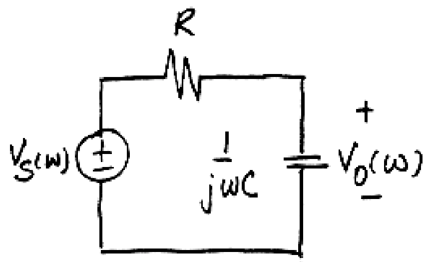
Chapter 13: Frequency Response

Ex. 13.3-1



$$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{1}{1 + j\omega CR}$$

$$\text{gain} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

$$\text{phase shift} = -\tan^{-1}\omega CR$$

When $R = 10^4$, $\omega = 100$, and $C = 10^{-6}$, then

$$\text{gain} = \frac{1}{\sqrt{2}} = 0.707, \text{ phase shift} = -45^o$$

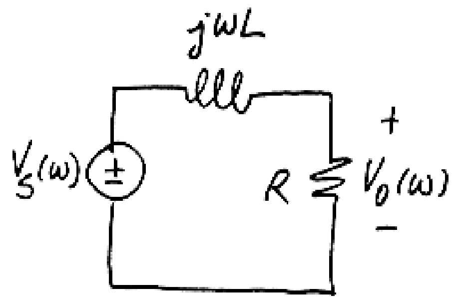
Ex. 13.3-2

$$H(\omega) = \frac{V_0(\omega)}{V_s(\omega)} = \frac{R}{R + j\omega L}$$

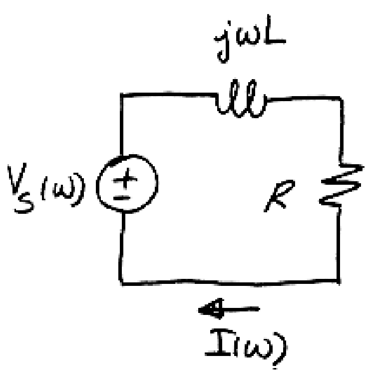
$$\text{gain} = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

$$0.6 = \frac{30}{\sqrt{30^2 + (2\omega)^2}}$$

$$\omega = \frac{\sqrt{\left(\frac{30}{.6}\right)^2 - 30^2}}{2} = 20 \text{ rad/s}$$



Ex. 13.3-3



$$H(\omega) = \frac{I(\omega)}{V_s(\omega)} = \frac{1}{R + j\omega L}$$

$$\text{gain} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}$$

$$\text{phase shift} = -\tan^{-1}\frac{\omega L}{R}$$

When $R = 30$, $L = 2$, and $\omega = 20$, then

$$\text{gain} = \frac{1}{\sqrt{30^2 + 40^2}} = 0.02\ \frac{A}{V}$$

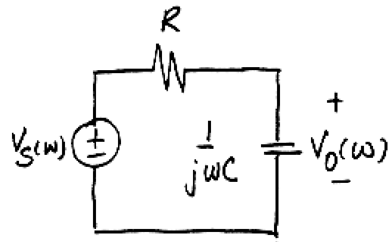
$$\text{phase shift} = -\tan^{-1}\left(\frac{40}{30}\right) = -53.1^\circ$$

Ex. 13.3-4

$$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{1}{1 + j\omega CR}$$

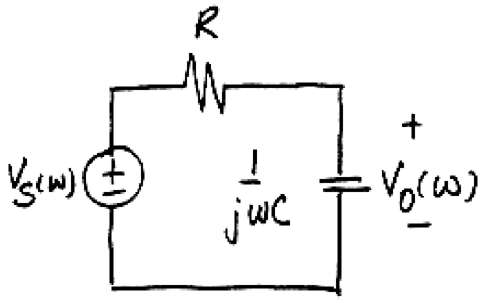
$$\text{gain} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

$$\text{phase shift} = -\tan^{-1}\omega CR$$



$$-45^\circ = -\tan^{-1}(20\cdot 10^{-6}\cdot R) \quad \Rightarrow \quad R = \frac{\tan(45^\circ)}{20\cdot 10^{-6}} = 50\cdot 10^3\ \Omega$$

Ex. 13.3-5



$$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{1}{1 + j\omega CR}$$

$$\text{gain} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

ω, C, and R are all positive, or at least nonnegative, so gain ≤ 1. These specifications cannot be met.

Ex. 13.4-1

(a) $\text{dB} = 20\log(.5) = \underline{-6.02\text{ dB}}$

(b) $\text{dB} = 20\log 2 = \underline{6.02\text{ dB}}$

Ex. 13.4-2

$$20\log H = 20\log\left(\frac{1}{\omega^2}\right) = 20\log(\omega)^{-2} = -40\log\omega$$

$$\text{slope} = 20\log H(\omega_2) - 20\log H(\omega_1) = -40\log\omega_2 + 40\log\omega_1 = -40\log\left(\frac{\omega_2}{\omega_1}\right)$$

let $\omega_2 = 10\,\omega_1$ to consider 1 decade, then

$$\underline{\text{slope} = -40\log 10 = -40\ \text{dB}/\text{decade}}$$

Ex. 13.4-3

$\underline{\textit{When } \omega C >> B,}$ $H(\omega) \simeq \dfrac{j\omega A}{j\omega C} = \dfrac{A}{C}$

(d) $|H(\omega)|$, dB $= 20\log_{10}|H(\omega)| = \underline{20\log_{10}\left(\frac{A}{C}\right)}$

(b) $|H(\omega)|$ does not depend on ω so $\underline{\text{slope} = 0}$

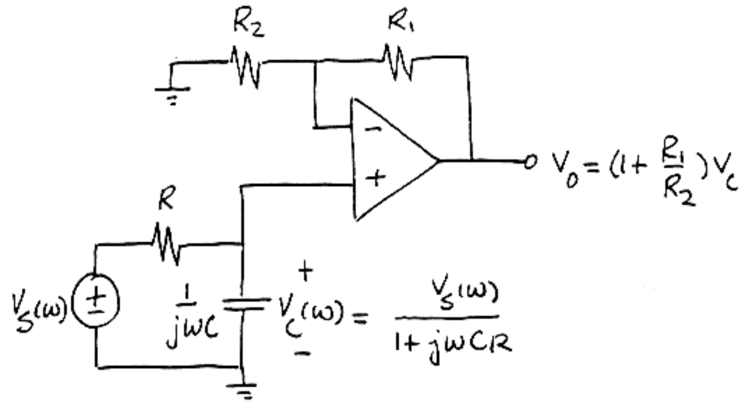
$\underline{\textit{When } \omega C << B,}$ $H(\omega) \simeq \dfrac{j\omega A}{B} = j\omega\left(\dfrac{A}{B}\right)$

$|H(\omega)|$, dB $= 20\log_{10}|H(\omega)| = 20\log_{10}\omega + 20\log_{10}\left(\frac{A}{B}\right)$

(c) The slope is the coefficient of $20\log_{10}\omega$, that is, $\underline{\text{slope} = 20}$ dB/decade

(a) The break frequency is the frquency at which $\omega C = B$, that is, $\underline{\omega = \frac{B}{C}}$

Ex. 13.4-4

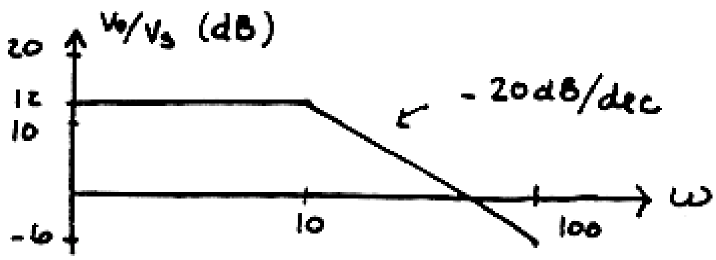


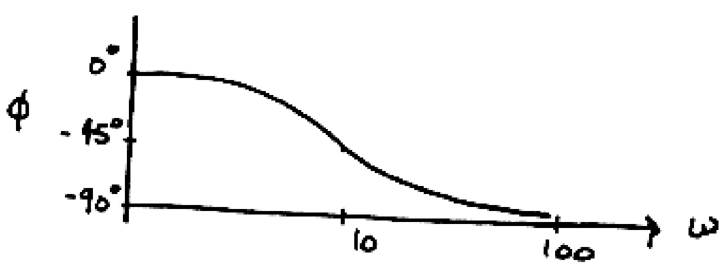
$$V_o(\omega) = \left(1+\frac{R_1}{R_2}\right)V_c(\omega) = \left(1+\frac{R_1}{R_2}\right)\frac{1}{1 + j\omega CR}V_s(\omega)$$

$$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \left(1+\frac{R_1}{R_2}\right)\frac{1}{1+ j\omega CR}$$

