Chapter 10 – Sinusoidal Steady-State Analysis

Exercises

Ex. 10.3-1 (a) $T = 2\pi / \omega = \underline{2\pi/4}$

(b) v leads i by $30-(-70) = \underline{100^\circ}$

Ex. 10.3-2

$$v = 3\cos 4t + 4\sin 4t = \sqrt{(3)^2+(4)^2}\cos\left(4t-\tan^{-1}\tfrac{4}{3}\right) = \underline{5\cos(4t-53^\circ)}$$

Ex. 10.3-3

$$i = -5\cos 5t + 12\sin 5t = \sqrt{(-5)^2+(12)^2}\cos\left(5t-(180+\tan^{-1}\tfrac{12}{-5})\right) = \underline{13\cos(5t-112.6^\circ)}$$

Ex. 10.4-1

KCL: $i_s = v/R + C\,dv/dt$

$$\frac{dv}{dt}+\frac{v}{RC} = \frac{I_m}{C}\cos\omega t$$

Try $v_f(t) = A\cos\omega t + B\sin\omega t$ & plug into above D.E.

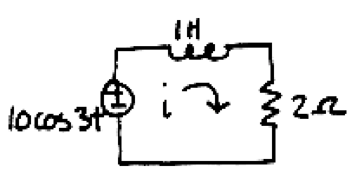
$$\Rightarrow -\omega A\sin\omega t + \omega B\cos\omega t + \tfrac{1}{RC}(A\cos\omega t + B\sin\omega t) = \frac{I_m}{C}\cos\omega t$$

equating $\sin\omega t$ & $\cos\omega t$ terms yields $A = \dfrac{RI_m}{1+\omega^2R^2C^2}$ and $B = \dfrac{\omega R^2 C\, I_m}{1+\omega^2R^2C^2}$

$$\therefore v_f(t) = \frac{RI_m}{1+\omega^2R^2C^2}\cos\omega t + \frac{\omega R^2 C\, I_m}{1+\omega^2R^2C^2}\sin\omega t$$

$$\underline{v_f(t) = \frac{RI_m}{\sqrt{1+\omega^2R^2C^2}}\cos\left[\omega t-\tan^{-1}(\omega RC)\right]}$$

Ex. 10.4-2



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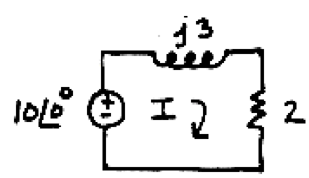


$j\omega L = j\,3\times 1 = j\,3$

KVL : $-10 + j3I + 2I = 0$

$$\Rightarrow I = \frac{10}{2+j3} = \frac{10\angle 0^\circ}{\sqrt{13}\,\angle 56.3^\circ} = \frac{10}{\sqrt{13}}\angle -56.3^\circ$$

$$\therefore \underline{i(t) = \frac{10}{\sqrt{13}}\cos(3t-56.3^\circ)}$$

Ex. 10.5-1

$$\frac{10}{2.36e^{j45}} = 4.24e^{-j45} = \underline{3-j3}$$

Ex. 10.5-2

$$\frac{j32}{-3+j8} = \frac{32e^{j90}}{8.54e^{j111}} = \frac{32}{8.54}e^{j(90-111)} = \underline{3.75\, e^{-j21}}$$

Ex. 10.6-1

(a) $i = 4\cos(\omega t - 80^\circ) = \mathrm{Re}\{4e^{j\omega t}e^{-j80^\circ}\}$

$\therefore\ \underline{I = 4e^{-j80^\circ} = 4\angle -80^\circ}$

(b) $i = 10\cos(\omega t + 20^\circ) = \mathrm{Re}\{10e^{j\omega t}e^{j20^\circ}\}$

$\therefore\ \underline{I = 10e^{j20^\circ} = 10\angle 20^\circ}$

(c) $i = 8\sin(\omega t - 20^\circ) = 8\cos(\omega t - 110^\circ) = 8\mathrm{Re}\{e^{j\omega t}e^{-j110^\circ}\}$

$\therefore\ \underline{I = 8e^{-j110^\circ} = 8\angle -110^\circ}$

Ex. 10.6-2

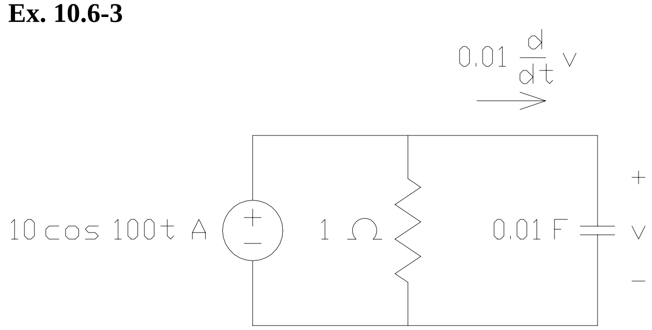
(a) $V = 10\angle -140^\circ = 10e^{-j140^\circ}$

$\therefore\ v(t) = \mathrm{Re}\{10e^{-j140^\circ}e^{jwt}\} = \underline{10\cos(\omega t - 140^\circ)}$

(b) $V = 80 + j75 = 109.7\angle 43.2^\circ = 109.7e^{j43.2^\circ}$

$\therefore\ v(t) = \mathrm{Re}\{109.7e^{j43.2^\circ}e^{j\omega t}\} = \underline{109.7\cos(\omega t + 43.2^\circ)}$

Ex. 10.6-3



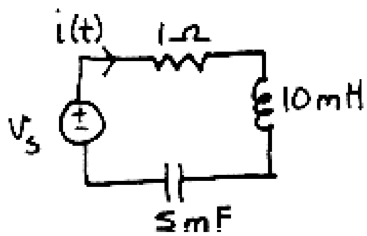
$$0.01\frac{d}{dt}v + v = 10\cos 100\,t$$

$$(0.01)(j\,100)\mathbf{V} + \mathbf{V} = 10$$

$$\mathbf{V} = \frac{10}{1+j} = 7.071\angle -45^\circ$$

$$v = 7.071\cos 100\,t \quad \text{A}$$

Ex. 10.6-4



$v_s = 40\cos 100t = \text{Re}\left\{4e^{j100t}\right\}$

KVL : $i(t) + 10\times10^{-3}\dfrac{di(t)}{dt} + \dfrac{1}{5\times10^{-3}}\displaystyle\int_{-\infty}^{t} i(t)dt = v_S$

Assume $i(t) = Ae^{j100t}$ where i_s is complex number to be determined

Plugging into D.E. yields

$Ae^{j100t} + jAe^{j100t} + (-j2A)e^{j100t} = 4e^{j100t} \Rightarrow A = \dfrac{4}{1-j} = 2\sqrt{2}e^{j45^\circ}$

so $\beta = \tan^{-1}\dfrac{1}{1} = 45^\circ$

$\underline{i(t) = \text{Re}\,2\sqrt{2}e^{j100t}e^{j45^\circ} = \text{Re}\left\{2\sqrt{2}e^{j(100t\text{-}45^\circ)}\right\}} = 2\sqrt{2}\cos(100t + 45^\circ)$

Ex. 10.7-1

(a) $v = Ri = 10(5\cos 100t) = \underline{50\cos 100t}$

(b) $v = L\dfrac{di}{dt} = 0.01\left[5(-100)\sin 100t\right] = -5\sin 100t = \underline{5\cos(100t + 90^\circ)}$

(c) $v = {}^{1}\!/_{C}\displaystyle\int idt = 10^3\int 5\cos 100tdt = 50\sin 100t = \underline{50\cos(100t - 90^\circ)}$

Ex. 10.7-2

$i = C\dfrac{dv}{dt} = 10\times10^{-6}[100(-500)\sin(500t+30^\circ)]$

$= -0.5\sin(500t+30^\circ) = 0.5\sin(500t+210^\circ) = \underline{0.5\cos(500t+120^\circ)}$

Ex. 10.7-3

From Figure Ex. 10.7-3 we get $i(t) = I_m \sin\omega t$; $I = I_m\angle -90^\circ$ A

$v(t) = V_m \cos\omega t$; $V = V_m\angle 0^\circ$ V

$i(t) = I_m\sin\omega t = I_m\cos(\omega t - 90^\circ)$

The voltage leads the current by 90°, $\therefore$ it is an inductor

$\Rightarrow Z_{eq} = \dfrac{V}{I} = \dfrac{V_m\angle 0^\circ}{I_m\angle -90^\circ} = \dfrac{V_m}{I_m}\angle 90^\circ\,\Omega$

also $Z_{eq} = j\omega L = \omega L\angle 90^\circ \Rightarrow \omega L = \dfrac{V_m}{I_m}$ or $\underline{L = \dfrac{V_m}{\omega I_m}}$ (H)

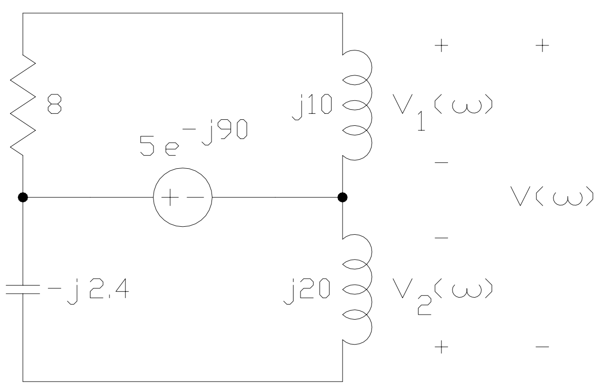
Ex. 10.8-1 $\mathbf{Z}_R$ = 8 Ω, $\mathbf{Z}_C$ = $\dfrac{1}{j5\frac{1}{12}} = \dfrac{2.4}{j} = \dfrac{j\,2.4}{j\times j} = -j\,2.4$ Ω, $\mathbf{Z}_{L1}$ = j 5 (2) = j 10 Ω,

$\mathbf{Z}_{L2}$ = j 5 (4) = j 20 Ω and $\mathbf{V}_S$ = 5 ∠-90° V.

Ex. 10.8-2 $\mathbf{Z}_R$ = 8 Ω, $\mathbf{Z}_C$ = $\dfrac{1}{j3\frac{1}{12}} = \dfrac{4}{j} = \dfrac{j\,4}{j\times j} = -j\,4$ Ω, $\mathbf{Z}_{L1}$ = j 3 (2) = j 6 Ω,

$\mathbf{Z}_{L2}$ = j 3 (4) = j 12 Ω and $\mathbf{I}_S$ = 4 ∠15° V.

Ex 10.9-1

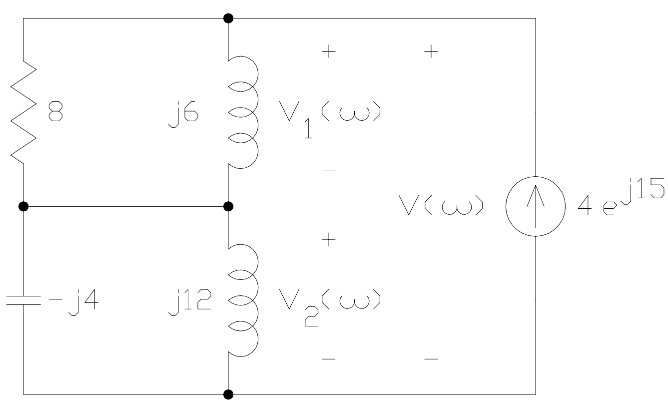


$$\mathbf{V}_1(\omega) = \frac{j10}{8+j10}\,5e^{-j90} = 3.9e^{-j51}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20-j2.4}\,5e^{-j90} = 5.68e^{-j90}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9e^{-j51} - 5.68e^{-j90}$$
$$= 3.58e^{j47}$$

Ex 10.9-2

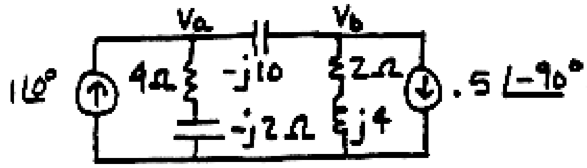


$$\mathbf{V}_1(\omega) = \frac{8(j6)}{8+j6}\,4e^{j15} = 19.2e^{j68}$$

$$\mathbf{V}_2(\omega) = \frac{j12(-j4)}{j12-j4}\,4e^{j15} = 24e^{-j75}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) + \mathbf{V}_2(\omega) = 14.4e^{-j22}$$

Ex. 10.10-1



KCL at V_a: $\dfrac{V_a}{4-j2}+\dfrac{V_a-V_b}{-j10}=1$

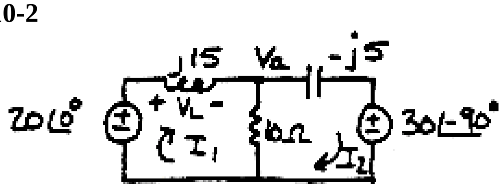
$\Rightarrow \underline{(4-j12)V_a+(-4+j2)V_b=-20-j40}$

KCL at V_b : $\dfrac{V_b-V_a}{-j10}+\dfrac{V_b}{2+j4}+.5\angle-90^\circ = 0 \Rightarrow \underline{(-2-j4)V_a+(2-j6)V_b=10+j20}$

Using Cramer's rule $V_a=\dfrac{\begin{vmatrix}(-20-j40) & (-4+j2)\\ (10+j20) & (2-j6)\end{vmatrix}}{\begin{vmatrix}(4-j12) & (-4+j2)\\ (-2-j4) & (2\text{-}j6)\end{vmatrix}}=\dfrac{-200+j100}{-80-j60}=\sqrt{5}\angle 296.5^\circ$

$\therefore \underline{v_a(t)=\sqrt{5}\,\cos(100t+296.5^\circ)=\sqrt{5}\,\cos(100t-63.5^\circ)}$

Ex. 10.10-2



KVL aI_1 : $j15I_1+10(I_1-I_2) = 20$

$\Rightarrow (10+j15)I_1-10I_2 = 20 \qquad (1)$

KVL aI_2 : $-j5I_2+10(I_2-I_1) = -30\angle-90^\circ$

$\Rightarrow -10I_1+(10-j5)I_2 = j30 \qquad (2)$

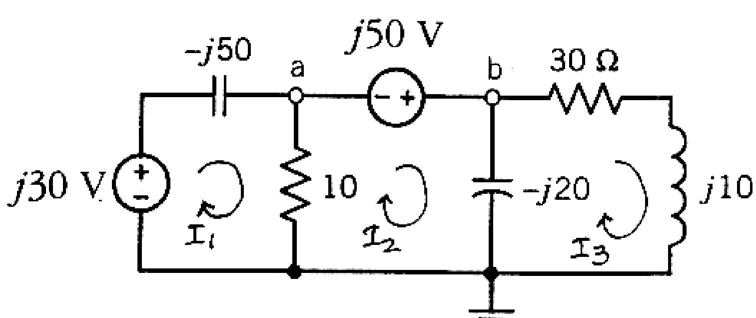
From Cramer's rule

$$I_1 = \frac{\begin{vmatrix}20 & -10\\ j30 & 10-j5\end{vmatrix}}{\begin{vmatrix}10+j15 & -10\\ -10 & 10-j5\end{vmatrix}} = \frac{200+j200}{75+j100} = 2.263\angle-8.1^\circ$$

Now $V_L = (j15)I_1 = (15\angle 90^\circ)(2.263\angle-8.1^\circ) = 24\sqrt{2}\angle 82^\circ$

$\therefore \underline{v_L(t) = 24\sqrt{2}\cos(\omega t+82^\circ)V}$

Ex. 10.10-3



Writing mesh equations:

