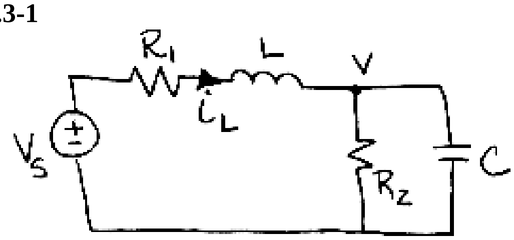
Problems

Section 9-3: Differential Equations for Circuits with Two Energy Storage Elements

P9.3-1



KCL: $i_L = \frac{v}{R_2} + C\frac{dv}{dt}$

KVL: $V_s = R_1 i_L + L\frac{di_L}{dt} + v$

$$v_s = R_1\left[\frac{v}{R_2} + C\frac{dv}{dt}\right] + \frac{L}{R_2}\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v$$

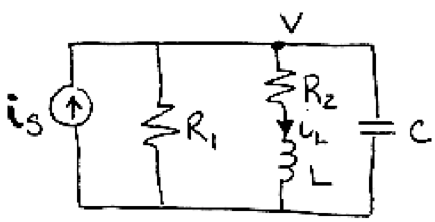
$$v_s = \left[\frac{R_1}{R_2} + 1\right]v + \left[R_1C + \frac{L}{R_2}\right]\frac{dv}{dt} + [LC]\frac{d^2v}{dt^2}$$

$R_1 = 2\Omega,\ R_2 = 100\Omega,\ L = 1mH,\ C = 10\mu F$

$$v_s = 1.02v + .00003\frac{dv}{dt} + 1\times10^{-8}\frac{d^2v}{dt^2}$$

$$\underline{1\times10^8 v_s = 1.02\times10^8 v + 3000\frac{dv}{dt} + \frac{d^2v}{dt^2}}$$

P9.3-2



KCL: $i_s = \frac{v}{R_1} + i_L + Csv$

KVL: $v = R_2 i_L + Lsi_L$

Solving Cramer's rule for i_L:

$$i_L = \frac{i_s}{\frac{R_2}{R_1} + \frac{Ls}{R_1} + R_2Cs + LCs^2 + 1}$$

$$\left[1 + \frac{R_2}{R_1}\right]i_L + \left[\frac{L}{R_1} + R_2C\right]si_L + [LC]s^2 i_L = i_s$$

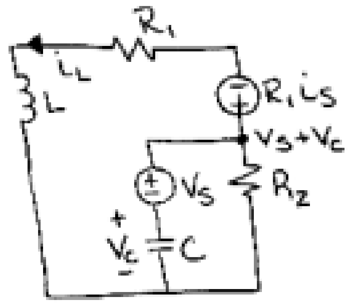
$R_1 = 100\Omega,\ R_2 = 10\Omega,\ L = 1mH,\ C = 10\mu F$

$$1.1i_L + .00011si_L + 1\times10^{-8}s^2i_L = i_s$$

$$\underline{1.1\times10^8 i_L + 11000si_L + s^2i_L = 1\times10^8 i_s}$$

P9.3-3

t > 0



$$\text{KCL: } i_L + C\frac{dv_c}{dt} + \frac{v_s + v_c}{R_2} = 0$$

$$\text{KVL: } R_1 i_s + R_1 i_L + L\frac{di_L}{dt} - v_c - v_s = 0$$

Solving for i_L:

$$\frac{d^2 i_L}{dt^2} + \left[\frac{R_1}{L} + \frac{1}{R_2 C}\right]\frac{di_L}{dt} + \left[\frac{R_1}{LR_2C} + \frac{1}{LC}\right] i_L = \frac{-R_1}{LCR_2} i_S - \frac{R_1}{L}\frac{di_s}{dt} + \frac{1}{L}\frac{dv_s}{dt}$$

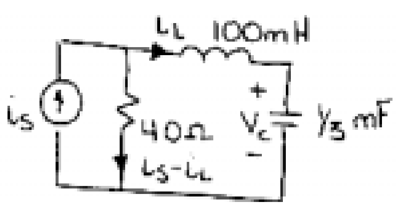
Section 9-4: Solution of the Second Order Differential Equation - The Natural Response

P9.4-1 From Problem P 9.3-2 the characteristic equation is

$$1.1\times10^8 + 11000s + s^2 = 0 \quad \Rightarrow$$

$$s_1,\ s_2 = \frac{-11000 \pm \sqrt{(11000)^2 - 4(1.1\times10^8)}}{2} = -5500 \pm j8930$$

P9.4-2



$$\text{KVL: } 40(i_s - i_L) = 100m\frac{di_L}{dt} + v_c \qquad \text{where } m = 10^{-3}$$

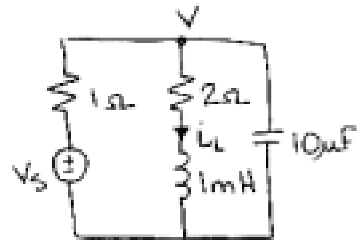
$$i_L = i_c = \frac{1}{3}m\frac{dv_c}{dt}$$

$$i_L = \frac{40}{3}m\frac{di_S}{dt} - \frac{40}{3}m\frac{di_L}{dt} - \frac{100}{3}m^2\frac{d^2 i_L}{dt^2}$$

$$\frac{d^2 i_L}{dt^2} + 400\frac{di_L}{dt} + 30000 i_L = 400\frac{di_S}{dt}$$

$$s^2 + 400s + 30000 = 0 \quad \Rightarrow \quad (s+100)(s+300) = 0 \quad \Rightarrow \quad s_1 = -100, \quad s_2 = -300$$

P9.4-3



KCL: $\frac{v - v_s}{1} + i_L + 10\mu \frac{dv}{dt} = 0$ where $\mu = 10^{-6}$

KVL:

$v = 2i_L + 1m \frac{di_L}{dt}$ where $m = 10^{-3}$

$$0 = 2i_L + 1m \frac{di_L}{dt} - v_s + i_L + 10\mu \cdot 2 \frac{di_L}{dt} + 10\mu \cdot 1m \frac{d^2 i_L}{dt}$$

$$v_s = 3i_L + .00102 \frac{di_L}{dt} + 1 \times 10^{-8} \frac{d^2 i_L}{dt^2}$$

$$\frac{d^2 i_L}{dt} + 102000 \frac{di_L}{dt} + 3 \times 10^{-8} i_L = 1 \times 10^{8} v_s$$

$$\underline{s^2 + 102000s + 3 \times 10^8 = 0}, \quad \therefore \underline{s_1 = 3031, \quad s_2 = -98969}$$