When $RC = 0.1$ and $R_1 / R_2 = 3$, then

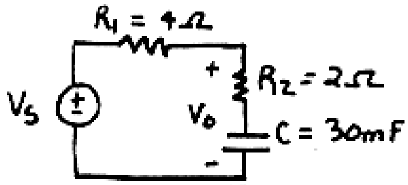
$$H(\omega) = \frac{4}{1 + j\dfrac{\omega}{10}}$$





Ex. 13.4-5

a)

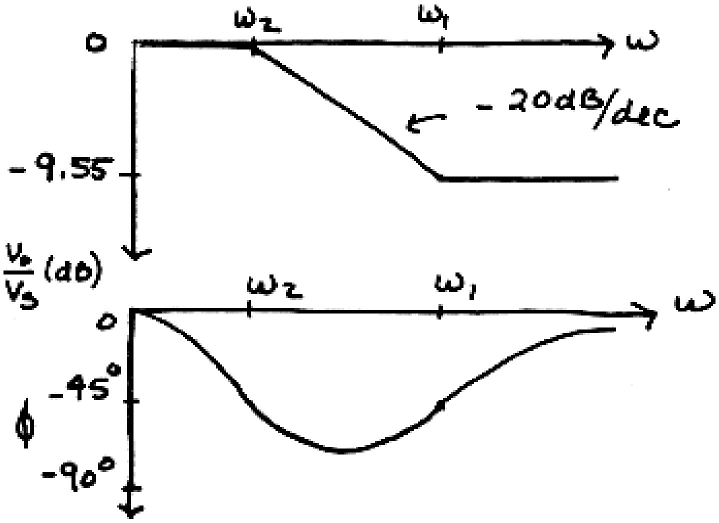


$$Z_o = R_2 + \frac{1}{j\omega C}$$

$$\frac{V_o}{V_s} = \frac{Z_o}{R_1 + Z_o} = \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{1 + \frac{j\omega}{\omega_1}}{1 + \frac{j\omega}{\omega_2}}$$

where $\omega_1 = 1/R_2C = 16.7 \text{ rad/s}$

$$\omega_2 = \frac{1}{(R_1 + R_2)C} = 5.56 \text{ rad/s}$$

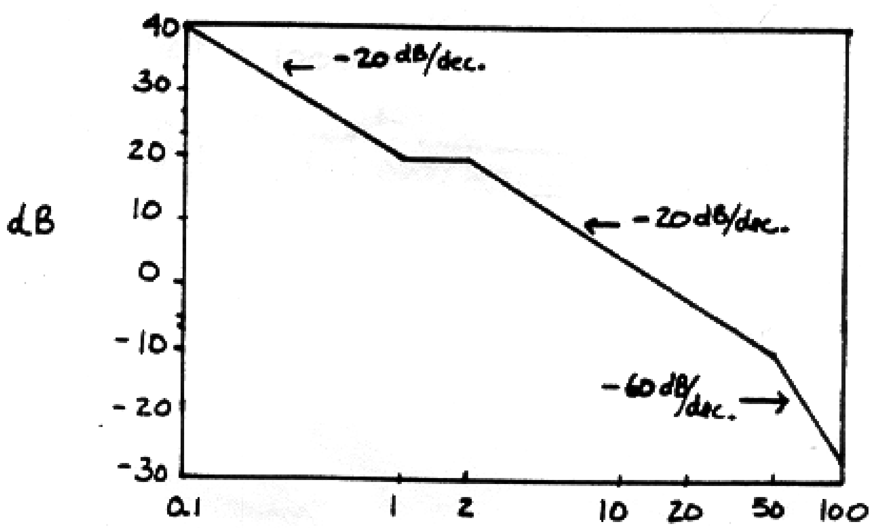


$v_s = 10 \cos 20t$ or $V_s = 10\angle 0^\circ$

$$\therefore \frac{V_o}{V_s} = \frac{1 + j\left(\frac{20}{16.7}\right)}{1 + j\left(\frac{20}{5.56}\right)} = \frac{1 + j\,1.20}{1 + j\,3.60} = 0.417\angle -24.3^\circ$$

b) So $v_o = 4.17\angle -24.3^\circ \Rightarrow$ $\underline{v_0(t) = 4.17 \cos(20t - 24.30) \text{ V}}$

Ex. 13.4-6



Ex. 13.5-1

(a) $Q = \omega_o RC = R\sqrt{C/L} = 8000\sqrt{\dfrac{2.5\times 10^{-7}}{40\times 10^{-3}}} = \underline{20}$

(b) $BW = \dfrac{\omega_o}{Q} = \dfrac{1}{Q\sqrt{LC}} = \dfrac{1}{20\sqrt{(40\times10^{-3})\,(2.5\times10^{-7})}} = \underline{500\ \text{rad/s}}$

Ex. 13.5-2

$Q = \omega_0/BW = 10^7/2\times 10^5 = \underline{50}$

$\text{Now } \omega_o = \dfrac{1}{\sqrt{LC}} \Rightarrow L = \dfrac{1}{\omega_o^{\,2}C} = \dfrac{1}{(10^7)^2\ (10\times 10^{-12})} = \underline{1\text{ mH}}$

Ex. 13.5-3

$\omega_o = 1/\sqrt{LC} = \dfrac{1}{\left[(10^{-3})(10^{-5})\right]^{1/2}} = 10^4\ \text{rad/s}$

$Q = \omega_0/BW = 10^4/2\pi\,(15.9) = \underline{100}$

$R = \dfrac{\omega_o L}{Q} = \dfrac{(10^4)(10^{-3})}{100} = \underline{0.1\Omega}$

Ex. 13.5-4

(a) $\omega_o = 1/\sqrt{LC} \Rightarrow C = 1/(10^6)^2\ (0.01) = \underline{100\text{pF}}$

$Q = \omega_o/BW = 1/\omega_0 RC \Rightarrow R = \dfrac{BW}{\omega_o^{\,2}\ C} = \dfrac{10^3}{(10^6)^2(10^{-10})} = \underline{10\Omega}$

(b) $Q = \omega_o/BW = 10^6/10^3 = 1000$

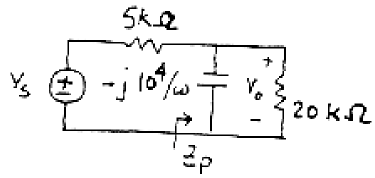
$H = \dfrac{1}{1+jQ\left(\dfrac{\omega}{\omega_o}-\dfrac{\omega_o}{\omega}\right)} = \dfrac{1}{1+j1000\left[\dfrac{1.05\times10^6}{10^6}-\dfrac{10^6}{1.05\times10^4}\right]}$

$\underline{H = \dfrac{1}{1+j\,97.6}}$

Problems

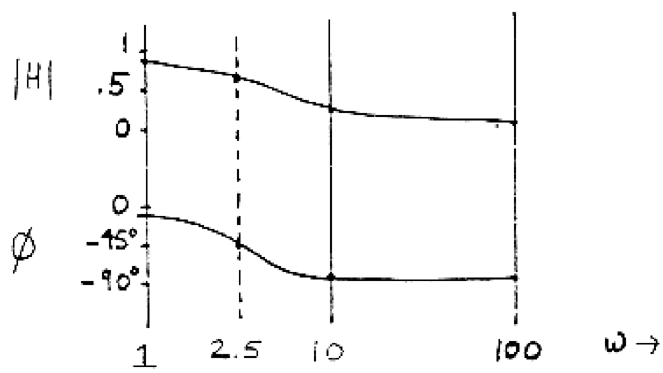
Section 13-3: Gain, Phase Shift, and the Network Function

P13.3-1



$$Z_p = \frac{(2\times10^4)(-j10^4/\omega)}{2\times10^4 - j10^4/\omega} = \frac{2\times10^4}{1+j2\omega}$$

$$\therefore\ H(j\omega) = V_0/V_s = \frac{Z_P}{Z_P+5000} = \frac{4}{[5+j2\omega]} \Rightarrow |H\ (j\omega)| = \frac{4}{\left[(2\omega)^2+5^2\right]^{1/2}} \Rightarrow \phi(\omega) = -\tan^{-1}\frac{2\omega}{5}$$

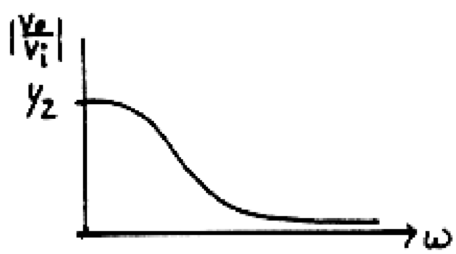


P13.3-2

a)
$$\frac{V_o}{V_i} = \frac{\dfrac{R/sC}{R+1/sC}}{R+\dfrac{R/sC}{R+1/sC}} = \frac{1}{2+RsC} = \frac{1}{2+j\omega RC}$$

b)
$$\Rightarrow \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{4+(\omega RC)^2}}$$