$(10+j50)I_1-10I_2 = j30$

$-10I_1+(10-j20)I_2+j20I_3 = j50$

$j20I_2+(30-j10)I_3 = 0$

Solving these equations gives

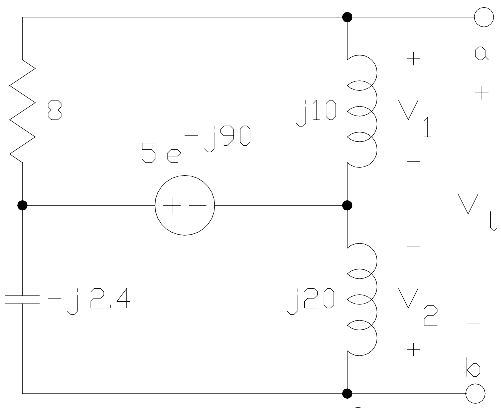
$I_1 = -0.87-j0.09, \quad I_2 = -1.32+j1.27, \quad I_3 = 0.5+j1.05$

Then

$V_a = 10(I_1-I_2) = 14.3\angle-72^\circ$ V

$V_b = V_a + j50 = 36.6\angle 83^\circ$ V

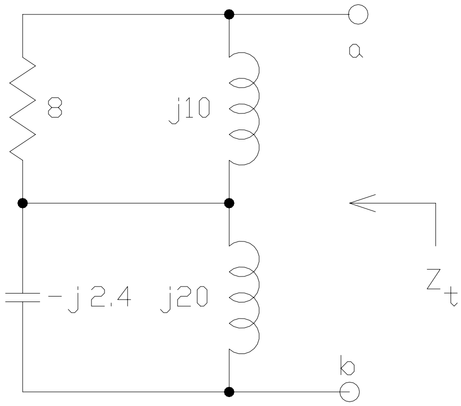
Ex 10.11-1



$$\mathbf{V}_1 = \frac{j10}{8+j10}\,5e^{-j90} = 3.9\,e^{-j51}$$

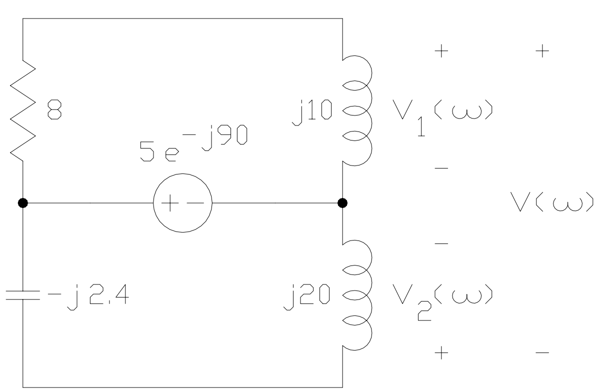
$$\mathbf{V}_2 = \frac{j20}{j20-j2.4}\,5e^{-j90} = 5.68\,e^{-j90}$$

$$\mathbf{V}_t = \mathbf{V}_1 - \mathbf{V}_2 = 3.9\,e^{-j51} - 5.68\,e^{-j90}$$
$$= 3.58\,e^{j47}$$



$$\mathbf{Z}_t = \frac{8(j10)}{8+j10} + \frac{-j2.4(j20)}{-j2.4+j20} = 4.9 + j\,1.2$$

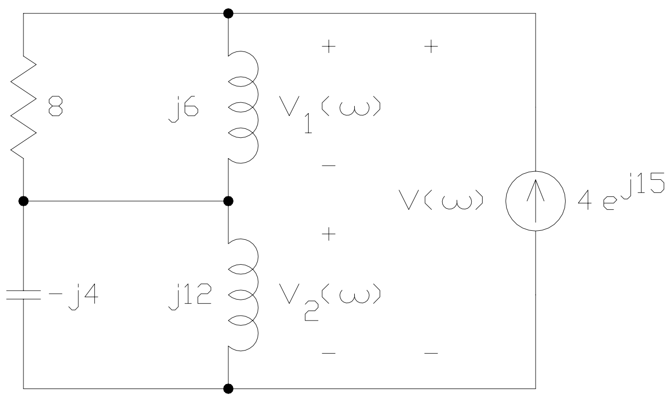
Ex 10.11-2



$$\mathbf{V}_1(\omega) = \frac{j10}{8+j10}\,5e^{-j90} = 3.9\,e^{-j51}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20-j2.4}\,5e^{-j90} = 5.68\,e^{-j90}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9\,e^{-j51} - 5.68\,e^{-j90}$$
$$= 3.58\,e^{j47}$$



$$\mathbf{V}_1(\omega)=\frac{8(j6)}{8+j6}4e^{j15}=19.2e^{j68}$$

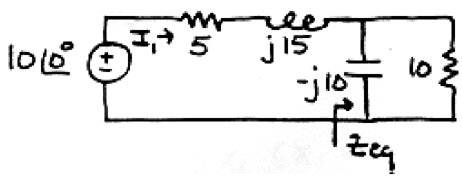
$$\mathbf{V}_2(\omega)=\frac{j12(-j4)}{j12-j4}4e^{j15}=24e^{-j75}$$

$$\mathbf{V}(\omega)=\mathbf{V}_1(\omega)+\mathbf{V}_2(\omega)=14.4e^{-j22}$$

Using superposition: $v(t)$ = 3.58 cos (5t + 47°) + 14.4 cos (3t - 22°)

Ex. 10.11-3

a) Turn off current source, use phasors with ω = 10 rad/sec



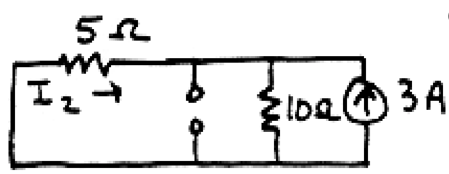
$$Z_{eq} = -j\frac{10\cdot10}{10-j10} = 5(1-j)$$

$$\text{KVL a: } -10+5I_1+j15I_1+5(1-j)I_1 = 0$$

$$\Rightarrow I_1 = \frac{10}{10+j10} = 0.707\angle-45^\circ$$

$$\therefore\ \underline{i_1(t) = 0.707\cos(10t-45^\circ)\ \text{A}}$$

b) Turn off voltage source, ω = 0 rad/sec



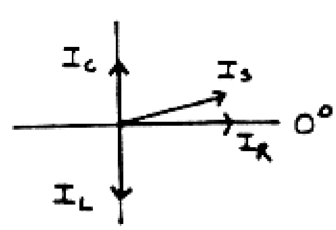
Current divider $I_2 = -\frac{10}{15}3 = -2\ \text{A}$

So by superposition $\underline{i(t) = 0.707\cos(10t-45^\circ)-2\ \text{A}}$

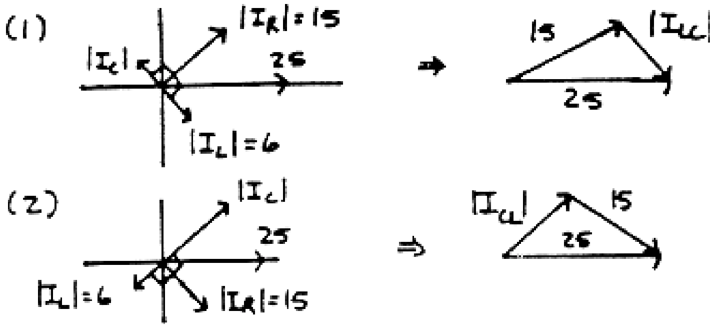
Ex. 10.12-1 $\omega^2 = \frac{1}{LC} = \frac{1}{(1\times10^{-3})(1\times10^{-3})} = 10^6 \quad \therefore\ \underline{\omega = 1000\text{rad/sec}}$

Ex. 10.12-2

Diagram drawn with relative magnitudes arbitrarily chosen



Ex. 10.12-3 Two possible phasor diagrams for currents



$$\therefore\ |I_{LC}| = \sqrt{(25^2)-(15^2)} = 20 = |I_{CL}|$$

Now if $|I_{LC}| = |I_L|-|I_C| \Rightarrow |I_C| = 6-20 = -14$ (impossible)

$\therefore$ from case (2) $|I_{CL}| = |I_C|-|I_L| \Rightarrow \underline{|I_C| = 20+6 = 26}$

Ex. 10.14-1

$$Z_1 = \frac{R_1X_1(X_1-jR_1)}{R_1^2+X_1^2} \quad \text{and} \quad R_1 = 1\text{k}\Omega\ ,\ X_1 = \frac{1}{\omega C_1} = \frac{1}{(1000)(10^{-6})} = 1\text{k}\Omega$$

$$\therefore\ Z_1 = \frac{(1)(1)(1-j1)}{1+1} = \frac{1}{2}-j\frac{1}{2}\ \text{k}\Omega$$

$$Z_2 = R_2 = 1\text{k}\Omega$$

$$\underline{\therefore\ \frac{V_o}{V_s} = -\frac{Z_2}{Z_1} = \frac{-1}{1/2-j1/2} = -1-j}$$

Problems

Section 10-3: Sinusoidal Sources

P10.3-1

(a)
$$\begin{aligned} i(t) &= 2\cos(6t+120^\circ)+4\sin(6t-60^\circ) \\ &= 2\ (\cos 6t\cos 120^\circ-\sin 6t\sin 120^\circ)+4\ (\sin 6t\cos 60^\circ-\cos 6t\sin 60^\circ) \\ &= 2.46\cos 6t+0.27\sin 6t = \underline{2.47\cos(6t-6.26^\circ)} \end{aligned}$$

(b)
$$\begin{aligned} v(t) &= 5\sqrt{2}\cos 8t+10\sin(8t+45^\circ) \\ &= 5\sqrt{2}\cos 8t+10[\sin 8t\cos 45^\circ+\cos 8t\sin 45^\circ] \\ &= 10\sqrt{2}\cos 8t+5\sqrt{2}\sin 8t \end{aligned}$$

$$v(t) = \sqrt{250}\cos(8t-26.56^\circ) = \underline{5\sqrt{10}\sin(8t+63.4^\circ)\ \text{V}}$$

P10.3-2

$$\omega = 2\pi f = 2\pi/T = \frac{2\pi}{1\times10^{-3}} = 6283\ rad/\sec$$

$$v(t) = V_m \sin(\omega t+\phi) = 100\sin(6283t+\phi)$$

$$v(0) = 10 = 100\sin\phi \Rightarrow \phi = \sin^{-1}(0.1) = 6^\circ$$

So $\underline{v(t) = 100\sin(6283t+6^\circ)V}$

P10.3-3

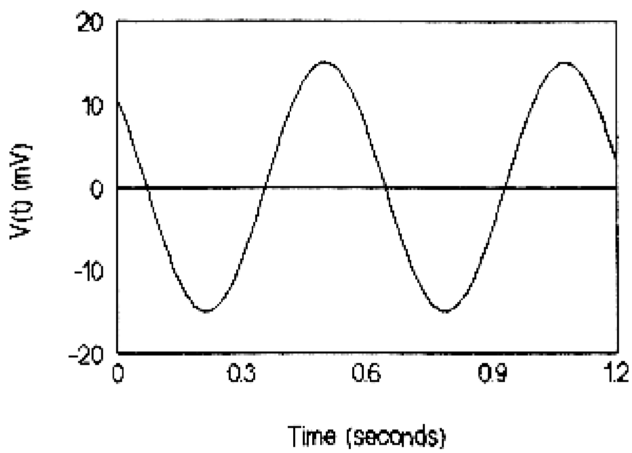
$$f = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} = 600\text{Hz}$$

$$i(2\times10^{-3}) = 300\cos(1200\pi(2\times10^{-3})+55^\circ) = 3\cos(2.4\pi+55^\circ)$$

$$\text{but } 2.4\pi\times\left(\frac{180^\circ}{\pi}\right) = 432^\circ$$

So $i(2\times10^{-3}) = 300\cos(432^\circ+55^\circ) = 300\cos(127^\circ) = \underline{-180.5\text{ mA}}$

P10.3-4



P10.3-5

a)

$A = 10$

$T = 3.9\text{ms} - 0.6\text{ms} = 3.3\text{ms}$

$\omega = \frac{2\pi}{T} = 1900\ \text{rad/s}$

$10\cos(\theta) = 0.87 \Rightarrow \theta = 30^\circ$

$\underline{v_s(t) = 10\cos(1900t+30^\circ)\ V}$

b)

$A = 10$

$T = \frac{1}{2}(10.9\text{ms} - 0.9\text{ms}) = 5\text{ms}$

$\omega = \frac{2\pi}{T} = 1260\ \text{rad/s}$

$10\cos(\theta) = 0.87 \Rightarrow \theta = 30^\circ$

$\underline{v(t) = 10\cos(1260+30^\circ)\ V}$

Section 10-4: **Steady-State Response of an RL Circuit for a Sinusoidal Forcing Function**

P10.4-1

$$L\frac{di}{dt} + R_i = -v_s \text{ yields } \frac{di}{dt} + 120i = -400\cos 300t$$

Try $i_f = A\cos 300t + B\sin 300t$

$$\frac{di_f}{dt} = -300\,A\sin 300t + 300B\cos 300t$$

$$\left.\begin{aligned} &\text{yields } -300A + 120B = 0 \\ &\text{and } \quad 300B + 120A = -400 \end{aligned}\right\} \begin{aligned} A &= -0.46 \\ B &= -1.15 \end{aligned}$$

so $i(t) = -0.46\cos 300t - 1.15\sin 300t = \underline{1.24\cos(300t - 68^\circ)\ A}$

P10.4-2

$$\text{KCL}: \ -i_s + \frac{v}{2} + C\frac{dv}{dt} = 0 \Rightarrow \quad \frac{dv}{dt} + 500v = 500\cos 1000t$$

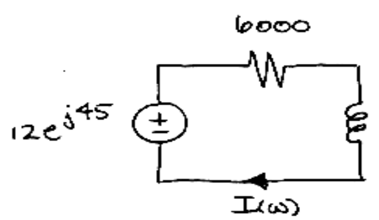
Try $v_f = A\cos 1000t + B\sin 1000t$

$$\frac{dv_f}{dt} = -1000A\cos 1000t + 1000B\cos 1000t$$

$$\left.\begin{aligned} &\text{yields } -1000A + 500B = 0 \\ &\text{and } 1000B + 500A = 500 \end{aligned}\right\} \text{solving} \quad \begin{aligned} A &= 0.2 \\ B &= 0.4 \end{aligned}$$

so $v(t) = 0.2\cos 1000t + 0.4\sin 1000t = \underline{0.447\cos(1000t - 63^\circ)\ V}$

P10.4-3



$(j4)(.05) = j(0.2)$

$$I(\omega) = \frac{12e^{j45}}{6000 + j(0.2)} \simeq \frac{12e^{j45}}{6000} = (2\cdot 10^{-3})e^{j45} \quad \Rightarrow \quad \underline{i(t) = (2)\cos(4t + 45^\circ)\ \text{mA}}$$

Section 10.5: Complex Exponential Forcing Function

a) Complex Numbers

P10.5-1

$$\frac{(5\angle 36.9^\circ)\,(10\angle -53.1^\circ)}{(4+j3)(6-j8)} = \frac{50\angle -16.2^\circ}{10-j5} = \frac{10\angle -16.2^\circ}{\sqrt{5}\angle -26.56^\circ} = \underline{2\sqrt{5}\angle 10.36^\circ}$$

P10.5-2

$$5\angle + 81.87^\circ \left[4 - j3 + \frac{3\sqrt{2}\angle -45^\circ}{5\sqrt{2}\angle -8.13^\circ}\right] = 5\angle + 81.87^\circ\,[4 - j3 + \tfrac{3}{5}\angle - 36.87^\circ]$$

$$= 5\angle + 81.87^\circ (4.48 - j3.36) = 5\angle + 81.87^\circ (5.6\angle -36.87^\circ) = 28\angle + 45^\circ = \underline{14\sqrt{2} + j14\sqrt{2}}$$

