0^-) = v_c(0^+) = 2i_2 = 7/4\ V$



0 < t < .35 Closing the rightmost switch shorts out 1Ω in parallel with 7A source and isolates 7A source leaving



KCL at a : $\dfrac{-v_x}{4} + 5i_c + i_c + \dfrac{v_c}{2} = 0$ (1)

KCL at b : $\dfrac{v_x}{4} - 5_{ic} + \dfrac{v_c + v_x}{1} = 0$ (2)

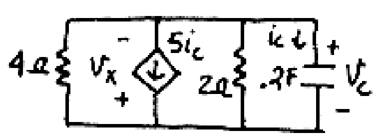
also: $i_c = .2\ {dV_c}/{dt}$ (3)

Plugging (3) into (1) & (2) & then eliminating v_c yields $\Rightarrow \dfrac{dv_c}{dt} + \dfrac{7}{10}v_c = 0$

So $v_c(t) = v_c(0)\ e^{-0.7t} = \underline{7/4\ e^{-0.7t}}$, $i_c = .2\dfrac{dv_c}{dt} = -.245e^{-.7t}$

So from (1) we have $v_x(t) = 24i_c + 2v_c = \underline{-2.38\ e^{-.7t}}$

t > .35



$v_c(.35) = 7/4\, e^{-(0.7)(.35)} = 1.37V$

now = $\underline{v_x = -v_c}$

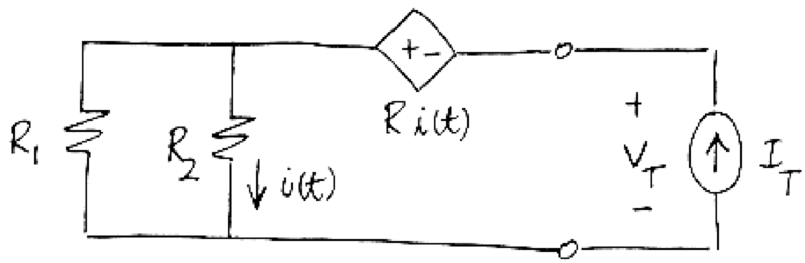
KCL: ${v_c}/{4} + 5i_c + {v_c}/{2} + i_c = 0$ (1)

also : $i_c = 0.2\ {dV_c}/{dt}$ (2)

From (1) & (2) $\Rightarrow dv_c/dt + 5/8\ v_c = 0 \Rightarrow v_c(t) = v_c(.35)e^{-(t-.35)5/8} = \underline{-1.37\ e^{-.625(t-.35)}}$

Section 8-5: Stability of First Order Circuits

P8.5-1 This circuit will be stable if the Thèvenin equivalent resistance of the circuit connected to the inductor is positive.

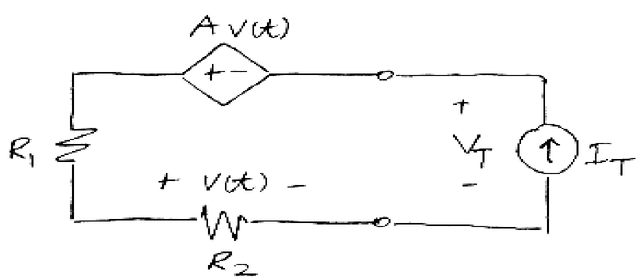


$R_{th} = \frac{V_T}{I_T}$

$$\left.\begin{aligned} i(t) &= \frac{R_1}{R_1+R_2} I_T \\ V_T &= R_2 i(t) - R i(t) \end{aligned}\right\} \Rightarrow R_{th} = \frac{(R_2 - R)R_1}{R_1+R_2}$$

Then $R_{th} > 0$ requires $R_2 > R$. In this case $R_2 = 400\Omega$ so $400 > R$ is required to guarantee stability.