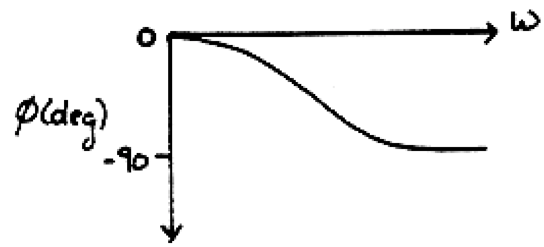
(c)



Above gives correct limiting results. At ω = 0, the cap is open, so dc voltage divider yields

$V_o/V_i = 1/2$. At $\omega = \infty$

Cap is a short $\Rightarrow V_o/V_i = 0$



At $\omega = 0$, V_i and V_0 are in phase. At $\omega = \infty$, V_0 logs $I_C = V_i / R$ by 90°

P13.3-3

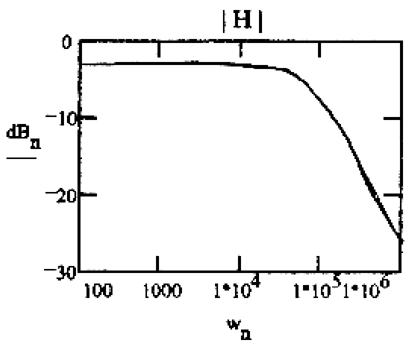
$$\frac{V_o(\omega)}{V_s(\omega)} = \frac{50}{20+j\omega(10^{-3})+50} = \frac{50{,}000}{j\omega+70{,}000}$$

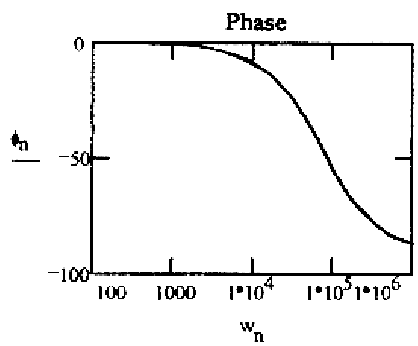
MathCad Analysis

$N := 100 \qquad n := 0 .. N$

$\omega min := 10^2 \quad \omega max := 10^6 \quad m := \ln\left(\frac{\omega max}{\omega min}\right) \qquad \omega_n := \omega min \cdot e^{\frac{n}{N}\cdot m}$

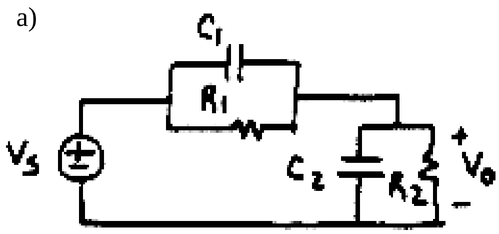
$H_n := \frac{50000}{j\cdot\omega_n+70000} \qquad dB_n := 20\log\left(\left|H_n\right|\right) \qquad \phi_n := \arg\left(H_n\right)\cdot\frac{180}{\pi}$





P13.3-4

a)



From voltage divider

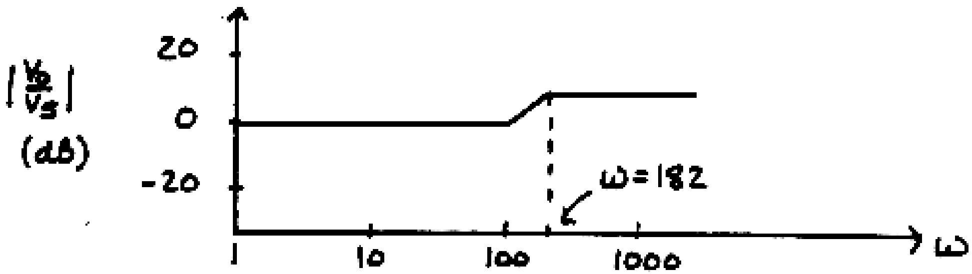
$$V_o = \frac{\dfrac{R_2/sC_2}{R_2+1/sC_2}}{\dfrac{R_2/sC_2}{R_2+1/sC_2}+\dfrac{R_1/sC_1}{R_1+1/sC_1}}V_s$$

$$\Rightarrow \frac{V_o}{V_s} = \left(\frac{R_2}{R_1+R_2}\right)\frac{1+R_1C_1s}{1+\left(\dfrac{R_1R_2}{R_1+R_2}\right)(C_1+C_2)\,s}$$

$$\therefore \frac{V_o}{V_s} = \frac{R_2}{R_1+R_2} = K \quad \text{when } R_1C_1 = \frac{R_1R_2}{R_1+R_2}(C_1+C_2)$$

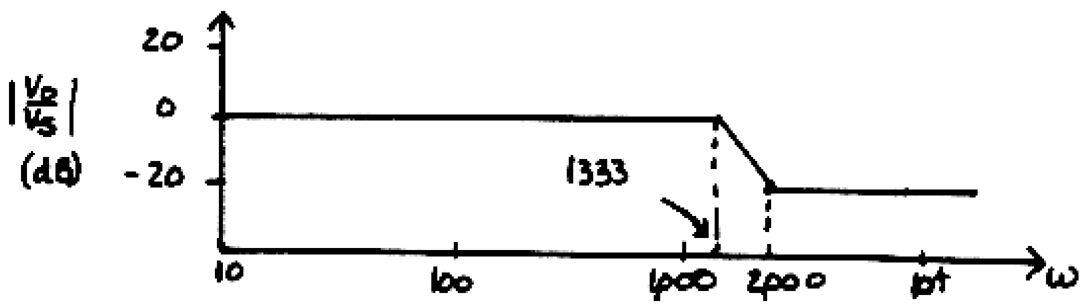
$$\Rightarrow \underline{R_1C_1 = R_2C_2}$$

(b) $\underline{C_1 = 1\mu F} \qquad V_o/V_s = \frac{1}{2}\,\frac{1+j\omega\,(10^{-2})}{1+j\omega\,(5.5\times10^{-3})}$

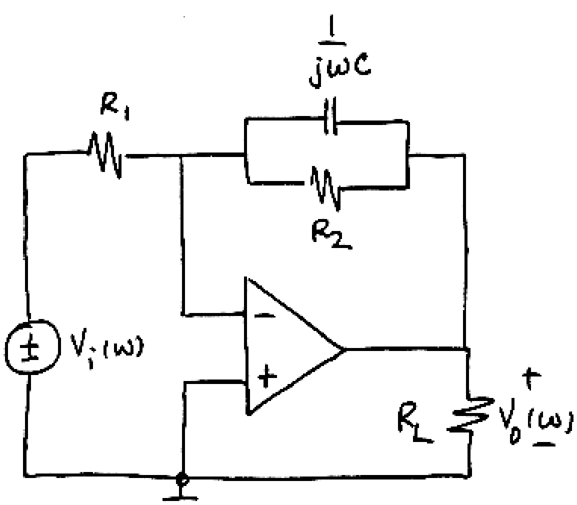


$\underline{C_1 \; = \; 0.1\mu\text{ F}}\,, \; \dfrac{V_o}{V_s} \; = \; 1/2 \; = \; \text{constant} \; \therefore \; 20 \; \log \; \left|\dfrac{V_o}{V_s}\right| \; = \; \underline{-\,3\text{dB} \; (\text{constant})}$

$\underline{C_1 \; = \; 0.05\mu\text{ F}}\,, \qquad 2\,\dfrac{V_o}{V_s} \; = \; \dfrac{1 + j\omega(5\times10^{-4})}{1 + j\omega(.75\times10^{-3})}$



P13.3-5



$$H(\omega) \; = \; \frac{V_o(\omega)}{V_i(\omega)}$$

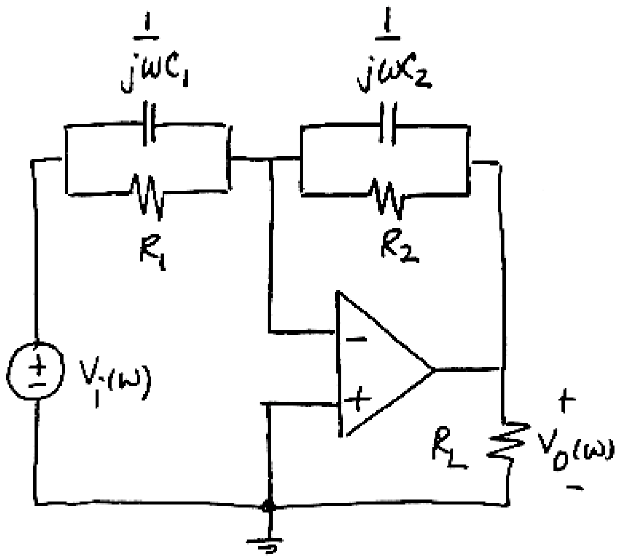
$$= \; -\frac{R_2 \,\|\, \frac{1}{j\omega C}}{R_1}$$

$$= \; \frac{-\left(R_2/R_1\right)}{1 + j\omega C R_2}$$

When $R_1 = 10\text{ k}\Omega$, $R_2 = 50\text{k}\Omega$, and $C = 2\mu\text{F}$, then