P10.5-3 $\frac{A^*C^*}{B} = \frac{(3-j7)5e^{-j2.3}}{6e^{j15}} = \underline{0.65-j6.31}$

P10.5-4

$(6\angle 120^\circ)\ (-4+j3+2e^{j15}) = -12.1-j21.3$

so $\underline{a=-12.1}$ and $\underline{b=-21.3}$

P10.5-5

a) $Ae^{j120} = -4+j(3-b) = \sqrt{4^2+(3-b)^2}\ e^{j\tan^{-1}\left(\frac{3-b}{-4}\right)}$

$120 = \tan^{-1}\left(\frac{3-b}{-4}\right) \Rightarrow b = 3+4+\tan(120^\circ) = \underline{-3.93}$

$A = \sqrt{4^2+(3-b)^2} = \sqrt{4^2+(3-(-3.93))^2} = \underline{8.00}$

b) $-4+8\cos\theta + j(b+8\sin\theta) = 3e^{-j120} = -1.5-j2.6$

$-4+8\cos\theta = -1.5 \Rightarrow \underline{\theta = \cos^{-1}\frac{2.5}{8}} = \underline{72^\circ}$

$b+8\sin(72^\circ) = -2.6 \Rightarrow \underline{b = -10.2}$

c) $-10+j2a = Ae^{j60} = A\cos 60^\circ - jA\sin 60^\circ$

$A = \frac{-10}{\cos 60^\circ} = \underline{-20};\quad a = \frac{-20\sin 60}{2} = -\underline{8.66}$

b) Response of a circuit

P10.5-6

$Z_R = 100,\ Z_L = j(10^7)(1\times10^{-3}) = j\,10{,}000,\ Z_c = \frac{-j}{(10^7)(10\times10^{-12})} = -j\,10{,}000$

$I(\omega) = \frac{V_s}{Z_R+Z_L+Z_C} = \frac{0.1\angle 90^\circ}{100+j\,10000-j\,10000} = 0.001\angle 90^\circ$

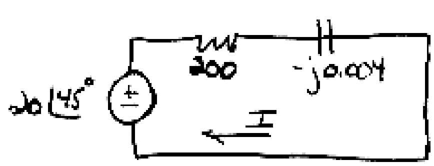
$\underline{i(t) = 1\cos(\omega t+90^\circ)\text{ mA},\ \omega = 10^7\text{ rad/sec.}}$

P10.5-7

$Z_L = j(25\times10^6)(160\times10^{-6}) = j\,4000$

$Z_L = \frac{-j}{(25\times10^6)(10\times10^{-6})} = -j\,0.004$

$Z_L // Z_c = -j\,0.004$



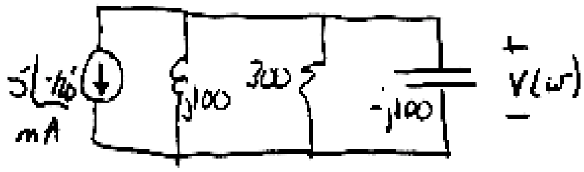
$I(\omega) = \frac{V}{Z_R+Z_c} = \frac{20\angle 45^\circ}{200-j\,0.004} = 0.1\angle 45^\circ$

$i(t) = 0.1\cos(\omega t+45^\circ)\ A$

Section 10-6: The Phasor Concept

P10.6-1 Phasor ckt: $Z_R = R = 300, \quad Z_L = j\omega_L = j(1\times10^5)(1\times10^{-3}) = j100$

$$Z_c = \frac{-j}{\omega C} = \frac{-j}{(1\times10^5)(0.1\times10^{-6})} = -j100$$



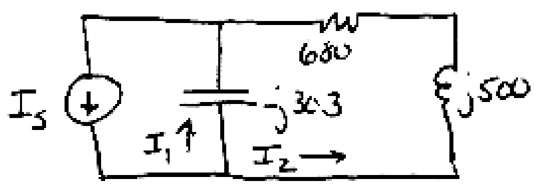
KCL: $\frac{V}{Z_L} + \frac{V}{Z_R} + \frac{V}{Z_C} = -I$

yields $V(\omega) = 1.5\angle 60^\circ$

so $\underline{v(t) = 1.5 \cos(\omega t + 60^\circ) \text{ V}, \ \omega = 10^5 \text{ rad/sec}}$

P10.6-2 $Z_R = R = 680\Omega, \ Z_L = j\omega L = j(1000)(500\times10^{-3}) = j500$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{(1000)(3.3\times10^{-6})} = -j303, \ I_s(\omega) = 25\times10^{-3}\angle{-120^\circ}\text{ A} = 25\angle{-120^\circ}\text{ mA}$$



$$I_2(\omega) = \left(\frac{Z_C}{Z_T}\right) I_S$$

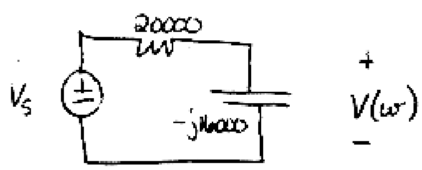
$$Z_T = Z_1 + Z_2 = (680 + j500) + (-j303)$$

$$I_2(\omega) = \left(\frac{-j303}{680 + j197}\right)\left(25\angle{-120^\circ}\right) = \frac{7575\angle{-210^\circ}}{708\angle 16^\circ} = 10.7\angle{-226^\circ} = 10.7\angle 134^\circ$$

so $\underline{i(t) = 10.7 \cos(1000t + 134^\circ) \text{ mA}}$

P10.6-3

Convert to phasor circuit: $Z_R = R; \ Z_C = \frac{-j}{\omega C} = \frac{-j}{(500)(0.125\times10^{-6})} = -j16000$



$V_S = 2\angle{-90^\circ}$

voltage divider

$$V(\omega) = \left(\frac{-j16000}{20000 - j16000}\right)\left(2\angle{-90^\circ}\right) = \frac{\left(16000\angle{-90^\circ}\right)\left(2\angle{-90^\circ}\right)}{25612\angle{-39^\circ}} = 1.25\angle{-141^\circ}$$

so $\underline{v(t) = 1.25 \cos(500t - 141^\circ) \text{ V}}$

Section 10-7: Phasor Relationships for R, L, and C Elements

P10.7-1 (a) Rotate 45° $\Rightarrow$ $I = 6 + j8 = 10\angle 53.1^\circ$

subtract 45°

$\underline{I' = 10\angle 8.1^\circ = 7\sqrt{2} + j\sqrt{2}}$

(b) Rotate 90° $\Rightarrow$ $I = 10\angle 53.1^\circ$

add 90°

$\underline{I' = 10\angle 143.1^\circ = -8 + j6}$

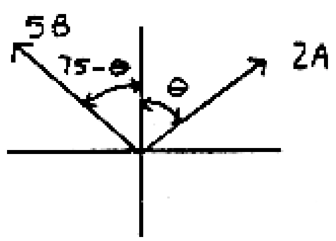
P10.7-2 (a)

$$V_1 = 3\angle 60^\circ = 1.5 + j2.598$$
$$V_2 = 8\angle -22.5^\circ = 7.391 - j3.061$$
$$\therefore V_1 + V_2 = 8.891 - j0.463 = 8.90\angle -2.98^\circ$$
$$\Rightarrow \underline{v_1 + v_2 = 8.90\cos(2t - 2.98^\circ)}$$

(b)

$$v_1 = 2\sqrt{2}\cos(4t - 90^\circ) \Rightarrow V_1 = 2\sqrt{2}\angle -90^\circ = -j2\sqrt{2}$$
$$V_2 = 10\angle 30^\circ = 5\sqrt{3} + j5$$
$$\therefore V_1 + V_2 = 5\sqrt{3} + j(5 - 2\sqrt{2}) = 8.93\angle 14.1^\circ$$
$$\Rightarrow \underline{v_1 + v_2 = 8.93\cos(4t + 14.1^\circ)}$$

P10.7-3



$2A + 5B$ is pure imaginary and on the '+' imaginary axis

$$\therefore 2|A|\sin\theta = 5B\sin(75^\circ - \theta)$$
$$2\left(5\sqrt{2}\right)\sin\theta = 5B\sin(75^\circ - \theta)$$
$$= 20[\sin 75^\circ\cos\theta - \cos 75^\circ\sin\theta)$$
$$\Rightarrow \tan\theta = \frac{\sin 75^\circ}{1/\sqrt{2} + \cos 75^\circ} = 1 \qquad \therefore \theta = 45^\circ$$

so $\underline{A = 5\sqrt{2}\angle 45^\circ}$ and $\underline{B = 4\angle 90^\circ + (75^\circ - 45^\circ) = 4\angle 120^\circ}$

P10.7-4

(a) $v = 15\cos(400t + 30^\circ)$

$i = 3\sin(400t + 30^\circ) = 3\cos(400t - 60^\circ)$

$\therefore v$ leads i by 90° $\Rightarrow$ element is an <u>inductor</u>

Now $|Z_L| = \dfrac{v_{peak}}{i_{peak}} = \dfrac{15}{3} = 5 = \omega L = 400L \Rightarrow \underline{L = 0.0125\text{ H} = 12.5\text{ mH}}$

(b) i leads v by 90° $\therefore$ <u>capacitor</u>

$$|Z_c| = \frac{v_{peak}}{i_{peak}} = \frac{8}{2} = 4 = \frac{1}{\omega C} = \frac{1}{900C} \Rightarrow \underline{C = 277.77\mu F}$$

(c) $v = 20\cos(250t + 60^\circ)$

$i = 5\sin(250t + 150^\circ) = 5\cos(250t + 60^\circ)$

Since v & i are in phase $\Rightarrow$ element is a <u>resistor</u>

$$\therefore R = \frac{v_{peak}}{i_{peak}} = \frac{20}{5} = \underline{4\Omega}$$

P10.7-5

For algebraic addition, the rectangular form is convenient,

$V_1 = 150\cos(-30^\circ) + j150\sin(-30^\circ) = 130 - j75\text{ V}$

$V_2 = 200\cos 60^\circ + j200\sin 60^\circ = 100 + j173\text{ V}$

By the rules for equality and addition

$$V = V_1 + V_2 = 230 + j98 = 250\angle 23.1^\circ \text{ V}$$

Thus $\underline{v(t) = v_1(t) + v_2(t) = 250\cos(377t + 23.1^\circ)\text{ V}}$

<u>Section 10-8:</u> Impedance and Admittance

P10.8-1 $\omega = 2\pi f = 2\pi(10\times 10^3) = 62830\text{ rad/sec}$

$Z_R = R = 36\Omega$ $\qquad Y_R = \dfrac{1}{36} = 0.0278\text{ S}$

$Z_L = j\omega L = j(62830)(160\times 10^{-6}) = j10\Omega$ $\qquad Y_L = \dfrac{1}{Z_L} = -0.1j\text{ S}$

$Z_C = \dfrac{-j}{\omega C} = \dfrac{-j}{(62830)(1\times 10^{-6})} = -j\,16\Omega$ $\qquad Y_C = \dfrac{1}{Z_C} = 0.0625j\text{ S}$

$Y_{eq} = Y_P = Y_R + Y_L + Y_C = \underline{0.0278 - j0.00375\quad S} = 0.027\angle 9^\circ$

$Z_{eq} = \dfrac{1}{Y_{eq}} = 36.5\angle 9^\circ = \underline{36 - j5.86\ \Omega}$

P10.8-2

$$Z = \frac{V}{-I} = \frac{-10\angle 40^\circ}{2\times 10^{-3}\angle 195^\circ} = -5000\angle -155^\circ\ \Omega = 4532 + 2113j = R + j\omega L$$

so $\underline{R = 4532\ \Omega}$ and $L = \dfrac{2113}{\omega} = \dfrac{2113}{2\times 10^6} = \underline{1.06\text{ m H}}$

P10.8-3

$$Z(\omega)=\frac{-\frac{j}{\omega C}(R+j\omega L)}{-\frac{j}{\omega C}+(R+j\omega L)}=\frac{\frac{L}{C}-j\frac{R}{\omega C}}{R+j\left(\omega L-\frac{1}{\omega C}\right)}$$

$$=\frac{\left(\frac{L}{C}-j\frac{R}{\omega C}\right)\left(R-j\left(\omega L-\frac{1}{\omega C}\right)\right)}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}=\frac{\frac{RL}{C}-\frac{R}{\omega C}\left(\omega L-\frac{1}{\omega C}\right)-j\left(\frac{R^2}{\omega C}+\frac{L}{C}\left(\omega L-\frac{1}{\omega C}\right)\right)}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

So $Z(\omega)$ will be purely resistive when

$$\frac{R^2}{\omega C}+\frac{L}{C}\left(\omega L-\frac{1}{\omega C}\right)=0 \Rightarrow \omega^2=\frac{1}{CL}-\left(\frac{R}{L}\right)^2$$

when $R=6\Omega$, $C=22\mu F$, and $L=27$ mH, then $\omega=1278$ rad/s.

P10.8-4 $\omega = 2\pi f = 6283$ rad/sec

$$Z = Z_L + \frac{Z_c R}{R+Z_c} = j\omega L + \frac{\frac{R}{j\omega C}}{R+\frac{1}{j\omega C}}$$

becomes (after manipulation)

$$Z=\frac{R+j(\omega L-\omega R^2C+\omega^3R^2LC^2)}{1+(\omega RC)^2}$$

Set real part equal to 100Ω to get C

$$\frac{R}{1+(\omega RC)^2}=100 \Rightarrow \underline{C=0.158\mu F}$$

Set imaginary part of numerator equal to 0 to get L

$$L-R^2C+\omega^2R^2LC^2=0 \Rightarrow \underline{L=0.1587\text{ H}}$$

P10.8-5

$$\omega = 2\pi f = 6.28\times10^6 \text{ rad/sec}$$

$$Z_L = j\omega L = j\,(6.28\times10^6)(47\times10^{-6}) = j\,300$$

$$Z_{eq}=Z_c \| (Z_R+Z_L)=\frac{\left(\frac{1}{j\omega C}\right)(300+j300)}{\frac{1}{j\omega C}+300+j300}=590.7$$

Rearranging $590.7=\dfrac{300+300j}{1+300j\omega C-300\omega C}$

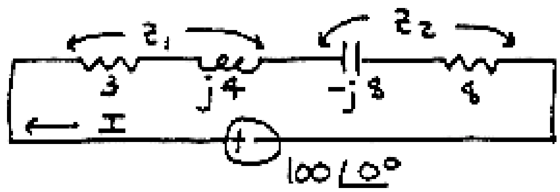
or $590.7-(590.7)(300\omega C)+j(590.7)(300\omega C)=300+j300$

equating imaginary terms

$$(590.7)(300\omega C)=300 \Rightarrow \underline{C=0.27\text{ nF}}$$

Section 10-9: Kirchhoff's Laws Using Phasors

P10.9-1



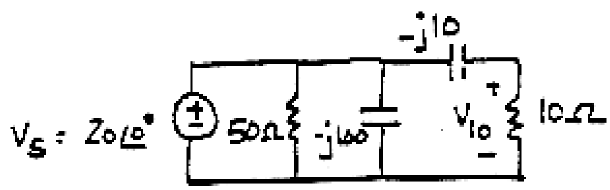
(a) $\underline{Z_1 = 3 + j4 = 5\angle 53.1^\circ}$ $\qquad$ $\underline{Z_2 = 8 - j8 = 8\sqrt{2}\ \angle -45^\circ}$

