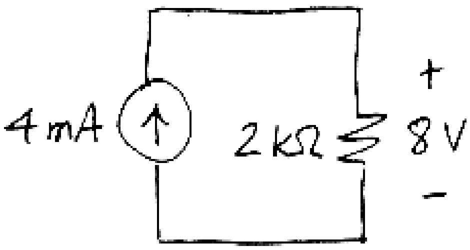
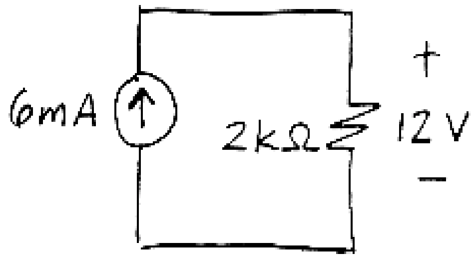
**P8.6-3**

$$4+2u(t)=\begin{cases}4+2(0)=4 & t<0\\ 4+2(1)=6 & t<0\end{cases}$$

t < 0:



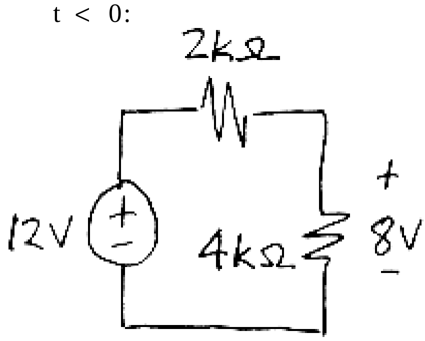

t > 0:



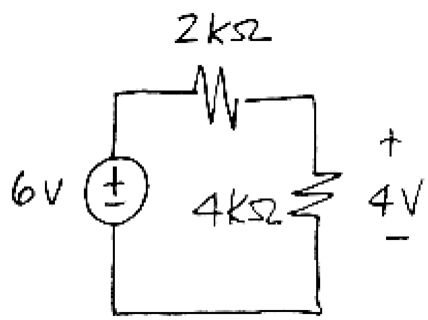

**P8.6-4**

$$6u(-t)+6=\begin{cases}6(1)+6=12 & t<0\\ 6(0)+6=6 & t>0\end{cases}$$

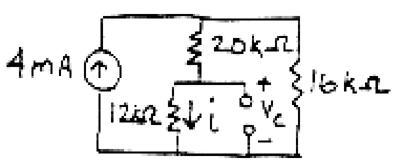
t < 0:




t > 0:




**P8.6-5** $\underline{t<0}$ (steady-state)

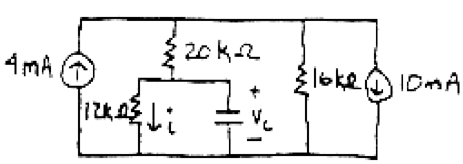



Current divider $i(0^-) = 4\left(\frac{16}{16+32}\right) = 4/3$ mA

$\therefore v_c(0^-) = 12\, i(0^-) = 16 \text{ V} = v_c(0^+) = 12\, i(0^+)$

$\underline{\therefore i(0^+) = 4/3 \text{ mA}}$

$\underline{t>0}$



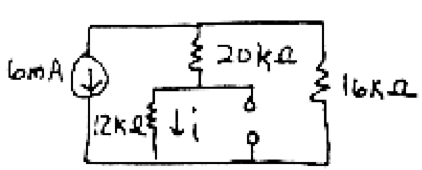

$i(t) = i(\infty) + Ae^{-t/Rc}$

$= i(\infty) + Ae^{-2000t}$

$R = 12||32 = 9\text{ k}\Omega$

$C = 1/18\,\mu\text{F}$

$\underline{t=\infty}$ (steady-state)




$$i(\infty) = -6\left[\frac{16}{16+32}\right] = -2\text{mA}$$

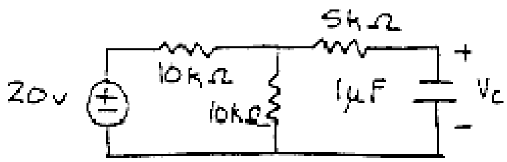
$\therefore i(t) = -2 + Ae^{-2000t}$

Now $i(0) = 4/3 = -2 + A \Rightarrow A = 10/3$

$\underline{\therefore i(t) = (10/3)\, e^{-2000t} - 2 \text{ mA} \qquad t>0}$

**P8.6-6**

t > 0




$v_c(t) = v_c(\infty) + Ae^{-t/RC}$ where $R = 5\,k\Omega + 10k\Omega \,||\, 10k\Omega = 10k\Omega$

from voltage divider $v(\infty) = 20\left(\frac{10}{10+10}\right) = 10$ V

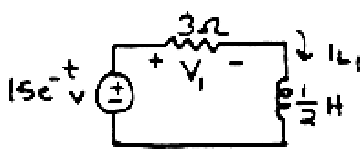
$\therefore v_c(t) = 10 + Ae^{-100t}$

now $v_c(0) = 0 = 10 + A \Rightarrow A = -10$

$v_c(t) = 10(1 - e^{-100\,t})$ V

**P8.6-7** Use superposition, first look at $15e^{-t}$ u(t) source

t > 0

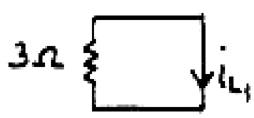



KVL a: $-15e^{-t} + 3i_{L_1} + 1/2\frac{di_{L_1}}{dt} = 0 \Rightarrow \frac{di_{L_1}}{dt} + 6i_{L_1} = 30e^{-t}$

$\therefore\ i_{L_1}(t) = Ae^{-6t} + i_{L_p}(t) \Rightarrow$ try $i_{L_p}(t) = Be^{-t}$ & plug into D.E. $\Rightarrow B = 6$

$i_{L_1}(t) = Ae^{-6t} + 6e^{-t}$

$t = 0^-$ (steady-state)




$i_{L_1}(0^-) = 0 = i_{L_1}(0^+)$

$\therefore i_{L_1}(t) = -6e^{-6t} + 6e^{-t}$

Now $v_1(t) = 3i_{L_1}(t) = -18e^{-6t} + 18e^{-t}$ V u(t)