$R_2/R_1 \; = \; 5$ and $R_2 C = \dfrac{1}{10}$ so $H(\omega) \; = \; \dfrac{-5}{1 + j\frac{\omega}{10}}$

P13.3-6



$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = -\frac{R_2 \left\| \frac{1}{j\omega C_2} \right.}{R_1 \left\| \frac{1}{j\omega C_1} \right.} = -\frac{\frac{R_2}{1+j\omega C_2 R_2}}{\frac{R_1}{1+j\omega C_1 R_1}}$$

$$H(\omega) = -\left(\frac{R_2}{R_1}\right)\left(\frac{1+j\omega C_1 R_1}{1+j\omega C_2 R_2}\right)$$

When $R = 10\text{k}\Omega$, $R_2 = 50\text{k}\Omega$, $C_1 = 4\mu\text{F}$ and

$C_2 = 2\mu\text{F}$, then $R_2/R_1 = 5$, $C_1R_1 = \frac{1}{25}$ and $C_2R_2 = \frac{1}{10}$

so $$H(\omega) = -5\left(\frac{1+j\frac{\omega}{25}}{1+j\frac{\omega}{10}}\right)$$

$$\text{gain} = |H(\omega)| = 5\,\frac{\sqrt{1+\frac{\omega^2}{625}}}{\sqrt{1+\frac{\omega^2}{100}}}$$

$$\text{phase shift} = \angle H(\omega) = 180 + \tan^{-1}\frac{\omega}{25} - \tan^{-1}\frac{\omega}{10}$$

P13.3-7

$$R_3 \left\| \frac{1}{j\omega C} \right. = \frac{R_3 \frac{1}{j\omega C}}{R_3 + \frac{1}{j\omega C}} = \frac{R_1}{1+j\omega C R_3}$$

$$T(\omega) = -\frac{R_2 + \frac{R_3}{1+j\omega C R_3}}{R_1} = -\frac{R_2+R_3+j\omega R_2R_3C}{R_1+j\omega R_1R_3C}$$

$$5 = \lim_{\omega\to 0} |T(\omega)| = \frac{R_2+R_3}{R_1}$$

$$2 = \lim_{\omega\to\infty} |T(\omega)| = \frac{R_2}{R_1} \Rightarrow R_2 = 2R_1 = 20\text{k}\Omega$$

then $R_3 = 5R_1 - R_2 = 30\text{k}\Omega$

P13.3-8

$$T(\omega) = -\frac{R_2 + \frac{1}{j\omega C}}{R_1} = -\frac{1 + j\omega\, CR_2}{j\omega CR_1}$$

$$\angle T(\omega) = 180^\circ + \tan^{-1}\omega CR_2 - 90$$

$$\angle T(\omega) = 135^\circ \;\Rightarrow\; \tan^{-1}\omega CR_2 = 45$$

$$\Rightarrow \omega CR_2 = 1$$

$$\Rightarrow R_2 = \frac{1}{10^3 10^{-7}} = 10\text{k}\Omega$$

$$10 = \lim_{\omega\to\infty} |T(\omega)| = \frac{R_2}{R_1} \;\Rightarrow\; R_1 = \frac{R_2}{10} = 1\text{k}\Omega$$

P13.3-9

$$T(\omega) = \frac{-R_2}{+\frac{1}{j\omega C}} = -\frac{j\omega CR_2}{1 + j\omega CR_1}$$

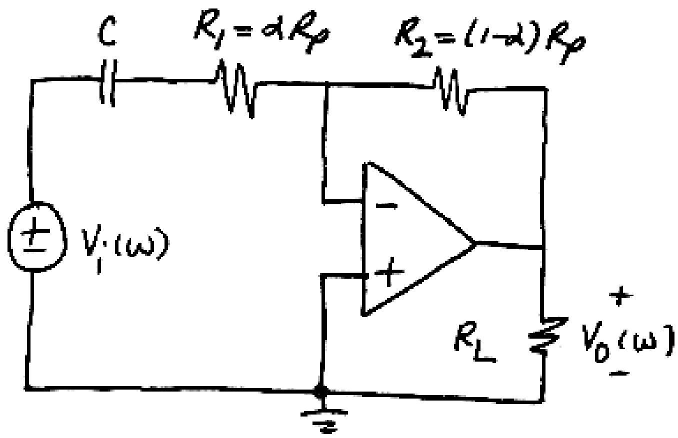
$$10 = \lim_{\omega\to\infty} |T(\omega)| = \frac{R_2}{R_1} \Rightarrow R_2 = 10R_1$$

$$\angle T(\omega) = 180 + 90 - \tan^{-1}\omega CR_1$$

$$\Rightarrow R_1 = \frac{\tan(270 - \angle T(\omega))}{\omega C} = 10^4 \cdot \tan(270 - \angle T(\omega)) = 10^4 = 10\text{ k}\Omega$$

$$\Rightarrow R_2 = 100\text{k}\Omega$$

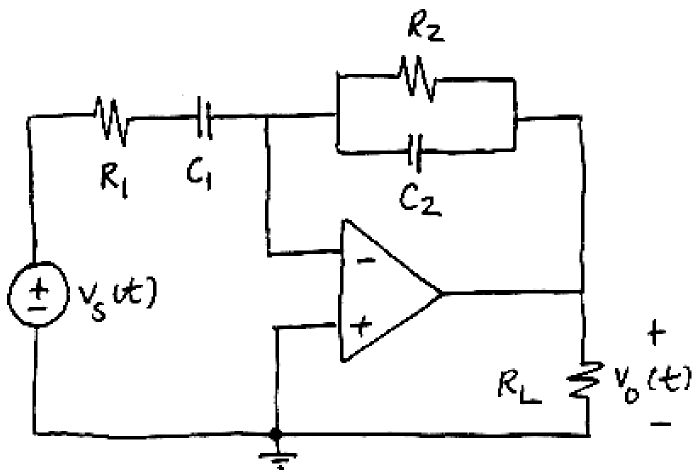
P13.3-10



$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = -\frac{R_2}{R_1 + \frac{1}{j\omega C}} = -\frac{j\omega CR_2}{1 + j\omega CR_1} = -\frac{j\omega C(1-\alpha)R_p}{1 + j\omega C\alpha R_p}$$

$$4 = \lim_{\omega\to\infty} |H(\omega)| = \frac{1-\alpha}{\alpha} \Rightarrow \alpha = 0.2$$

P13.3-11



$$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = -\frac{R_2 \Big\| \frac{1}{j\omega C_2}}{R_1 + \frac{1}{j\omega C_1}} = -C_1 R_2 \frac{j\omega}{(1+j\omega R_1 C_1)\,(1+j\omega R_2 C_2)}$$

When $R_1 = 5\text{k}\Omega$, $C_1 = 1\mu\text{F}$, $R_2 = 10\text{k}\Omega$ and $C_2 = 0.1\mu\text{F}$, then

$$H(\omega) = -0.01 \frac{j\omega}{\left(1+j\frac{\omega}{200}\right)\left(1+j\frac{\omega}{1000}\right)}$$

So

ω	$\lvert H(\omega)\rvert$	$\angle H(\omega)$
0	0	-90°
500	1.66	175°
2500	0.74	116°

Then

$$v_o(t) = (0)\,50 + (1.66)\,30\cos(500t+115^\circ+175^\circ) - (0.74)\;20\cos\;(2500t+30^\circ+116^\circ)$$

$$= 49.8\cos(500t-70^\circ) - 14.8\cos(2500t+146^\circ)\text{ mV}$$

When R_1=5kΩ, C_1=1μF, R_2=10k and C_2=0.01μF, then

$$H(\omega) = -0.01 \frac{j\omega}{\left(1+j\frac{\omega}{200}\right)\left(1+j\frac{\omega}{10{,}000}\right)}$$

So

ω	$\lvert H(\omega)\rvert$	$\angle H(\omega)$
0	0	-90°
500	1.855	-161°
2500	1.934	170°

