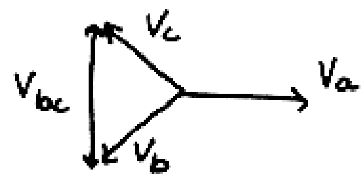
Chapter 12: Three-Phase Circuits

Exercises

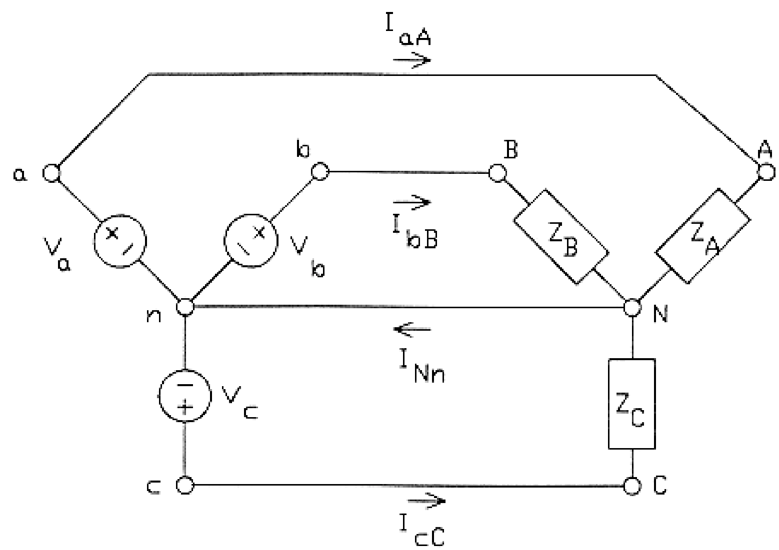
Ex. 12.3-1

$V_C = 120\angle -240^\circ$ so $V_A = 120\angle 0^\circ$ and $V_B = 120\angle -120^\circ$



$V_{bc} = \sqrt{3}(120)\angle -90^\circ$

Ex. 12.4-1 Four-wire Y-to-Y Circuit



Mathcad analysis

Describe the three-phase source: $Va := 120 \cdot e^{j \cdot \frac{\pi}{180} \cdot 0}$ $Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120}$ $Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$

Describe the three-phase load: $ZA := 80 + j \cdot 50$ $ZB := 80 + j \cdot 80$ $ZC := 100 - j \cdot 25$

Calculate the line currents: $IaA := \frac{Va}{ZA}$ $IbB := \frac{Vb}{ZB}$ $IcC := \frac{Vc}{ZC}$

$IaA = 1.079 - 0.674i$ $IbB = -1.025 - 0.275i$ $IcC = -0.809 + 0.837i$

$|IaA| = 1.272$ $|IbB| = 1.061$ $|IcC| = 1.164$

$\frac{180}{\pi} \cdot \arg(IaA) = -32.005$ $\frac{180}{\pi} \cdot \arg(IbB) = -165$ $\frac{180}{\pi} \cdot \arg(IcC) = -134.036$

Calculate the current in the neutral wire: $INn := IaA + IbB + IcC$ $INn = -0.755 - 0.112i$

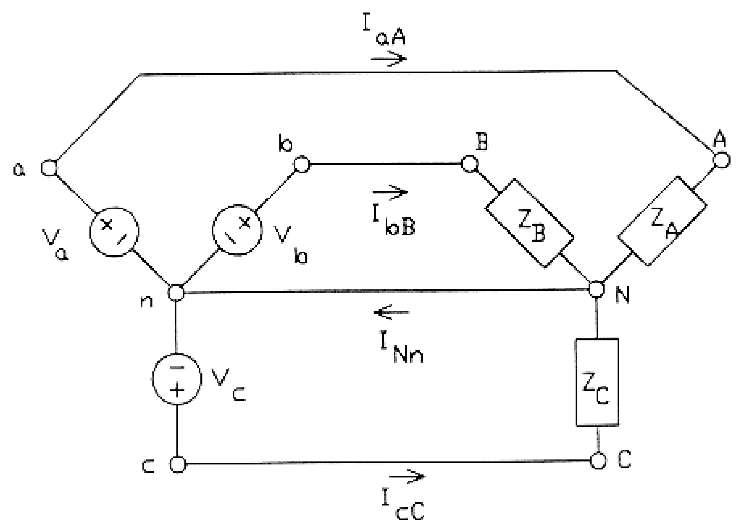
Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot Va$ $SB := \overline{IbB} \cdot Vb$ $SC := \overline{IcC} \cdot Vc$

$SA = 129.438+80.899i$ $SB = 90+90i$ $SC = 135.529-33.883i$

$SA + SB + SC = 354.968 + 137.017i$

Ex. 12.4-2 Four-wire Y-to-Y Circuit



Mathcad analysis

Describe the three-phase source: $Va := 120 \cdot e^{j \cdot \frac{\pi}{180} \cdot 0}$ $Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120}$ $Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$

Describe the three-phase load: $ZA := 40 + j \cdot 30$ $ZB := ZA$ $ZC := ZA$

Calculate the line currents: $IaA := \frac{Va}{ZA}$ $IbB := \frac{Vb}{ZB}$ $IcC := \frac{Vc}{ZC}$

$IaA = 1.92 - 1.44i$ $IbB = -2.207 - 0.943i$ $IcC = 0.287 + 2.383i$

$|IaA| = 2.4$ $|IbB| = 2.4$ $|IcC| = 2.4$

$\frac{180}{\pi} \cdot \arg(IaA) = -36.87$ $\frac{180}{\pi} \cdot \arg(IbB) = -156.87$ $\frac{180}{\pi} \cdot \arg(IcC) = 83.13$

Calculate the current in the neutral wire: $INn := IaA + IbB + IcC$ $INn = 0$

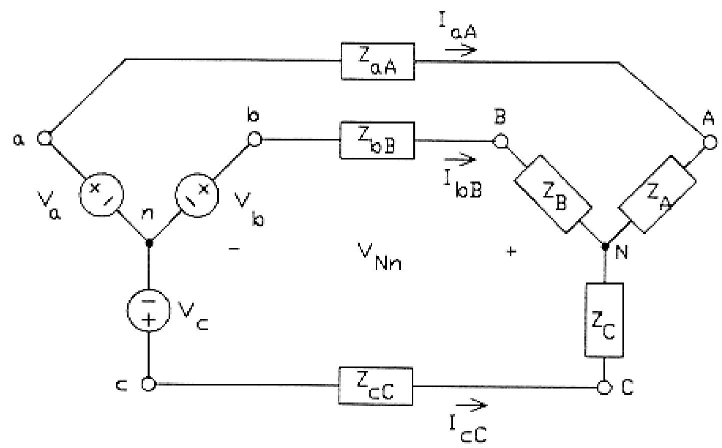
Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot Va$ $SB := \overline{IbB} \cdot Vb$ $SC := \overline{IcC} \cdot Vc$

$SA = 230.4+172.8i$ $SB = 230.4+172.8i$ $SC = 230.4+172.8i$

$SA + SB + SC = 691.2 + 518.4i$

Ex. 12.4-3 Three-wire Y-to-Y Circuit with line impedances



Mathcad analysis

Describe the three-phase source: $Vp := 120$

$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0}$ $\quad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120}$ $\quad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$

Describe the three-phase load: $ZA := 80 + j \cdot 50$ $\quad ZB := 80 + j \cdot 80$ $\quad ZC := 100 - j \cdot 25$

Describe the three-phase line: $ZaA := 0$ $\quad ZbB := 0$ $\quad ZcC := 0$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA+ZA)\cdot(ZcC+ZC)\cdot e^{j\cdot\frac{4}{3}\cdot\pi} + (ZaA+ZA)\cdot(ZbB+ZB)\cdot e^{j\cdot\frac{2}{3}\cdot\pi} + (ZbB+ZB)\cdot(ZcC+ZC)}{(ZaA+ZA)\cdot(ZcC+ZC) + (ZaA+ZA)\cdot(ZbB+ZB) + (ZbB+ZB)\cdot(ZcC+ZC)} \cdot Vp$$

$VnN = -25.137 - 14.236i$ $\quad |VnN| = 28.888$ $\quad \frac{180}{\pi} \cdot arg(VnN) = -150.475$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA + ZaA}$ $\quad IbB := \frac{Vb - VnN}{ZB + ZbB}$ $\quad IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = 1.385 - 0.687i$ $\quad IbB = -0.778 - 0.343i$ $\quad IcC = -0.606 + 1.03i$

$|IaA| = 1.546$ $\quad |IbB| = 0.851$ $\quad |IcC| = 1.195$

$\frac{180}{\pi} \cdot arg(IaA) = -26.403$ $\quad \frac{180}{\pi} \cdot arg(IbB) = -156.242$ $\quad \frac{180}{\pi} \cdot arg(IcC) = 120.475$

Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA$ $\quad SB := \overline{IbB} \cdot IbB \cdot ZB$ $\quad SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 191.168 + 119.48i$ $\quad SB = 57.87 + 57.87i$ $\quad SC = 142.843 - 35.711i$

$SA + SB + SC = 391.88 + 141.639i$

Ex. 12.4-4

Mathcad analysis

Describe the three-phase source: $Vp := 120$

$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0}$ $\quad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120}$ $\quad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$