(b) Total impedance = $Z_1 + Z_2 = 3 + j4 + 8 - j8 = 11 - j4 = \underline{11.7\angle -20.0^\circ}$

(c)
$$I = \frac{100\angle 0^\circ}{Z_1 + Z_2} = \frac{100}{11.7\ \angle -20^\circ} = \frac{100}{11.7}\angle 20.0^\circ$$

$\therefore\ \underline{i(t) = 8.55\cos(1250t + 20.0^\circ)\ \text{A}}$

P10.9-2



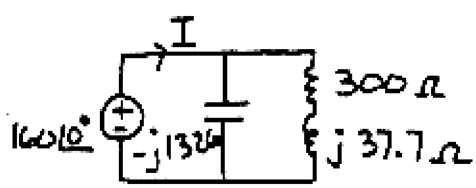
Using voltage divider

$$V_{10} = V_s\left(\frac{10}{10 - j10}\right) = 20\angle 0^\circ\left(\frac{10}{10\sqrt{2}\angle -45^\circ}\right)$$
$$= 0\sqrt{2}\angle 45^\circ$$

$\therefore v_{10}(t) = \underline{10\sqrt{2}\ \cos(100t + 45^\circ)\ \text{V}}$

P10.9-3

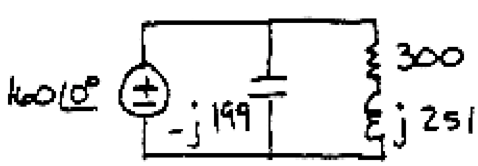
(a)



$$I = \frac{160\ \angle 0^\circ}{\dfrac{(-1326)\ (300 + j37.7)}{-j1326\ +\ 300 + j\ 37.7}}$$
$$= \frac{160\ \angle 0^\circ}{303\ \angle -5.9^\circ} = 0.53\ \angle 5.9^\circ$$

$\therefore\ \underline{i(t) = 0.53\cos(120\pi t + 5.9^\circ)\ \text{A}}$

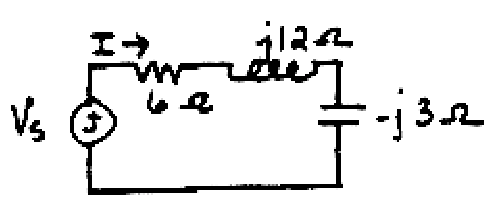
(b)



$$I = \frac{160\angle 0^\circ}{\dfrac{(-j199)(300 + j251)}{-j199 + 300 + j251}}$$
$$= \frac{160\angle 0^\circ}{256\angle -59.9^\circ} = 0.625\angle 59.9^\circ$$

$\therefore\ \underline{i(t) = 0.625\cos(800\pi t + 59.9^\circ)\ \text{A}}$

P10.9-4

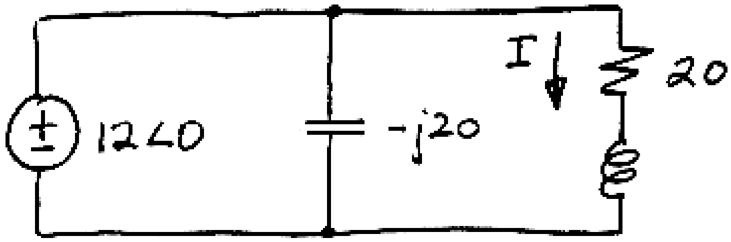


$V_s = 2\ \angle 30^\circ$

$$I = \frac{2\ \angle 30^\circ}{6 + j12 + 3/j} = .185\ \angle -26.3^\circ$$

$\therefore\ \underline{i(t) = 0.185\cos(4t - 26.3^\circ)\ \text{A}}$

P10.9-5



$$j15 = j(2\pi \cdot 796)(3 \cdot 10^{-3})$$

$$I = \frac{12}{20 + j15} = 0.48 \angle -37$$

$$i(t) = 0.48 \cos(2\pi \cdot 796t - 37^\circ)$$

P10.9-6

$$\text{current divider } I = \left(\frac{Z_1}{Z_1 + Z_2}\right) I_s \qquad Z_1 = R = 8,\ Z_2 = j\omega L = j3L$$

$$I = B \angle -51.87^\circ,\ I_s = 2 \angle -15^\circ$$

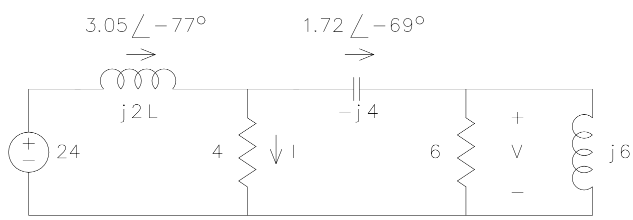
$$\text{so } \frac{I}{I_s} = \frac{B \angle -51.87^\circ}{2 \angle -15^\circ} = \frac{Z_1}{Z_1 + Z_2} = \frac{8}{8 + j3L} = \frac{8 \angle 0^\circ}{\sqrt{8^2 + (3L)^2} \angle \tan^{-1}\left(\frac{3L}{8}\right)}$$

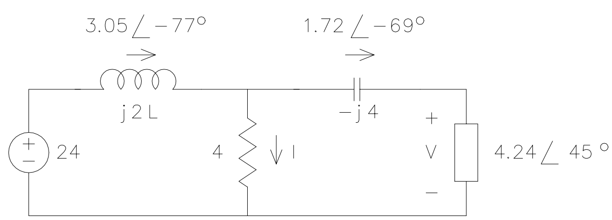
Set angles and magnitude equal

$$\text{angles: } +36.87 = +\tan^{-1}\left(\frac{3L}{8}\right) \Rightarrow \underline{L = 2\text{ H}}$$

$$\text{magnitude: } \frac{8}{\sqrt{64 + 9L^2}} = \frac{B}{2} \Rightarrow \underline{B = 1.6}$$

P10.9-7





The voltage ***V*** can be calculated using Ohm's Law.

$$\mathbf{V} = (1.72 \angle\text{-}69)\ (4.24 \angle 45) = 7.29 \angle\text{-}24$$

The current ***I*** can be calculated using KCL.

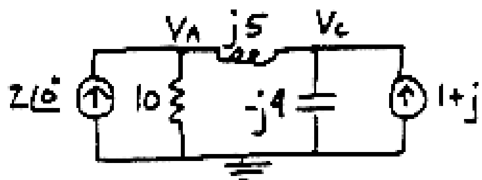
$$I = (3.05 \angle\text{-}77) - (1.72 \angle\text{-}69) = 1.34 \angle\text{-}87$$

Using KVL to calculate the voltage across the inductor and then Ohm's Law gives:

$$j2L = \frac{24 - 4(1.34 \angle\text{-}87)}{3.05 \angle\text{-}77} \Rightarrow L = 4$$

Section 10-10: Node Voltage and Mesh Current Analysis Using Phasors
(a) Node Voltage Analysis

P10.10-1 Draw phasor circuit and use nodal analysis



KCL at V_A : $-2+V_A/10+(V_A-V_C)/j5 = 0$

$$\Rightarrow (2+j)V_A - 2V_C = j20 \quad (1)$$

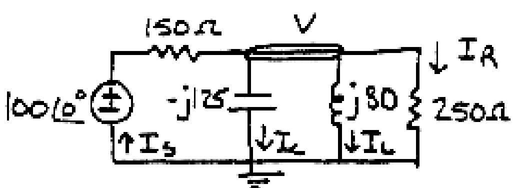
KCL at V_C : $\frac{(V_C-V_A)}{j5}+\frac{V_C}{-j4} - (1+j) = 0$

$$\Rightarrow 4V_A + V_C = 20 - j\,20 \quad (2)$$

Using Cramer's rule

$$V_c = \frac{\begin{vmatrix}(2+j) & j20 \\ 4 & 20-j20\end{vmatrix}}{\begin{vmatrix}(2+j) & -2 \\ 4 & 1\end{vmatrix}} = \frac{60-j100}{10+j} = \frac{116.6\ \angle -59^\circ}{\sqrt{101}\ \angle 5.7^\circ} = \underline{11.6\ \angle -64.7^\circ\ V}$$

P10.10-2



KCL at V: $\frac{(V-100)}{150}+\frac{V}{-j125}+\frac{V}{j80}+\frac{V}{250} = 0$

$$\Rightarrow V = 57.6\ \angle 22.9^\circ$$

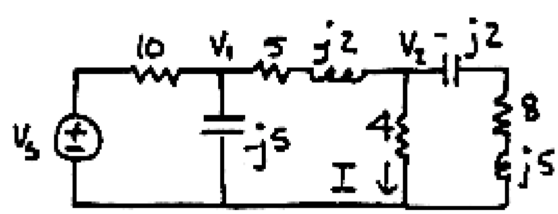
$$\therefore\ I_S = \frac{100-V}{150} = .667 - .384\ \angle 22.9^\circ = \underline{.347\ \angle -25.5^\circ}$$

$$I_C = \frac{V}{125\ \angle -90^\circ} = \underline{0.461\ \angle 112.9^\circ}$$

$$I_L = \frac{V}{80\angle 90^\circ} = \underline{0.720\ \angle -67.1^\circ}$$

$$I_R = \frac{V}{250} = \underline{0.230\angle 22.9^\circ}$$

P10.10-3



KCL at V_1 : $\frac{V_1-V_s}{10}+\frac{V_1}{-j5}+\frac{V_1-V_2}{5+j2} = 0$

$$\Rightarrow (11+j2)V_1 - (5+j2)V_s = 10V_2 \quad (1)$$

KCL at V_2: $\frac{V_2-V_1}{5+j2}+I+\frac{V_2}{8+j3} = 0$

$$\Rightarrow (8+j3)V_1 = (13+j5)V_2 + (34+j31)I \quad (2)$$

also $V_2 = 4I = 4(3\angle 45^\circ) = 12\angle 45^\circ = 6\sqrt{2}+j6\sqrt{2}$ (3)

Plugging I and (3) into (2) yields

$$(8+j3)V_1 = 74.24 + j\,290.62$$

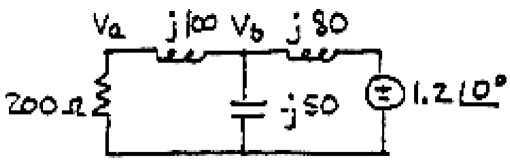
$$\therefore V_1 = \frac{300\angle 75.7^\circ}{8.54\angle 20.6^\circ} = 35.1\angle 55.1^\circ = 20.1 + j28.8$$

Now plugging V_1 and V_2 into (1) yields

$$(5+j2)V_s = -209.4 + j\,473.1$$

$$\therefore V_s = \frac{517.4\angle 113.9^\circ}{5.38\angle 21.8^\circ} = 96.1\angle 92.1^\circ \text{ V}$$

P10.10-4



KCL at V_a: $\dfrac{V_a}{200} + \dfrac{V_a - V_b}{j100} = 0$ (1)

KCL at V_b: $\dfrac{V_b - V_a}{j100} + \dfrac{V_b}{-j50} + \dfrac{V_b - 1.2}{j80} = 0$

$\Rightarrow V_a = (1/4)\,V_b - 3/2$ (2)

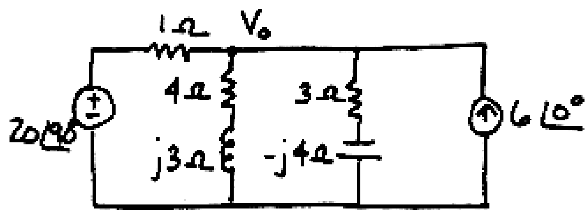
(2) into (1) yields $V_b = 2.21\angle -144^\circ$

and from (2) $V_a = 0.55\angle -144^\circ - 1.5 = 1.97\angle -171^\circ$

$\therefore v_a(t) = 1.97\cos(5000t - 171^\circ)\,\text{V}$

$v_b(t) = 2.21\cos(4000t - 144^\circ)\,\text{V}$

P10.10-5



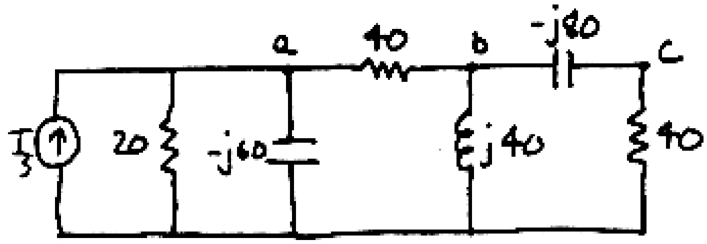
KCL at V_o:

$$\frac{V_o}{4+j3} + \frac{V_o}{3-j4} + \frac{V_o - j20}{1} = 6\angle 0^\circ$$

$\Rightarrow V_o = 16.31\angle 71.5^\circ$ V

$v_o(t) = 16.31\cos(10^5 t + 71.5^\circ)$ V

P10.10-6



$\omega = 10^4$ rad/s

$I_s = 20\angle 53^\circ$

KCL at a: $\left(\dfrac{1}{20} + \dfrac{1}{40} + \dfrac{j}{60}\right)V_a + \left(-\dfrac{1}{40}\right)V_b = 20\angle 53.13^\circ$ (1)

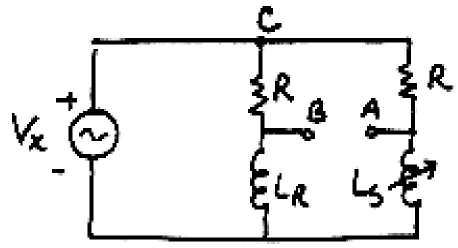
KCL at b: $\left(-\dfrac{1}{40}\right)V_a + \left(\dfrac{1}{40} - \dfrac{j}{40} + \dfrac{j}{80}\right)V_b - {}^{j}\!/_{80}\,V_c = 0$ (2)

KCL at c: $\dfrac{-j}{80}V_b + \left(\dfrac{1}{40} + \dfrac{j}{80}\right)V_c = 0$ (3)

Solving (1) - (3) simultaneously for V_a

$V_a = \sqrt{2}\cdot 240\angle 45^\circ$; thus $v_a(t) = 339.4\cos(\omega t + 45^\circ)$

P10.10-7



$v_X = \sin(2\pi\cdot 400t),\quad \omega = 2\pi\cdot 400$

$R = 100\Omega$

$L_R = 40\text{mH}$

$L_S = 40\text{mH}$, door opened

$= 60\text{mH}$, door closed

with the door open $\rightarrow$ $|V_A - V_B| = 0$ since bridge circuit is balanced

with the door closed $\rightarrow$ $Z_{L_R} = j(800\pi)(0.04) = j100.5\Omega$

$Z_{L_S} = j(800\pi)(0.06) = j150.8\Omega$

using nodal analysis

node B: $\frac{V_B - V_C}{R} + \frac{V_B}{Z_{L_R}} = 0 \Rightarrow V_B = \frac{j100.5}{j100.5+100} V_C$

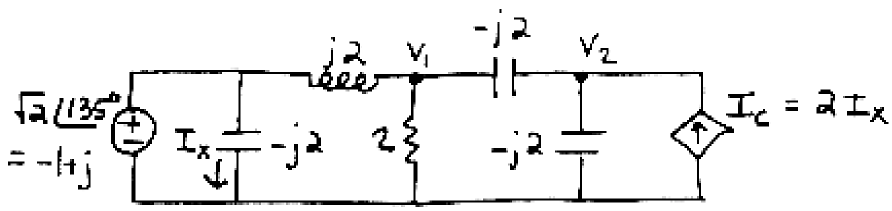
for $V_C = |V_X| = 1$ $\quad V_B = 0.709\angle 44.86^\circ$

node A : $\frac{V_A - V_C}{R} + \frac{V_A}{Z_{L_S}} = 0 \Rightarrow V_A = 0.833\angle 33.55^\circ$ for $V_C = |V_x| = 1$