Then

$$v_o(t) = (0)\,50 + (1.855)\,30\cos(500t+115^\circ-161^\circ) - (1.934)\;20\cos(2500t+30^\circ+170^\circ)$$

$$= 55.65\cos(500t-46^\circ) - 38.68\cos(2500t+190^\circ)\text{ mV}$$

P13.3-12

a) $$|V_s| = \frac{(8\text{ div})\left(\frac{2\text{ V}}{\text{div}}\right)}{2} = 8\text{ V}$$

$$|V_o| = \frac{(6.2\text{div})\left(\frac{2\text{V}}{\text{div}}\right)}{2} = 6.2\text{ V}$$

$$\text{gain} = \frac{|V_o|}{|V_s|} = \frac{6.2}{8} = 0.775$$

b) $$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$$

Let $g = |H(\omega)| = \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$

Then

$$C = \frac{1}{\omega R}\sqrt{\left(\frac{1}{g}\right)^2 - 1}$$

In this case $\omega = 2\,\pi \cdot 500 = 3142$ rad/s, $|H(\omega)| = 0.775$ and R=1000Ω so C = 0.26μF.

c) $$\angle H(\omega) = -\tan^{-1}\omega RC$$

so

$$\omega = \frac{\tan(-\angle H(\omega))}{RC}$$

Recalling that R = 1000 and C=0.26μF, we calculate

ω	$\lvert H(\omega)\rvert$	$\angle H(\omega)$
$2\pi(200)$	0.95	-18°
$2\pi(2000)$	0.26	-73°

Next , $\angle H(\omega) = -45^\circ$ requires $\omega = \frac{\tan\left(-\left(-45^\circ\right)\right)}{(1000)\left(.26 \cdot 10^{-6}\right)} = 3846$ rad/s

Similarly , $\angle H(\omega) = -135^\circ$ requires $\omega = \frac{\tan(-(-135))}{(1000)\,(0.26 \cdot 10^{-6})} = -3846$ rad/s

A negative frequency is not acceptable. We conclude that this circuit cannot produce a phase shift equal to –135°.

d) $C = \dfrac{\tan(-\angle H(\omega))}{\omega R}$

$$C = \frac{\tan(-(60^\circ))}{(2\pi \cdot 500)(1000)} = 0.55\mu F$$

$$C = \frac{\tan(-(-300^\circ))}{(2\pi \cdot 500)(1000)} = -0.55\mu F$$

A negative value of capacitance is not acceptable and indicates that this circuit cannot be designed to produce a phase shift at -300° at a frequency of 500 Hz.

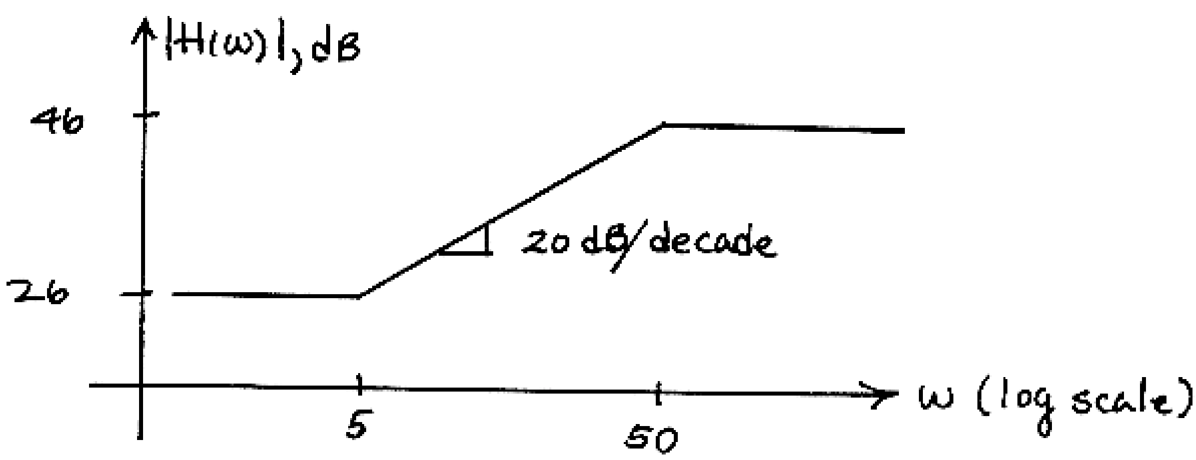
e) $C = \dfrac{\tan(-(-120^\circ))}{(2\pi \cdot 500)(100)} = -0.55\mu F$

This circuit cannot be designed to produce a phase shift of -120° at 500 Hz.

Section 13-4: Bode Plots

P13.4-1

$$H(\omega) = \frac{20\left(1 + j\dfrac{\omega}{5}\right)}{\left(1 + j\dfrac{\omega}{50}\right)} \qquad H(\omega) \simeq \begin{cases} 20 & \omega < 5 \\ 20\left(j\dfrac{\omega}{5}\right) = j4\omega & 5 < \omega < 50 \\ 20\left(\dfrac{j\frac{\omega}{5}}{j\frac{\omega}{50}}\right) = 200 & 50 < \omega \end{cases}$$

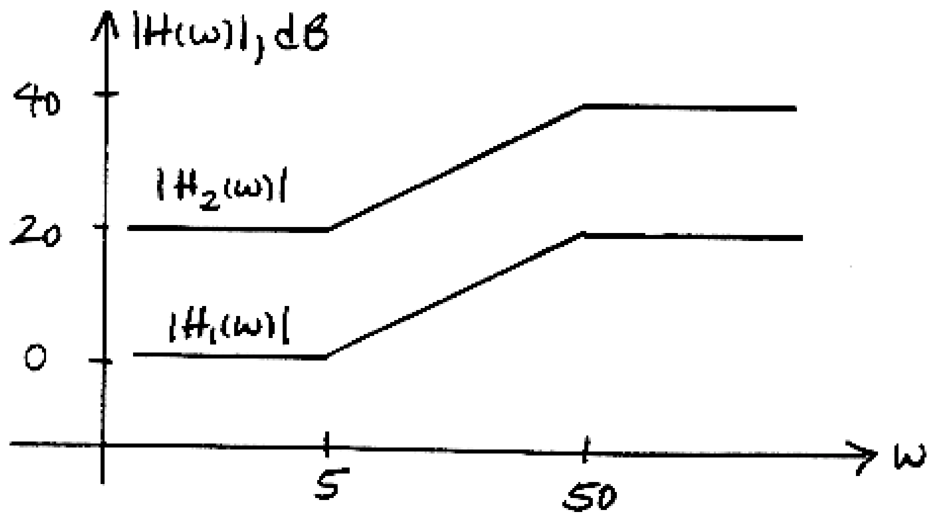


P13.4-2

$$H_1(\omega) = \frac{1+j\frac{\omega}{5}}{1+j\frac{\omega}{50}} \qquad H_2(\omega) = 10\,\frac{1+j\frac{\omega}{5}}{1+j\frac{\omega}{50}}$$

Both $H_1(\omega)$ and $H_2(\omega)$ have a pole at ω = 50rad/s and a zero at ω = 5rad/s. The slopes of both magnitude Bode plots increase by 20dB/decade at ω = 5rad/s and decrease by 20dB/decade at ω = 50rad/s. The difference is that for ω < 5rad/s

and
$$|H_1(\omega)| \simeq 1 = 0 \text{ dB}$$
$$|H_2(\omega)| \simeq 10 = 20 \text{ dB}$$