Now consider $-15e^{-t}$ u(t−1) source

Just take previous result and change t→t−1 and flip signs

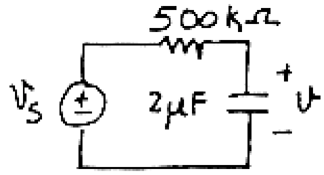
$\therefore v_2(t) = 18\,e^{-6(t-1)} - 18e^{-(t-1)}$ u(t−1)

$\therefore v(t) = \left[-18e^{-6t} + 18e^{-t}\right] u(t) + \left[18e^{-6(t-1)} - 18e^{-(t-1)}\right] u(t-1)$

**P8.6-8** $v(t) = 4u(t) - u(t-1) - u(t-2) + u(t-4) - u(t-6)$

**P8.6-9**

$$v_s = \begin{cases} 0 & t < 1 \\ 4 & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$



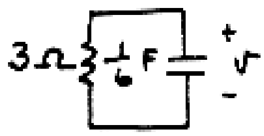

$RC = (5\times10^5)(2\times10^{-6}) = 1$

Now $v(2)=4-4e^{-1}$ so $v(t) = (4-4e^{-1})e^{-(t-2)}$ $t > 2$

$$\therefore \quad v(t)=\begin{cases} 0 & t<1 \\ 4-4e^{-(t-1)} & 1<t<2 \\ (4-4e^{-1})e^{-(t-2)} & t>2 \end{cases}$$

**P8.6-10** $v(0) = 10\text{V}$, need solve for $v(t)$ for $0 < t < 2$

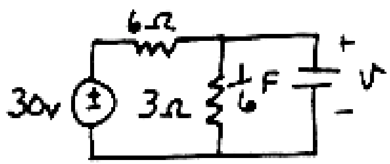
$\underline{0 < t < .5}$




KCL : $v/3 + \frac{1}{6}\frac{dv}{dt} = 0$ or $(s+2)\,v = 0$

so $\underline{v(t) = 10e^{-2t}}$

$\underline{t \ge .5}$




$v(.5) = 10e^{-1} = 3.68$

KCL : $\frac{1}{6}(v-30)+\frac{v}{3}+\frac{1}{6}\frac{dv}{dt} = 0$

or $(s+3)\,v = 30$

$v_n = Be^{-3t}$ & $v_f = 10 \Rightarrow v(t)=10+Be^{-3(t-.5)} \Rightarrow v(.5)=3.68=10+B \Rightarrow B=6.32$

$$\text{so } v(t) = \begin{cases} 10\text{ V} & t = 0 \\ 10e^{-2t} & 0 < t < .5 \\ 10\left[1-.632e^{-3(t-.5)}\right] & t > .5 \end{cases}$$

**P8.6-11** $i_s = 40\,\left[u(t)-u(t-t_0)\right]$ A $t_0 = 1\text{ms}$

$\underline{\text{for } t<0}$ $i_s=0$

so $\underline{\text{for } t>0}$ $v = v_n + v_f$ where $v_n = Ae^{-t/\tau}$ and $\tau = L/R = \frac{50\times10^{-3}}{50\Omega} = 1\times10^{-3}\text{s} = 1\text{ms}$

and where $v_f = 20\left(\frac{30}{30+20}\right)40 = 480$ V

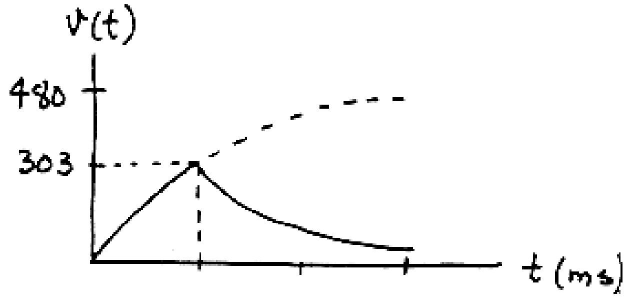
$\therefore\ v(t) = Ae^{-1000t} + 480$

now $i_L(0)=0$ so $v(0)=0 \Rightarrow A=-480$ so $v=\begin{cases} 0 & t<0 \\ 480(1-e^{-1000t}) & 0<t<1\text{ms} \end{cases}$

$\underline{\text{for } t > 1\text{ms}}$ $v = Be^{-1000(t-t_0)}$

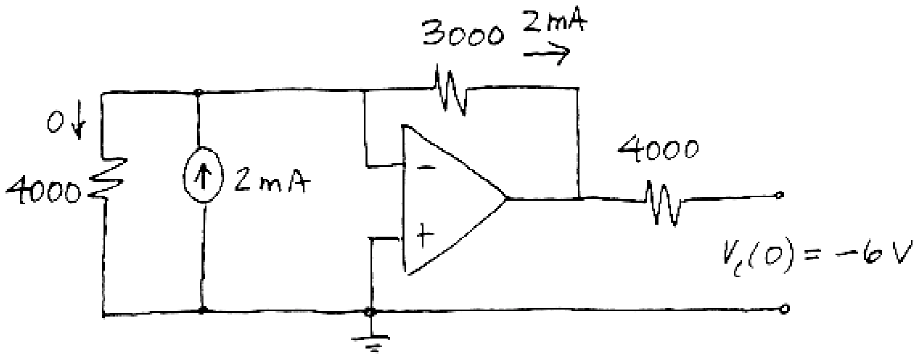
$v(1\text{ms}) = B = 480(1-e^{-1})$

$\Rightarrow v(t) = 480(1-e^{-1})\,e^{-1000(t-t_0)}$

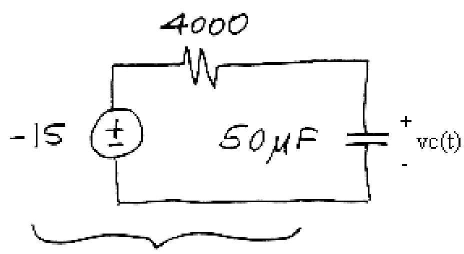



**P8.6-12**

t < 0




t ≥ 0:



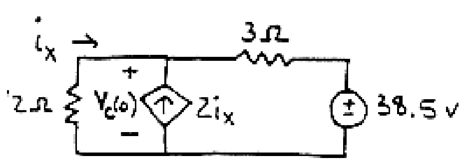

$\left(v_C(t)\right) = -15 + 9\,e^{-5t} \quad t \geq 0$

Thèvenin equivalent of the circuit connected to the capacitor

## Section 8-7 The Response of an RL or RC Circuit to a Nonconstant Source

**P8.7-1**

$\underline{t = 0^-}$ (steady-state)

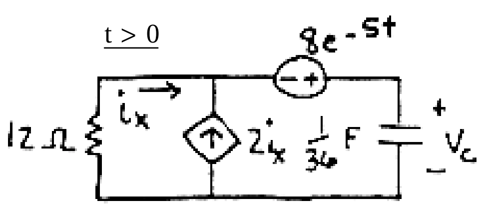



KVL a: $12i_x + 3(3i_x) + 38.5 = 0$

$\Rightarrow i_x = 01.8\overline{3}\text{ A}$

also $\underline{v_c(0^-) - 12i_x = 22\text{ V} = v_c(0^+)}$

$\underline{t > 0}$




KVL a: $\underline{12i_x - 8e^{-5t} + v_c = 0}$ (1)

KCL : $-i_x - 2i_x + (1/36)\dfrac{dv_c}{dt} = 0 \Rightarrow \underline{i_x = \dfrac{1}{108}\dfrac{dv_c}{dt}}$ (2)

(2) into (1) yields $\frac{dv_c}{dt} + 9v_c = 72e^{-5t} \Rightarrow v_{Cn}(t) = Ae^{-9t}$

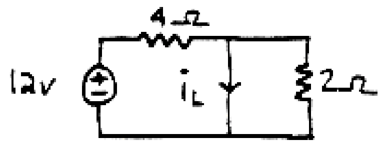
*Try* $v_{Cf}(t) = Be^{-5t}$ & plug into D.E. $\Rightarrow$ B = 18

$\therefore v_c(t) = Ae^{-9t} + 18\,e^{-5t}$

$v_c(0) = 22 = A + 18 \Rightarrow A = 4 \quad \underline{\therefore v_c(t) = 4e^{-9t} + 18e^{-5t}\ V}$

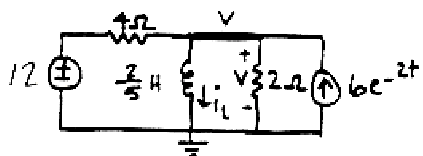
**P8.7-2**

<u>$t = 0^-$</u> (steady-state)




$i_L(0^-) = \frac{12\text{ V}}{4\Omega} = 3\text{ A} = i_L(0^+)$

<u>$t > 0$</u>




KCL at v: $\frac{(v-12)}{4} + i_L + \frac{v}{2} = 6e^{-2t}$ (1)

also: $v = (2/5)\,di_L/dt$ (2)

Plugging (2) into (1) $\Rightarrow di_L/dt + (10/3)\,i_L = 10 + 20e^{-2t}$

$\therefore i_{L_n}(t) = Ae^{-10/3t}$, try $i_{L_f}(t) = B + Ce^{-2t}$ & plugging into D.E.

& equating like terms $\Rightarrow$ B=3, C=15

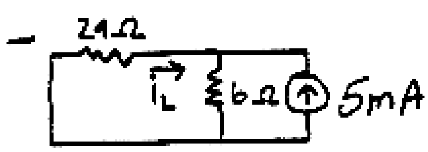
$\therefore i_L(t) = Ae^{-10/3t} + 3 + 15e^{-2t}, i_L(0) = 3 = A + 3 + 15 \Rightarrow A = -15$

$\therefore i_L(t) = -15e^{-(10/3)t} + 3 + 15e^{-2t}$

$\underline{\text{Now } v(t) = 2/5\,\frac{di_L}{dt} = 20e^{-(10/3)t} - 12e^{-2t}\text{ V}}$

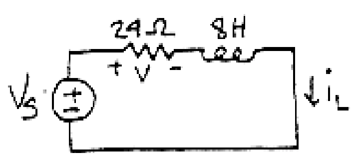
**P8.7-3**

$t = 0^-$




Current divider $\Rightarrow i_L(0^-) = -.5\left(\frac{6}{6+24}\right) = -1\text{mA}$

<u>$t > 0$</u>




$v_s = 25\sin 4000t$

KVL a: $-25\sin 4000t + 24i_L + .008\frac{di_L}{dt} = 0$

$\underline{\frac{di_L}{dt} + 3000i_L = \frac{25}{.008}\sin 4000t}$

$i_{L_n}(t) = Ae^{-3000t}$, *try* $i_{L_f}(t) = B\cos 4000t + C\sin 4000t$ &plug into D.E. and equate like terms $\Rightarrow$ B = −1/2, C=3/8

$\therefore i_L(t) = Ae^{-3000t} - (1/2)\cos 4000t + (3/8)\sin 4000t$

$i_L(0^+) = i_L(0^-) = -1 = A - 1/2 \Rightarrow A = -1/2$

$\therefore i_L(t) = -(1/2)e^{-3000t} - (1/2)\cos 4000t + (3/8)\sin 4000t$ mA

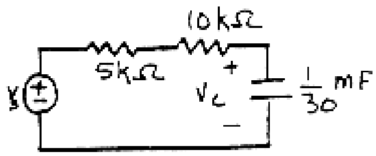
but $\underline{v(t) = 24i_L(t) = 12\,e^{-3000t} - 12\cos 4000t + 9\sin 4000t\ V}$

**P8.7-4**

<u>$t = 0^-$</u>    Steady-state


$v_c(0^-) = 0 = v_c(0^+)$

<u>$t > 0$</u>




$v_s = 10\cos 2t$

KVL a: $-10\cos 2t + 15\left(\frac{1}{30}\frac{dv_c}{df}\right) + v_c = 0$

$$\frac{dv_c}{dt} + 2v_c = 20\cos 2t$$

$v_{c_n}(t) = Ae^{-2t}$, try $v_{c_f}(t) = B\cos 2t + C\sin 2t$ & plug into D.E. $\Rightarrow B = C = 5$