<u>Section 9.5:</u> Natural Response of the Unforced Parallel RLC Circuit

P9.5-1 $v(0) = 6, \ \frac{dv(0)}{dt} = -3000$

Using operators, the node equation is: $Csv + \frac{v}{R} + \frac{(v - v_S)}{sL} = 0$ or $\left(LCs^2 + \frac{L}{R}s + 1\right)v = v_S$

So the characteristic equation is: $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$

$$\Rightarrow s_{1,2} = -250 \pm \sqrt{250^2 - 40{,}000} = -100, \ -400$$

So $v(t) = Ae^{-100t} + Be^{-400t}$

$v(0) = 6 = A + B$

$$\left.\frac{dv(0)}{dt} = -3000 = -100A - 400B\right\} \begin{array}{l} A = -2 \\ B = 8 \end{array}$$

$\therefore \ \underline{v(t) = -2e^{-100t} + 8e^{-400t}}$ $\quad t > 0$

P9.5-2

$v(0) = 2, \ i(0) = 0$

Characteristic equation $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 4s + 3 = 0 \Rightarrow s = -1, \ -3$

$v(t) = Ae^{-t} + Be^{-3t}$

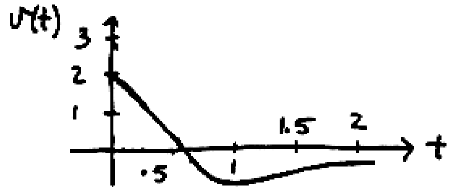
Use eq. 9.5−12 $\Rightarrow s_1 A + s_2 B = -\frac{v(0)}{RC} - \frac{i(0)}{C}$

$$-1A - 3B = -\frac{2}{1/4} - 0 = -8 \qquad (1)$$

also have $v(0) = 2 = A + B$ $\qquad (2)$

From (1) & (2) get $A = -1, \ B = 3$

$\therefore \ \underline{v(t) = -1e^{-t} + 3e^{-3t} \text{ V}}$



P9.5-3

$$\text{KVL}: \; i_1 + 5\frac{di_1}{dt} - 3\frac{di_2}{dt} = 0 \qquad (1)$$

$$\text{KVL}: \; -3\frac{di_1}{dt} + 3\frac{di_2}{dt} + 2i_2 = 0 \qquad (2)$$

in operator form

$$\left.\begin{array}{l}(1+5s)i_1 + (-3s)i_2 = 0 \\ (-3s)\, i_1 + (3s+2)\, i_2 = 0\end{array}\right\} \text{ thus } \Delta = (1+5s)\,(3s+2) - 9s^2 = 6s^2 + 13s + 2 = 0 \;\Rightarrow\; s = -\tfrac{1}{6}, -2$$

$$\text{Thus } i_1(t) = Ae^{-t/6} + Be^{-2t}$$

$$i_2(t) = Ce^{-t/6} + De^{-2t}$$

Now $i_1(0) = 11 = A + B; i_2(0) = 11 = C + D$

from (1) & (2) get

$$\frac{di_1(0)}{dt} = -\frac{33}{2} = -\frac{A}{6} - 2B\,; \quad \frac{di_2(0)}{dt} = -\frac{143}{6} = -\frac{C}{6} - 20$$

which yields $A = 3,\ B = 8,\ C = -1,\ 0 = 12$

$\underline{i_1(t) = 3e^{-t/6} + 8e^{-2t}\ \text{A}}$ & $\underline{i_2(t) = -e^{-t/6} + 12e^{-2t}\ \text{A}}$

<u>Section 9.6:</u> Natural Response of the Critically Damped Unforced Parallel RLC Circuit

P9.6-1

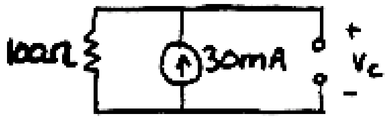
$\underline{t > 0}$

$$\text{KVL a: } 100i_c + .025\frac{di_c}{dt} + v_c = 0,\; i_c = 10^{-5}\frac{dv_c}{dt}$$

$$\therefore \; \frac{d^2v_c}{dt^2} + 4000\frac{dv_c}{dt} + 4\times10^6 v_c = 0$$

$$s^2 + 4000s + 4\times10^6 = 0 \Rightarrow s = -2000,\ -2000 \;\therefore\; v_c(t) = A_1e^{-2000t} + A_2te^{-2000t}$$

$\underline{t = 0^-}$ (Steady – State)



$$i_L = i_c(0^-) = 0 = i_c(0^+) \Rightarrow \frac{dv_c(0^+)}{dt} = 0$$

$$v_c(0^-) = 3\text{ V} = v_c(0^+)$$

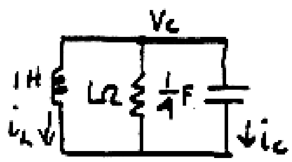
so $v_c(0^+) = 3 = A_1$

$$\frac{dv_c(0^+)}{dt} = 0 = -2000A_1 + A_2 \Rightarrow A_2 = 6000$$

$$\therefore \underline{v_c(t) = (3+6000t)e^{-2000t}\text{ V}}$$

P9.6-2

$\underline{t > 0}$

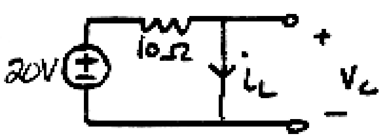


KCL at v_c: $\int_{-\infty}^{t} v_c\,dt + v_c + \frac{1}{4}\frac{dv_c}{dt} = 0$

$$\Rightarrow \frac{d}{dt}\;\frac{d^2v_c}{dt} + 4\frac{dv_c}{dt} + 4v_c = 0$$

$s^2 + 4s + 4 = 0,\ s = -2,\ -2 \quad \therefore v_c(t) = A_1e^{-2t} + A_2te^{-2t}$

$\underline{t = 0^-}$ (Steady − State)



$$v_c(0^-) = 0 = v_c(0^+) \;\&\; i_L(0^-) = \frac{20\text{ V}}{10\Omega} = 2\text{ A} = i_L(0^+)$$

Since $v_c(0^+) = 0$ then $i_c(0^+) = -i_L(0^+) = -2\text{ A}$

$$\therefore \frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{1/4} = -8\,\text{V/S}$$