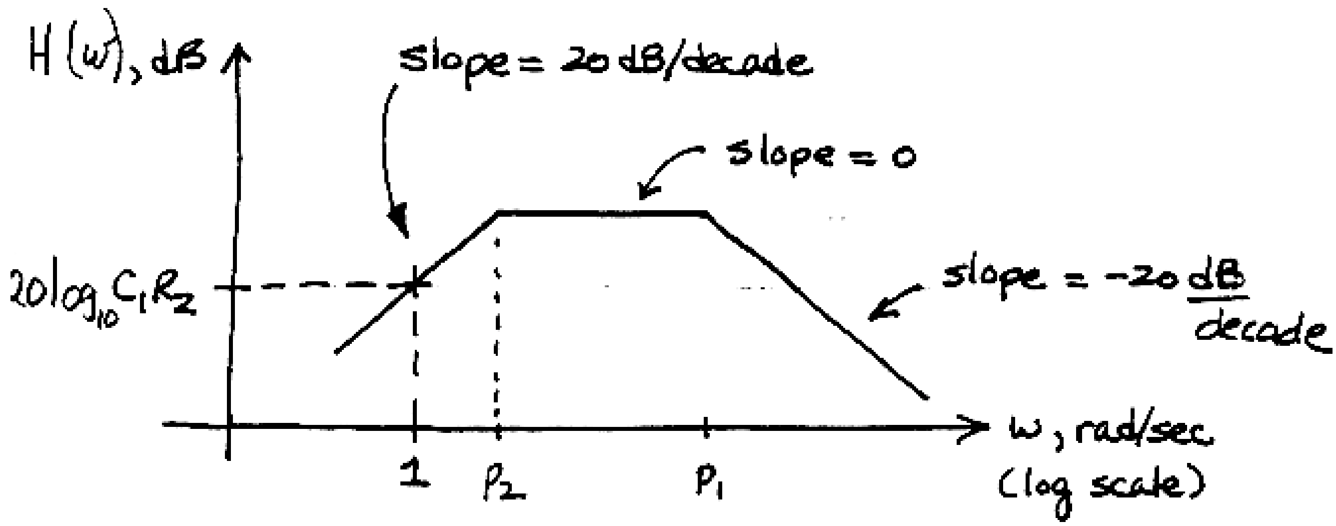
P13.4-3

$$H(\omega) = -\frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{R_1 + \dfrac{1}{j\omega C_1}} = -C_1 R_2 \frac{j\omega}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$$

This network function has poles at

$$p_1 = \frac{1}{R_1 C_1} = 2000\,\text{rad/s} \qquad p_2 = \frac{1}{R_2 C_2} = 1000\,\text{rad/s}$$

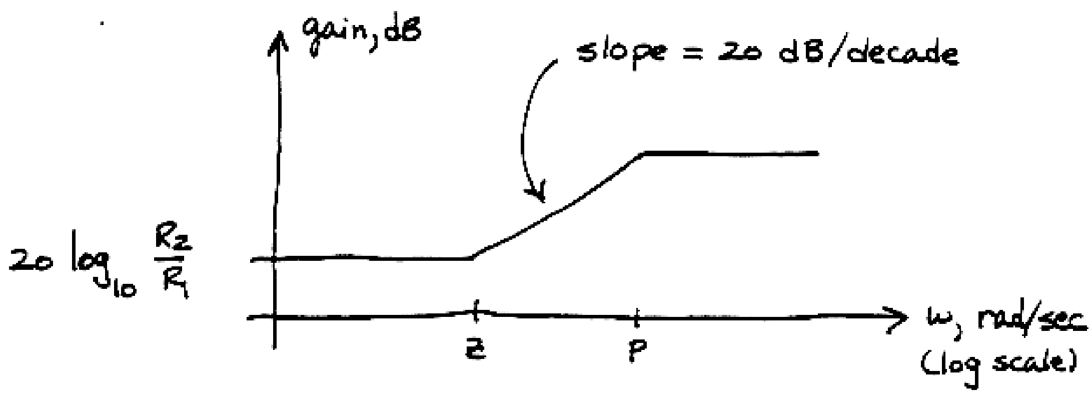
$$H(\omega) \simeq \begin{cases} (C_1 R_2) j\omega & \omega < p_1 \\ (C_1 R_2)\dfrac{j\omega}{j\omega C_1 R_1} = \dfrac{R_2}{R_1} = 2 & p_1 < \omega < p_2 \\ (C_1 R_2)\dfrac{j\omega}{(j\omega C_1 R_1)(j\omega C_2 R_2)} = \dfrac{1}{j\omega C_2 R_1} & \omega > p_2 \end{cases}$$



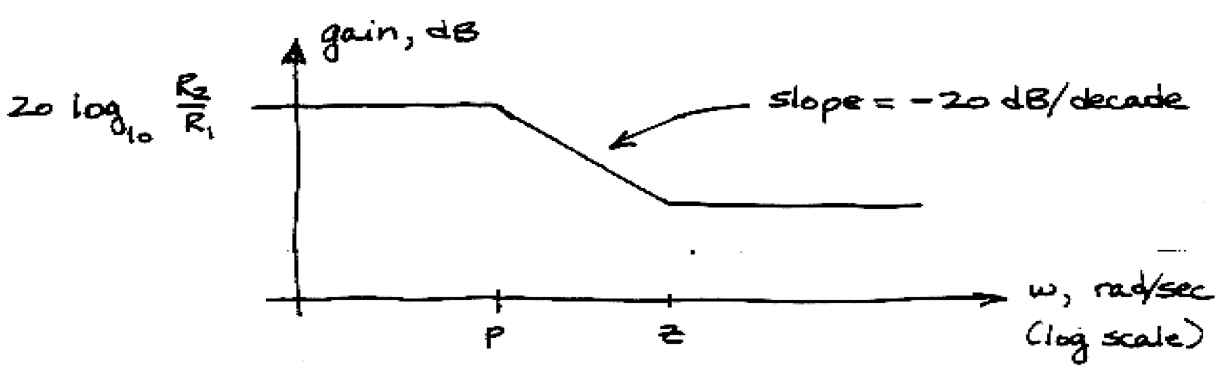
P13.4-4

$$H(\omega) = -\frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{\dfrac{R_1}{1+j\omega C_1 R_1}} = -\frac{R_2(1+j\omega C_1 R_1)}{R_1(1+j\omega C_2 R_2)} \text{ so } K = -\frac{R_2}{R_1},\ z = \frac{1}{C_1 R_1} \text{ and } p = \frac{1}{C_2 R_2}$$

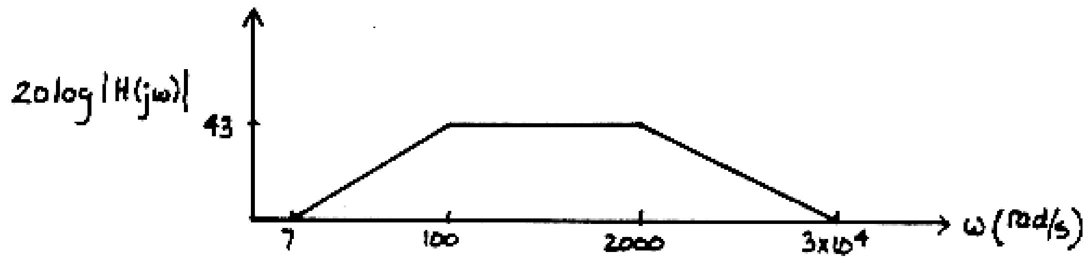
When $z<p$, then



When $p<z$, then



P13.4-5 A reasonable approximation (asymptotic) for the Bode diagram is:



$$\therefore\ H(j\omega) = \frac{A\left(1+{j\omega}/{7}\right)\left(1+{j\omega}/{3\times10^4}\right)}{\left(1+{j}/{100}\right)\left(1+{j\omega}/{2000}\right)}$$

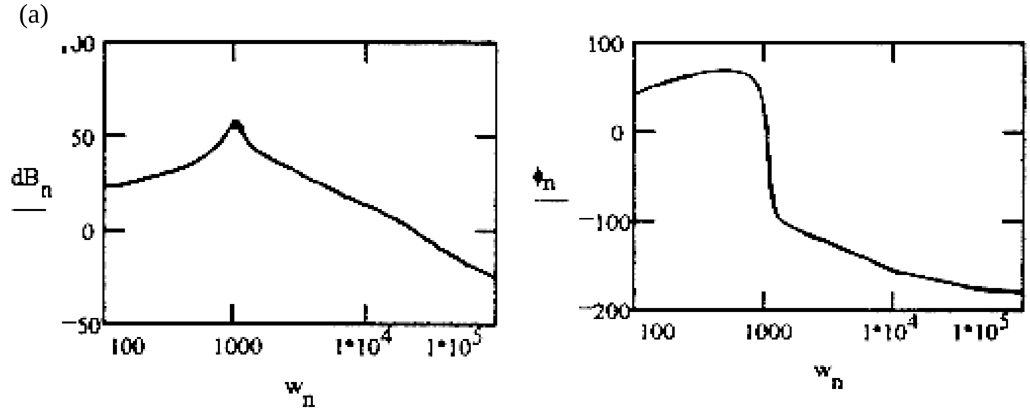
$$\text{At the peak,}\quad H(j\omega) \simeq \frac{A\ {j\omega}/{7}}{{j\omega}/{100}} = \frac{100}{7}A$$

$$\therefore\ 43 = 20\log\left(\frac{100}{7}A\right) \quad \Rightarrow\ A = 10$$

$$\Rightarrow H(j\omega) = \frac{10\left(1+{j\omega}/{7}\right)\left(1+{j\omega}/{3\times10^4}\right)}{\left(1+{j}/{100}\right)\left(1+{j\omega}/{2000}\right)}$$

P13.4-6

(a)