$\therefore v_c(t) = Ae^{-2t} + 5\cos 2t + 5\sin 2t$

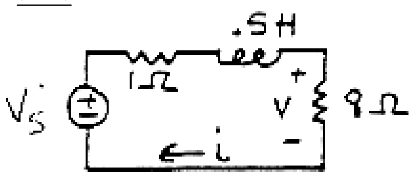
Now $v_c(0) = 0 = A + 5 \Rightarrow A = -5$

$\therefore$ <u>$v_c(t) = -5e^{-2t} + 5\cos 2t + 5\sin 2t$ V</u>

**P8.7-5**

<u>$t < 0$</u>    no sources present $\therefore i(0) = 0$

<u>$t < 0$</u>




$v_S = 10\sin 100t$

KVL a: $-10\sin 100t + i + 5di/dt + v = 0$    (1)

also : $i = v/8$    (2)

(2) into (1) yields <u>$\frac{dv}{dt} + 18v = 160\sin 100t$</u>

$\therefore v_n(t) = Ae^{-18t}$, try $v(t) = B\cos 100t + C\sin 100t$ & plug into the D.E. & equate like terms

$\Rightarrow B = -1.55$ & $C = 0.279$

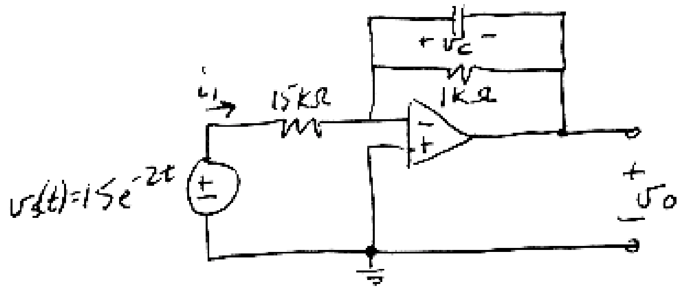
$\therefore v(t) = Ae^{-18t} - 1.55\cos 100t + 0.279\sin 100t$

Now from (2) : $v(0) = 8\, i\,(0) = 0$

$\therefore v(0) = 0 = A - 1.55 \Rightarrow A = 1.55$

so <u>$v(t) = 1.55e^{-18t} - 1.55\cos 100t + 0.279\sin 100t$ V</u>

**P8.7-6**




$v_o = -v_c$

so $v_c(0^-) = v_c(0^+) = -10\text{V}$

$$\left.\begin{aligned} \text{also } i &= \frac{v_s}{15\text{k}\Omega} = 0.001e^{-2t}\text{A} \\ i &= C\frac{dv_c}{dt} + \frac{v_c}{1\text{k}\Omega} \end{aligned}\right\} \text{yields } \frac{dv_c}{dt} + 4000v_c = 4000e^{-2t}$$

so $v_m = Ae^{-4000t}$ try $v_f = Be^{-2t}$ into D.E. $\Rightarrow B = 1.0005$

then $v_c(t) = v_f + v_m = 1.0005\,e^{-2t} + Ae^{-4000t}$

but $v_c(0) = -10 = 1.5000 + A \Rightarrow A = -11.000\,5$

so $v_L(t) = 1.0005\,e^{-2t} - 11.0005\,e^{-4000t}\text{V}$

but $\underline{v_o(t) = -v_c(t) = 11.000\,5\,e^{-4000t} - 1.0005\,e^{-2t}\text{ V}, t \ge 0}$

**P8.7-7** From graph $i_L(t) = -\frac{1}{4}t\text{ mA}$

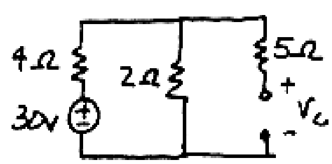
KVL in circuit yields $1i_L + 0.4\frac{di_L}{dt} = v_1 \Rightarrow \frac{di_L}{dt} + 2.5i_L = 2.5v_1$

but $i_L(t) = -\frac{1}{4}t\text{ mA}$

Solving gives $\underline{v_1 = -0.1 - 0.25t\text{ V}}$

**P8.7-8**

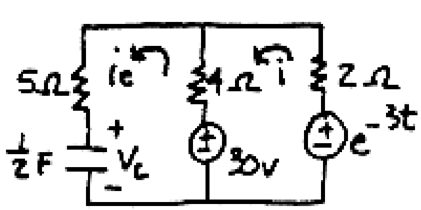
$\underline{t = 0^-\text{ (steady-state)}}$




$v_c(0^-) = \frac{2}{4+2}30 = 10\text{V}$

$\therefore\ v_c(0^+) = 10\text{V}$

$\underline{t > 0}$




$$\text{KVL}: 5/2\,\frac{dv_c}{dt} + v_c + 4\left(\frac{1}{2}\frac{dv_c}{dt} - i\right) = 30 \qquad (1)$$

$$2i + 4\left(i - \frac{1}{2}\frac{dv_c}{dt}\right) + 30 = e^{-3t} \qquad (2)$$

Solving for i in (2) and plugging into (1) yields

$dv_c/dt + (6/19)v_c = (6/19)\,(10 + (2/3)e^{-3t})$

$v_{c_n} = Ae^{-(6/19)t}$

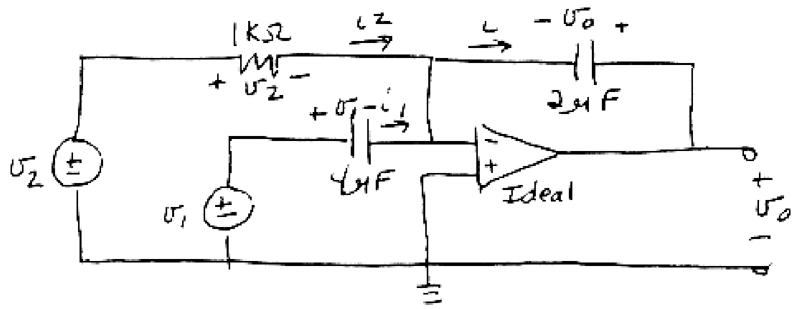
try $v_{c_f} = B + Ce^{-3t}$ & plug into D.E. $\Rightarrow -3Ce^{-3t} + \frac{6}{19}(B + Ce^{-3t}) = \frac{60}{19} + \frac{4}{19}e^{-3t}$

Equating coefficients and like terms yields $B = 10$, $C = -4/51$ $\Rightarrow v_c = 10 - \frac{4}{51}e^{-3t} + Ae^{-(6/19)t}$

When t=0, $v_c(0^+) = 10V$

$\therefore\ 10 = 10 - 4/51 + A$ $\therefore\ \underline{v_c(t) = 10 + \frac{4}{51}(e^{-(6/19)t} - e^{-3t})V}$ $\Rightarrow A = 4/51$

**P8.7-9** a)




Equations :

$$\left.\begin{aligned} v_o &= -v_{c2} \\ i &= C_2\frac{dv_{c2}}{dt} \\ i &= i_1 + i_2 \\ i &= C_1\frac{dv_{c1}}{dt} \\ i_2 &= \frac{v_2}{R} \\ v_{c1} &= v_1 \end{aligned}\right\} \Rightarrow \underline{v_0(t) = -2v_1(t) - 500\int_0^t v_2(t)dt}$$

b) Plugging in the sources gives

$$\underline{v_0(t) = -20e^{-2000t} + 2.5e^{-1000t} - 2.5\text{ V}}$$

**P8.7-10**

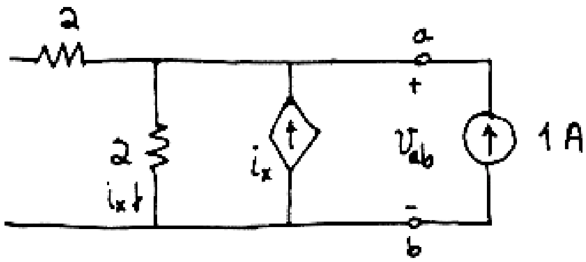
$$v_c(0^-) = v_c(0^+) = 3V$$

$$\left.\begin{aligned} v_c(\infty) &= 12 + 4000i \\ i &= 2mA \end{aligned}\right\} \Rightarrow v_L(\infty) = 12 - 8 = 4V$$

$$\tau = C\,R_{eq} = (1\times10^{-6})\,(4\times10^{3}) = 0.004\text{ sec},\quad 1/\tau = 250\text{ sec}^{-1}$$

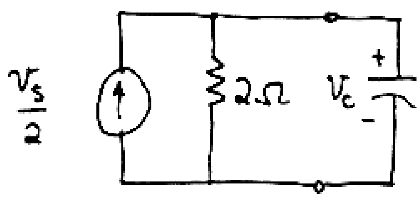
$$v(t) = 4 + (3-4)\,e^{-250t} = 4 - e^{-250t}\ \text{v},\ t \ge 0$$

**P8.7-11** Find the Norton equivalent; First $I_n = I_{sc} = \frac{v_s}{2}$. Next to find $R_T$ remove C & short $v_s$




$R_T = \frac{V_{ab}}{1}$ (note $i_x = 0$) so $R_T = 2$

Norton:

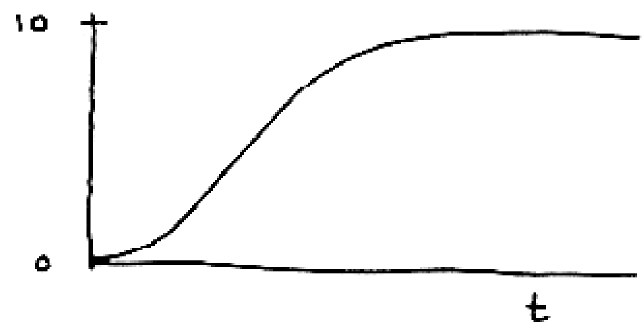



$v_c(0) = 0$

$$\frac{dv}{dt} + \frac{v}{2} = \frac{v_s}{2}$$

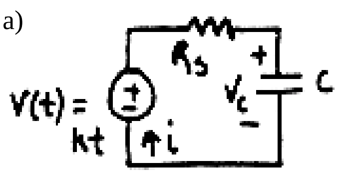
$$v_s = \begin{cases} 5t & 0 \le t \le 2 \\ 10 & t > 2 \end{cases}$$

| | | | |
|---|---|---|---|
| | $v = 5t + 10(e^{-t/2} - 1)$ | $0<t<2$ | |
| at t=2 | $v = 10e^{-1} = 3.68$ | | |
| t>2 | $v = Ae^{-t/2} + 10$ | | |
| at t=2 | $3.68 = A\,e^{-1} + 10$ | or | $A = 10 - 10e$ |
| | $v = 10e^{-t/2} - 10e^{-\frac{(t-2)}{2}} + 10$ | $t \ge 2$ | |




**P8.7-12**

a)




KVL a: $-kt + R_s i + v_s = 0$ and $i = c\, dv_c/dt$

$$\Rightarrow \frac{dv_c}{dt} + \frac{1}{R_sC} v_c = \frac{k}{R_sC} t$$

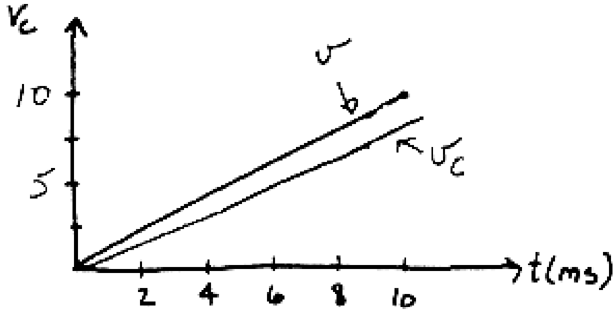
$v_c = v_{c_n} + v_{c_f}$, now $v_{c_n} = Ae^{-t/R_sC}$, try $v_{c_f} = B_0 + B_1 t$ & plug into D.E. $\Rightarrow B_1 + \frac{1}{R_sC}\left[B_0 + B_1 t\right] = \frac{k}{R_sC} t$

thus $B_0 = -kR_sC$, $B_1 = k$

so have $v_c(t) = Ae^{-t/R_sC} + k(t - R_sC)$, $v_c(0) = 0 = A - kR_sC \Rightarrow A = kR_sC$

$\therefore v_c(t) = k[t - R_sC(1 - e^{-t/R_sC})]$ plugging in k=1000, $R_s = 625\text{k}\Omega$ & C=2000pF get

$v_c(t) = 1000[t - 1.25 \times 10^{-5}(1 - e^{-800\,t})]$




$v$ & $v_c$ track well on millisecond time scale

b) $F=qE$ , $E={}^{v_c}\!/_{s}$ , $q=Cv_c$ $\Rightarrow$ $\therefore F=\frac{C}{s}v_c^2$

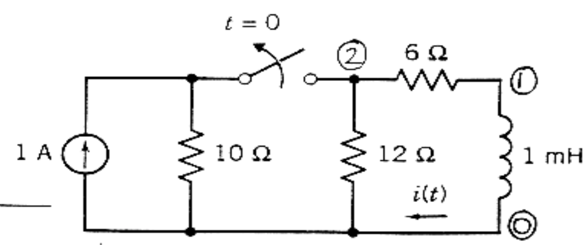
Now $F=m\ddot{y}$ where y = transverse deflection of beam and y(t) is described by the differential equation

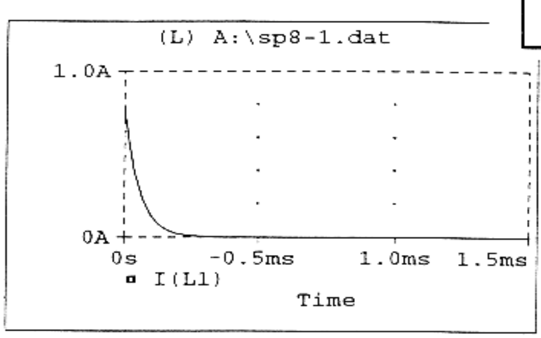
$$\frac{d^2y}{dt^2}=\frac{C}{ms}v_c^2(t)$$

where $v_c(t)$ is given in part a)

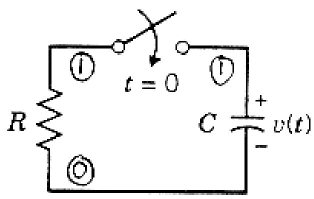
## PSpice Problems

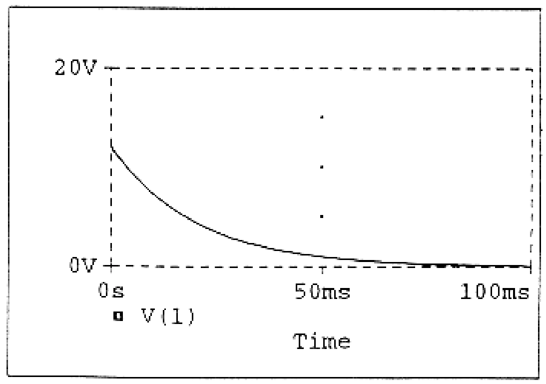
```
SP 8-1 Spice deck corresponding to
Problem SP 8-1
L1    1    0    1m    IC=.789
R1    1    2    6
R2    2    0    12
.TRAN .0001 1.5mS UIC
.PRINT TRAN I(L1)
.PROBE      I(L1)
.END
```



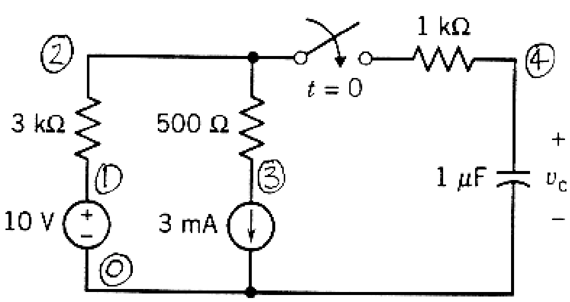



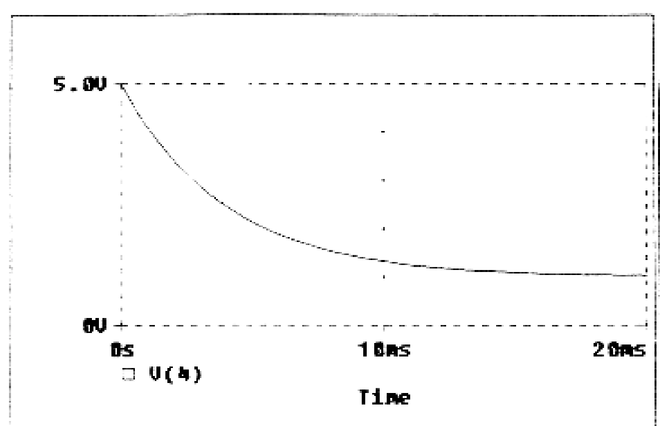
```
SP 8-2 Spice deck corresponding to Problem SP 8-2
C1    1     0     20u   IC=12
R1    1     0     1k
.TRAN 5mS   100mS UIC
.PRINT TRAN V(1)
.PROBE      V(1)
.END
```