So $v_c(0^+) = 0 = A_1$

$$\frac{dv_c(0^+)}{dt} = -8 = A_2$$

$$\therefore \underline{v_c(t) = -8te^{-2t}\text{ V}}$$

P9.6-3

Assume steady – state at $t = 0^-$ $\quad \therefore v_c(0^-) = 10^4\,\text{V} \;\&\; i_L(0^-) = 0$

t > 0

KVL a: $-v_c + .01\dfrac{di_L}{dt} + 10^6 i_L = 0 \qquad (1)$

Also : $i_L = -C\,dv_c/dt = -C\left[.01\dfrac{d^2 i_L}{dt^2} + 10^6\dfrac{di_L}{dt}\right] \qquad (2)$

$$\therefore \quad .01C\frac{d^2 i_L}{dt^2} + 10^6 C\frac{di_L}{dt} + i_L = 0$$

Characteristic eq. $\Rightarrow .01C\,s^2 + 10^6 s + 1 = 0 \Rightarrow s = \dfrac{-10^6 C \pm \sqrt{\left(10^6 C\right)^2 - 4(.01C)}}{2(.01C)}$

for critically damped: $10^{12}C^2 - .04C = 0$

$\Rightarrow C = 0.04\text{pF} \;\; \therefore s = -5\times10^7,\; -5\times10^7$

So $i_L(t) = A_1 e^{-5\times10^7 t} + A_2 t e^{-5\times10^7 t}$

Now from (1) $\Rightarrow \dfrac{di_L}{dt}(0^+) = 100\left[v_c(0^+) - 10^6 i_L(0^+)\right] = 10^6\,A/s$

So $i_L(0) = 0 = A_1$ and $\dfrac{di_{L(0)}}{dt} = 10^6 = A_2 \quad \therefore i_L(t) = 10^6 t e^{-5\times10^7 t}\,\text{A}$

Now $\underline{v(t) = 10^6 i_L(t) = 10^{12} t e^{-5\times10^7 t}\,\text{V}}$

P9.6-4

$s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC} = 0 \qquad$ with $\dfrac{1}{RC} = 500 \;\&\; \dfrac{1}{LC} = 62.5\times10^3$ yields $s = -250,\; -250$

$$v(t) = Ae^{-250t} + Bte^{-250t}$$

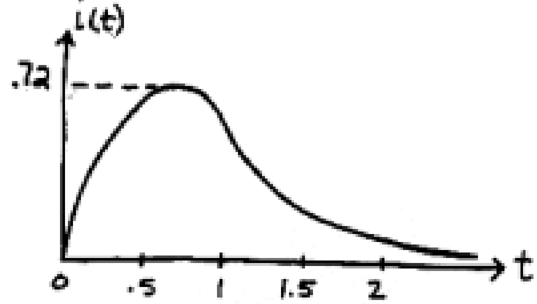
$$v(0) = 6 = A$$

$$\frac{dv(0)}{dt} = -3000 = -250A + B \Rightarrow B = -1500$$

$$\therefore \underline{v(t) = 6e^{-250t} - 1500te^{-250t}}$$

P9.6-5

KVL : $\frac{di}{dt} + Ri + \underbrace{2+4\int_0^t i\,dt}_{v(t)} = 6 \qquad (1)$



taking the derivative wrt t : $\frac{d^2i}{dt^2}+R\frac{di}{dt}+4i = 0$

Char. eq.: $s^2+Rs+4 = 0$

Let R=4 for critical damping $\Rightarrow$ $(s+2)^2 = 0$

So $i(t) = Ate^{-2t}+Be^{-2t}$

$i(0) = 0 \Rightarrow B = 0$

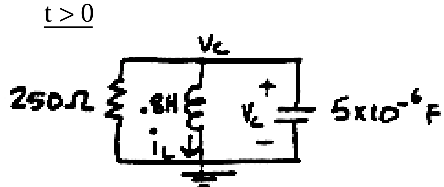
from (1) $\frac{di(0)}{dt} = 4-R(i(0)) = 4-R(0) = 4 = A$

$\therefore$ $\underline{i(t) = 4te^{-2t}A}$

Section 9-7: Natural Response of an Underdamped Unforced Parallel RLC Circuit

P9.7-1

t > 0



KCL at v_c: $v_c/250 + i_L + 5\times10^{-6}\frac{dv_c}{dt} = 0 \qquad (1)$

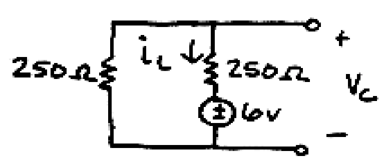
also : $v_c = .8\frac{di_L}{dt} \qquad (2)$

Solving for i_L in (1) & plugging into (2)

$$\frac{d^2v_c}{dt^2}+800\frac{dv_c}{dt}+2.5\times10^5 v_c = 0 \quad \Rightarrow s^2+800s+250{,}000 = 0,\ \ s = -400\pm j\,300$$

$$\therefore\ v_c(t) = e^{-400t}\left[A_1\cos 300t+A_2\sin 300t\right]$$

$t = 0^-$ (Steady − State)



$i_L(0^-) = \frac{-6V}{500\Omega} = -6/500\,A = i_L(0^+)$

$v_c(0^-) = 250(-6/500)+6 = 3V = v_c(0^+)$

Now from (1): $\frac{dv_c(0^+)}{dt} = -2\times10^5 i_L(0^+) - 800v_c(0^+) = 0$

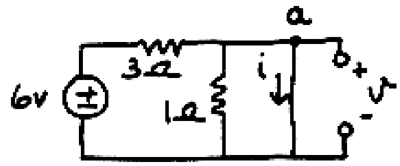
So $v_c(0^+) = 3 = A_1$

$\frac{dv_c(0^+)}{dt} = 0 = -400A_1 + 300A_2 \Rightarrow A_2 = 4$

$\therefore$ $\underline{v_c(t) = e^{-400t}\left[3\cos 300t + 4\sin 300t\right] V}$

P9.7-2

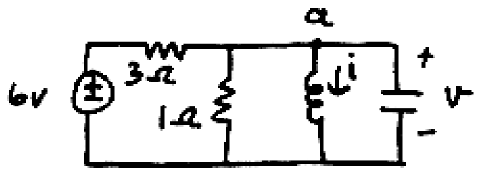
t = 0^-



$i(0) = 2A$

$v(0) = 0$

t = 0^-



KCL at node a:

$$\frac{v}{1} + C\frac{dv}{dt} + \frac{1}{L}\int_0^t v\,dt \; + \; i(0) \; = \; 0 \quad (1)$$

in operator form have $v + Csv + \frac{1}{Ls}v + i(0) = 0$ or $\left(s^2 + \frac{1}{C}s + \frac{1}{LC}\right)v = 0$

with $s^2 + 4s + 8 = 0 \quad \Rightarrow \; s = -2 \pm j2$

$$v(t) = e^{-2t}\left[B_1\cos 2t \; + \; B_2 \sin 2t\right]$$

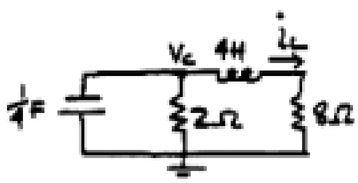
$$v(0) = 0 = B_1$$

From (1), $\frac{dv(0)}{dt} = \frac{1}{C}\left[-i(0) - v(0)\right] = -4[2] = -8 = 2B_2$ or $B_2 = -4$