(b) $BW = {\omega_0}/{Q} = {1000}/{10} = \underline{100}$

(c) From the Bode diagram, it is clear that the overall Q of the circuit is dictated by the Q of the second order factor. Thus the overall Q=10

(d) From, the plot, the gain at ω_0 = 40dB

P13.4-7

- The slope is 40dB/decade for low frequencies, so the numerator will include the factor $(j\omega)^2$.
- The slope decreases by 40dB/decade at ω = 0.7rad/sec. So there is a second order pole at ω_0 = 0.7rad/sec. The damping factor of this pole cannot be determined from the asymptotic Bode plot; call it δ_1. The denominator of the network function will contain the factor

$$1+2\delta_1 j\frac{\omega}{0.7}-\left(\frac{\omega}{0.7}\right)^2$$

- The slope increases by 20dB/decade at ω = 10rad/s, indicating a zero at 10rad/s.
- The slope decreases by 20dB/decade at ω = 100rad/s, indicating a pole at 100rad/s.
- The slope decreases by 40dB/decade at ω = 600rad/s, indicating a second order pole at ω_0 = 600rad/s. The damping factor of this pole cannot be determined from an asymptotic Bode plot; call it δ_2. The denominator of the network function will contain the factor

$$1+2\delta_2 j\frac{\omega}{600}-\left(\frac{\omega}{600}\right)^2$$

$$H(\omega) = \frac{K(1+j\frac{\omega}{10})(j\omega)^2}{\left(1+2\delta_1 j\frac{\omega}{0.7}-\left(\frac{\omega}{0.7}\right)^2\right)\left(1+2\delta_2 j\frac{\omega}{600}-\left(\frac{\omega}{600}\right)^2\right)\left(1+j\frac{\omega}{100}\right)}$$

To determine K, notice that $|H(\omega)|=0\text{dB}=1$ when $0.7 < \omega < 10$. That is

$$1 = \frac{K(1)(j\omega)^2}{-\left(\frac{\omega}{0.7}\right)^2(1)(1)} = K(0.7)^2 \;\Rightarrow\; K = 2$$

P13.4-8 By inspection,

$$H(j\omega) = \frac{A\left(1+j^{\omega}/_{100}\right)}{j\omega\left(1+j^{\omega}/_{1000}\right)}$$

from the magnitude plot for $100<\omega<1000$

$$|H(j\omega)| \simeq Aj\frac{^{\omega}/_{100}}{j\omega}={}^{A}/_{100}$$

$$\therefore\; 20\log {}^{A}/_{100}=0 \quad \Rightarrow A=100$$

$$\text{So}\;\; H(j\omega) = \frac{100\left(1+j^{\omega}/_{100}\right)}{j\omega\left(1+j^{\omega}/_{1000}\right)}$$

P13.4-9

(a) $T(\omega)=\dfrac{K\left(1+j\dfrac{\omega}{z}\right)}{j\omega}$

$$|T(\omega)|=\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

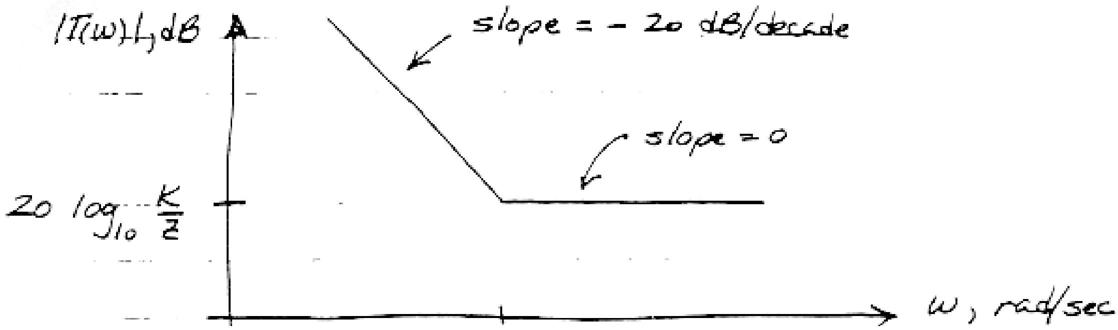
$$|T(\omega)|\text{ dB} = 20\log_{10}\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2} = 20\log_{10}K - 20\log_{10}\omega + 20\log_{10}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

Let $|T_L(\omega)|\text{ dB} = 20\log_{10}K - 20\log_{10}\omega$

and $|T_H(\omega)|\text{ dB} = 20\log_{10}\dfrac{K}{z}$

Then $|T(\omega)|\text{ dB} \simeq \begin{cases} |T_L(\omega)|\text{ dB} & \omega \ll z \\ |T_H(\omega)|\text{ dB} & \omega \gg z \end{cases}$

So $|T_L(\omega)|$ dB and $|T_H(\omega)|$ dB are the required low and high-frequency asymptotes.



The Bode plot will be within 1% of |T(ω)| dB both for ω<<z and for ω>>z. The range when ω<<z is characterized by

$|T_L(\omega)| = .99|T(\omega)|$ (gains not in dB)

$\Rightarrow 20\log_{10}.99 = |T_L(\omega)|\text{ dB} - |T(\omega)|\text{ dB}$ (gains in dB)

$$= 20\log_{10}K - 20\log_{10}\omega - 20\log_{10}\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= -20\log_{10}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= 20\log_{10}\frac{1}{\sqrt{1+\left(\frac{\omega}{z}\right)^2}}$$

$$\Rightarrow \; .99 = \frac{1}{\sqrt{1+\left(\frac{\omega}{z}\right)^2}} \;\Rightarrow\; \omega = z\sqrt{\left(\frac{1}{.99}\right)^2 - 1} = .14z \simeq \frac{z}{7}$$

The range when $\omega >> z$ is characterized by

$|T_H(\omega)| = .99|T(\omega)|$ (gains not in dB)

$$\Rightarrow \; 20\log_{10}.99 = |T_H(\omega)|\text{ dB} - |T(\omega)|\text{ dB} \quad \text{(gains in dB)}$$

$$= 20\log_{10}K - 20\log_{10}z - 20\log_{10}\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= -20\log_{10}\frac{z}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= 20\log_{10}\frac{1}{\sqrt{\left(\frac{Z}{\omega}\right)^2+1}}$$

$$\Rightarrow \; \frac{z}{\omega} = \sqrt{\left(\frac{1}{.99}\right)^2 - 1}$$

$$\Rightarrow \; \omega = \frac{z}{\sqrt{\left(\frac{1}{.99}\right)^2 - 1}} = \frac{z}{.14} \simeq 7z$$

The error is less than 1% when $\omega < \frac{z}{7}$ and when $\omega > 7z$.

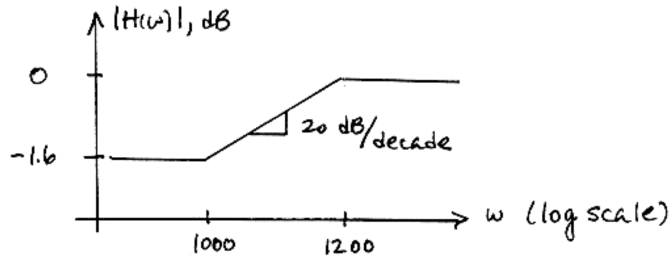
P13.4-10

$$H(\omega) = \frac{V_0(\omega)}{V_s(\omega)} = \frac{R_t}{R_t + R_1 \| \frac{1}{jwC}} = \frac{R_t}{R_t + \frac{R_1}{1+jwCR_1}}$$

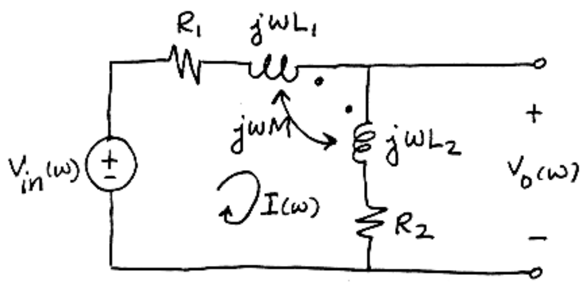
$$H(\omega) = \frac{R_t(1+j\omega CR_1)}{R_1 + R_t + j\omega CR_1R_t} = \left(\frac{R_t}{R_1+R_t}\right)\frac{1+j\omega CR_1}{1+j\omega\left(\frac{CR_1R_t}{R_1+R_t}\right)}$$

When $R_1 = 1k\Omega$, $C = 1\mu F$ and $R_t = 5k\Omega$

$$H(\omega) = \frac{5}{6}\left(\frac{1+j\frac{\omega}{1000}}{1+j\frac{\omega}{1200}}\right) \;\Rightarrow\; H(\omega) \cong \begin{cases} \frac{5}{6} & \omega<1000 \\ \left(\frac{5}{6}\right)j\frac{\omega}{1000} & 1000<\omega<1200 \\ 1 & \omega<1200 \end{cases}$$