$\therefore V_A - V_B = .833\angle 33.55^\circ - .709\angle 44.86^\circ = (.694 + j.460) - (.503 + j0.500) = .191 - j0.040$

$V_A - V_B = \underline{0.195\angle -11.83^\circ}$

P10.10-8



$$\text{node } V_1 : \frac{V_1 - (-1+j)}{j2} + \frac{V_1}{2} + \frac{V_1 - V_2}{-j2} = 0 \qquad (1)$$

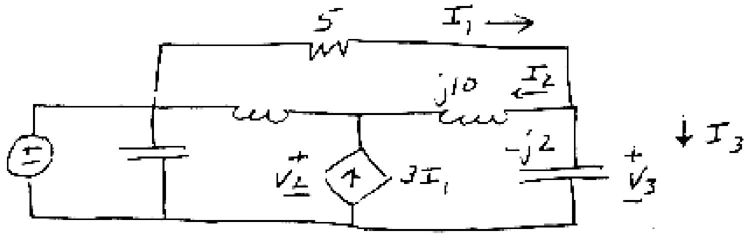
$$\text{node } V_2 : \frac{V_2 - V_1}{-j2} + \frac{V_2}{-j2} - I_C = 0 \qquad (2)$$

$$\text{also : } I_C = 2I_x = 2\left[\frac{-1+j}{-2j}\right] = -1 - j \qquad (3)$$

$$\text{Solving (1) through (3) yields } V_2 = \frac{-3-j}{1+j2} = \sqrt{2}\angle -135^\circ$$

$$\therefore\ v(t) = v_2(t) = \underline{\sqrt{2}\cos(40t - 135^\circ)\text{ V}}$$

P10.10-9



$V_2 = 0.7571\angle 66.7^\circ$

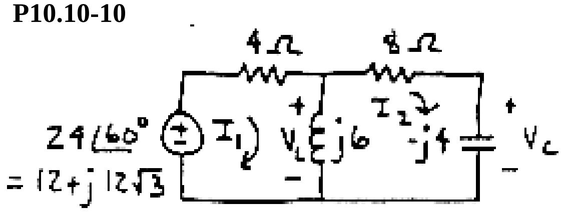
$V_3 = 0.6064\angle -69.8^\circ$

$$\left.\begin{array}{l}\text{Using: } I_1 = I_2 + I_3 \\ I_2 = \dfrac{V_3 - V_2}{j10} \\ I_3 = \dfrac{V_3}{-j2}\end{array}\right\} \text{ yields } \left\{\begin{array}{l} I_3 = 0.3032\ \angle 20.2^\circ \\ I_2 = 0.1267\angle -184^\circ \\ I_1 = 0.195\angle 36^\circ \end{array}\right.$$

so $\underline{i_1(t) = 0.195 \cos(2t + 36^\circ)\ \text{A}}$

(b): Mesh Current Analysis

P10.10-10



KVL loop1: $(4 + j6)I_1 - j6I_2 = 12 + j12\sqrt{3}$ (1)

KVL loop2: $-j6I_1 + (8 + j2)I_2 = 0$ (2)

Using Cramer's rule to solve I_1

$$I_1 = \frac{(12 + j12\sqrt{3})(8 + j2)}{(4 + j6)(8 + j2) - (-j6)(-j6)} = \underline{2.5\angle 29^\circ = 2.2 + j1.2}\ \ \text{A}$$

from (2) $I_2 = \dfrac{j6}{8 + j2}(2.5\angle 29^\circ) = \dfrac{6\angle 90^\circ}{\sqrt{68}\angle 14^\circ}(2.5\angle 29^\circ) = \underline{1.82\angle 105^\circ}$

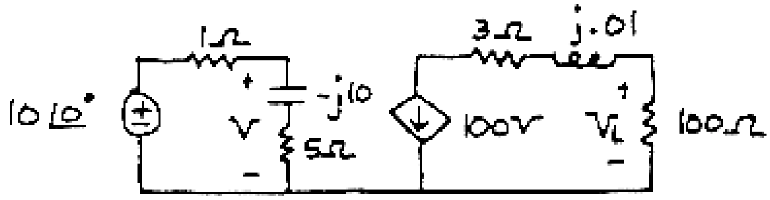
Now $V_L = j6(I_1 - I_2) = (6\angle 90^\circ)(2.5\angle 29^\circ - 1.82\angle 105^\circ)$

$= (6\angle 90^\circ)(2.71\angle -11.3^\circ)$

$\underline{V_L = 16.3\angle 78.7^\circ\ \text{V}}$

and $V_c = -j4I_2 = (4\angle -90^\circ)(1.82\angle 105^\circ) = \underline{7.28\angle 15^\circ\ \text{V}}$

P10.10-11



Using voltage divider

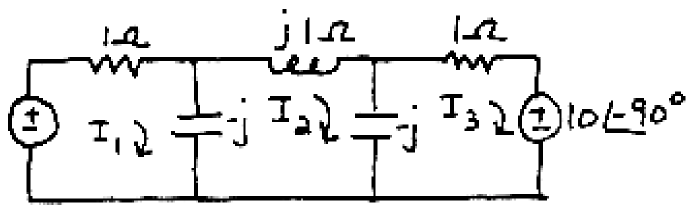
$$V = 10\angle 0^\circ \left[\frac{5 + -j10}{1 + 5 - j10}\right]$$

$= 9.59\angle -4.4^\circ$

Now $V_L = -100(100V) = -9.59 \times 10^4 \angle -4.4 = 9.59 \times 10^4 \angle 175.6^\circ$

$\therefore\ \underline{v_L(t) = 9.59 \times 10^4\ \cos(10^8 t + 175.6^\circ)\ \text{V}}$

P10.10-12



Three mesh equations: $(1-j)\,I_1 + (j)\,I_2 + 0I_3 = 10$

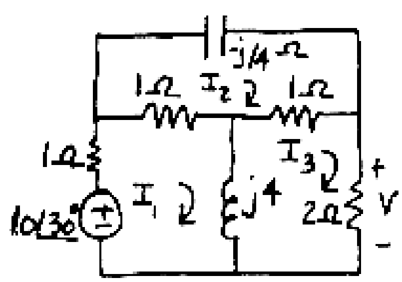
$$j\,I_1 - jI_2 + jI_3 = 0$$

$$0I_1 + jI_2 + (1-j)\,I_3 = j10$$

$$\text{So } I_2 = \frac{\begin{vmatrix} (1-j) & 10 & 0 \\ j & 0 & j \\ 0 & j\,10 & (1-j) \end{vmatrix}}{\begin{vmatrix} (1-j) & j & 0 \\ j & -j & j \\ 0 & j & (1-j) \end{vmatrix}} = \frac{10-j10}{1-j} = 10$$

$\therefore$ $\underline{i\,(t) = 10\cos 10^3 t\ \text{A}}$

P10.10-13



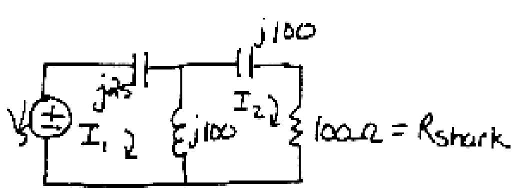
$$\begin{bmatrix} (2+j4) & -1 & -j4 \\ -1 & (2+1/j4) & -1 \\ -j4 & -1 & (3+j4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 30^\circ \\ 0 \\ 0 \end{bmatrix}$$

Using Cramer's rule $\quad I_3 = \dfrac{2+j8}{12+j\,22.5}\ 10\angle 30^\circ = 3.225\ \angle 44^\circ$

$$V = 2\,I_3$$

$$\therefore \underline{v(t) = 6.45\cos(10^5 t + 44^\circ)\ \text{V}}$$

P10.10-14



$\omega = 400$ rad/sec $\qquad V_s = 375\angle 0^\circ$

KVL : $j\,75I_1 - j100I_2 = 375$ (1)

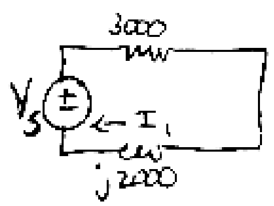
KVL : $-j\,100I_1 + (100+j100)I_2 = 0$ (2)

Solving for I_2 yields $I_2 = 4.5 + j1.5 \Rightarrow \underline{i_2(t) = 4.74\ \angle 18.4^\circ\,\text{A}}$

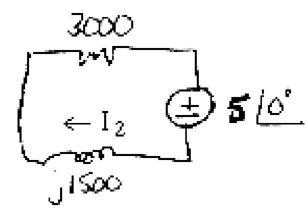
<u>Section 10-11:</u> Superposition, Thèvenin and Norton Equivalents and Source Transformations

(a) Superposition Principle

P10.11-1 Use superposition



$V_s = 12\angle 45^\circ$
$\omega = 4000$ rad/sec
$Z_L = j2000$

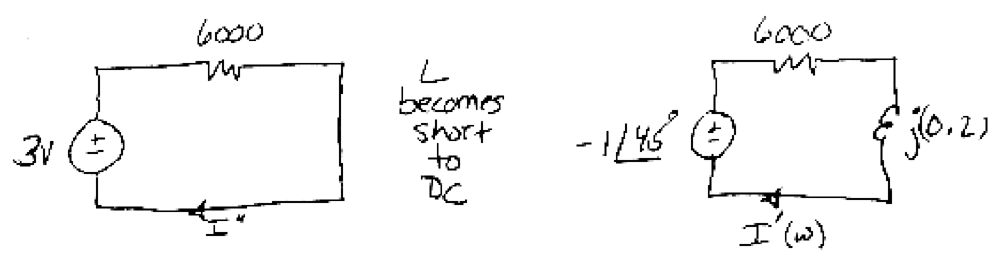


$\omega = 3000$ rad/sec
$Z_L = j1500$

$$I_1 = \frac{V_1}{Z_T} = \frac{12\angle 45^\circ}{3000+j2000} = 3.3\angle 11.3^\circ \text{ mA}; \quad I_2 = \frac{-V_2}{Z_T} = \frac{-5\angle 0^\circ}{3000+j1500} = 1.5\angle 153^\circ \text{ mA}$$

$$\underline{i(t) = 3.3\cos(4000t + 11.3^\circ) + 1.5\cos(3000t + 153^\circ) \text{ mA}}$$

P10.11-2 Use superposition



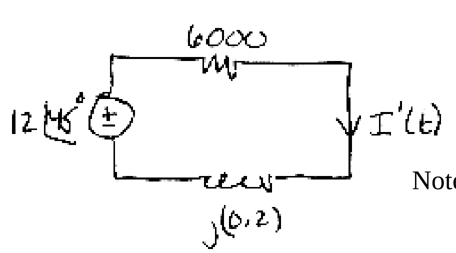
$$I'(\omega) = \frac{-1\angle 45^\circ}{6000 + j0.2} = -0.166 \times 10^{-3}\angle 45^\circ \qquad I'' = \frac{3}{6000} = 0.5\text{mA}$$

$$i(t) = i'(t) + i'' = (-0.166\cos(4t + 45^\circ) + 0.5) \text{ mA}$$

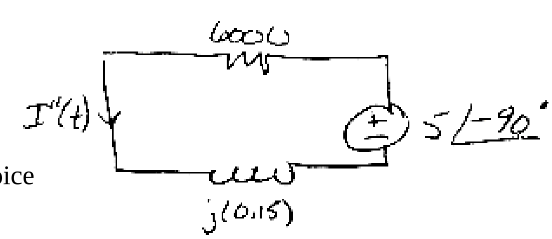
$$= \underline{[(0.166\cos(4t - 135^\circ) + 0.5)] \text{ mA}}$$

P10.11-3 Use superposition

$$v_2 = 5\sin 3t = 5\cos(3t - 90^\circ)$$



Note direction choice



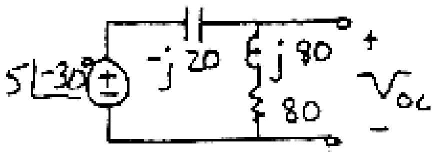
$$I'(t) = \frac{12\angle 45^\circ}{6000 + j0.2} = 2\angle 45^\circ \text{ mA} \qquad I''(t) = \frac{5\angle -90^\circ}{6000 + j0.15} = 0.833\angle -90^\circ \text{ mA}$$

$$i(t) = i'(t) - i''(t) = 2\cos(4t + 45^\circ) - 0.833\cos(3t - 90^\circ) \text{ mA}$$

Section 10-11 (b): Thévenin and Norton Equivalent Circuits

P10.11-4

Find V_{oc}

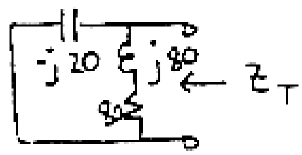


Using voltage divider

$$V_{oc} = 5\angle{-30^\circ}\left(\frac{80+j80}{80+j80-j20}\right) = 5\angle{-30^\circ}\left(\frac{80\sqrt{2}\angle{-21.9^\circ}}{100\angle 36.90^\circ}\right)$$

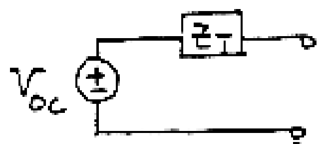
$$= \underline{4\sqrt{2}\angle{-21.9^\circ}\ \text{V}}$$

Find Z_T (Kill volatage source)



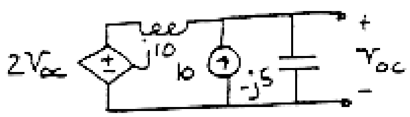
$$= \frac{(-j20)(80+j80)}{-j20+80+j80} = \underline{23\ \angle{-81.9^\circ}\ \Omega}$$

∴ have the equivalent circuit



P10.11-5

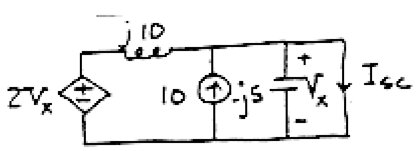
Find V_{oc}



KCL at top: $\dfrac{V_{oc}-2V_{oc}}{j\ 10} - 10 + \dfrac{V_{oc}}{-j5} = 0$

$$\Rightarrow \quad V_{oc} = -j\,{}^{100}\!/_{3} = {}^{100}\!/_{3}\angle{-90^\circ}\ \text{V}$$

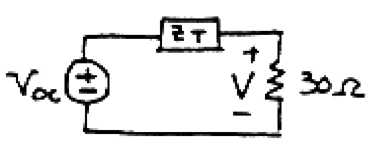
Find I_{sc}



$V_x = 0 \ \Rightarrow\ I_{sc} = 10\angle 0^\circ\ \text{A}$

$$Z_T = \frac{V_{oc}}{I_{sc}} = \frac{{}^{100}\!/_{3}\angle{-90^\circ}}{10\angle 0^\circ} = \underline{{}^{10}\!/_{3}\angle{-90^\circ}\ \Omega}$$

So have the equivalent circuit
Using voltage divider

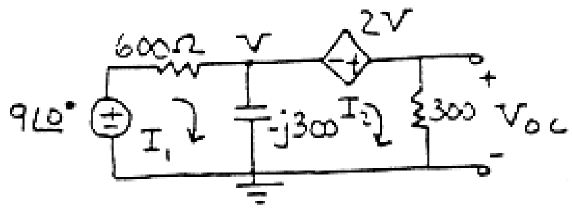


$$V = V_{oc}\left(\frac{30}{30+Z_t}\right) = -j\frac{300}{9-j} = 33.13\angle{-83.66^\circ}\ \text{V}$$