P8.5-2 The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



$R_{th} = \frac{V_T}{I_T}$

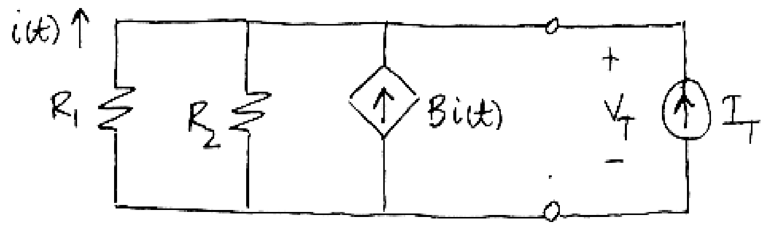
$$\left.\begin{aligned} v(t) &= R I_T \\ V_T &= I_T(R_1+R_2) - ARI_T \end{aligned}\right\} \Rightarrow R_{th} = R_1 + R_2 - AR_2$$

The circuit will be stable when $R_{th} > 0$, that is,

$$R_{th} > 0 \Rightarrow \frac{R_1+R_2}{R_2} > A$$

When $R_1 = 4k\Omega$ and $R_2 = 1k\Omega$, then $A < 5$ is required to guarantee stability.

P8.5-3 The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



$R_{th} = \frac{V_T}{I_T}$

$$V_T = -R_1 i(t) = R_2 (i(t) + Bi(t) + I_T)$$

$$i(t) = \frac{-R_2}{R_1 + R_2 + R_2 B} I_T$$

$$V_T = -R_1 \frac{-R_2}{R_1 + R_2 + R_2 B} I_T$$

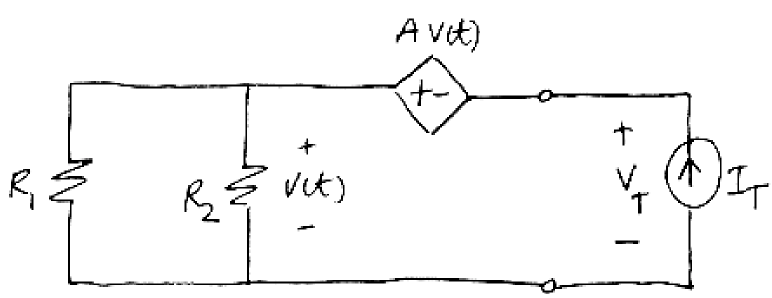
$$R_{th} = \frac{R_1 R_2}{R_1 + R_2 + R_2 B}$$

The circuit is stable when $R_{th} > 0$, that is

$$R_1 + R_2 + R_2 B > 0 \Rightarrow B > -\frac{R_1 + R_2}{R_2}$$

when $R_1 = 6\text{k}\Omega$ and $R_2 = 3\text{k}\Omega$, $B > -3$ is required to guarantee stability.

P8.5-4 The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



$R_{th} = \frac{V_T}{I_T}$

$$\left.\begin{aligned} v(t) &= \frac{R_1 R_2}{R_1 + R_2} I_T \\ V_T &= v(t) - Av(t) \end{aligned}\right\} R_{th} = \frac{R_1 R_2}{R_1 + R_2}(1 - A)$$

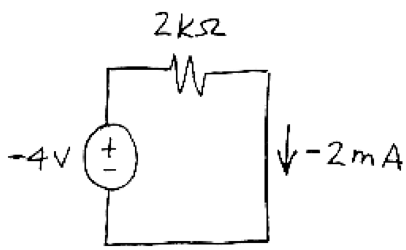
The circuit will be stable when $R_{th} > 0$, that is, when $A < 1$.

Section 8-6: The Unit Step Response

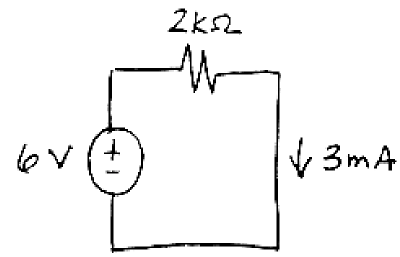
P8.6-1

$$10u(t)-4=\begin{cases}10(0)-4=-4 & t<0\\ 10(1)-4=6 & t>0\end{cases}$$

t < 0:



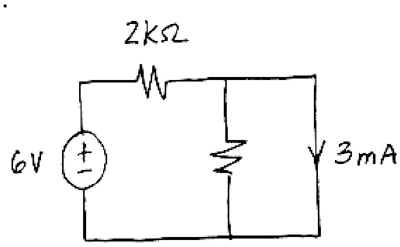
t > 0:



P8.6-2

$$6u(-t)+4u(t)=\begin{cases}6(1)+4(0)=6 & t<0\\ 6(0)+4(1)=4 & t>0\end{cases}$$

t < 0:



t > 0: