P13.4-16 Pick the appropriate circuit from Table 13.4-2.


$$
H(\omega)=-\frac{k}{\left(1+j \frac{\omega}{p_{1}}\right)\left(1+j \frac{\omega}{p_{2}}\right)}
$$

where

$$
\begin{aligned}
\mathrm{k} & =\mathrm{C}_{1} \mathrm{R}_{2} \\
\mathrm{p}_{1} & =\frac{1}{\mathrm{C}_{1} \mathrm{R}_{1}} \\
\mathrm{p}_{2} & =\frac{1}{\mathrm{C}_{2} \mathrm{R}_{2}}
\end{aligned}
$$

We require

$$
\begin{aligned}
& 200=\mathrm{p}_{1}=\frac{1}{\mathrm{C}_{1} \mathrm{R}_{1}} \\
& 500=\mathrm{p}_{2}=\frac{1}{\mathrm{C}_{2} \mathrm{R}_{2}} \\
& 34 \mathrm{~dB}=50=\mathrm{k}=\mathrm{C}_{1} \mathrm{R}_{2}
\end{aligned}
$$

Pick $\mathrm{C}_{1}=1 \mu \mathrm{~F}$. Then $\mathrm{R}_{1}=5 \mathrm{k} \Omega, \mathrm{R}_{2}=50 \mathrm{k} \Omega$ and $\mathrm{C}_{2}=0.04 \mu \mathrm{~F}$.

P13.4-17

$$
H(j \omega)=\frac{10(j \omega / 50+1)}{(j \omega / 2+1)(j \omega / 50+1)(j \omega / 80+1)}
$$



$$
\varphi=\tan ^{-1} \omega / 50-\tan ^{-1} \omega / 2-\tan ^{-1} \omega / 20-\tan ^{-1} \omega / 80
$$



P13.4-18

$$
H_{1,2}=-\frac{R_{2} / R_{1}}{1+j \omega R_{2} C} \Rightarrow H_{\text {Total }}=H_{1,2}^{2}=\left(R_{2} / R_{1}\right)^{2}\left(\frac{1}{1+j \omega R_{2} C}\right)^{2}
$$

(a) At low frequency,
$\mathrm{H}_{\mathrm{T}}=\left(\mathrm{R}_{2} / \mathrm{R}_{1}\right)^{2} \quad \therefore$ for $\mathrm{H}_{\mathrm{T}}=1$, need $\mathrm{R}_{1}=\mathrm{R}_{2}$
Now also $\omega_{1}=1000=1 / R_{2} C$, so let $C=1 \mu \mathrm{~F}$

$$
\therefore \mathrm{R}_{1}=\mathrm{R}_{2}=1 /(1000)\left(10^{-6}\right)=1 \mathrm{k} \Omega
$$

(b) At $\omega=10,000$

$$
\begin{aligned}
|\mathrm{H}| & =\frac{1}{1+\left(\omega \mathrm{R}_{2} \mathrm{C}\right)^{2}}=\frac{1}{1+\left[\left(10^{4}\right)\left(10^{3}\right)\left(10^{-6}\right)\right]^{2}}=10^{-2} \\
& \Rightarrow 20 \log |\mathrm{H}|=20 \log 10^{-2}=\underline{-40 \mathrm{~dB}}
\end{aligned}
$$

P13.4-19

(a) $\quad H(\omega)=\frac{V_{0}(\omega)}{V_{s}(\omega)}=-\frac{R_{2} / R_{1}}{1+j \omega R_{2} C}=-\frac{10}{1+j \frac{\omega}{10,000}}$
(b) $10=20 \mathrm{~dB}$
(c) $10,000 \mathrm{rad} / \mathrm{sec}$

P13.4-20


Assume $\mathrm{V}_{\mathrm{i}} \simeq 0$, then $\mathrm{V}_{+}=\mathrm{V}_{-}=\mathrm{V}_{\mathrm{o}}$ Use $s=j \omega$

Voltage divider yields : $\mathrm{V}_{\mathrm{o}}=\mathrm{V}_{1} \frac{1 / \mathrm{sC}}{\mathrm{R}+1 / \mathrm{sC}} \Rightarrow \mathrm{V}_{1}=(1+\mathrm{sRC}) \mathrm{V}_{0}$
KCL at $\mathrm{V}_{1}:\left(\mathrm{V}_{1}-\mathrm{V}_{\mathrm{s}}\right) / \mathrm{mR}+\left(\mathrm{V}_{1}-\mathrm{V}_{\mathrm{o}}\right) / \mathrm{R}+\left(\mathrm{V}_{1}-\mathrm{V}_{\mathrm{o}}\right) \operatorname{snC}=0$
Plugging $\mathrm{V}_{1}$ into above yields
$\mathrm{V}_{\mathrm{o}}\left[\frac{1}{\mathrm{mR}}+\mathrm{sC}+\frac{\mathrm{sC}}{\mathrm{m}}+\mathrm{s}^{2} \mathrm{nRC}^{2}\right]=\frac{\mathrm{V}_{\mathrm{s}}}{\mathrm{mR}}$
$\therefore \frac{\mathrm{V}_{\mathrm{o}}}{\mathrm{V}_{\mathrm{s}}}=\frac{1}{1+\mathrm{s}(\mathrm{m}+1) \mathrm{RC}+\mathrm{nmR}^{2} \mathrm{C}^{2} \mathrm{~s}^{2}}$
$\Rightarrow \underline{H(j \omega)=V_{0} / V_{s}=\frac{1}{1-\left(\omega / \omega_{\mathrm{o}}\right)^{2}+\mathrm{j}\left(\omega / \mathrm{Q} \omega_{\mathrm{o}}\right)}}$ where $\omega_{0}=\frac{1}{\sqrt{\mathrm{mn} R C}}$

$$
\mathrm{Q}=\frac{\sqrt{\mathrm{mn}}}{\mathrm{~m}+1}
$$



P13.4-21


$$
\left.\begin{array}{l}
V_{o}(\omega)=\frac{\frac{1}{j \omega C_{2}}}{R+\frac{1}{j \omega C_{2}}} V_{a}(\omega) \\
=\frac{V_{a}(\omega)-V_{s}(\omega)}{R_{1}}+j \omega C_{1}\left(V_{a}(\omega)-V_{o}(\omega)\right)
\end{array}\right\} \Rightarrow V_{0}\left(1+j \omega C_{1} R_{1}\right)\left(1+j \omega C_{2} R_{2}\right)=j \omega C_{1} R_{1} V_{o}+V_{s}
$$

$T(\omega)=\frac{\mathrm{V}_{\mathrm{o}}(\omega)}{\mathrm{V}_{\mathrm{s}}(\omega)}=\frac{1}{1+\mathrm{C}_{2} \mathrm{R}_{2} \mathrm{j} \omega-\omega^{2} \mathrm{C}_{1} \mathrm{C}_{2} \mathrm{R}_{1} \mathrm{R}_{2}}=\frac{1}{-\omega^{2}+.8 \mathrm{j} \omega+1}$
Second order poles with $\omega_{\mathrm{o}}=0$ and $\delta=.4$


Section 13-5: Resonant Circuits
P13.5-1

$$
\begin{aligned}
& \omega_{0}=\frac{1}{\sqrt{\mathrm{LC}}}=\frac{1}{\sqrt{\left(\frac{1}{120}\right)\left(\frac{1}{30} \times 10^{-6}\right)}}=60 \mathrm{k} \mathrm{rad} / \mathrm{sec} \\
& \mathrm{Q}=\mathrm{R} \sqrt{\frac{\mathrm{C}}{\mathrm{~L}}}=10,000 \sqrt{\frac{\frac{1}{30} \times 10^{-6}}{\frac{1}{120}}}=20 \\
& \omega_{1}=-\frac{\omega_{0}}{2 \mathrm{Q}}+\sqrt{\left(\frac{\omega_{0}}{2 \mathrm{Q}}\right)^{2}+\omega_{0}^{2}}=58.52 \mathrm{k} \mathrm{rad} / \mathrm{s} \\
& \omega_{2}=\frac{\omega_{0}}{2 \mathrm{Q}}+\sqrt{\left(\frac{\omega_{0}}{2 \mathrm{Q}}\right)^{2}+\omega_{0}^{2}}=61.52 \mathrm{k} \mathrm{rad} / \mathrm{s} \\
& \mathrm{BW}=\frac{1}{\mathrm{RC}}=\frac{1}{(10000)\left(\frac{1}{30} \times 10^{-6}\right)}=3 \mathrm{k} \mathrm{rad} / \mathrm{s} \\
& \text { Notice that } \mathrm{BW}=\omega_{2}-\omega_{1}=\frac{\omega_{0}}{\mathrm{Q}}
\end{aligned}
$$

## P13.5-2

$\mathrm{R}=\mathrm{k}=\left|\mathrm{H}\left(\omega_{0}\right)\right|=\frac{8}{20 \cdot 10^{-3}}=400 \Omega$
$\omega_{0}=1000 \mathrm{rad} / \mathrm{s}$
$|H(\omega)|=\frac{k}{\sqrt{1+Q^{2}\left(\frac{\omega}{\omega_{0}}-\frac{\omega_{0}}{\omega}\right)^{2}}}$
At $\omega=897.6 \mathrm{rad} / \mathrm{s},|H(\omega)|=\frac{4}{20.10^{-3}}=200$, so
$200=\frac{400}{\sqrt{1+\mathrm{Q}^{2}\left(\frac{897.6}{1000}-\frac{1000}{897.6}\right)^{2}}} \Rightarrow \mathrm{Q}=8$