Describe the three-phase load: $ZA := 40 + j \cdot 30$ $\quad ZB := ZA$ $\quad ZC := ZB$

Describe the three-phase line: $ZaA := 0$ $\quad ZbB := 0$ $\quad ZcC := 0$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA+ZA)\cdot(ZcC+ZC)\cdot e^{j\cdot\frac{4}{3}\cdot\pi} + (ZaA+ZA)\cdot(ZbB+ZB)\cdot e^{j\cdot\frac{2}{3}\cdot\pi} + (ZbB+ZB)\cdot(ZcC+ZC)}{(ZaA+ZA)\cdot(ZcC+ZC)+(ZaA+ZA)\cdot(ZbB+ZB)+(ZbB+ZB)\cdot(ZcC+ZC)} \cdot Vp$$

$VnN = -4.075 \cdot 10^{-15} + 1.397 \cdot 10^{-14} i$ $\quad |VnN| = 1.455 \cdot 10^{-14}$ $\quad \frac{180}{\pi} \cdot \arg(VnN) = 106.26$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA + ZaA}$ $\quad IbB := \frac{Vb - VnN}{ZB + ZbB}$ $\quad IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = 1.92 - 1.44i$ $\quad IbB = -2.207 - 0.943i$ $\quad IcC = 0.287 + 2.383i$

$|IaA| = 2.4$ $\quad |IbB| = 2.4$ $\quad |IcC| = 2.4$

$\frac{180}{\pi} \cdot \arg(IaA) = -36.87$ $\quad \frac{180}{\pi} \cdot \arg(IbB) = -156.87$ $\quad \frac{180}{\pi} \cdot \arg(IcC) = 83.13$

Calculate the power delivered to the load:

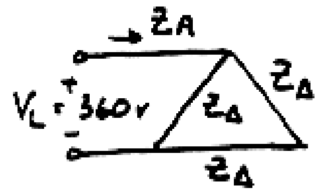
$SA := \overline{IaA} \cdot IaA \cdot ZA$ $\quad SB := \overline{IbB} \cdot IbB \cdot ZB$ $\quad SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 230.4 + 172.8i$ $\quad SB = 230.4 + 172.8i$ $\quad SC = 230.4 + 172.8i$

$SA + SB + SC = 691.2 + 518.4i$

Ex. 12.6-1

balanced



(See Table 12.5-1)

$Z_{\Delta} = 180\angle -45^{\circ}$

phase currents: $I_{AB} = \frac{V_{AB}}{Z} = \frac{360\angle 0^{\circ}}{180\angle -45^{\circ}} = 2\angle 45^{\circ}$ A

$I_{BC} = \frac{V_{BC}}{Z} = \frac{360\angle -120^{\circ}}{18\angle -45^{\circ}} = 2\angle -75^{\circ}$ A

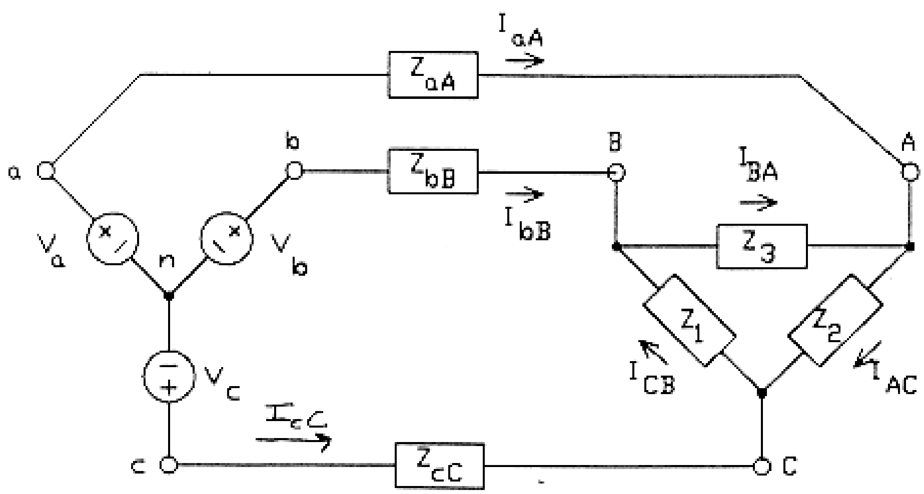
$I_{CA} = \frac{V_{CA}}{Z} = \frac{360\angle 120^{\circ}}{180\angle -45^{\circ}} = 2\angle 165^{\circ}$ A

line currents: $I_{A} = I_{AB} - I_{CA} = 2\angle 45^{\circ} - 2\angle 165^{\circ} = 2\sqrt{3}\angle 15^{\circ}$ A

$I_{B} = 2\sqrt{3}\angle -105^{\circ}$ A

$I_{C} = 2\sqrt{3}\angle 135^{\circ}$ A

Ex. 12.7-1 <u>and</u> **Ex. 12.8-**1 Three-wire Y-to-Delta Circuit with line impedances



<u>Mathcad analysis</u>

Describe the three-phase source: $Vp := 110$

$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0}$ $Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120}$ $Vc := Va.e^{j \cdot \frac{\pi}{180} \cdot 120}$

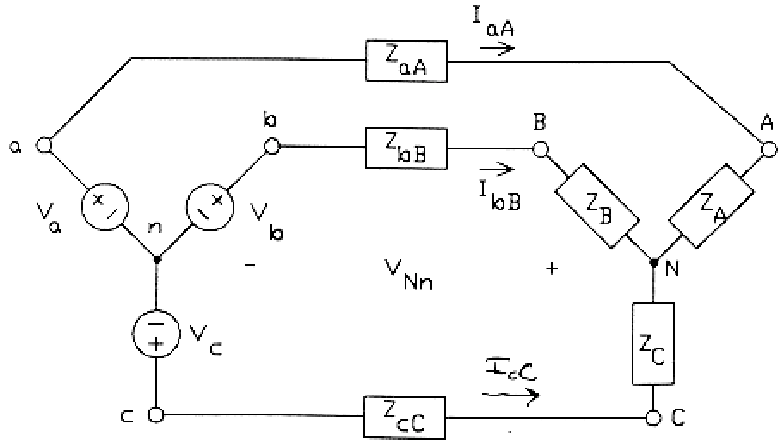
Describe the delta-connected load: $Z1 := 150 + j \cdot 270$ $Z2 := Z1$ $Z3 := Z1$

Convert the delta load to the equivalent Y load:

$ZA := \frac{Z1 \cdot Z3}{Z1 + Z2 + Z3}$ $ZB := \frac{Z2 \cdot Z3}{Z1 + Z2 + Z3}$ $ZC := \frac{Z1 \cdot Z2}{Z1 + Z2 + Z3}$

· $ZA = 50 + 90i$ $\quad$ $ZB := 50 + 90i$ $\quad$ $ZC = 50 + 90i$

Describe the three-phase line: $\quad ZaA := 10 + j \cdot 25 \quad ZbB := ZaA \quad ZcC := ZaA$



Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA+ZA)\cdot(ZcC+ZC)\cdot e^{j\cdot\frac{4}{3}\cdot\pi}+(ZaA+ZA)\cdot(ZbB+ZB)\cdot e^{j\cdot\frac{2}{3}\cdot\pi}+(ZbB+ZB)\cdot(ZcC+ZC)}{(ZaA+ZA)\cdot(ZcC+ZC)+(ZaA+ZA)\cdot(ZbB+ZB)+(ZbB+ZB)\cdot(ZcC+ZC)}\cdot Vp$$

$VnN = -4.235\cdot 10^{-15} + 7.787\cdot|VnN| = 8.864\cdot 10^{-15}$ $\quad$ $\frac{180}{\pi}\cdot \arg(VnN) = 118.541$

Calculate the line currents: $\quad IaA := \frac{Va - VnN}{ZA + ZaA} \quad IbB := \frac{Vb - VnN}{ZB + ZbB} \quad IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = 0.392 - 0.752i$ $\quad$ $IbB = -0.847 + 0.036i$ $\quad$ $IcC = 0.455 + 0.716i$

$|IaA| = 0.848$ $\quad$ $|IbB| = 0.848$ $\quad$ $|IcC| = 0.848$

$\frac{180}{\pi}\cdot \arg(IaA) = -62.447$ $\quad$ $\frac{180}{\pi}\cdot \arg(IbB) = 177.553$ $\quad$ $\frac{180}{\pi}\cdot \arg(IcC) = 57.553$

Calculate the phase voltages of the Y-connected load:

$VAN := IaA \cdot ZA$ $\quad$ $VBN := IbB \cdot ZB$ $\quad$ $VCN := IcC \cdot ZC$

$|VAN| = 87.311$ $\quad$ $|VBN| = 87.311$ $\quad$ $|VCN| = 87.311$

$\frac{180}{\pi}\cdot \arg(VAN) = -1.502$ $\quad$ $\frac{180}{\pi}\cdot \arg(VBN) = -121.502$ $\quad$ $\frac{180}{\pi}\cdot \arg(VCN) = 118.498$

Calculate the line-to-line voltages at the load:

$VAB := VAN - VBN$ $\quad$ $VBC := VBN - VCN$ $\quad$ $VCA := VCN - VAN$

$|VAB| = 151.227$ $\quad$ $|VBC| = 151.227$ $\quad$ $|VCA| = 151.227$

$\frac{180}{\pi}\cdot \arg(VAB) = 28.498$ $\quad$ $\frac{180}{\pi}\cdot \arg(VBC) = -91.502$ $\quad$ $\frac{180}{\pi}\cdot \arg(VCA) = 148.49$

Calculate the phase currents of the Δ-connected load:

$IAB := \frac{VAB}{Z3}$ $\quad IBC := \frac{VBC}{Z1}$ $\quad ICA := \frac{VCA}{Z2}$

$|IAB| = 0.49$ $\quad |IBC| = 0.49$ $\quad |ICA| = 0.49$

$\frac{180}{\pi}\cdot \arg(IAB) = -32.447$ $\quad \frac{180}{\pi}\cdot \arg(IBC) = -152.447$ $\quad \frac{180}{\pi}\cdot \arg(ICA) = 87.55$

Calculate the power delivered to the load:

$SA := \overline{IaA}\cdot IaA\cdot ZA$ $\quad SB := \overline{IbB}\cdot IbB\cdot ZB$ $\quad SC := \overline{IcC}\cdot IcC\cdot ZC$

$SA = 35.958+64.725i$ $\quad SB = 35.958+64.725i$ $\quad SC = 35.958+64.725i$

$SA + SB + SC = 107.875 + 194.175i$

Ex. 12.9-1

$$P_1 = V_{AB}\, I_A \cos(\theta+30^\circ) + V_{CB}\, I_C \cos(\theta-30^\circ) = P_1 + P_2$$

$$\text{pf} = .4 \text{ lagging } \Rightarrow \theta = 61.97^\circ$$

$$\text{So } P_T = 450(24)\left[\cos 91.97^\circ + \cos 31.97^\circ\right] = 8791 \text{ W}$$

$$\therefore P_1 = -371 \text{ W} \quad P_2 = 9162 \text{ W}$$

Ex. 12.9-2 See Fig. 12.9-1

$P_1 = 60\text{kW} \quad P_2 = 40\text{kW}$

a) $P = P_1 + P_2 = 100\text{kW}$

b) use eqn. 12.9-7

$$\tan\theta = \sqrt{3}\,\frac{P_2 - P_1}{P_L + P_2} = \sqrt{3}\,\frac{40-60}{100} = -.346$$

$$\therefore \theta = -19.11^\circ$$

so pf = cos (−19.110) = 0.945 leading

Problems

Section 12-3: Three Phase Voltages

P12.3-1

Given $V_C = 277\angle 45^\circ$

ABC reference $\quad V_A = 277\angle .75^\circ$

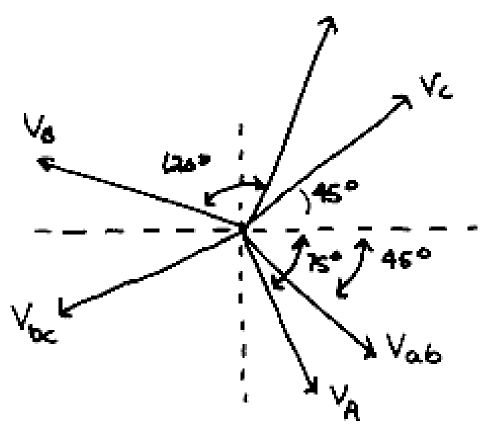
$V_B = 277\angle 45^\circ + 120^\circ = 277\angle 165^\circ$

$V_L = \sqrt{3}(277) = 480 \text{ V}$

$V_{ab} = 480\angle -75^\circ + 30^\circ = 480\angle -45^\circ$

$V_{bc} = 480\angle -165^\circ$

$V_{ca} = 480\angle 75^\circ$



P12.3-2 $V_l = 12470\text{V} \quad V_p = \dfrac{12470}{\sqrt{3}} = 7200\text{V} \quad V_{BA} = 12470\angle -35^\circ$

$$V_b = \frac{12470}{\sqrt{3}}\angle(-35^\circ + 30^\circ) = 7200\angle -5^\circ \text{ V}$$

$$V_a = 7200\angle(-5^\circ + 120^\circ) = 7200\angle -115^\circ$$

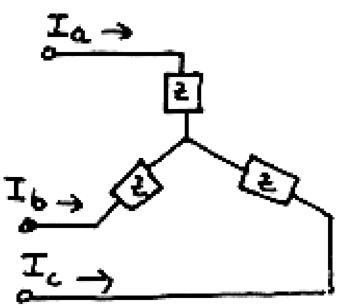
$$V_c = 7200\angle(-5^\circ - 120^\circ) = 7200\angle -125^\circ$$

P12.3-3 $V_{ab} = 1500\angle 30^\circ = V_L$

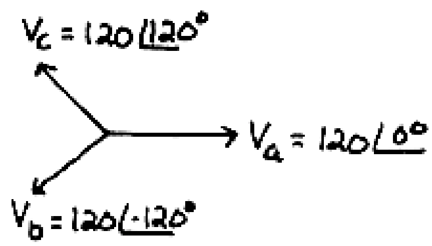
$$V_P = \frac{V_L}{\sqrt{3}}\angle(\theta - 30^\circ) = \frac{1500}{\sqrt{3}}\angle(30^\circ - 30^\circ) = \underline{866\angle 0^\circ \text{ V}}$$

Section 12-4: The Y-to-Y Circuit

P12.4-1 $V_L = 208\text{V}$ balanced Y load , $Z = 12\angle 30^\circ \;\; 12\angle\theta$



$$V_p = \frac{V_L}{\sqrt{3}} = 120$$



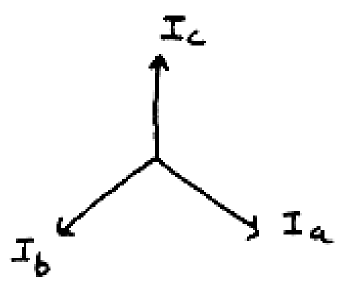
$$I_a = \frac{V_a}{Z} = \frac{120\angle 0^\circ}{12\angle 30^\circ} = 10\angle -30^\circ$$

$$I_b = 10\angle(-30^\circ - 120^\circ) = 10\angle -150^\circ$$

$$I_c = 10\angle 90^\circ$$

$$P = \sqrt{3}V_L I_L \cos\theta$$

$$= \sqrt{3}(120)(10)\cos 30^\circ = \underline{1800 \text{ W}}$$



P12.4-2 $Z_T = Z_L + Z_l = 10 + j\omega 100 + 2 = 12 + j37.7$

$V_p = 120\text{V} \;\; \therefore V_L = 120\sqrt{3}$ so $V_A = 208\angle 0^\circ$

$$V_B = 208\angle -120^\circ$$

$$V_C = 208\angle +120^\circ$$

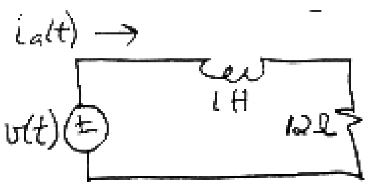
$$I_p = \frac{V_p}{Z_T} = \frac{120\angle 0^\circ}{12 + j37.7} = \frac{120\angle 0^\circ}{40\angle 72^\circ} = 3\angle -72^\circ = I_A$$

$$I_B = 3\angle -192^\circ$$

$$I_C = 3\angle 48^\circ$$

P12.4-3

a) look @ one phase



$v(t) = 10\cos(16t - 120^\circ)$

$= V_p \cos(\omega t + \theta)$

$V_A = V_P\angle\theta_V = 10\angle -120^\circ$

$Z_A = j\omega L + R = 12 + j16 = 20\angle -53^\circ\,\Omega$

$I_A = \dfrac{V_A}{Z_A} = \dfrac{10\angle -120^\circ}{20\angle -53^\circ} = 0.5\angle -173^\circ = I_P\angle\theta_I$

$\underline{i_a(t) = 0.5\cos(16t - 173^\circ)\text{ A}}$

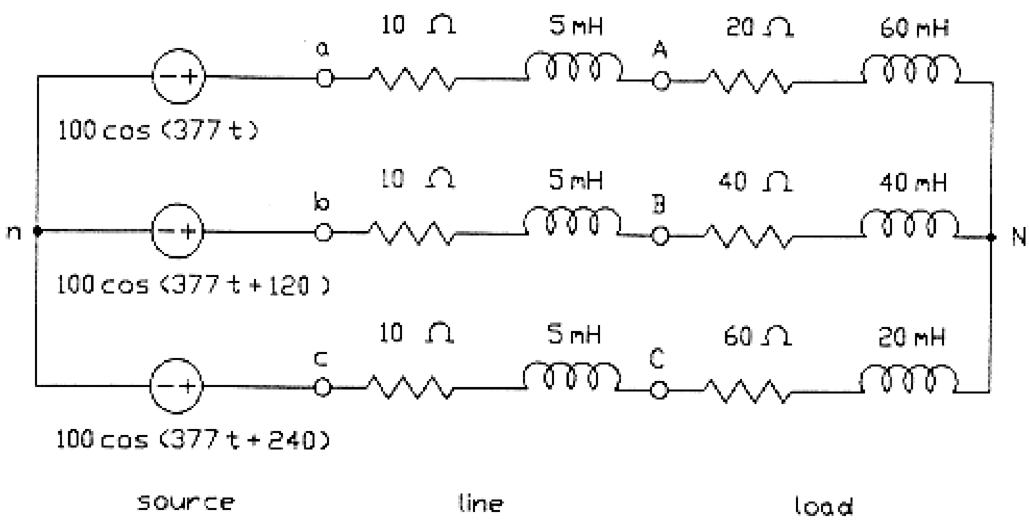
$\text{rms} = \dfrac{|i_a(t)|}{\sqrt{2}} = 0.353$

b) average power $P = 3V_pI_p\cos\theta$

$\theta = \theta_V - \theta_I = -120 - (-173) = 53^\circ$ (also $\theta = \theta_Z$)