So $\underline{v(t) = -4e^{-2t}\sin 2t\text{V}}$

P9.7-3

t > 0



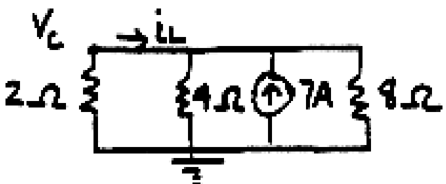
KCL at v_c : $\frac{1}{4}\frac{dv_c}{dt} + \frac{v_c}{2} + i_L \; = \; 0 \qquad (1)$

KVL : $v_c \; = \; \frac{4di_L}{dt} + 8i_L \qquad (2)$

(2) into (1) yields $\frac{d^2i_L}{dt^2} + 4\frac{di_L}{dt} + 5i_L \; = \; 0 \Rightarrow \; s^2 + 4s + 5 = 0 \; \Rightarrow \; s = -2 \pm i$

$$\therefore i_L(t) \; = \; e^{-2t}\left[A_1\cos t + A_2 \sin t\right]$$

t = 0^- (Steady – State)



$$\frac{v_c(0^-)}{2} \; = \; 7\left(\frac{4\|8}{4\|8+2}\right)$$

$$\Rightarrow v_c(0^-) \; = \; 8\text{ V} \; = \; v_c(0^+)$$

$$i_L(0^-) \; = \; \frac{-8\text{ V}}{\Omega} \; = \; -4\text{ A} \; = \; i_L(0^+)$$

$$\therefore \text{ from (2) } \frac{di_L(0^+)}{dt} \; = \; \frac{v_c(0^+)}{4} - 2i_L(0^+) \; = \; \frac{8\text{ V}}{4} - 2(-4) \; = \; 10\;\frac{\text{A}}{\text{s}}$$

So $i_L(0^+) \; = \; -4 \; = \; A_1$

$$\frac{di_L(0^+)}{dt} \; = \; 10 \; = \; -2A_1 + A_2 \; \Rightarrow A_2 \; = \; 2$$

$\therefore \; \underline{i_L(t) \; = \; e^{-2t}\left[-4\cos t + 2\sin t\right]\text{A}}$

P9.7-4 Have underdamped response

$$\therefore\ v(t) = e^{-\alpha t}\left[k_1 \cos \omega t + k_2 \sin\omega t\right] + k_3$$

$$v(\infty) = 0 \Rightarrow k_3 = 0,\ v(0) = 0 \Rightarrow k_1 = 0$$

$$\therefore\ v(t) = k_2 e^{-\alpha t}\sin\omega t$$

from Fig. P 9.7-6

$t \approx 5\text{ms} \leftrightarrow v \approx 260\text{mV}$ (max)

$t \approx 7.5\text{ms} \leftrightarrow v \approx -200\text{ mV}$ (min)

$\therefore$ distance between adjacent maxima is $\approx 2(7.5\text{ms}) = T \quad \Rightarrow \omega = \dfrac{2\pi}{T} = 1257\ \text{rad}/\text{s}$

$$\text{So } .26 = k_2 e^{-\alpha(.005)}\sin 1257(.005) \qquad (1)$$

$$-.2 = k_2 e^{-\alpha(.0075)}\sin 1257(.0075) \qquad (2)$$

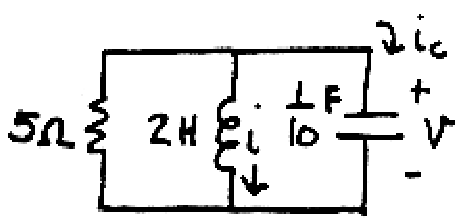
dividing (1) by (2)

$$-1.3 = e^{\alpha(.0025)}\frac{\sin(6.29\text{ rad})}{\sin(9.43\text{ rad})} \Rightarrow e^{.0025\alpha} = 1.95 \Rightarrow \alpha = 267$$

From (1) get $k_2 = 544$

$$\therefore\ \underline{v(t) = 544e^{-267t}\sin 1257t} \qquad (\text{approx. answer})$$

P9.7-7



$v(0) = 2\text{V}$

$i(0) = 1/10\ \text{A}$

Char. eq.$\Rightarrow s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC} = 0$ or $s^2 + 2s + 5 = 0$ thus roots are $s = -1 \pm j2$

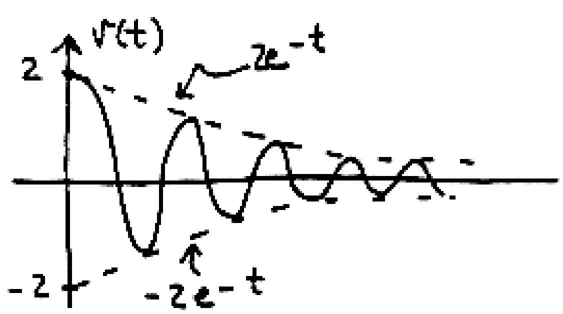
So have $v(t) = e^{-t}\left[B_1 \cos 2t + B_2 \sin 2t\right]$

now $v(0^+) = \underline{2 = B_1}$

$$\text{Need } \frac{dv(0^+)}{dt} = \frac{1}{C}i_c(0^+) \qquad \text{KCL yields } i_c(0^+) = -\frac{v(0^+)}{5} - i(0^+) = -\frac{1}{2}\ \frac{\text{V}}{\text{s}}$$

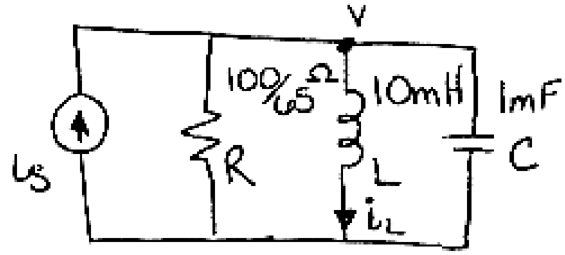
$$\text{So } \frac{dv(0^+)}{dt} = 10\left(-\frac{1}{2}\right) = -B_1 + 2B_2 \Rightarrow \underline{B_2 = -3/2}$$

$$\text{Finally, have } \underline{v(t) = 2e^{-t}\cos 2t - \frac{3}{2}e^{-t}\sin 2t\ \text{V}} \qquad t > 0$$