P13.4-11



$$V_{in}(\omega) = I(\omega)\,[R_1 + (j\omega l_1 - j\omega M) + (-j\omega M + j\omega L_2) + R_2]$$

$$V_0(\omega) = I(\omega)\,[(-j\omega M + j\omega L_2) + R_2]$$

$$T(\omega) = \frac{V_0(\omega)}{V_{in}(\omega)} = \frac{R_2 + j\omega(L_2 - M)}{R_1 + R_2 + j\omega(L_1 + L_2 - 2M)}$$

$$K_1 = \lim_{w\to\infty} |T(\omega)| = \frac{L_2 - M}{L_1 + L_2 - 2M} = 0.75$$

$$K_2 = \lim_{w\to 0} |T(\omega)| = \frac{R_2}{R_1 + R_2} = 0.2$$

$$z = \frac{R_2}{L_2 - M} = 333 \text{ rad/s}$$

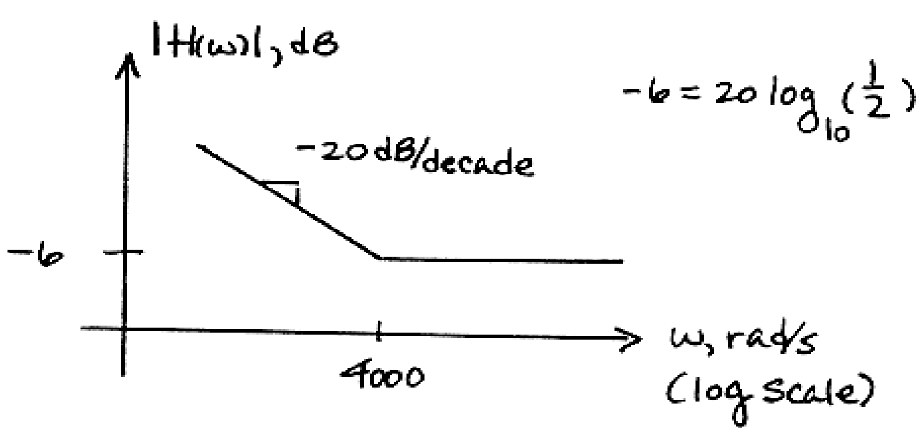
$$p = \frac{R_1 + R_2}{L_1 + L_2 - 2M} = 1250 \text{ rad/s}$$

P13.4-12

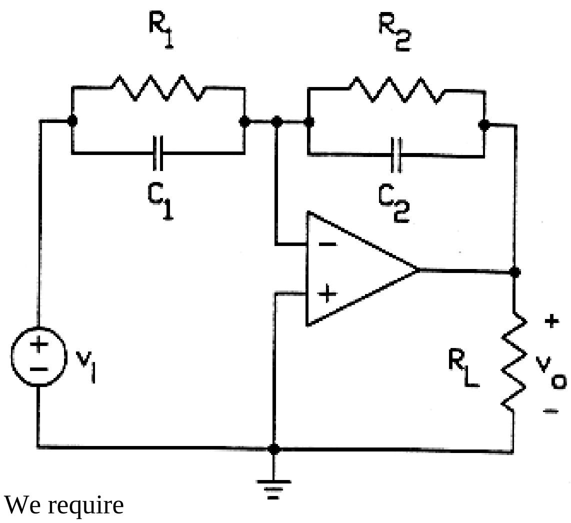
$$H(\omega) = -\frac{\dfrac{1}{j\omega C_2}}{R_1 \left\| \dfrac{1}{j\omega C_1}\right.} = -\frac{1 + j\omega R_1 C_1}{j\omega R_1 C_2} = -\frac{1}{R_1 C_2}\,\frac{(1 + j\omega R_1 C_1)}{j\omega}$$

$$H(\omega) \simeq \begin{cases} -\dfrac{1}{R_1 C_2}\left(\dfrac{1}{j\omega}\right) & \omega < \dfrac{1}{R_1 C_1} \\ -\dfrac{1}{R_1 C_2}(R_1 C_1) = -\dfrac{C_1}{C_2} & \omega > \dfrac{1}{R_1 C_1} \end{cases}$$

$$\frac{C_1}{C_2} = \frac{1}{2}, \frac{1}{R_1 C_1} = 4000 \text{ rad / s}$$



P13.4-13 Pick the appropriate circuit from Table 13.4-2.



$$H(\omega) = -k\frac{1+j\frac{\omega}{z}}{1+j\frac{\omega}{p}}$$

where

$$k = \frac{R_1}{R_2}$$

$$z = \frac{1}{C_1R_1}$$

$$p = \frac{1}{C_2R_2}$$

We require

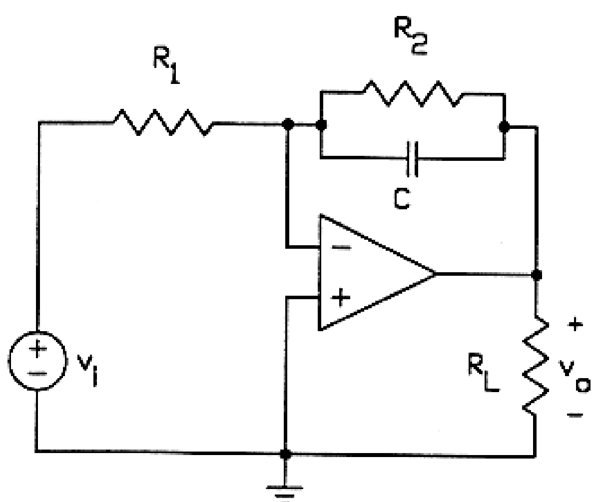
$$200 = z = \frac{1}{C_1R_1}$$

$$500 = p = \frac{1}{C_2R_2}$$

$$14\text{ dB} = 5 = k\frac{p}{z} = \frac{C_1}{C_2}$$

Pick $C_1 = 1\mu F$, then $C_2 = 0.2\mu F$, $R_1 = 5k\Omega$ and $R_2 = 10k\Omega$

P13.4-14 Pick the appropriate circuit from Table 13.4-2.



$$H(\omega) = -\frac{k}{1+j\frac{\omega}{p}}$$

where

$$k = \frac{R_2}{R_1}$$

$$p = \frac{1}{CR_2}$$

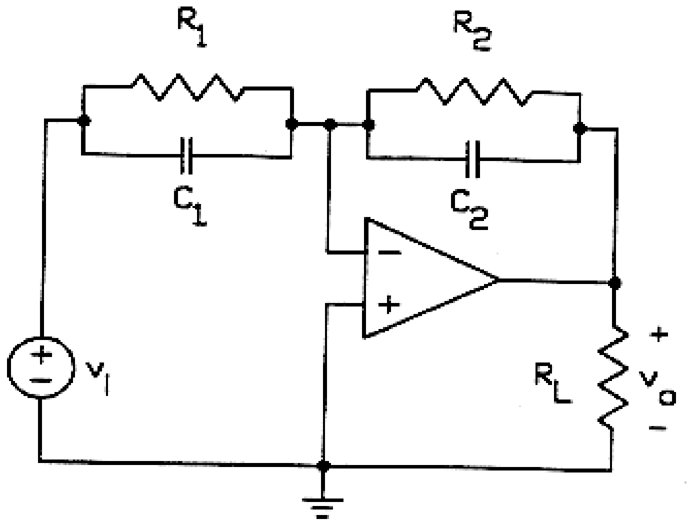
We require

$$500 = p = \frac{1}{CR_2}$$

$$34\text{dB} = 50 = \frac{R_2}{R_1}$$

Pick $C = .1\mu F$, then $R_2 = 20k\Omega$, $R_1 = 400\Omega$

P13.4-15 Pick the appropriate circuit from Table 13.4-2.



$$H(\omega) = -k\frac{1+j\frac{\omega}{z}}{1+j\frac{\omega}{p}}$$

.

where

$$k = \frac{R_2}{R_1}$$

$$z = \frac{1}{C_1R_1}$$

$$p = \frac{1}{C_2R_2}$$

We require

$$200 = p = \frac{1}{C_2R_2}$$

$$500 = z = \frac{1}{C_1R_1}$$

$$14\text{dB} = 5 = k = \frac{R_2}{R_1}$$

Pick $C_1 = 0.1\mu\text{F}$. Then $R_1 = 20\text{k}\Omega$, $R_2 = 100\text{k}\Omega$ and $C_2 = 0.05\mu\text{F}$.