$P = 3(10)(0.5)\cos(53^\circ) = \underline{9.0\text{W}}$

P12.4-4



<u>Mathcad analysis</u>

Describe the three-phase source: $Vp := 100 \quad \omega := 377$

$Va := Vp \cdot e^{j\cdot\frac{\pi}{180}\cdot 0}$ $\quad Vb := Va \cdot e^{j\cdot\frac{\pi}{180}\cdot 120}$ $\quad Vc := Va \cdot e^{j\cdot\frac{\pi}{180}\cdot 120}$

Describe the three-phase load: $ZA := 20 + j\cdot\omega\cdot 0.06$ $\quad ZB := 40 + j\cdot\omega\cdot 0.04$ $\quad ZC := 60 + j\cdot\omega\cdot 0.02$

Describe the three-phase line: $ZaA := 10 + j\cdot\omega\cdot 0.005$ $\quad ZbB := 10 + j\cdot\omega\cdot 0.005$ $\quad ZcC := 10 + j\cdot\omega\cdot 0.005$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$\text{VnN} := \frac{(\text{ZaA}+\text{ZA})\cdot(\text{ZcC}+\text{ZC})\cdot e^{j\cdot\frac{4}{3}\cdot\pi}+(\text{ZaA}+\text{ZA})\cdot(\text{ZbB}+\text{ZB})\cdot e^{j\cdot\frac{2}{3}\cdot\pi}+(\text{ZbB}+\text{ZB})\cdot(\text{ZcC}+\text{ZC})}{(\text{ZaA}+\text{ZA})\cdot(\text{ZcC}+\text{ZC})+(\text{ZaA}+\text{ZA})\cdot(\text{ZbB}+\text{ZB})+(\text{ZbB}+\text{ZB})\cdot(\text{ZcC}+\text{ZC})}\cdot\text{Vp}$$

$\text{VnN} = 12.209-24.552i$ $\quad |\text{VnN}| = 27.42$ $\quad \frac{180}{\pi}\cdot\arg(\text{VnN}) = -63.561$

Calculate the line currents: $\quad \text{IaA:} = \frac{\text{Va}-\text{VnN}}{\text{ZA}+\text{ZaA}}$ $\quad \text{IbB:} = \frac{\text{Vb}-\text{VnN}}{\text{ZB}+\text{ZbB}}$ $\quad \text{IcC:} = \frac{\text{Vc}-\text{VnN}}{\text{ZC}+\text{ZcC}}$

$\text{IaA} = 2.156-0.943i$ $\quad \text{IbB} = -0.439+2.372i$ $\quad \text{IcC} = -0.99-0.753i$

$|\text{IaA}| = 2.353$ $\quad |\text{IbB}| = 2.412$ $\quad |\text{IcC}| = 1.244$

$\frac{180}{\pi}\cdot\arg(\text{IaA}) = -23.619$ $\quad \frac{180}{\pi}\cdot\arg(\text{IbB}) = 100.492$ $\quad \frac{180}{\pi}\cdot\arg(\text{IcC}) = -142.741$

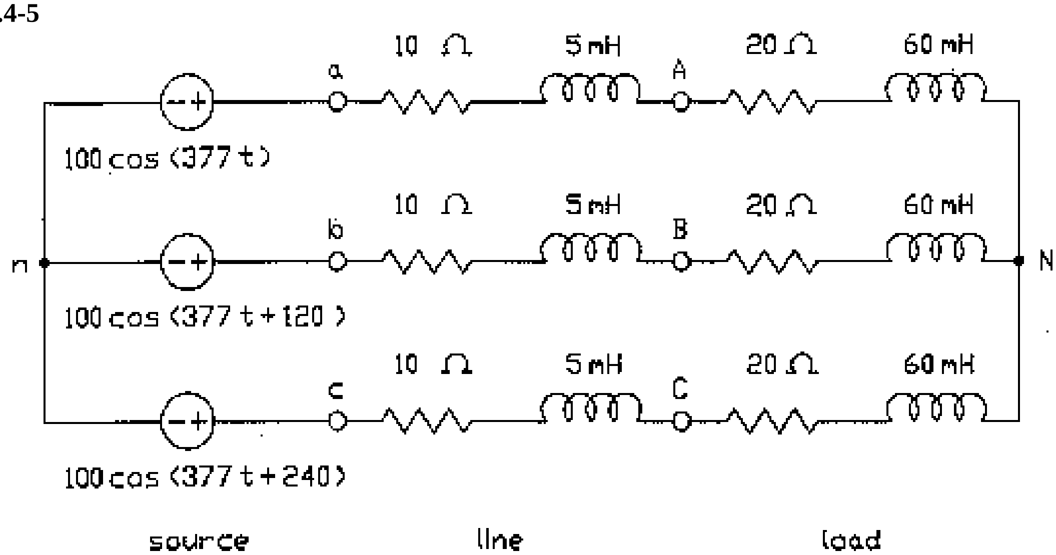
Calculate the power delivered to the load:

$\text{SA:} = \overline{\text{IaA}}\cdot\text{IaA}\cdot\text{ZA}$ $\quad \text{SB:} = \overline{\text{IbB}}\cdot\text{IbB}\cdot\text{ZB}$ $\quad \text{SC:} = \overline{\text{IcC}}\cdot\text{IcC}\cdot\text{ZC}$

$\text{SA} = 110.765+125.275i$ $\quad \text{SB} = 232.804+87.767i$ $\quad \text{SC} = 92.85+11.668i$

$\text{SA}+\text{SB}+\text{SC} = 436.418+224.71i$

P12.4-5



Mathcad analysis

Describe the three-phase source: Vp: = 100 $\quad \omega := 377$

$\text{Va:} = \text{Vp}\cdot e^{j\cdot\frac{\pi}{180}\cdot 0}$ $\quad \text{Vb:} = \text{Va}\cdot e^{j\cdot\frac{\pi}{180}\cdot 120}$ $\quad \text{Vc:} = \text{Va}\cdot e^{j\cdot\frac{\pi}{180}\cdot 120}$

Describe the three-phase load: ZA := 20+j ·ω·0.06 $\quad$ ZB := 20+j ·ω·0.06 $\quad$ ZC := 20+j ·ω·0.06

Describe the three-phase line: ZaA := 10+j ·ω·0.005 $\quad$ ZbB := 10+j ·ω·0.005 $\quad$ ZcC := 10+j ·ω·0.005

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$\text{VnN:} = \frac{(\text{ZaA}+\text{ZA})\cdot(\text{ZcC}+\text{ZC})\cdot e^{j\cdot\frac{4}{3}\pi}+(\text{ZaA}+\text{ZA})\cdot(\text{ZbB}+\text{ZB})\cdot e^{j\cdot\frac{2}{3}\pi}+(\text{ZbB}+\text{ZB})\cdot(\text{ZcC}+\text{ZC})}{(\text{ZaA}+\text{ZA})\cdot(\text{ZcC}+\text{ZC})+(\text{ZaA}+\text{ZA})\cdot(\text{ZbB}+\text{ZB})+(\text{ZbB}+\text{ZB})\cdot(\text{ZcC}+\text{ZC})}\cdot\text{Vp}$$

$\text{VnN} = -6.966\cdot 10^{-15}+8.891\cdot 10^{-15}i$ $\quad |\text{VnN}| = 1.129\cdot 10^{-14}$ $\quad \frac{180}{\pi}\cdot\arg(\text{VnN}) = -128.079$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA + ZaA}$ $IbB := \frac{Vb - VnN}{ZB + ZbB}$ $IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = 1.999 - 1.633i$ $IbB = 0.415 + 2.548i$ $IcC = -2.414 - 0.915i$

$|IaA| = 2.582$ $|IbB| = 2.582$ $|IcC| = 2.582$

$\frac{180}{\pi} \cdot \arg(IaA) = -39.243$ $\frac{180}{\pi} \cdot \arg(IbB) = 80.757$ $\frac{180}{\pi} \cdot \arg(IcC) = -159.243$