<u>Section 9-8:</u>

P9.8-1



$$\text{KCL}: i_s = \frac{v}{R} + i_L + C\frac{dv}{dt}$$

$$\text{KVL}: v = L\frac{di_L}{dt}$$

$$i_s = \frac{L}{R}\frac{di_L}{dt} + i_L + LC\frac{d^2 i_L}{dt^2}$$

(a)

$$i_s = 1\,u(t) \quad \therefore \quad \text{assume } i_f = A$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{1}{LC} i_L = i_s$$

$$0 + 0 + A\frac{1}{(.01)(1\times10^{-3})} = 1$$

$$A = \underline{1\times10^{-5} = i_f}$$

(b)

$$i_s = .5t\,u(t) \quad \therefore \quad \text{assume } i_f = At + B$$

$$0 + A\frac{65}{(100)\,(.001)} + (At + B)\frac{1}{(.01)(.001)} = .5t$$

$$650A + 100000B = 0$$

$$100000At = .5t$$

$$A = 5\times10^{-6}$$

$$B = 3.25\times10^{-8}$$

$$\underline{i_f = 5\times10^{-6}t - 3.25\times10^{-8}\,A}$$

(c)

$$i_s = 2e^{-250t} \quad \therefore \quad \text{assume } i_f = Ae^{-250t}$$

This does not work $\therefore \; i_f = Bte^{-250t}$

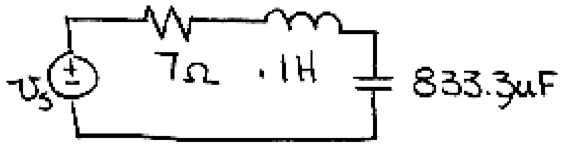
$$\frac{Be^{-250t}}{RC} + \frac{-250Bte^{-250t}}{RC} + \frac{Bte^{-250t}}{LC} = 2e^{-250t}$$

$$150\,B = 2$$

$$B = .0133$$

$$\underline{i_f = .0133\,te^{-250t}\,A}$$

P9.8-2



$$\frac{d^2v}{dt} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = v_s$$

(a) $v_s = 2 \quad \therefore \text{ assume } v_f = A$

$0 + 0 + 12000A = 2$

$A = \frac{1}{6000} = v_f$

(b) $v_s = .2t \quad \therefore \text{ assume } v_f = At + B$

$70A + 12000At + 12000B = .2t$

$70A + 12000\,B = 0$

$12000\,At = .2t$

$A = \frac{1}{60000}\ ,\ B = \frac{70A}{12000}\ ,\ B = 350$

$\therefore v_f = \frac{t}{60000} + 350\text{ V}$

(c) $v_s = e^{-30t} \qquad \therefore \text{ assume } Ae^{-30t}$

$900A - 2100Ae^{-30t} + 12000Ae^{-30t} = e^{-30t}$

$10800Ae^{-30t} = e^{-30t}$

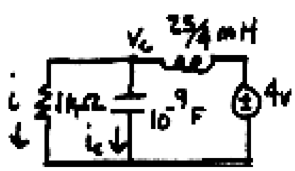
$A = \frac{1}{10800}$

$v_f = \frac{e^{-30t}}{10800}\text{V}$

Section 9-9: Complete Response of an RLC Circuit

P9.9-1

t > 0



KCL at v_c : $\frac{v_c}{1000} + 10^{-9}\frac{dv_c}{dt} + \frac{1}{25/4 \times 10^{-3}}\int_{-\infty}^{t}(v_c - 4)\,dt = 0$

$$\frac{d}{dt} \quad 10^{-3}\frac{dv_c}{dt} + 10^{-9}\frac{d^2v_c}{dt^2} + 160(v_c - 4) = 0$$

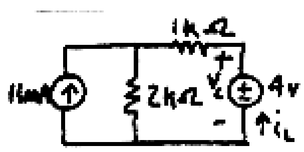
$$\Rightarrow \frac{d^2v_c}{dt^2} + 10^6\frac{dv_c}{dt} + 1.6\times10^{11}v_c = 6.4\times10^{11}$$

$$\therefore\ s^2 + 10^6 s + 1.6\times10^{11} = 0 \Rightarrow s = -2\times10^5,\ -8\times10^5 \Rightarrow v_{cn}(t) = A_1e^{-2\times10^{5}t} + A_2e^{-8\times10^{5}t}$$

Try $v_{cf} = B$ & plug into D.E. $\Rightarrow B = 4$

$\therefore\ v_c(t) = A_1e^{-2\times10^{5}t} + A_2e^{-8\times10^{5}t} + 4$

$t = 0^-$ (steady-state)



$v_c(0^-) = 4V = v_c(0^+)$ & from KVL : $-4 + i_L + 2(11 + i_L) = 0$

$\Rightarrow i_L(0^-) = i_L(0^+) = -6 \text{ mA}$

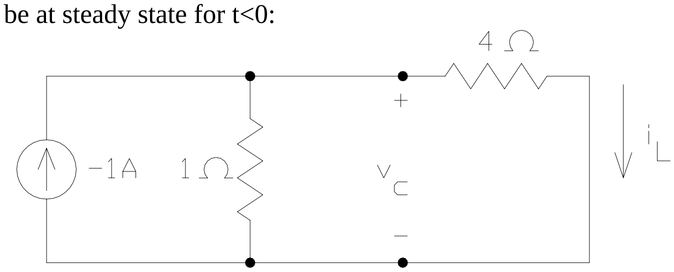
$\underline{t = 0^+}$

$$i_c(0^+) = i_L(0^+) - \frac{v_c(0^+)}{1} = -6 - 4 = -10\text{mA} \quad \therefore \quad \frac{dv_c(0^+)}{dt} = -\frac{.01}{10^{-9}} = -10^7 \text{ V/s}$$

$$\left.\begin{array}{l} \text{So } v_c(0) = 4 = A_1 + A_2 + 4 \Rightarrow A_1 + A_2 = 0 \\ \frac{dv_c(0)}{dt} = -10^7 = -2\times10^5 A_1 - 8\times10^5 A_2 \end{array}\right\} \begin{array}{l} A_1 = 50/3 \\ A_2 = -50/3 \end{array}$$

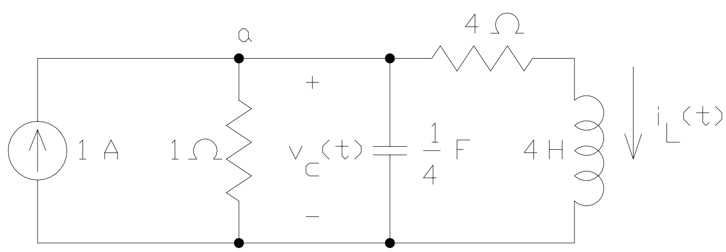
$$\text{So } i(t) = \frac{v_c(t)}{1} = -\frac{50}{3}e^{-2\times10^5 t} + \frac{50}{3}e^{-8\times10^{5t}} + 4 \text{ mA}$$

9.9-2 The circuit will be at steady state for t<0:



so $i_L(0+) = i_L(0-) = -0.2$ A and $v_C(0+) = v_C(0-) = -0.8$ V.