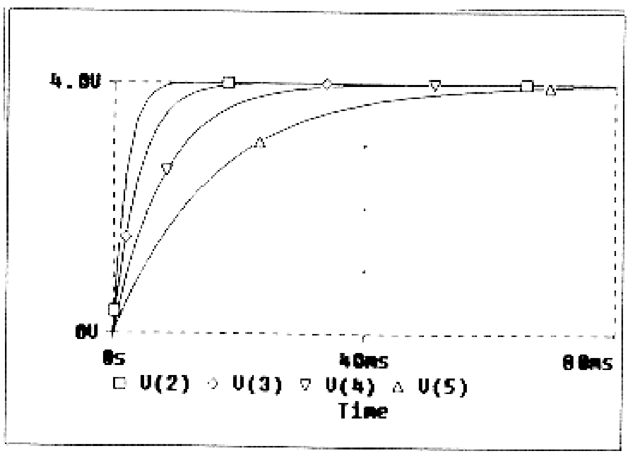




```
SP 8-3 Spice deck corresponding to Problem SP 8-3
V1          1     0     dc              10
R2          2     1                      3K
R3          2     3                     500
I4          3     0     dc                 3m
R5          2     4                    1000
C6          4     0                   1000n        IC=5v

.tran 1ms   20ms  UIC
.PROBE V(4)
.END
```



```
SP 8-4     Spice deck corresponding to Problem SP 8-4
V1    1     0     dc    4
R1    1     2     2K
C1    2     0     1000n IC=0
R2    1     3     4K
C2    3     0     1000n IC=0
R3    1     4     8K
C3    4     0     1000n IC=0
R4    1     5     16K
C4    5     0     1000n IC=0

.tran 2ms   80ms  UIC
.PROBE
.END
```




Verification Problems

**VP 8-1** First look at the circuit. The initial capacitor voltage is $v_c(0) = 8$. The steady-state capacitor voltage is $v_c = 4$. We expect an exponential transition from 8 volts to 4 volts. That's consistent with the plot.

Next, let's check the shape of the exponential transition. From the solution to Problem 1b

$$v_c(t) = 4e^{-\frac{t}{.67}} + 4$$

where t has units of ms. To check the point labeled on the plot, let $t_1 = 133$ms. Then

$$v_c(t_1) = 4e^{-\left(\frac{1.33}{.67}\right)} + 4 = 4.541 \underset{-}{\sim} 4.5398$$

So the plot is correct.

**VP 8-2**

The initial and steady-state inductor currents shown on the plot agree with the values obtained from the circuit.

The solution to Problem 1b gives

$$i_L(t) - 2e^{-\frac{t}{3.75}} + 5 \text{ mA}$$

where t has units of ms. Let $t_1 = 3.75$ms; then

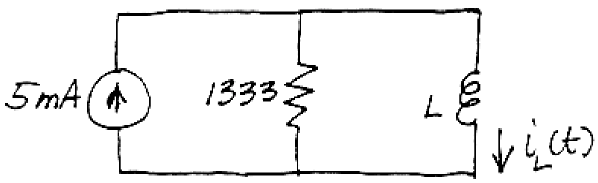
$$i_L(t_1) = -2e^{-\left(\frac{3.75}{3.75}\right)} + 5 = 4.264 \neq 4.7294$$

so the plot does not correspond to this circuit.

**VP 8-3**

Notice that the steady-state inductor current does not depend on the inductance, L. The initial and steady-state inductor currents shown on the plot agree with the values obtained from the circuit.

After t = 0




So $I_{sc} = 5$mA

$$\tau = \frac{L}{1333}$$

The inductor current is given by

$$i_L(t) = -2e^{-\frac{1333t}{L}} + 5 \text{ mA}$$

where t has units of seconds and L has units of henries.

Let $t_1 = 3.75$ms; then

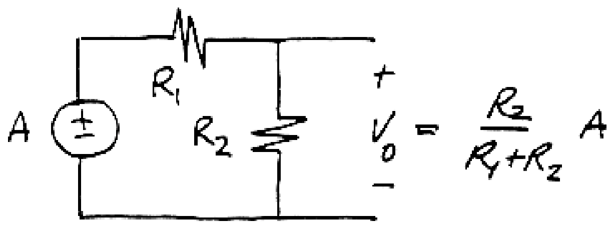
$$4.836 = i_L(t_1) = -2e^{-\frac{(1333)\cdot(.00375)}{L}} + 5 = -2e^{-\frac{5}{L}} + 5$$

so $\dfrac{4.836-5}{-2} = e^{-\frac{5}{L}}$ and $L = \dfrac{-5}{\ln\left(\dfrac{4.836-5}{-2}\right)} = 2$

is the required inductance.

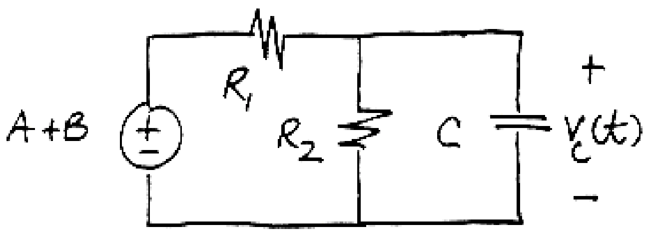
First consider the circuit. When t < 0 and the circuit is at steady-state

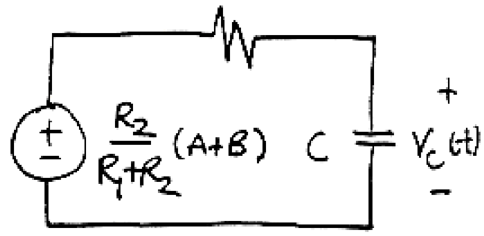
**VP 8-4**




For t > 0

$$R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$




So

$$V_{oc} = \frac{R_2}{R_1 + R_2}(A + B) \ , \quad R_{th} = \frac{R_1 R_2}{R_1 + R_2} \quad \text{and} \quad \tau = \frac{R_1 R_2 C}{R_1 + R_2}$$

Next, consider the plot. The initial capacitor voltage is ($V_o$ =) –2 and the steady-state capacitor voltage is ($V_{oc}$ =) 4

So $\quad v_C(t) = -6e^{-\frac{t}{\tau}} + 4$

At $t_1 = 1.333$ ms

$$3.1874 = v_C(t_1) = -6e^{-\frac{1.333\text{ ms}}{\tau}} + 4$$

So $\quad \tau = \dfrac{-1.333\text{ ms}}{\ln\left(\dfrac{-4+3.1874}{-6}\right)} = 0.67$ ms

Combining the information obtained from the circuit with the information obtained from the plot gives

$$\frac{R_2}{R_1 + R_2} A = -2$$

$$\frac{R_2}{R_1 + R_2}(A + B) = 4$$

$$\frac{R_1 R_2 C}{R_1 + R_2} = 0.67 \text{ ms}$$

There are many ways that A, B, $R_1$, $R_2$, and C can be chosen to satisfy these equations. Here is one convenient way. Pick $R_1$ = 3000 and $R_2$ = 6000. Then

$$\frac{2A}{3} = -2 \Rightarrow A = -3$$

$$\frac{2(A + B)}{3} = 4 \Rightarrow B - 3 = 6 \Rightarrow B = 9$$

$$2000 \cdot C = \frac{2}{3} \text{ ms} \Rightarrow \frac{1}{3} \mu\text{F} = C$$

## Design Problems

**DP 8-1**

6 V = the steady state response when the switch is open = $\frac{R_3}{R_1+R_2+R_3}12\text{ V} \quad\Rightarrow\quad R_1+R_2=R_3$.