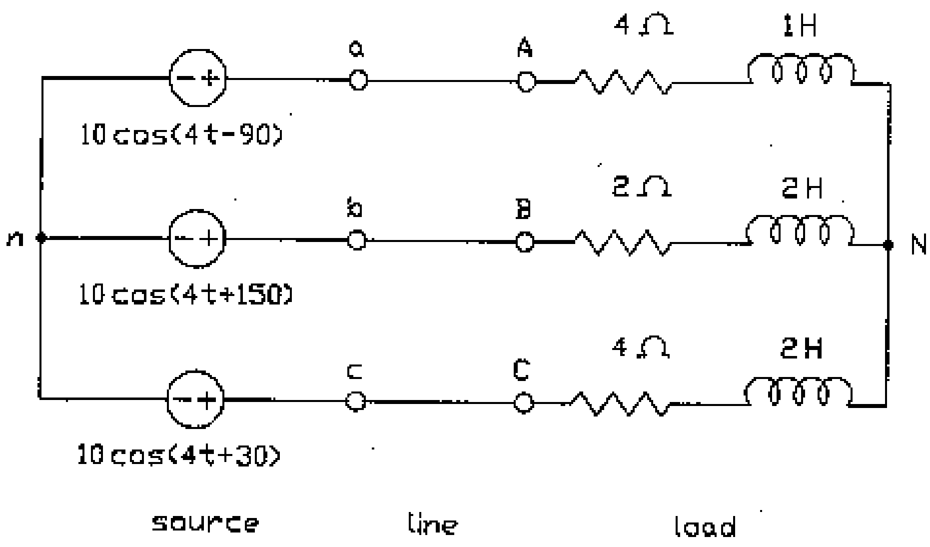
Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA$ $SB := \overline{IbB} \cdot IbB \cdot ZB$ $SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 133.289 + 150.75i$ $SB = 133.289 + 150.75i$ $SC = 133.289 + 150.75i$

$SA + SB + SC = 399.868 + 452.251i$

P12.4-6



<u>Mathcad analysis</u>

Describe the three-phase source: $Vp := 10$ $\omega := 4$

$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 90}$ $Vb := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 150}$ $Vc := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 30}$

Describe the three-phase load: $ZA := 4 + j \cdot \omega \cdot 1$ $ZB := 2 + j \cdot \omega \cdot 2$ $ZC := 4 + j \cdot \omega \cdot 2$

Describe the three-phase line: $ZaA := 0$ $ZbB := 0$ $ZcC := 0$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA + ZA) \cdot (ZcC + ZC) \cdot Vb + (ZaA + ZA) \cdot (ZbB + ZB) \cdot Vc + (ZbB + ZB) \cdot (ZcC + ZC) \cdot Va}{(ZaA + ZA) \cdot (ZcC + ZC) + (ZaA + ZA) \cdot (ZbB + ZB) + (ZbB + ZB) \cdot (ZcC + ZC)}$$

$VnN = 1.528 - 0.863i$ $|VnN| = 1.755$ $\frac{180}{\pi} \cdot \arg(VnN) = -29.466$

Calculate the line currents: $\quad IaA := \frac{Va - VnN}{ZA + ZaA} \qquad IbB := \frac{Vb - VnN}{ZB + ZbB} \qquad IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = -1.333 - 0.951i \qquad IbB = 0.39 + 1.371i \qquad IcC = 0.943 - 0.42i$

$|IaA| = 1.638 \qquad |IbB| = 1.426 \qquad |IcC| = 1.032$

$\frac{180}{\pi} \cdot \arg(IaA) = -144.495 \qquad \frac{180}{\pi} \cdot \arg(IbB) = 74.116 \qquad \frac{180}{\pi} \cdot \arg(IcC) = -24.011$

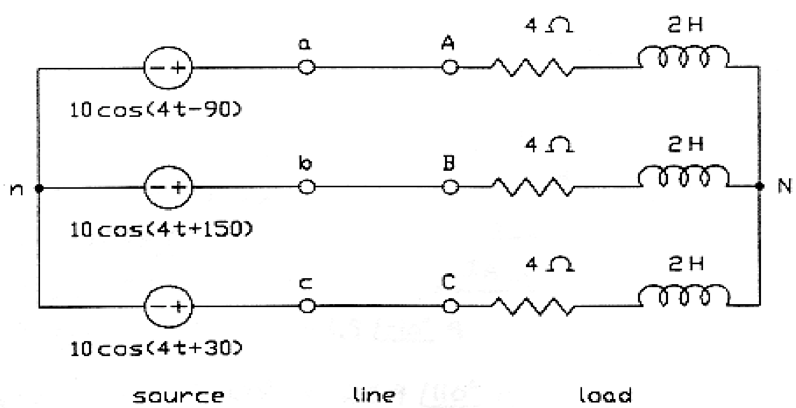
Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA \qquad SB := \overline{IbB} \cdot IbB \cdot ZB \qquad SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 10.727 + 10.727i \qquad SB = 4.064 + 16.257i \qquad SC = 4.262 + 8.525i$

$SA + SB + SC = 19.053 + 35.508i$

P12.4-7



Mathcad analysis

Describe the three-phase source: $Vp := 10 \quad \omega := 4$

$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot -90} \qquad Vb := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 150} \qquad Vc := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 30}$

Describe the three-phase load: $ZA := 4 + j \cdot \omega \cdot 2 \qquad ZB := 4 + j \cdot \omega \cdot 2 \qquad ZC := 4 + j \cdot \omega \cdot 2$

Describe the three-phase line: $ZaA := 0 \qquad ZbB := 0 \qquad ZcC := 0$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA + ZA) \cdot (ZcC + ZC) \cdot Vb + (ZaA + ZA) \cdot (ZbB + ZB) \cdot Vc + (ZbB + ZB) \cdot (ZcC + ZC) \cdot Va}{(ZaA + ZA) \cdot (ZcC + ZC) + (ZaA + ZA) \cdot (ZbB + ZB) + (ZbB + ZB) \cdot (ZcC + ZC)}$$

$VnN = 0 \qquad |VnN| = 0 \qquad \frac{180}{\pi} \cdot \arg(VnN) = -36.87$

Calculate the line currents: $\quad IaA := \frac{Va - VnN}{ZA + ZaA} \qquad IbB := \frac{Vb - VnN}{ZB + ZbB} \qquad IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = -1 - 0.5i$ $\quad$ $IbB = 0.067 + 1.116i$ $\quad$ $IcC = 0.933 - 0.616i$

$|IaA| = 1.118$ $\quad$ $|IbB| = 1.118$ $\quad$ $|IcC| = 1.118$

$\frac{180}{\pi} \cdot arg(IaA) = -153.435$ $\quad$ $\frac{180}{\pi} \cdot arg(IbB) = 86.565$ $\quad$ $\frac{180}{\pi} \cdot arg(IcC) = -33.435$

Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA$ $\quad$ $SB := \overline{IbB} \cdot IbB \cdot ZB$ $\quad$ $SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 5+10i$ $\quad$ $SB = 5+10i$ $\quad$ $SC = 5+10i$

$SA + SB + SC = 15 + 30i$

Section 12-6: The Δ- Connected Source and Load

P12.5-1

Given $I_B = 50\angle -40^\circ \text{ A} = I_L \angle\phi$

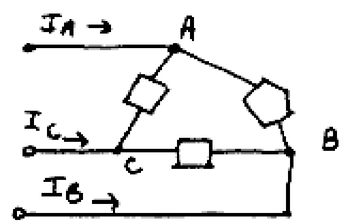
$I_B = \sqrt{3} I \angle(\phi - 30^\circ)$ eqn. 19–15

$\therefore I_P = \frac{I_L}{\sqrt{3}} \angle(\phi + 30^\circ)$

$I_{BC} = \frac{50}{\sqrt{3}} \angle(-40^\circ + 30^\circ) = 28.9\angle -10^\circ \text{ A}$

$I_{AB} = 28.9\angle(-10^\circ + 120^\circ) = 28.9\angle 110^\circ \text{ A}$

$I_{CA} = 28.9\angle(-10^\circ - 120^\circ) = 28.9\angle -130^\circ \text{ A}$



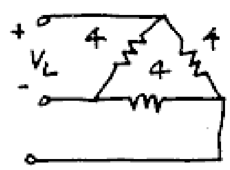
P12.5-2

2 delta loads in parallel so 5||20 = 4Ω

$V_L = V_p = 480\text{V}$

phase current $I_p = \frac{480}{4} = 120 \text{ A}$

line current $I_L = \sqrt{3} I_p = 208 \text{ A}$



Section 12-6: The Y- to Δ- Circuit

P12.6-1

delta load $Z = 12\angle 30^\circ = 12\angle\theta$

$V_L = 208$

$I_P = \frac{208}{|Z|} = \frac{208}{12} = 17.32$

Let $V_{ab} = 208\angle 0^\circ \quad \rightarrow \quad I_{ab} = 17.32\angle -30^\circ$

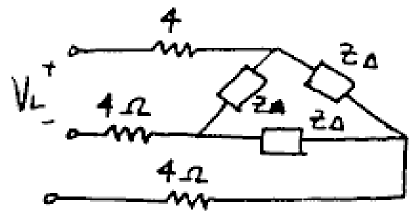
$I_A = \sqrt{3} I_P \angle(\theta - 30^\circ) = \sqrt{3}(17.32)\angle(-30^\circ - 30^\circ) = 30\angle -60^\circ$

then $I_B = 30\angle(-60^\circ - 120^\circ) = 30\angle -180^\circ$

$I_C = 30\angle(-60^\circ + 120^\circ) = 30\angle 60^\circ$

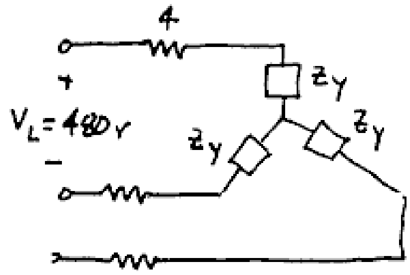
$P = \sqrt{3} V_L I_L \cos\theta = \sqrt{3}(208)(30)\cos 30^\circ = 9360W$