For t>0:



Apply KCL at node a to get:
$$1 = \frac{v_C(t)}{1} + \frac{1}{4}\frac{d}{dt}v_C(t) + i_L(t)$$

Apply KVL to the right-most mesh to get:
$$v_c(t) = 4\,i_L(t) + 4\frac{d}{dt}i_L(t)$$

Use the substitution method to get

$$1 = \left(4i_L(t) + 4\frac{d}{dt}i_L(t)\right) + \frac{1}{4}\frac{d}{dt}\left(4i_L(t) + 4\frac{d}{dt}i_L(t)\right) + i_L(t)$$

or

$$1 = \frac{d^2}{dt^2}i_L(t) + 5\frac{d}{dt}i_L(t) + 5i_L(t)$$

The forced response will be a constant, i_L= B so $1 = \frac{d^2}{dt^2}B + 5\frac{d}{dt}B + 5B \quad \Rightarrow \quad B = 0.2 \text{ A}$.

To find the natural response, consider the characteristic equation:

$$0 = s^2 + 5s + 5 = (s+3.62)(s+1.38)$$

The natural response is

$$i_n = A_1 e^{-3.62t} + A_2 e^{-1.38t}$$

so

$$i_L(t) = A_1 e^{-3.62t} + A_2 e^{-1.38t} + 0.2$$

Then

$$v_C(t) = \left(4\, i_L(t) + 4\frac{d}{dt} i_L(t)\right) = -10.48 A_1 e^{-3.62t} - 1.52 A_2 e^{-1.38t} + 0.8$$

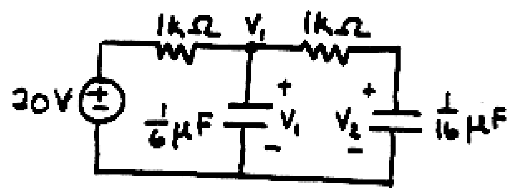
At t=0+

$$-0.2 = i_L(0+) = A_1 + A_2 + 0.2$$
$$-0.8 = v_C(0+) = -10.48 A_1 - 1.52 A_2 + 0.8$$

so A_1 = 0.246 and A_2 = -0.646. Finally

$$i_L(t) = 0.246\, e^{-3.62\,t} - 0.646\, e^{-1.38\,t} \text{ A}$$

P9.9-3 t > 0



KCL at v_1 : $\frac{v_1 - 20}{1000} + \frac{1}{6}\times 10^{-6}\frac{dv_1}{dt} + \frac{v_1 - v_2}{1000} = 0$

yields $2v_1 + \frac{1}{6}\times 10^{-3}\frac{dv_1}{dt} - 20 = v_2$

also : $\frac{v_1 - v_2}{1000} = \frac{1}{16}\times 10^{-6}\frac{dv_2}{dt}$

yields $v_1 - v_2 - \frac{1}{16}\times 10^{-3}\frac{dv_2}{dt} = 0$

(1) into (2)Yields: $\frac{d^2 v_1}{dt^2} + 2.8\times 10^4 \frac{dv_1}{dt} + 9.6\times 10^7 v_1 = 1.92\times 10^9$

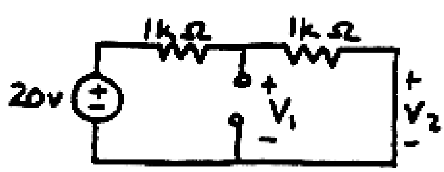
$$\Rightarrow s^2 + 2.8\times 10^4 s + 9.6\times 10^7 = 0 \quad \Rightarrow s = -4\times 10^3,\ -2.4\times 10^4$$

$\therefore\ v_{1n}(t) = A_1 e^{-4\times 10^3 t} + A_2 e^{-2.4\times 10^4 t}$

Try $v_{1f} = B$ & plug into D.E. $\Rightarrow B = 20$

So $v_1(t) = A_1 e^{-4\times 10^3 t} + A_2 e^{-2.4\times 10^4 t} + 20$

$t = 0^-$ (steady−state)



$$v_2(0^-) = 0 = v_2(0^+) \;\; \& \;\; v_1(0^-) = 10V = v_1(0^+)$$

$$\therefore \text{ from } (1) \;\; \frac{dv_1(0^+)}{dt} = -1.2\times10^4 v_1(0^+) + 1.2\times10^5$$

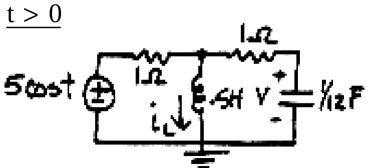
$$= -1.2\times10^4(10) + 1.2\times10^5 = 0$$

$$\left.\begin{aligned} &\text{So } v_1(0^+) = 10 = A_1 + A_2 + 20 \\ &\frac{dv_1(0^+)}{dt} = 0 = -4\times10^3 A_1 + 2.4\times10^4 A_2 \end{aligned}\right\} \begin{aligned} A_1 &= -12 \\ A_2 &= 2 \end{aligned}$$

$$\therefore \;\; \underline{v_1(t) = -12e^{-4\times10^3 t} \;\; 2e^{-2.4\times10^4 t} + 20 \text{ V}}$$