$\underline{\text{So } v(t) = 33.13\cos\left(20t-83.66^\circ\right)\ \text{V}}$

P10.11-6

Find V_{oc}



KVL loop I_1: $600I_1 - j300(I_1 - I_2) = 9$

$\Rightarrow (600-j300)I_1 + j300I_2 = 9 \qquad (1)$

KVL loop I_2: $-2V + 300I_2 - j300(I_2 - I_1) = 0$

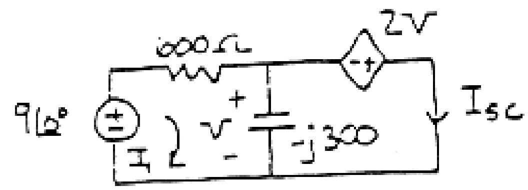
also $V = -j300(I_1 - I_2)$

$\Rightarrow j3I_1 + (1-j3)I_2 = 0 \qquad (2)$

Using Cramer's rule for equations (1) and (2) $\Rightarrow I_2 = .0124\angle{-16^\circ}$

$\therefore V_{oc} = 300I_2 = 3.71\angle{-16^\circ}\ \text{V}$

Find I_{sc}

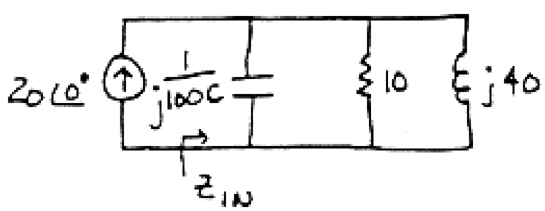


KVL loop I_{sc}: $-2V - V = 0 \qquad \Rightarrow V = 0$

$\therefore I_{sc} = \dfrac{9\angle 0^\circ}{600} = .015\angle 0^\circ$

So $Z_T = \dfrac{V_{oc}}{I_{sc}} = \dfrac{3.71\angle{-16^\circ}}{0.015} = 247\angle{-16^\circ}\ \Omega$

P10.11-7



$+$

$V = 100 - j100 = 100\sqrt{2}\angle{-45^\circ}$

$-$

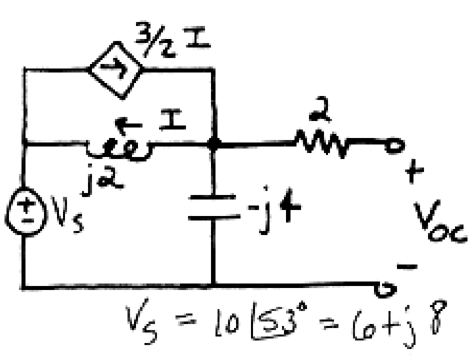
$Y_{IN} = 1/Z_{IN} = I/V = \dfrac{20\angle 0^\circ}{100\sqrt{2}\angle{-450^\circ}} = (.1)\sqrt{2}\angle 45^\circ = .1 + .1j$

Also $Y_{IN} = j100c + \dfrac{1}{j40} + \dfrac{1}{10} = j\left(100C - 1/40\right) + 1/10$

Equating the imaginary terms

$100C - 1/40 = .1 \qquad \Rightarrow C = 1/800\,\text{F}$

P10.11-8

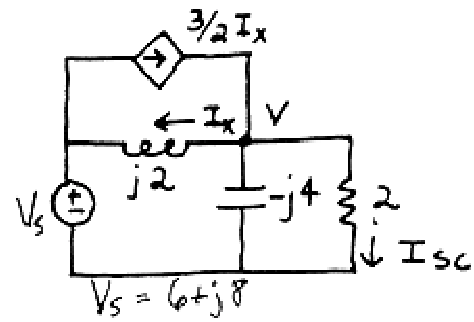


a) V_{oc} Node equation :

$$\frac{V_{oc}}{-j4} + \frac{V_{oc}-(6+j8)}{j2} - \frac{3}{2}\left(\frac{V_{oc}-(6+j8)}{j2}\right) = 0$$

yields $V_{oc} = 3 + j4 = 5\angle 53.1^\circ$

b) $\underline{I_{sc}}$



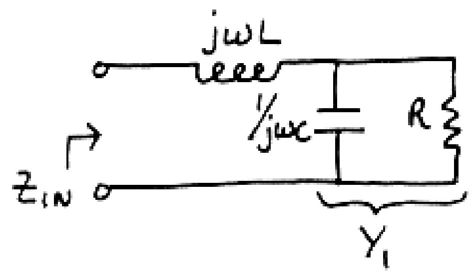
Node equation:

$$\frac{V}{2}+\frac{V}{-j4}+\frac{V-(6+j8)}{j2}-\frac{3}{2}\left[\frac{V-(6+j8)}{j2}\right]=0$$

yields $V=\dfrac{3+j4}{1-j}$

Thus $I_{sc}=\dfrac{V}{2}=\dfrac{3+j4}{2-j2}$ $\quad\therefore Z_T=\dfrac{V_{oc}}{I_{sc}}=3+j4\left(\dfrac{2-j2}{3+j4}\right)=\underline{2-j2}$

P10.11-9



$$Y_1=\frac{1}{R}+j\omega C$$

$$Z_1=\frac{1}{Y_1}=\frac{1}{1/R+j\omega C}=\frac{R}{1+j\omega RC}$$

Now $Z_{IN}=j\omega L+\dfrac{R}{1+j\omega RC}=\dfrac{R(1-\omega^2 LC)+j\omega L}{1+j\omega RC}$

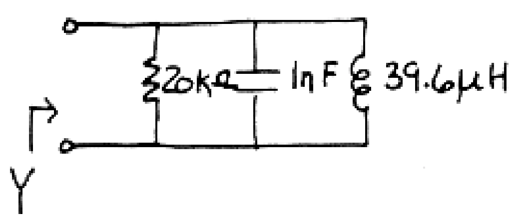
with L=97.5nH, C=39pF, $\omega=10^8$ rad/sec

$$Z_{IN}=\frac{R(1-.038)+j9.75}{1+j0.0039R}=\frac{.962R+j9.75}{1+j0.0039R}$$

$\underline{\text{for } R=25\Omega}$ $\quad Z_{IN}=\underline{25.8\angle 16.5^\circ=24.7+j7.33\Omega}$

for R = 50Ω $\quad Z_{IN}=\underline{48.2\angle 0.43^\circ=48.2+j0.36\Omega}$

P10.11-10



$$Y=G+Y_L+Y_C$$

$Y=G$ when $Y_L+Y_C=0$ or $\dfrac{1}{j\omega L}+j\omega C=0$

$$\omega_O=\frac{1}{\sqrt{LC}},\ f_O=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{39.6\times 10^{-15}}}$$

$= .07998\times 10^7$ Hz=800 KHz

(80 on the dial of the radio)

P10.11-11 $Z_1 = 50;\ \ |I_1| = \dfrac{|V_1|}{|Z_1|} = \dfrac{25}{50} = 0.5\text{ A}$

also $Z_2 = \dfrac{1}{j\omega C} = \dfrac{1}{j(2000)(2.5\times10^{-6})} = -j200$

$|I_2| = \dfrac{|V_2|}{|Z_2|} = \dfrac{100}{200} = 0.5\text{ A}$

and $Z_3 = j\omega L = j(2000)(50\times10^{-3}) = j100$

$|I_3| = \dfrac{|V_3|}{|Z_3|} = \dfrac{50}{100} = 0.5\text{A}$

Since $|I|$ is the same for all three cases, Z_{thev} and Z_n must also be equal.

So $|Z_t + Z_1| = |Z_t + Z_2| = |Z_t + Z_3|$

or $(R+50)^2 + X^2 = R^2 + (X-200)^2 = R^2 + (X+100)^2$

which requires that $(X-200)^2 = (X+100)^2 \Rightarrow X = 50\Omega$

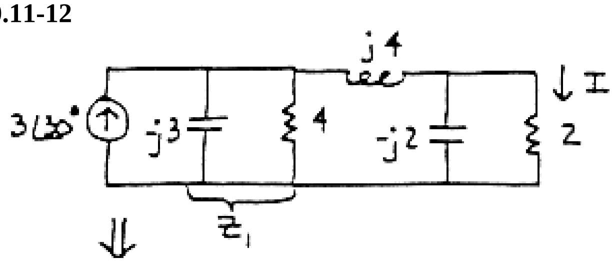
Using this in $(R+50)^2 + (50)^2 = R^2 + (-150)^2 \Rightarrow R = 175\Omega$

so $\underline{Z_t = 175 + j50\Omega}$ and if $V_t = |V_t|\angle 0^\circ$

we get $|V_t| = |I_1||Z_t + R_1| = (0.5)[(175+50)^2 + (50)^2]^{1/2} = \underline{115.25\text{V}}$

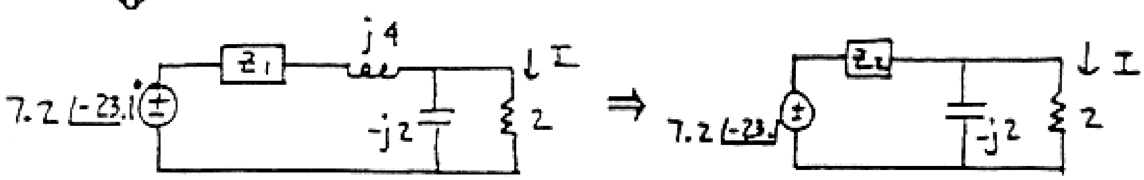
Section 10-11: (c) Source Transformation

P10.11-12

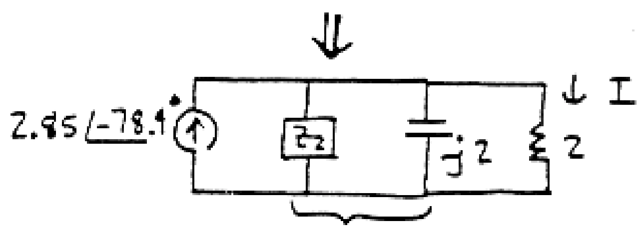


$Z_1 = \dfrac{(-j3)(4)}{-j3+4} = 2.4\angle -53.1^\circ$

$= 1.44 - j1.92$



$Z_2 = Z_1 + j4 = 1.44 + j2.08 = 2.53\angle 55.3^\circ$

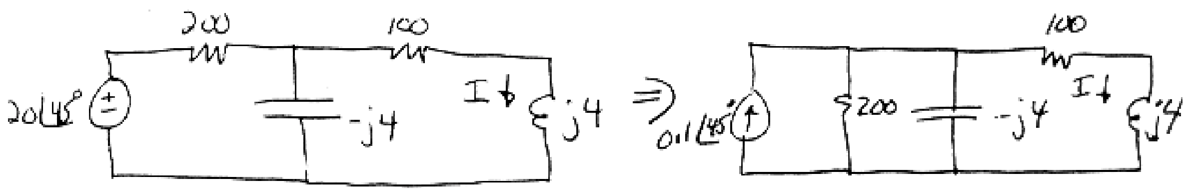


$Z_3 = 3.51\angle -37.9^\circ = 2.77 - j2.16$

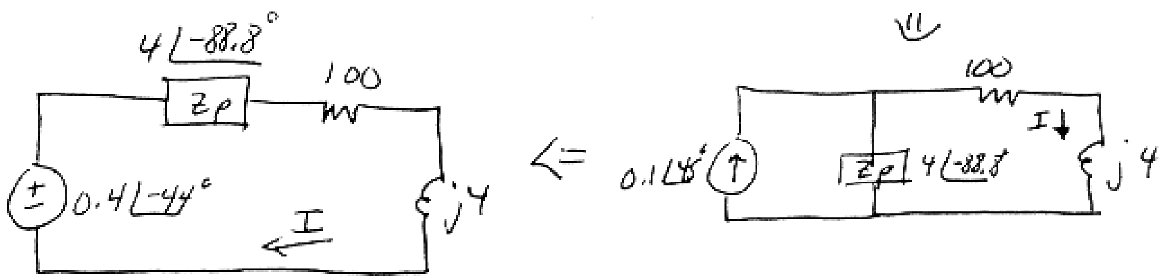
$$\therefore \mathrm{I} = 2.85\angle -78.4^\circ \left(\frac{3.51\angle -37.9^\circ}{2.77 - j2.16 + 2} \right)$$

$$\mathrm{I} = 2.85\angle -78.4^\circ \; \frac{\left(3.51\angle -37.9^\circ\right)}{\left(5.24\angle -24.4^\circ\right)} = \underline{1.9\angle -92^\circ \text{ A}}$$

P10.11-13



$$\mathrm{Z_P} = \frac{(200)(-j4)}{200 - j4} = 4\angle -88.8^\circ$$

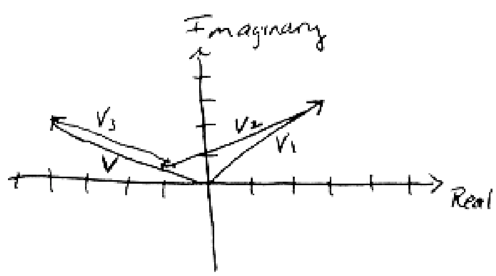


$$\mathrm{I} = \frac{0.4\angle -44^\circ}{-4j + 100 + j4} = 4\angle -44^\circ \text{ mA}; \quad \underline{i(t) = 4\cos(25000t - 44^\circ) \text{ mA}}$$

<u>Section 10-12:</u> Phasor Diagrams

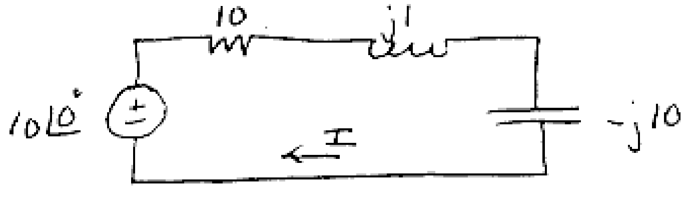
P10-12-1

$$\mathrm{V_1} = 3 + j3$$
$$-\mathrm{V_2} = -4 - j2$$
$$\mathrm{V_3^*} = -3 + j3$$

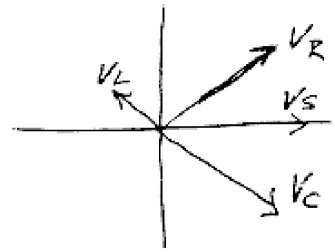


$$\mathrm{V} = -4 + j3$$
$$= 5\angle 180^\circ - \tan^{-1}\left(\frac{3}{4}\right)$$
$$= 5\angle 143^\circ$$

P10.12-2



$$\mathrm{I} = \frac{10\angle 0^\circ}{10 + j1 - j10}$$
$$= 0.74\angle 42^\circ$$



$$\mathrm{V_R} = \mathrm{RI} = 7.4\angle 42^\circ$$
$$\mathrm{V_L} = \mathrm{Z_L I} = (1\angle 90^\circ)(0.74\angle 42^\circ) = 0.74\angle 132^\circ$$
$$\mathrm{V_C} = \mathrm{Z_C I} = (10\angle -90^\circ)(0.74\angle 42^\circ) = 7.4\angle -48^\circ$$
$$\mathrm{V_S} = 10\angle 0^\circ$$

P10.12-3 $I = 72\sqrt{3} + 36\sqrt{3}\angle(140^\circ - 90^\circ) + 144\angle 210^\circ + 25\angle\phi$

$= 40.08 - j24.23 + 25\angle\phi$

$= 46.83\angle -31.15^\circ + 25\angle\phi$

Clearly for $|I|$ to be maxima, the above 2 terms must add in same direction (in phase) $\Rightarrow \underline{\phi = -31.15^\circ}$