P12.6-2



$V_L = 480V$ Transform $Z_\Delta \Rightarrow Z_Y$

$Z_\Delta = 39\angle -40^\circ \Omega$ $Z_Y = \frac{Z_\Delta}{3} = 13\angle -40^\circ = 9.96 - j8.36$



$Z_T = Z_Y + 4 = 13.96 - j8.36 = 16.3\angle -30.9^\circ$

$\text{then } I_p = I_L = \frac{V_P}{Z_T}$ where $V_P = \frac{V_L}{\sqrt{3}}$

$V_a = \frac{480}{\sqrt{3}}\angle -30^\circ$ $I_A = \frac{\frac{480}{\sqrt{3}}\angle -30^\circ}{16.3\angle -30.9^\circ} = 17\angle 0.9^\circ$

P12.6-3

$Z_Y = 3 + j4$ $V_L = 380 \Rightarrow V_p = V_L / \sqrt{3} = 220V$

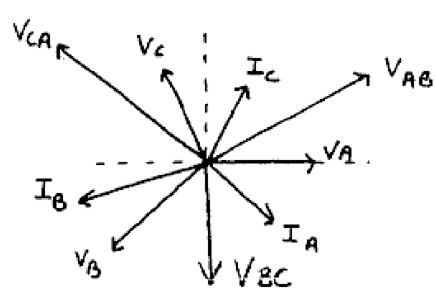
$V_A = 220\angle 0^\circ$ $V_{AB} = 380\angle 30^\circ$

$V_B = 220\angle -120^\circ$ $V_{BC} = 380\angle -90^\circ$

$V_C = 220\angle 120^\circ$ $V_{CA} = 380\angle 150^\circ$

$I_A = \frac{220}{1 + j4} = 44\angle -53.1^\circ$ $I_B = 44\angle -173.1^\circ$ $I_C = 44\angle 66.9^\circ$

and $I_A = I_a$



P12.6-4 Delta load $Z_\Delta = 9 + j12 \qquad V_L = 380V$

$V_{AB} = 380\angle 0^\circ$

$V_L = V_P \qquad\qquad$ So $I_{AB} = \dfrac{380}{9+j12} = 25.33\angle -53.1^\circ$

$$I_L = \sqrt{3}\, I_P \angle(\phi - 30^\circ) = 43.9\angle -83^\circ$$

Section 12-7: Balanced Three-Phase Circuits

P12.7-1 $V_\ell = 25\,kV$

$V_P = \dfrac{25}{\sqrt{3}}\times 10^3\,V \qquad$ phase A: $I_A = \dfrac{25/\sqrt{3}\times 10^3}{150\angle 25^\circ}\angle 0^\circ = 96\angle -25^\circ\ A$

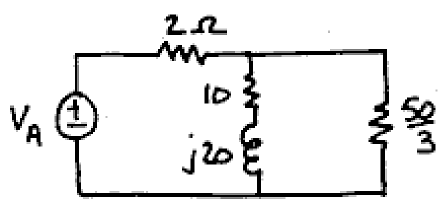
$$P = 3V_A I_A \cos\theta = 3\left(\frac{25}{\sqrt{3}}\times 10^3\right) 96\cos(25^\circ) = \underline{3.77mW}$$

P12.7-2 $V_L = 45kV \qquad Z_y = 10 + j20$

$Z_L = 2\Omega \qquad Z_\Delta = 50\Omega \qquad Z_\Delta \Rightarrow \hat{Z}_y = {}^{50}\!/_{3}$

One per-phase circuit is:

$V_P = \dfrac{45}{\sqrt{3}}\,kV = 26kV$



Use $V_A = 26kV\angle 0^\circ$

$$Z_{eq} = \frac{(10+j20)(50/3)}{10+50/3 + j20} = 10 + j5$$

$$Z_T = 2 + Z_{eq} = 12 + j5 = 13\angle 22.6^\circ$$

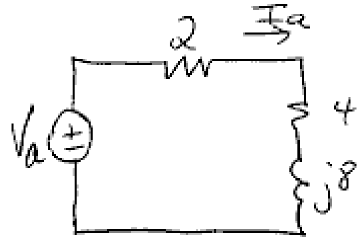
$I_L = \dfrac{26kV}{13} = 2kA$ and $V_L = 45kV$

$P_{\text{loss in lines}} = |I_L|^2 (2\Omega) = 8mW$ for each line

$P_T = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3}(45x10^3)(2x10^3)\cos 22.6^\circ = 144mW = 3P_{phase}$

$\therefore$ % lost $= \dfrac{8\times 3\times 100\%}{144} = \underline{16.6\%}$

P12.7-3



$$V_a = 5\angle 30^\circ \text{ V} \qquad |V_L| = 5\text{V}$$

$$I_a = \frac{V_a}{Z_T} = \frac{5\angle 30^\circ}{6+j8} = 0.5\angle -23^\circ \text{ A} \quad \therefore |I_a| = 0.5\text{A}$$

$$P_T = \sqrt{3}\, V_L I_L \cos\theta$$

$$= \sqrt{3}\,(5)\,(0.5)\cos(-30-23^\circ) = 2.6\text{W}$$

$$P_{line} = |I_L|^2\,(2\Omega) = (0.5)^2\,(2) = 0.5\text{W}$$

$$\therefore P_{load} = P_{total} - P_{line} = 2.6 - 0.5 = \underline{1.9\text{W}}$$

Section 12-8: Power in a Balanced Load

P12.8-1 $P = \sqrt{3}\, V_L I_L \cos\theta \qquad V_L = 208, \qquad I_L = 3$

Need power factor & need angle between I_B and V_B

Assuming a Y load (or transformed to a Y load)

V_B leads V_{CB} by $120^\circ + 30^\circ = 150^\circ$

So $V_B = |V_B|\angle\theta = |V_B|\ \angle 165^\circ$ and $I_B = 3\angle 110^\circ$

$$\text{So}\ \theta = 165 - 110 = 55^\circ$$

Then $P = \sqrt{3}\,(208)\,(3)\cos 55^\circ \ = \underline{620\text{W}}$

(OR)

$V_{CB} = 208\angle 15^\circ = V_L$

$I_B = 3\angle 110^\circ = I_L = I_P$

$V_B = \dfrac{208}{\sqrt{3}}\angle 15 - 30^\circ = 120\angle -15^\circ = V_P$

$P = 3V_P I_P \cos\theta = 3(120)\,(3)\ \cos(125) = \underline{619\text{W}}$

P12.8-2 $V_L = 480 \qquad \eta = .85 \qquad \text{pf} = .8 = \cos\theta \qquad \text{so}\ \ \theta = 36.9^\circ$

$P_{in} = \dfrac{P_{out}}{\eta} = \dfrac{20(745.7)}{.85} = 17.55kW \quad where\ \ 1hp = 745.7kW = \sqrt{3}\, V_L I_L \cos\theta$

Thus $I_L = \dfrac{17.55\times 10^3}{\sqrt{3}(480)(.8)} = 26.4\ A$

Assume Y connected load $I_A = 26.4\angle -36.9^\circ$ if $V_A = 480\angle 0^\circ$

P12.8-3

$V_L = 220V \quad P_T = 1500W \quad pf = .8 \text{ lagging}$

a) Δ connected : $P_T = \sqrt{3}\, V_L I_L pf \quad \Rightarrow I_L = \dfrac{1500}{\sqrt{3}(220)(.8)} = 4.92$

so $|Z_{ph}| = \dfrac{220}{2.84} = 77.44 \qquad I_P = \dfrac{I_L}{\sqrt{3}} = 2.84$

$\underline{Z_\Delta = 77.44\ \angle\cos^{-1}(.8) = 77.44\ \angle 36.9^\circ\ \Omega}$

b) Y connected : $I_L = 4.92$ as above $I_L = I_P$

$$V_p = V_L/\sqrt{3} = 127\text{ V}$$

$\therefore |Z_{ph}| = \dfrac{127}{4.92} = 25.8$

So $\underline{Z_y = 25.8\ \angle 36.9^\circ\ \Omega}$

P12.8-4

Parallel Δ loads

$Z_\Delta = \dfrac{Z_1 Z_2}{Z_1+Z_2} = \dfrac{(40\angle 30^\circ)\,(50\angle -60^\circ)}{40\angle 30^\circ + 50\angle -60^\circ} = 31.2\ \angle -8.7^\circ\ \Omega$

$V_L = V_P,\ I_P = \dfrac{V_P}{|Z_\Delta|} = \dfrac{600}{31.2} = 19.2\text{ A}, \quad I_L = \sqrt{3}\, I_P = 33.3\text{ A}$

So $P = \sqrt{3}\, V_L I_L pf = \sqrt{3}\,(600)\,(33.3)\cos(-8.7^\circ) = \underline{34.2\text{ kW}}$