8 V = the steady state response when the switch is open = $\frac{R_3}{R_1+R_3}12\text{ V} \quad\Rightarrow\quad R_1=\frac{R_3}{2}$.

10 ms = 5 τ = $\frac{R_3}{2}C$

Let $C$ = 1 μF. Then $R_3$ = 20 kΩ, $R_1$ = 10 kΩ and $R_2$ = 10 kΩ.

**DP 8-2**

1 mA = the steady state response when the switch is open = $\frac{12\text{ V}}{R_1+R_2} \quad\Rightarrow\quad R_1+R_2=12\text{ k}\Omega$.

4 mA = the steady state response when the switch is open = $\frac{12\text{ V}}{R_1} \quad\Rightarrow\quad R_1=3\text{ k}\Omega$.

Therefore, $R_2$ = 9 kΩ.

10 ms = 5 τ = $5\frac{L}{R_1+R_2}=\frac{L}{2400} \quad\Rightarrow\quad L=240\text{ H}$

**DP 8-3**

$R_t$ = 50 kΩ when the switch is open and $R_t$ = 49 kΩ ≈ 50 kΩ when the switch is closed so use $R_t$ = 50 kΩ.

(a) $C=\frac{10^{-6}}{5\left(5\times10^3\right)}=4\text{ pF}$

(b) $\Delta t=5\left(5\times10^3\right)\left(2\times10^{-6}\right)=0.5\text{ s}$

**DP 8-4**

$R_t$ = 50 kΩ when the switch is open and $R_t$ = 49 kΩ ≈ 50 kΩ when the switch is closed so use $R_t$ = 50 kΩ.

When the switch is open: $5e^{-\frac{\Delta t}{\tau}} = (1-k)5 \quad\Rightarrow\quad \ln(1-k) = -\frac{\Delta t}{\tau} \quad\Rightarrow\quad \Delta t = -\tau\ln(1-k)$

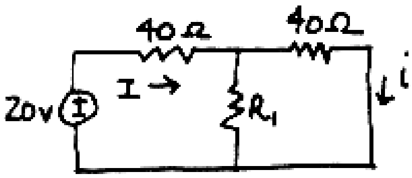
When the switch is open: $5-5e^{-\frac{\Delta t}{\tau}} = k5 \quad\Rightarrow\quad \Delta t = -\tau\ln(1-k)$

(a) $C = \frac{10^{-6}}{-\ln(1-.95)\left(5\times10^3\right)} = 6.67\text{ pF}$

(b) $\Delta t = -\ln(1-.95)\left(5\times10^3\right)\left(2\times10^{-6}\right) = 0.3\text{ s}$

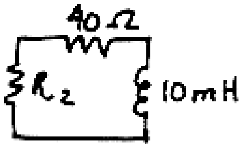
**DP 8-5**

$\underline{t = 0^-}$




Current divisions $i(o^-) = \dfrac{20V}{40+\dfrac{40R_1}{40+R_1}} \cdot \dfrac{R_1}{R_1+40}$

$\underline{t > 0}$




$i(t) = i(0)e^{-t/\tau}$ where $\tau = \dfrac{L}{R} = \dfrac{10^{-2}}{40+R_2}$

at $t < 200\mu s$ need $i>60mA$ and $i<180mA$

Find $R_1$ to set $i(0) < 180$ mA

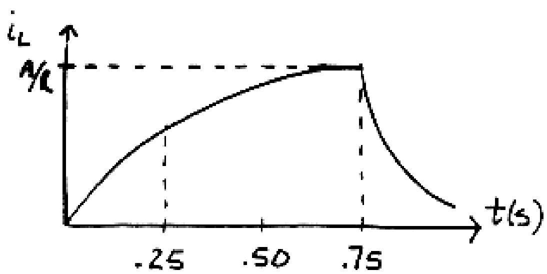
Try a solution: if $\underline{R_1 = 40\Omega}$ , then $i(0) = \frac{1}{6}A = 166.6mA$

Then $i(t) = 166.6$ mA $e^{-t/\tau}$ at $t_1 = 200\mu s = .2ms \rightarrow i(t_1) > 60mA$ required

Try $\underline{R_2 = 10\Omega}$, then $\tau = \dfrac{10^{-2}}{50} = .2ms = \dfrac{1}{5000}s$

$$i(t_1) = 166.6\times10^{-3}e^{-5000t_1} = 166.6\times10^{-3}e^{-1} = 61.2mA$$

**DP 8-6** The current wave form will look like:




We need only consider the rise time

where $i_L(t) = \dfrac{V_s}{R}(1-e^{-t/\tau})$

$= \dfrac{A}{R}(1-e^{-t/\tau})$

where $\tau = \dfrac{L}{R_T} = \dfrac{.2}{3} = \dfrac{1}{15}$

$\therefore i_L(t) = \dfrac{A}{3}(1-e^{-15t})$

Now find A so that $i_L^2 R_{fuse} \geq 10W$ during $.25 \leq t \leq .75$

$\therefore$want $[i_L^2(.25)]R_{fuse} = 10W$

$\dfrac{A^2}{9}(1-e^{-15(.25)})^2(1) = 10 \Rightarrow \underline{A = 9.715\text{ V}}$