Section 10-14: Phasor Circuits and the Operational Amplifier

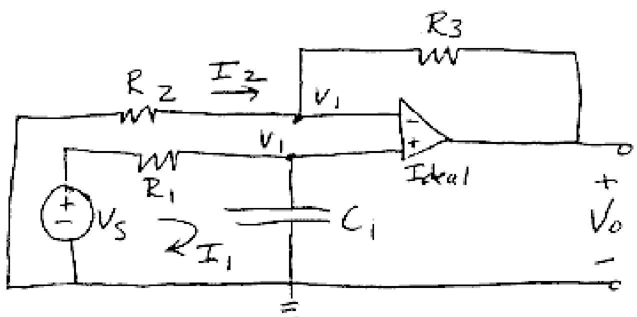
P10.14-1

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = -\left(\frac{10^4 \parallel -j10^4}{1000}\right) = -10\frac{-j}{1-j} = \frac{10}{\sqrt{2}}e^{-j225}$$

$$\mathbf{V}_s(\omega) = \sqrt{2} \quad \Rightarrow \quad \mathbf{V}_o(\omega) = \left(\frac{10}{\sqrt{2}}e^{-j225}\right)\sqrt{2} = 10e^{-j225}$$

$$v_o(t) = 10\cos(1000t - 225°) \quad \text{V}$$

P10.14-2



$$I_1 = \frac{V_S - V_1}{R_1} \qquad (1)$$

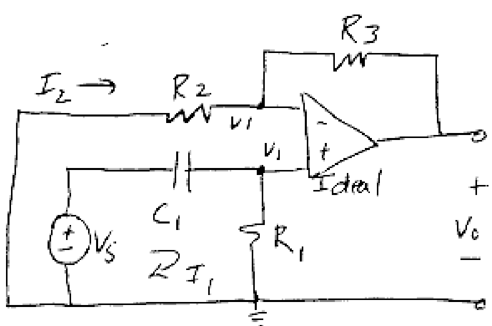
$$I_1 Z_C = V_1 \qquad (2)$$

$$\frac{-(V_1 - 0)}{R_2} = I_2 \qquad (3)$$

$$\frac{V_1 - V_0}{R_3} = I_2 \qquad (4)$$

Using equations (1) through (4) yields $\dfrac{V_0}{V_S} = \dfrac{1 + R_3/R_2}{1 + j\omega R_1 C_1}$

P10.14-3



The equations are:

$$\frac{V_S - V_1}{Z_C} = I_1 \qquad (1)$$

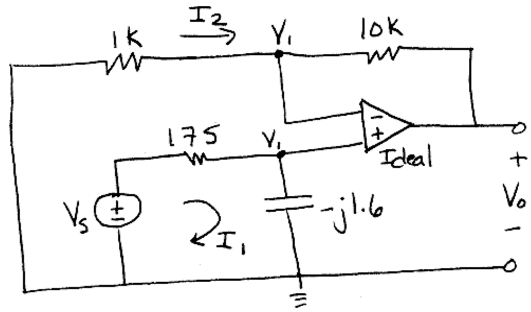
$$\frac{V_1}{R_1} = I_1 \qquad (2)$$

$$\frac{-(V_1 - 0)}{R_2} = I_2 \qquad (3)$$

$$\frac{V_1 - V_0}{R_3} = I_2 \qquad (4)$$

Solving these four equations yields $\dfrac{V_0}{V_S} = \dfrac{j\omega CR_1(1 + R_3/R_2)}{j\omega CR_1 + 1}$

P10.14-4



$\omega = 62832 \text{ rad/sec}$
$= 2\pi f$
$V_S = 5\angle 0^\circ \text{ mV}$
$Z_C = -j1.6$

Equations:

$\frac{V_S - V_1}{175} = I_1$ (1) $\qquad \frac{-(V_1 - 0)}{1000} = I_2$ (3)

$\frac{V_1 - 0}{-j1.6} = I_1$ (2) $\qquad \frac{V_1 - V_0}{10K} = I_2$ (4)

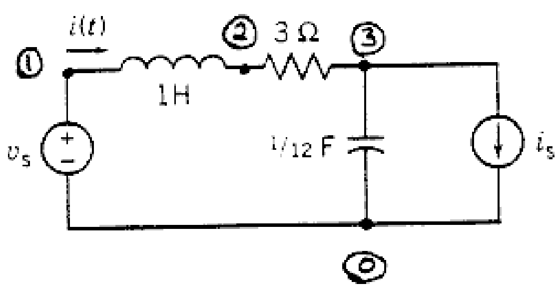
Using equations (1) through (4) yields $V_0 = 0.5\angle -89^\circ$

or $\underline{v_0(t) = 0.5\cos(\omega t - 89^\circ) \text{ mV}}$

PSpice Problems

SP 10-1

Circuit:



$v_S = 10\cos(6t + 45^\circ) \text{ V}$
$i_S = 2\cos(6t + 60^\circ) \text{ A}$

Input file:

```
Vs        1    0    ac       10      45
L2        1    2              1
R3        2    3              3
C4        3    0          83.3m
Is        3    0    ac        2      60

.ac lin  1 0.9549 0.9549 ; ω = 6
.print ac Im(L2) Ip(L2)

.END
```

Output:

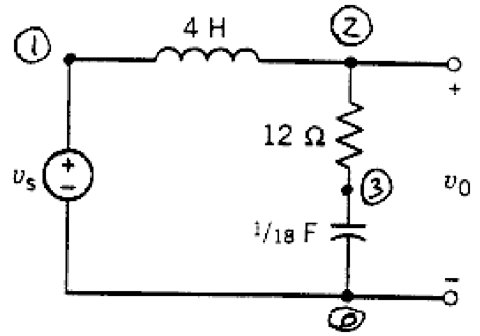
```
FREQ          IM(L2)         IP(L2)
9.549E-01   2.339E+00      -2.743E+01
```

So $\underline{i(t) = 2.34\cos(6t - 27^\circ)}$ A

SP 10-2

Circuit:

$v_S = 5\cos(5t - 30^\circ)V$



Input file:

```
Vs              1        0      ac          5     -30
L2              1        2                  4
R3              2        3                 12
C4              3        0              55.6m

.ac lin 1  0.7958  0.7958  ; ω = 5
.print ac Vm(2)  VP (2)

.END
```

Output:

```
  FREQ                VM(2)          VP (2)
7.958E-01          3.082E+00    -1.005E+02
```

So $\underline{v_0(t) = 3.08\cos(5t\text{-}100^\circ)}$ V

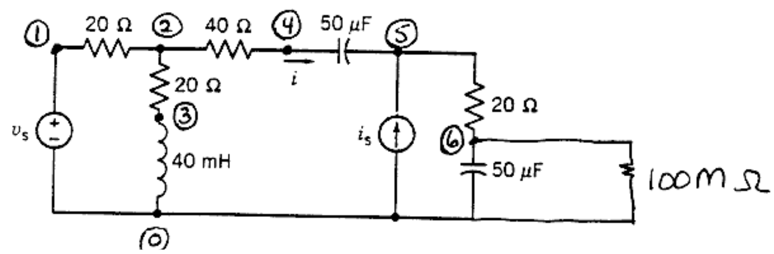
SP 10-3

Circuit:

$v_S = 200\cos\omega t$

$i_S = 8\cos(\omega t + 90^\circ)$

$\omega = 1000 \text{ rad/sec}$



Note: The 100mΩ resistor was added to provide the dc path to ground required by Spice. Since the resistance is so large, it has little effect on the solution calculated.

Input File:

```
 V1             1       0      ac        200
 R2             1       2                 20
 R3             2       3                 20
 L4             3       0                40m
 R5             2       4                 40
 C6             4       5                50u
 I7             0       5      ac          8      90
 R8             5       6                 20
 C9             6       0                50u
 R9             6       0             100MEG

.ac lin 1  159.15  159.15
.print ac Im(R5)  Ip(R5)

.END
```

Output:

```
   FREQ            IM(R5)           IP(R5)
 1.592E+02      1.335E+00        -7.018E+01
```

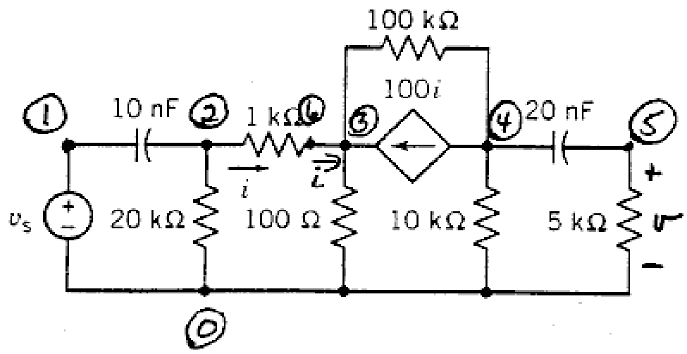
So $\underline{i(t) = 1.34\cos(1000t - 70^\circ)}$ A

SP 10-4

$v_S = 4\cos\omega t$

$\omega = 2\pi f$

$= 2\pi(1000)$ rad/sec




```
Input file:
 V1               1       0      ac           4      0
 C2               1       2                 10n
 R3               2       0                 20K
 R4               2       6                1000
 R5               3       0                 100
 R6               3       4                100K
 Vdummy           6       3                   0
 F7               4       3     Vdummy      100
 R8               4       0                 10K
 C9               4       5                 20n
 R10              5       0                  5K

.ac lin 1 1000 1000
.print ac Vm(5) VP (5)

Output:
 VM(5)        VP (5)
4.245E+01   -8.472E+01
```

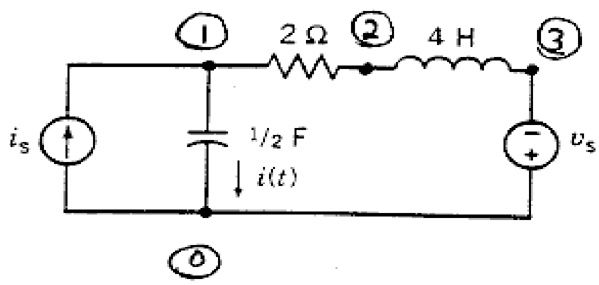
So $\underline{v(t) = 42.5\cos(\omega t - 85^\circ)\text{ V}}$

SP 10-5

Circuit:

$i_S = 2\cos(3t + 10^\circ)$ A

$v_S = 3\cos(2t + 30^\circ)$ V




```
Input file:

 Is          0      1     ac          2   10
 C2          1      0              500m
 R3          1      2                 2
 L4          2      3                 4
 Vs          0      3     ac          3   30

.ac lin 1 0.4776 0.4776
.print ac Im(C2) Ip(C2)

.END
 Output:
    FREQ         IM(C2)        IP(C2)
   4.776E-01    1.999E+00     1.706E+01
```

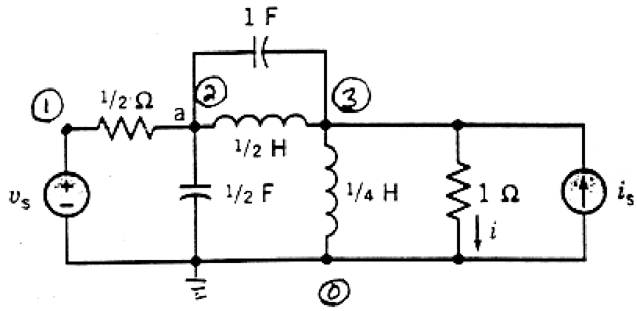
So $\underline{i(t) = 2\cos(2t + 17^\circ)\text{ A}}$

SP 10-6

Circuit:

v_S = 5cos2t

i_S = 5cos2t




```
Input file:
Vs              1     0     ac       5      0
R2              1     2           500m
C3              2     0           500m
C4              2     3              1
L5              2     3           500m
L6              3     0           250m
R7              3     0              1
Is              0     3     ac       5      0



.ac lin 1 0.3183 0.3183 ; ω = 2
.print ac Vm(2) VP (2)
.print ac Im(R7) Ip(R7)

.END
Output:

    FREQ          VM(2)          VP (2)
  3.183E-01     2.236E+00     -2.657E+01
    FREQ          IM(R7)         IP(R7)
  3.183E-01     4.472E+00      6.344E+01
```

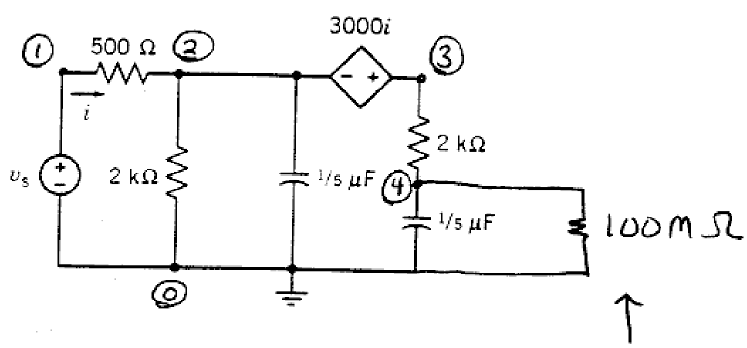
So $\underline{v_a(t) = 2.24 \cos(2t - 27^\circ) \text{ V}}$

and $\underline{i(t) = 4.47 \cos(2t + 63^\circ) \text{ A}}$

SP 10-7

Circuit:

v_S = 4cos 500t V



See note on problem SP 10-3 for explanation.

Input file:

```
 Vs          1     0      ac                4      0
 R2          5     2                      500
 R3          2     0                       2K
 C4          2     0                     200n
 Vdummy      1     5                        0
 H5          3     2     Vdummy           300
 R6          3     4                       2K
 C7          4     0                     200n
 R7          4     0                  100MEG

.ac lin 1 795.8  ; w = 5000 rad/sec
.print ac Im(R2)   Ip(R2)

.END
```

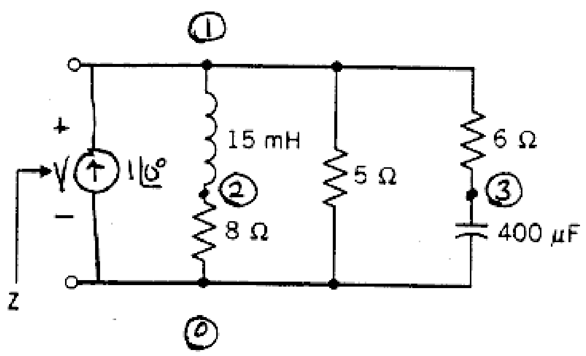
Output:

```
     FREQ          IM(R2)           IP(R2)
      7.958E+02    4.180E-03   3.103E+01
```

So $\underline{i(t) = 4.18 \cos(500t + 31^\circ) \text{ mA}}$

SP 10-8

Circuit:



$f = 60 \text{ Hz}$

Use $1\angle 0^\circ$ A test source

Then $Z = \dfrac{V}{1\angle 0^\circ}$

Input file:

```
 Is      0       1    ac         1       0
 L2      1       2              15m
 R3      2       0               8
 R4      1       0               5
 R5      1       3               6
 C6      3       0       400u
.ac lin 1 60 60  ;  f = 60Hz
.print ac Vm(1)  VP (1)

.END
```

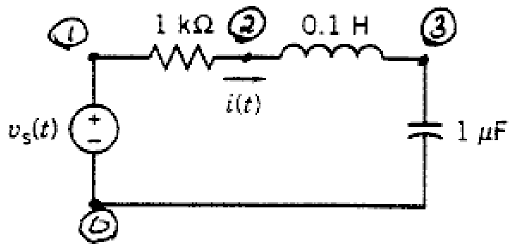
Output:

```
       FREQ        VM(1)              VP (1)
     6.000E+01        2.784E+00   -3.831E+00
```

So $\quad Z = \dfrac{V}{1\angle 0^\circ} = \underline{2.78\angle -4^\circ \, \Omega}$

SP 10-9

Circuit:



$v_S = 120\sin(\omega t + 30^\circ)$

$f = 10$ kHz

Input file:

```
Vs          1     0   ac       120    -60
R2          1     2           1000
L3          2     3           100m
C4          3     0           1000n


.ac lin 1 10e3 10e3
.print ac Im(R2) Ip(R2)

.END
```

Output:

```
     FREQ        IM(R2)        IP(R2)
   1.000E+04  1.891E-02    -1.409E+02
```

So i(t) 18.9 cos(ωt-141°)mA

Verification Problems

VP 10-1 Generally, it is more convenient to divide complex numbers in polar form. Sometimes, as in this case, it is more convenient to do the division in rectangular form.