P12.8-5

$\tilde{S}_1 = 27.3 + j\,27.85$

$\underline{\tilde{S}_2 = 15.0 - j\,70.57}$

$\tilde{S}_{3\phi} = 42.3 - j\,42.72\text{ kVA} \Rightarrow \tilde{S}_\phi = 14.1 - j14.24\text{ kVA} = \tilde{S}_{3\phi}/3$

$|\tilde{V}_{LP}| = 208/\sqrt{3} = 120\text{ V} = |V_P|$

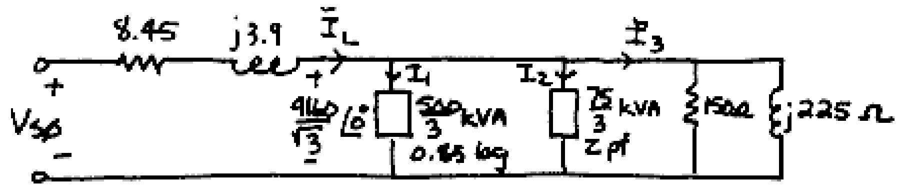
$\tilde{I}_L = \dfrac{(14100 + j\,14240)}{120} = 117.5 + j\,118.7\text{ A} = 167\ \angle 45.3^\circ\text{ A}$

Thus $\tilde{V}_{S\phi} = 120\angle 0^\circ + (0.038 + j\,0.072)(117.5 + 118.7) = 115.9 + j\,12.9$

$= 116.6\ \angle 6.4^\circ$ V (phase-neutral)

$\therefore |\tilde{V}_{SL}| = \sqrt{3}\,(116.6) = \underline{202.0\text{ V}}$

P12.8-6



$$\tilde{I}_1 = \frac{500/3\angle -\cos^{-1} .85}{2.402} = 58.98 - j\,36.56\text{ A},\ \tilde{I}_2 = \frac{25\angle 90^\circ}{2.402} = j\,10.4\text{ A}$$

$$\tilde{I}_3 = \frac{2402}{150} + \frac{2402}{j\,225} = 16 - j\,10.7\text{ A}$$

$$\tilde{I}_L = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3 = 75 - j\,36.8\text{ A}$$

$$\tilde{V}_{S\phi} = 2402\angle 0^\circ + (8.45 + j\,3.9)\,(75 - j\,36.8) = 3179\,\angle -0.3^\circ$$

$$\therefore\ |V_{SL}| = \sqrt{3}\,(3179) = \underline{5506\text{ V}}$$

P12.8-7

a) $\tilde{S}_1 = 1.125 + j\,0.9922 \qquad V_L = \frac{4160}{\sqrt{3}} \qquad \phi$ refers to per - phase

$\underline{\tilde{S}_2 = 2.000 + j\,1.500}$

$$\tilde{S}_L = 3.125 + j\,2.4922 \Rightarrow \tilde{S}_{L\phi} = 1.042 + j\,0.831\text{ MVA/phase}$$

$$\tilde{I}_L = \frac{(1.042 - j\,0.831)\times 10^6}{2402} = 4.337 - j\,345.9\text{ A} = 554.7\,\angle -38.6^\circ\text{ A}$$

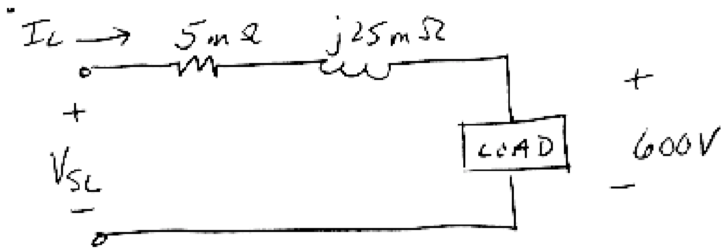
$$\tilde{V}_{S\phi} = 2402\,\angle 0^\circ + (0.4 + j\,0.8)\,(433.7 - j\,345.9) = 2859.6\,\angle 4.2^\circ\text{ V}$$

$$\therefore |V_{SL}| = \sqrt{3}\,(2859.6) = \underline{4953\text{ V}}$$

b) $P_S = \sqrt{3}\ (4953)\,(554.7)\cos\ (4.2^\circ + 38.6^\circ) = \underline{3.49\text{ MW}}$

c) efficiency $= \eta = \frac{3.125}{3.49}\times 100\% = \underline{89.5\%}$

P12.8–8



$$\theta = \cos^{-1}(0.8) = 37^\circ$$

$$P_{LOAD} = \sqrt{3}\, V_L I_L \text{ pf}$$

$$|I_L| = \frac{P_{LOAD}}{\sqrt{3}\, V_L \text{pf}} = \frac{480 \text{ k}}{\sqrt{3}\,(600)\,(0.8)} = 577 \angle{-37^\circ} = 461.6 - j\,346.2$$

$$V_{SL} = 600 \angle 0^\circ + (5\text{m} + j\,25\text{m})\,(461.6 - j\,346.2)$$

$$= \underline{610.1 + j\,9.8 = 611 \angle 0.92^\circ \text{ V}}$$

$$\text{pf} = \cos(\theta_V - \theta_I) = \cos(0.92 - (-37)) = \underline{0.789}$$

Section 12-9: Two-Wattmeter Power Measurement

P12.9-1 Assume motor is a balanced load $V_L = 440$, $I_L = 52.5$

$$P_{in} = \frac{P_{out}}{\eta} \quad \eta = .746$$

$$P_T = P_{in} = \frac{(20\text{hp})\,(746 \text{ W/hp})}{.746} = 20 \text{ kW}$$

also $P_T = \sqrt{3}\, V_L I_L \cos\theta$ then $\cos\theta = \dfrac{20 \times 10^3}{\sqrt{3}\,(440)\,(52.5)} = 0.50$

so $\theta = +60^\circ$

Use eqn. 12.9-5

$$P_T = V_L I_L \left[\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ)\right]$$

Then $W_A = 0$

$W_C = 20$ kW

P12.9-2 $V_L = 4000$ $\quad Z_\Delta = 40 + j\,30 = 50 \angle 36.9^\circ$

$$V_P = V_L = 4000$$

$$I_P = \frac{V_P}{50} = \frac{4000}{50} = 80\text{A} \qquad I_L = \sqrt{3}\, I_P = 138.6 \text{ A}$$

$$\text{pf} = \cos\theta = \cos(36.9^\circ) = .80$$

$$P_1 = V_L I_L \cos(\theta + 30^\circ) = 4000\,(138.6) \cos 66.9^\circ = 217.5 \text{ kW}$$

$$P_2 = V_L I_L \cos(\theta - 30^\circ) = 4000\,(138.6) \cos 6.9^\circ = 550.4 \text{ kW}$$

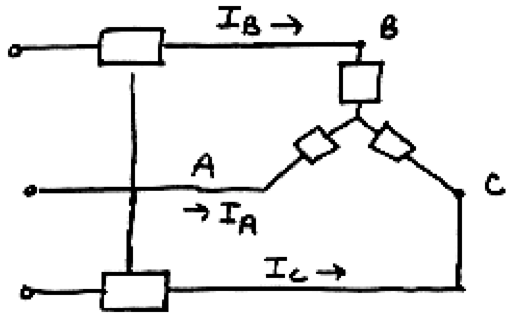
$$P_T = P_1 + P_2 = 767.9 \text{ kW}$$

Check : $P_T = \sqrt{3}\, I_L V_L \cos\theta = \sqrt{3}\,(4000)\,(138.6) \cos 36.9^\circ$

$= 768$ kW which checks

P12.9–3

$V_L = 200V,\ \text{Y load} \Rightarrow z = 70.7\angle 45^\circ$



$$V_p = \frac{200}{\sqrt{3}} = 115.47\ V$$

$V_A = 115.47\angle 0^\circ \qquad V_B = 115.47\angle -120^\circ$

$$I_A = \frac{V_A}{Z} = \frac{115.47\angle 0^\circ}{70.7\angle 45^\circ} = 1.633\angle -45^\circ$$

$I_B = 1.633\ \angle -165^\circ \qquad I_C = 1.633\ \angle 75^\circ$

$P_T = \sqrt{3}\ V_L I_L \cos\theta = \sqrt{3}\ (200)\ (1.633)\ \cos 45^\circ = 400\ W$

$P_B = V_{AC}\ I_A \cos\theta_1 = 200\ (1.633)\ \cos(45^\circ - 30^\circ) = 315.47\ W$

$P_C = V_{BC}\ I_B \cos\theta_2 = 200\ (1.633)\ \cos(45^\circ + 30^\circ) = 84.53\ W$

P12.9-4

$V_L = 208V$

$Z_Y = 10\angle -30^\circ \quad Z_\Delta = 15\angle 30^\circ$

Convert Z_Δ to $Z_{\hat{Y}} \rightarrow Z_{\hat{Y}} = \frac{Z_\Delta}{3} = 5\angle 30^\circ$

$$\text{then } Z_{eq} = \frac{(10\angle -30^\circ)\ (5\angle 30^\circ)}{10\angle -30^\circ + 5\angle 30^\circ} = \frac{50\angle 0^\circ}{13.228\ \angle -10.9^\circ} = 3.78\angle 10.9^\circ$$

$$V_p = \frac{208}{\sqrt{3}} = 120V$$

$$V_A = 120\angle 0 \quad \Rightarrow I_A = \frac{120\angle 0^\circ}{3.78\ \angle 10.9^\circ} = 31.75\ \angle -10.9^\circ$$