P9.9-4

t > 0



$$\text{KCL at top node}: \left(.5\frac{di_L}{dt} - 5\cos t\right) + i_L + \tfrac{1}{12}\, \tfrac{dv}{dt} = 0 \qquad (1)$$

$$\text{KVL at right loop}: .5\frac{di_L}{dt} = \tfrac{1}{12}\frac{dv}{dt} + v \qquad (2)$$

$$\tfrac{d}{dt} \text{ of } (1) \Rightarrow .5\frac{d^2 i_L}{dt^2} + \frac{di_L}{dt} + \frac{1}{12}\frac{d^2 v}{dt^2} = -5\sin t \qquad (3)$$

$$\tfrac{d}{dt} \text{ of } (2) \Rightarrow .5\frac{d^2 i_L}{dt^2} = \tfrac{1}{12}\frac{d^2 v}{dt^2} + \frac{dv}{dt} \qquad (4)$$

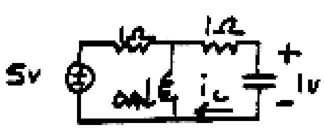
Solving for $\frac{d^2 i_L}{dt^2}$ in (4) and $\frac{di_L}{dt}$ in (2) & plugging into (3)

$$\underline{\frac{d^2 v}{dt^2} + 7\frac{dv}{dt} + 12v = -30\sin t} \qquad \Rightarrow s^2 + 7s + 12 = 0 \Rightarrow s = -3, -4$$

so $v(t) = A_1 e^{-3t} + A_2 e^{-4t} + v_f$

Try $v_f = B_1\cos t + B_2 \sin t$ & plug into D.E., equating like terms yields $B_1 = 21/17$, $B_2 = -33/17$

t = 0⁺

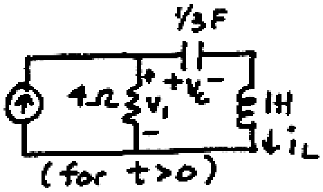


$$i_c(0^+) = \frac{5V - 1V}{1\Omega + 1\Omega} = 2A \qquad \therefore \frac{dv(0^+)}{dt} = \frac{2}{1/12} = 24 \, V/s$$

$$\left.\begin{aligned} &\text{So } v(0^+) = 1 = A_1 + A_2 + 21/17 \\ &\frac{dv(0^+)}{dt} = 24 = -3A_1 - 4A_2 - 33/17 \end{aligned}\right\} \begin{aligned} A_1 &= 25 \\ A_2 &= -429/17 \end{aligned}$$

$$\therefore \;\; \underline{v(t) = 25e^{-3t} - \tfrac{1}{17}\left(429e^{-4t} - 21\cos t + 33\sin t\right) \text{ V}}$$

P9.9-5 Use superposition – first consider 2u(t) source



KVL at right mesh : $v_c + si_L + 4(i_L - 2) = 0$ (1)

also : $i_L = (1/3)\, sv_c \;\Rightarrow v_c = (3/s)\; i_L$ (2)

Plugging (2) into (1) yields $(s^2 + 4s + 3)\, i_L = 0$, roots : $s = -1, -3$

So $i_L\ (t) = A_1 e^{-t} + A_2 e^{-3t}$

$\underline{t = 0^-}$ $\Rightarrow$ circuit is dead $\quad \therefore\ v_c(0) = i_L(0) = 0$

Now from (1) $\dfrac{di_L(0^+)}{dt} = 8 - 4i_L(0^+) - v_C(0^+) = 8\ \text{A/s}$

$$\left.\begin{aligned} &\text{So}\quad i_L(0) = 0 = A_1 + A_2 \\ &\frac{di_L(0)}{dt} = 8 = -A_1 - 3A_2 \end{aligned}\right\} A_1 = 4\ ,\ A_2 = -4$$

$\therefore\ i_L(t) = 4e^{-t} - 4e^{-3t}$

$\therefore\ \underline{v_1(t) = 8 - 4\, i_L(t) = 8 - 16e^{-t} + 16e^{-3t}\ \text{V}}$

Now for $2u(t-2)$ source, just take above expression and replace $t \to t-2$ and flip signs

$\therefore v_2(t) = -8 + 16e^{-(t-2)} - 16\, e^{-3(t-2)}\ \text{V}$

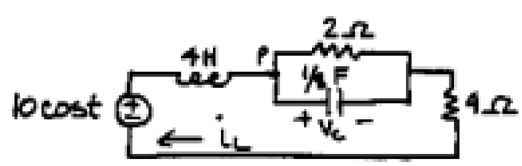
$\therefore\ v(t) = v_1(t) + v_2(t)$

$$\underline{v(t) = \left[8 - 16e^{-t} + 16e^{-3t}\right] u(t) + \left[-8 + 16e^{-(t-2)} - 16\, e^{-3(t-2)}\right] u\,(t-2)\ \text{V}}$$

P9.9-6 $\underline{t > 0}$

KVL: $-10\cos t + 4\dfrac{di_L}{dt} + v_c + 4i_L = 0$ (1)

KCL at P : $i_L = (1/8)\dfrac{dv_c}{dt} + v_c/2$ (2)



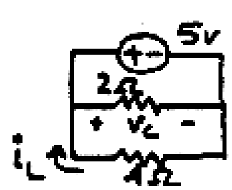
(2) into (1) yields $\dfrac{d^2v_c}{dt^2} + 5\dfrac{dv_c}{dt} + 6v_c = 20\cos t$

$\Rightarrow\ s^2 + 5s + 6 = 0\ ,\ s = -2, -3$ so $v_{c_n}(t) = A_1e^{-2t} + A_2e^{-3t}$

Try $v_{cf}\ (t) = B_1 \cos t + B_2 \sin t$ & plug into D.E.$\Rightarrow B_1 = B_2 = 2$

$\therefore\ v_c\ (t) = A_1e^{-2t} + A_2e^{-3t} + 2\cos t + \sin t$

$\underline{t = 0^-}$ (steady – state)



$v_c\ (0^-) = v_c\ (0^+) = 5\text{V}$ & $i_1\ (0^-) = {-5\text{V}}/{4\Omega} = \dfrac{-5}{4} = i_L\ (0^+)$

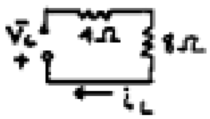
Now from (2) $\dfrac{dv_c\ (0^+)}{} = 8i_L\ (0^+) - 4v_c\ (0^+) = -30\dfrac{\text{V}}{}$

$$\left.\begin{aligned} &\text{So } v_c\ (0^+) = 5 = A_1 + A_2 + 2 \Rightarrow A_1 + A_2 = 3 \\ &\frac{dv_c\ (0^+)}{dt} = -30 = -2A_1 - 3A_2 + 2 \Rightarrow 2A_1 + 3A_2 = 32 \end{aligned}\right\} \begin{aligned} A_1 &= -23 \\ A_2 &= \ \ 26 \end{aligned}$$