Express $\mathbf{V}_1$ and $\mathbf{V}_2$ as: $\mathbf{V}_1 = -j\,20$ and $\mathbf{V}_2 = 20 - j\,40$

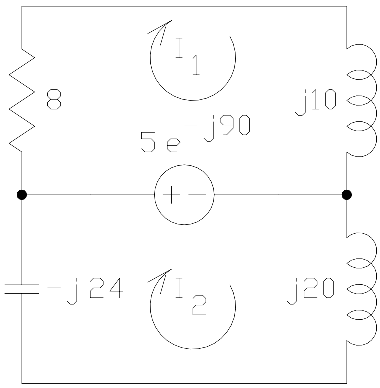
KCL at node 1: $2 - \frac{\mathbf{V}_1}{10} - \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 2 - \frac{-j\,20}{10} - \frac{-j\,20 - (20 - j\,40)}{j10} = 2 + j\,2 - 2 - j\,2 = 0$

KCL at node 2:

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} - \frac{\mathbf{V}_2}{10} + 3\left(\frac{\mathbf{V}_1}{10}\right) = \frac{-j\,20 - (20 - j\,40)}{j10} - \frac{20 - j\,40}{10} + 3\left(\frac{-j\,20}{10}\right) = (2 + j\,2) - (2 - j4) - j\,6 = 0$$

The currents calculated from $\mathbf{V}_1$ and $\mathbf{V}_2$ satisfy KCL at both nodes, so it is very likely that the $\mathbf{V}_1$ and $\mathbf{V}_2$ are correct.

VP 10-2



$$\mathbf{I}_1 = 0.390\angle 39° \text{ and } \mathbf{I}_2 = 0.284\angle 180°$$

Generally, it is more convenient to multiply complex numbers in polar form. Sometimes, as in this case, it is more convenient to do the multiplication in rectangular form.

Express $\mathbf{I}_1$ and $\mathbf{I}_2$ as: $\mathbf{I}_1 = 0.305 + j\,0.244$ and $\mathbf{I}_2 = -0.284$

KVL for mesh 1:

$$8\left(0.305 + j\,0.244\right) + j10\left(0.305 + j\,0.244\right) - (-j\,5) = j\,10$$

Since KVL is not satisfied for mesh 1, the mesh currents are not correct.

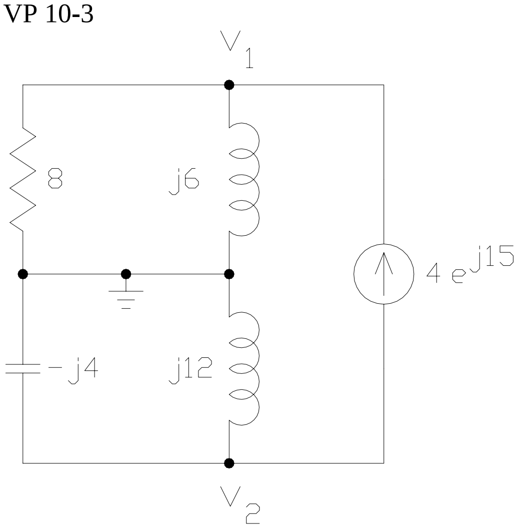
Here is a MATLAB file for this problem:

```
% Impedance and phasors for Figure VP 10-2
Vs = -j*5;
Z1 = 8;
Z2 = j*10;
Z3 = -j*2.4;
Z4 = j*20;

% Mesh equations in matrix form
Z = [  Z1+Z2     0;
         0     Z3+Z4 ];
V = [ Vs;
     -Vs  ];
I = Z\V
abs(I)
angle(I)*180/3.14159

% Verify solution by obtaining the algebraic sum of voltages for
% each mesh. KVL requires that both M1 and M2 be zero.
M1 = -Vs + Z1*I(1) +Z2*I(1)
M2 = Vs + Z3*I(2) + Z4*I(2)
```

VP 10-3



$$\mathbf{V}_1 = 19.2\angle 68° \text{ and } \mathbf{V}_2 = 24\angle 105° \text{ V}$$

KCL at node 1 :

$$\frac{19.2\angle 68°}{2} + \frac{19.2\angle 68°}{j\,6} - 4\angle 15 = 0$$

KCL at node 2:

$$\frac{24\angle 105°}{-j4} + \frac{24\angle 105°}{j12} + 4\angle 15 = 0$$

The currents calculated from $\mathbf{V}_1$ and $\mathbf{V}_2$ satisfy KCL at both nodes, so it is very likely that the $\mathbf{V}_1$ and $\mathbf{V}_2$ are correct.

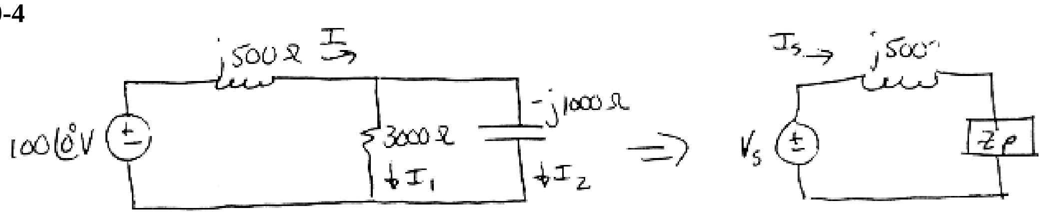
Here is a MATLAB file for this problem:

```
% Impedance and phasors for Figure VP 10-3
Is = 4*exp(j*15*3.14159/180);
Z1 = 8;
Z2 = j*6;
Z3 = -j*4;
Z4 = j*12;

% Mesh equations in matrix form
Y = [  1/Z1 + 1/Z2        0;
            0        1/Z3 + 1/Z4 ];
I = [ Is;
     -Is  ];
V = Y\I
abs(V)
angle(V)*180/3.14159

% Verify solution by obtaining the algebraic sum of currents for
% each node. KCL requires that both M1 and M2 be zero.
M1 = -Is + V(1)/Z1 + V(1)/Z2
M2 = Is + V(2)/Z3 + V(2)/Z4
```

VP 10-4



$$Z_P = \frac{(3000)(-j1000)}{3000-j1000} = 949\ \angle -72^\circ = 300 - j900\ \Omega$$

$$I = \frac{V_S}{j500+Z_P} = \frac{100\ \angle 0^\circ}{j500+300-j900} = \underline{0.2\angle 53^\circ\ A}$$

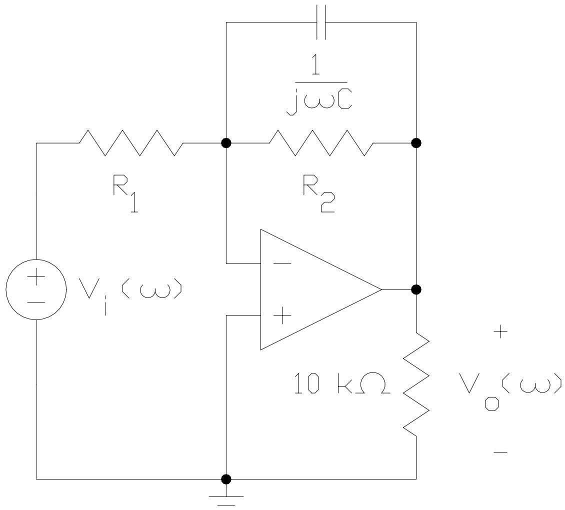
Now using current divider:

$$I_1 = \left(\frac{-j1000}{3000-j1000}\right)\left(0.2\ \angle 53^\circ\right) = \underline{63.3\ \angle -18.5\ mA}$$

$$I_2 = \left(\frac{3000}{3000-j1000}\right)\left(0.2\ \angle 53^\circ\right) = \underline{190\angle 71.4^\circ\ mA}$$

Design Problems

DP 10-1



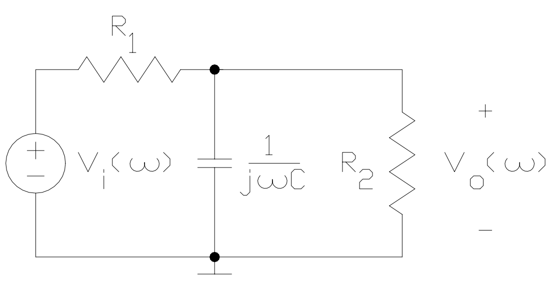
$$R_2 \left\| \frac{1}{j\omega C} = \frac{R_2}{1+j\omega CR_2} \right.$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = -\frac{\dfrac{R_2}{1+j\omega CR_2}}{R_1} = -\frac{\dfrac{R_2}{R_1}}{1+j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{R_1}}{\sqrt{1+(\omega CR_2)^2}} e^{j\left(180-\tan^{-1}\omega CR_2\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 104° so $CR_2 = \dfrac{\tan(180-76)}{1000} = 0.004$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{8}{2.5}$ so $\dfrac{\dfrac{R_2}{R_1}}{\sqrt{1+16}} = \dfrac{8}{2.5} \Rightarrow \dfrac{R_2}{R_1} = 132$. One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F},\ R_1 = 1515\ \Omega,\ R_2 = 20\ \text{k}\Omega$.

DP 10-2



$$R_2 \left\| \frac{1}{j\omega C} = \frac{R_2}{1+j\omega CR_2} \right.$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{1+j\omega CR_2}}{R_1 + \dfrac{R_2}{1+j\omega CR_2}} = -\frac{K}{1+j\omega CR_p}$$

where $K = \dfrac{R_1}{R_1+R_2}$ and $R_p = \dfrac{R_1 R_2}{R_1+R_2}$

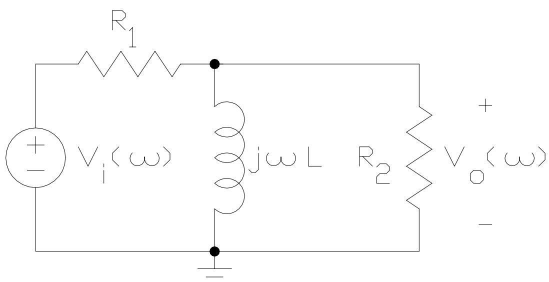
$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{K}{\sqrt{1+(\omega CR_p)^2}} e^{-j\tan^{-1}\omega CR_p}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be -76° so $C\,R_p = C\dfrac{R_1\,R_2}{R_1+R_2} = \dfrac{\tan(76)}{1000} = 0.004$

and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{12}$ so

$\dfrac{K}{\sqrt{1+16}} = \dfrac{2.5}{12} \quad \Rightarrow \quad 0.859 = K = \dfrac{R_2}{R_1+R_2}$. One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F},\ R_1 = 23.3\ \text{k}\Omega,\ R_2 = 142\ \text{k}\Omega$.

DP 10-3



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega\,L\,R_2}{R_2 + j\omega\,L}}{R_1 + \dfrac{j\omega\,L\,R_2}{R_2 + j\omega\,L}} = -\frac{j\omega\,\dfrac{L}{R_1}}{1 + j\omega\,\dfrac{L}{R_p}}$$

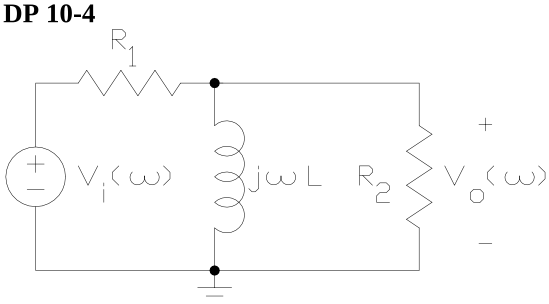
where $R_p = \dfrac{R_1\,R_2}{R_1+R_2}$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega\,\dfrac{L}{R_1}}{\sqrt{1+\left(\omega\,\dfrac{L}{R_p}\right)^2}}\,e^{j\left(90-\tan^{-1}\omega\frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 14° so $\dfrac{L}{R_p} = \dfrac{L\,(R_1+R_2)}{R_1\,R_2} = \dfrac{\tan(90-14)}{40} = 0.1$

and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{8}$ so $\dfrac{40\dfrac{L}{R_1}}{\sqrt{1+16}} = \dfrac{2.5}{8} \quad \Rightarrow \quad \dfrac{L}{R_1} = 0.0322$. One set of values that satisfies these two equations is $L = 1\ \text{H},\ R_1 = 31\ \Omega,\ R_2 = 14.76\ \Omega$.

DP 10-4



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = -\frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

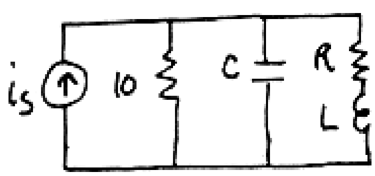
$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1+\left(\omega \dfrac{L}{R_p}\right)^2}} e^{j\left(90 - \tan^{-1}\omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be -14°. This requires

$$\frac{L}{R_p} = \frac{L(R_1 + R_2)}{R_1 R_2} = \frac{\tan(90+14)}{40} = -0.1$$

This condition cannot be satisfied with positive

DP 10-5



$Z_1 = 10$ $\quad Y_1 = 1/10$

$Z_2 = 1/j\omega C$ $\quad Y_2 = j\omega C$

$Z_3 = R + j\omega L$ $\quad Y_3 = 1/(R + j\omega L)$

Use the fact that admittances in parallel add and $V(\sum Y) = I$

So $V(Y_1 + Y_2 + Y_3) = I_S$ with $v(t) =$ 80 cos (1000t – θ) $\Rightarrow V = 8\angle -\theta$

$i_S(t) = 10 \cos 100t \quad \Rightarrow I_s = 10\angle 0^\circ$

So have $\quad 80\angle -\theta \left[\dfrac{1}{10} + \dfrac{1}{R + j\omega L} + j\omega c\right] = 10\angle 0^\circ$

$$\Rightarrow R + 10 - 10\omega^2 LC + j(\omega L + 10\omega RC) = 1.25R + j1.25\omega L$$

Equate real part: $40 - 40\omega^2 LC = R$ (1)

Equate imaginary part: $40 RC = L$ (2)

Plugging (2) into (1) yields $R = 40(1 - 4\times 10^7 RC^2)$ $\quad \omega = 1000$ rad/sec

Now try $\underline{R = 20\Omega}$ $\quad \Rightarrow 1 - 2(1 - 4\times 10^7 (20)C^2)$

which yields $C = 2.5\times 10^{-5} F = \underline{25\ \mu F}$

$\therefore L = 40\ RC = 0.02H = \underline{20mH}$

Now check θ : $Y_1 = 1/10 = 0.1$

$Y_2 = j0.25$

$Y_3 = 1/(20 + j20) = .025 - j.025$

$\therefore Y = Y_1 + Y_2 + Y_3 = .125$, so $V = YI_s = (.125\angle 0^\circ)(10\angle 0^\circ) = 1.25\angle 0^\circ$

$\therefore \underline{\theta = 0^\circ}$ meets the design spec