$I_B = 31.75\angle -130.9^\circ$

$I_C = 31.75\angle 109.1^\circ$

$P_T = \sqrt{3} V_L I_L\ \cos\theta = \sqrt{3}(208)(31.75)\cos(10.9) = 11.23\ \text{kW}$

$W_1 = V_L\ I_L\ \cos(\theta - 30^\circ) = 6.24\ \text{kW}$

$W_2 = V_L\ I_L\ \cos(\theta + 30^\circ) = 4.99\ \text{kW}$

P12.9-5 $W_1 = W_A$ Let $W_1 = 920$ $W_2 = 460$

$$P_T = W_1 + W_2 = 920 + 460 = 1380 \text{ W}$$

$$P_T = \sqrt{3} V_L \, I_L \, \cos\theta \;\text{ and } V_L = 120 \text{ V}$$

$$\tan\theta = \sqrt{3}\,\frac{P_2 - P_1}{P_2 + P_1} = \sqrt{3}\,\frac{(-460)}{1380} = -0.577 \qquad \Rightarrow \theta = -30^\circ$$

$$P_T = \sqrt{3}\, V_L \, I_L \, \cos\theta \;\text{ so } I_L = \frac{P_T}{\sqrt{3} V_L \cos\theta} = 7.67 \text{ A}$$

$$I_P = \frac{I_L}{\sqrt{3}} = 4.43 \;\therefore |Z_\Delta| = \frac{120}{4.43} = 27.1\,\Omega \;\text{ or } \underline{Z_\Delta = 27.1 \angle -30^\circ}$$

P12.9-6

$Z = 0.868 + j4.924 = 5\angle 80^\circ \quad \theta = 80^\circ$

$$V_L = 380\text{V},\; V_P = \frac{380}{\sqrt{3}} = 219.4\text{V}$$

$$I_L = I_P \text{ and } I_P = \frac{V_P}{Z} = 43.9 \text{ A}$$

$$W_1 = (380)(43.9)\cos\left(\theta - 30^\circ\right) = 10{,}723$$

$$W_2 = (380)(43.9)\cos\left(\theta + 30^\circ\right) = -5706$$

$\therefore\; P_T = 5017$ W

Verification Problems

VP 12-1 Y-Y connection

$$|V_P| = \frac{416}{\sqrt{3}} = 240 \text{ V} = |V_A|$$

$$Z = 10 + j4 = 10.77 \angle 21.8^\circ \;\Omega$$

$$|I_A| = \frac{|V_A|}{Z} = \frac{240}{10.77} = \underline{22.28 \text{ A}}$$

$$P = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3}\,(416)\,(22.28)\cos(-21.8^\circ) = \underline{14.9 \text{ kW}}$$

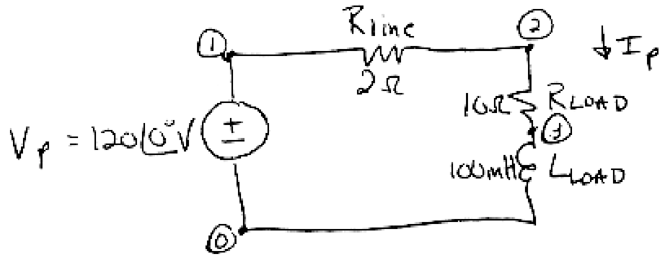
VP 12-2 Δ connection $V_L = V_P = 240$ V

$$Z = 40 + j30 = 50 \angle 36.9^\circ \;\Omega$$

$$I_P = \frac{V_P}{Z_\Delta} = \frac{240}{50\angle 36.9^\circ} = \underline{4.8 \angle -36.9^\circ \text{ A}}$$

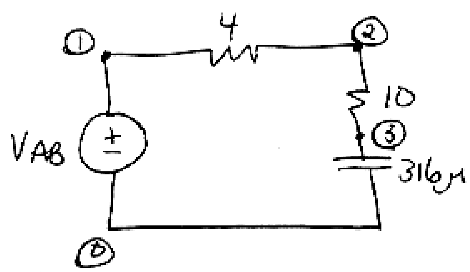
PSpice Problems

SP 12-1




```
Input:
      Vs    1    0    ac    120   0
      Rline    1    2    2
      Rload    2    3    10
      Lload    3    0    100m
      .ac lin 1 60 60
      .print ac Im(Rline) Ip(Rline)
      .end
Output:
       FREQ         IM(Rline)    IP(Rline)
       6.000E+01    3.033E+00    -7.234E+01
```

SP 12-2

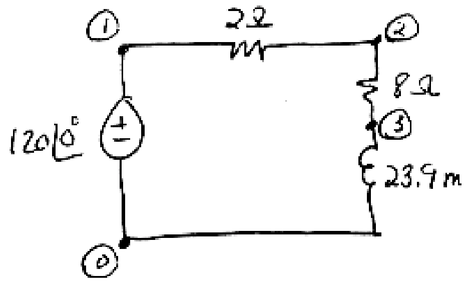


$$f = 60\text{Hz}$$

$$-j8.4 = \frac{-j}{\omega C} \Rightarrow C = 316\mu\text{F}$$

```
INPUT:
Vs      1   0   ac   277  -30
Rline   1   2   4
Rload   2   3   10
Cload   3   0   316u
.ac lin 1 60 60
.print ac Im(Rline) Ip(Rline)
.end
OUTPUT:
FREQ         IM(Rline)    IP(Rline)
6.000E+01    1.697E+01    9.464E-01
```

SP 12-3



$f = 60\text{ Hz},\ \omega = 377\frac{\text{rad}}{\text{sec}}$

$j9 = j\omega L$

$L = 23.9\text{ mH}$

```
Input:
      Vs    1   0   ac   120   0
      Rline   1   2  2
      Rload   2   3  8
      Lload   3   0  24m
      .ac lin 1 60 60
       .print ac Im(Rline) Ip(Rline)
       .end
Output:
       FREQ         IM(Rline)    IP(Rline)
       6.000E+01   8.898E+00   -4.214E+01
```

Design Problems

DP 12-1

$P_{\text{per phase}} = 400,\ \text{pf} = \cos\theta = \cos 20^\circ\ \ \theta = 20^\circ$

$400 = \frac{208}{\sqrt{3}} I_L \cos 20^\circ \quad \Rightarrow \qquad I_L = 3.5\text{ A}$

$\text{each } \Delta \text{ phase}: I_\Delta = \frac{I_L}{\sqrt{3}} = 2.04\text{ A}$

$$|Z| = \frac{V_L}{I_\Delta} = \frac{208}{2.04} = 101.8\ \Omega$$

so $\underline{Z = 101.8\angle 20^\circ\ \Omega}$

DP 12-2

$V_L = 240\text{V}$

$P_A = V_L I_L \cos(30^\circ + \theta) = 1440\text{ W}$

$P_C = V_L I_L \cos(30^\circ - \theta) = 0\text{ W}$

now $\cos\phi = 0$, when $\phi = 30 - \theta = 90^\circ$ or $\theta = -60^\circ$

then $1440 = 240\,(I_L)\cos(-30^\circ) \Rightarrow I_L = 6.93\text{ A}$

$I_L = I_P = \frac{V_P}{Z}$

So $|Z| = \frac{V_P}{I_P} = \frac{240/\sqrt{3}}{6.93} = 20\Omega$

Thus $\underline{Z = 20\angle -60^\circ\ \Omega}$

DP 12-3

$V_L = 480V$

$$P_{in} = \frac{P_0}{\eta} = \frac{100\ (746)}{.8} = 93.2\ \text{kW}$$

$$\text{correction C} = P_{in}\frac{\left[\tan\ (\cos^{-1}.75) - \tan\ (\cos^{-1}.9)\right]}{3\ (377)\ (480)^2} = \underline{160\ \mu F}$$

where pfc = .9 and pf = .75

DP 12-4

$V_L = 4\ \text{kV} \qquad Z_L = 4/3\ \Omega$

If $n_2 = 25 \rightarrow 25:1$ step down at load to 4 kV

then $V_2 = 100$ kV

$$\Rightarrow\ I_L \text{ at load} = \frac{4\times 10^3}{Z_L} = 3\ \text{kA}$$

$$\text{The line current in } 2.5\ \Omega \text{ is } |\,I\,| = \frac{3\ \text{kA}}{25} = 120\ \text{A}$$

Thus $V_1 = (R + jX)\, I + V_2$

$$= (2.5 + j40)\ (120) + 100\times 10^3 = 100.4\ \angle 2.7^\circ\ \text{kV}$$

$$\text{Step need : } n_1 = \frac{100.4\ \text{kV}}{20\ \text{kV}} = 5.02 \cong 5$$

$P_{loss} = |\,I\,|^2\ R = |\,120\,|^2\ (2.5) = 36\ \text{kW},\ \ P = (4\times 10^3)\ (3\times 10^3) = 12\ \text{MW}$

$$\eta = \frac{12 - .036}{12}\times 100\% = 99.7\ \%\ \text{to load}$$