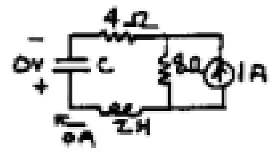
$\therefore\ \underline{v_c\ (t) = -23e^{-2t} + 26e^{-3t} + 2\cos t + 2\sin t\ \text{V}}$

P9.9-7

$\underline{t = 0^-}$ (steady−state) $\qquad\qquad$ $\underline{t = 0^+}$



$v_C(0) = 0$
$i_L(0) = 0$



Since $i_c(0^+)\ 0\ \dfrac{dv_c(0^+)}{dt} = 0$

also at $t = \infty \Rightarrow$ steady − state $\therefore\ v_c(\infty) = -8\Omega\,(1\text{ A}) = -8\text{ V}$

(a) $\underline{C = 1/18F}$ series RLC $\Rightarrow s = -R/2L \pm\sqrt{(R/2L)^2 - 1/LC} = -3,\ -3$

$\therefore\ v_c(t) = A_1e^{-3t} + A_2te^{-3t} + v_c(\infty) = A_1e^{-3t} + A_2te^{-3t} - 8$

Now $v_c(0) = 0 = A_1 - 8 \Rightarrow A_1 = 8\ \&\ \dfrac{dv_c(0)}{dt} = 0 = -3A_1,\ + A_2,\ \Rightarrow A_2 = 24$

So $\underline{v_c(t) = 8e^{-3t} + 24te^{-3t} - 8\text{ V}}$

(b) $\underline{C = 1/10F}$ $\Rightarrow s = -12/4 \pm\sqrt{9 - 10/2} = -5,\ -1\ \therefore\ v_c(t) = A_1e^{-t} + A_2e^{-5t} - 8$

$$\left.\begin{aligned} \text{Now } v_c(0) &= 0 = A_1 + A_2 - 8 \\ \frac{dv_c(0)}{dt} &= 0 = -A_1 + -5A_2 \end{aligned}\right\} A_1 = 10\ \&\ A_2 = -2$$

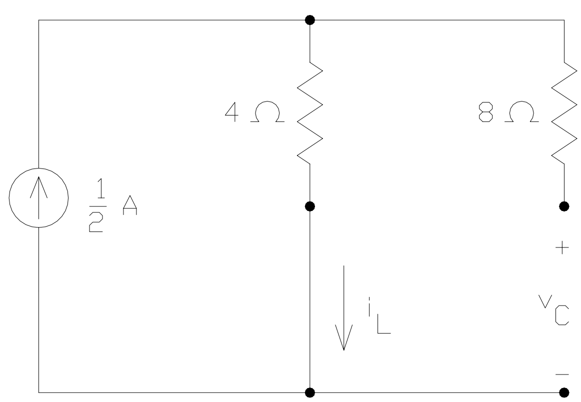
$\underline{\therefore v_c(t) = 10e^{-t} - 2e^{-5t} - 8\text{ V}}$

(c) $\underline{C = 1/20F}$ $\Rightarrow 5 = -3 \pm\sqrt{9 - 20/2} = -3 \pm j\ \therefore\ v_c(t) = e^{-3t}\left[A_1\cos t + A_2\sin t\right] - 8$

Now $v_c(0) = 0 = A_1 - 8\ \&\ \dfrac{dv_c(0)}{dt} = 0 = -3A_1 + A_2 \Rightarrow A_2 = 24$

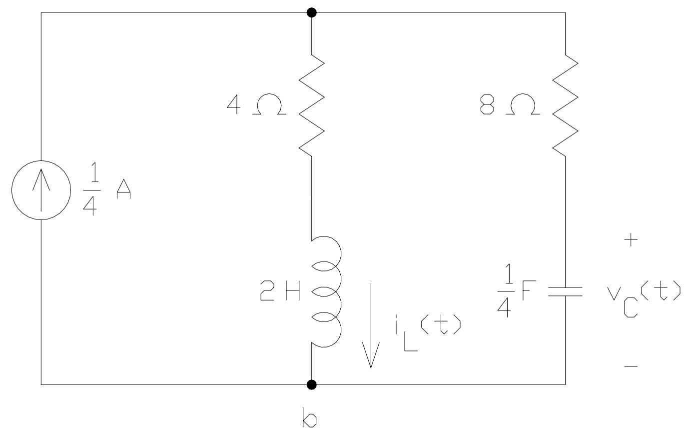
$\underline{\therefore\ v_c(t) = e^{-3t}\ [8\cos t + 24\sin t]\ - 8\text{ V}}$

9.9-8 The circuit will be at steady state for t<0:



so $i_L(0+) = i_L(0-) = 0.5$ A and $v_C(0+) = v_C(0-) = 2$ V.

For t>0:



Apply KCL at node b to get: $\frac{1}{4}=i_L(t)+\frac{1}{4}\frac{d}{dt}v_C(t) \quad \Rightarrow \quad i_L(t)=\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)$

Apply KVL to the right-most mesh to get: $4\,i_L(t)+2\frac{d}{dt}i_L(t)=8\left(\frac{1}{4}\frac{d}{dt}v_c(t)\right)+v_c(t)$

Use the substitution method to get

$$4\left(\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)\right)+2\frac{d}{dt}\left(\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)\right)=8\left(\frac{1}{4}\frac{d}{dt}v_c(t)\right)+v_c(t)$$

or
$$2=\frac{d^2}{dt^2}v_C(t)+6\frac{d}{dt}v_C(t)+2\,v_C(t)$$

The forced response will be a constant, v_C= B so $2=\frac{d^2}{dt^2}B+6\frac{d}{dt}B+2B \quad \Rightarrow \quad B=1\text{ V}$.

To find the natural response, consider the characteristic equation:

$$0=s^2+6\,s+2=(s+5.65)(s+0.35)$$

The natural response is
$$v_n=A_1\,e^{-5.65t}+A_2\,e^{-0.35t}$$

so
$$v_C(t)=A_1\,e^{-5.65t}+A_2\,e^{-0.35t}+1$$

Then
$$i_L(t)=\frac{1}{4}+\frac{1}{4}\frac{d}{dt}v_C(t)=\frac{1}{4}+1.41A_1\,e^{-5.65t}+0.0875A_2\,e^{-0.35t}$$

At t=0+
$$2=v_C(0+)=A_1+A_2+1$$
$$\frac{1}{2}=i_L(0+)=\frac{1}{4}+1.41A_1+0.0875A_2$$

so A_1 = 0.123 and A_2 = 0.877. Finally

$$v_C(t)=0.123\,e^{-5.65\,t}+0.877\,e^{-0.35\,t}+1\;V$$