Now

$$
\left.\begin{array}{c}
\frac{1}{\sqrt{\mathrm{LC}}}=\omega_{0}=1000 \\
400 \sqrt{\frac{\mathrm{C}}{\mathrm{~L}}}=\mathrm{Q}=8
\end{array}\right\} \Rightarrow \begin{aligned}
& \mathrm{C}=20 \mu \mathrm{~F} \\
& \mathrm{~L}=50 \mathrm{mH}
\end{aligned}
$$

## P13.5-3

$\omega_{0}=\frac{1}{\sqrt{\mathrm{LC}}}=10^{5} \mathrm{rad} / \mathrm{s}, \mathrm{Q}=\frac{1}{\mathrm{R}} \sqrt{\frac{\mathrm{L}}{\mathrm{C}}}=10, B W=\frac{\mathrm{R}}{\mathrm{L}}=10^{4} \mathrm{rad} / \mathrm{s}$

## P13.5-4

$$
\omega_{0}=\frac{1}{\sqrt{\mathrm{LC}}}=10^{4} \mathrm{rad} / \mathrm{s}, \mathrm{Q}=\frac{1}{\mathrm{R}} \sqrt{\frac{\mathrm{~L}}{\mathrm{C}}}=10, \mathrm{BW}=\frac{\mathrm{R}}{\mathrm{~L}}=10^{3} \mathrm{rad} / \mathrm{s}
$$

P13.5-5

$$
\begin{gathered}
\mathrm{R}=\mathrm{Z}\left(\omega_{0}\right)=100 \Omega \\
\frac{1}{100 \mathrm{C}}=\mathrm{BW}=500 \Rightarrow \mathrm{C}=20 \mu \mathrm{~F} \\
\frac{1}{\sqrt{\left(20 \cdot 10^{-6}\right) \mathrm{L}}}=\omega_{0}=2500 \Rightarrow \mathrm{~L}=8 \mathrm{mH}
\end{gathered}
$$

P13.5-6

$$
\begin{aligned}
& \mathrm{R}=\frac{1}{\mathrm{Y}\left(\omega_{0}\right)}=100 \Omega \\
& \frac{100 \Omega}{\mathrm{~L}}=\mathrm{BW}=500 \Rightarrow \mathrm{~L}=0.2 \mathrm{H} \\
& \frac{1}{\sqrt{(0.2) \mathrm{C}}}=\omega_{0}=2500 \Rightarrow \mathrm{C}=0.8 \mu \mathrm{~F}
\end{aligned}
$$

P13.5-7


$$
\begin{aligned}
Y(\omega) & =j \omega C+\frac{1}{R_{1}+j \omega L}+\frac{1}{R_{2}} \\
& =\frac{\left(R_{1}+R_{2}-\omega^{2} C L R_{2}\right)+j \omega\left(L+C R_{1} R_{2}\right)}{R_{2}\left(R_{1}+j \omega L\right)} \times \frac{R_{1}-j \omega L}{R_{1}-j \omega L} \\
& =\frac{R_{1}\left(R_{1}+R_{2}-\omega^{2} C L R_{2}\right)-\omega^{2} L\left(L+C R_{1} R_{2}\right)+j \omega R_{1}\left(L C R_{1} R_{2}\right)-j \omega L\left(R_{1}+R_{2}-\omega^{2} C L R_{2}\right)}{R_{2}\left(R_{1}-\omega^{2} L^{2}\right)}
\end{aligned}
$$

$\omega=\omega_{0}$ is the frequency at which the imaginary part of $\mathrm{Y}(\omega)$ is zero:

$$
\begin{aligned}
& \mathrm{R}_{1}\left(\mathrm{LCR}_{1} \mathrm{R}_{2}\right)-\mathrm{L}\left(\mathrm{R}_{1}+\mathrm{R}_{2}-\omega_{0}^{2} \mathrm{CLR}_{2}\right)=0 \\
& \omega_{0}=\sqrt{\frac{\mathrm{LR}_{2}-\mathrm{CR}_{1}^{2} \mathrm{R}_{2}}{\mathrm{CL}^{2} \mathrm{R}_{2}}}=12.9 \mathrm{M} \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

P13.5-8

(a) Using voltage divider

$$
\begin{aligned}
& V_{o}=\left(1000 \angle 0^{\circ}\right) \frac{\frac{(100)(-j 100)}{100-j 100}}{\frac{(100)(-j 100)}{100-j 100}+j 100}=\left(1000 \angle 0^{\circ}\right) \frac{100 / \sqrt{2} \angle-135^{\circ}}{100 / \sqrt{2} \angle-135^{\circ}+j 100}=\frac{10^{5} / \sqrt{2} \angle-135^{\circ}}{50 \sqrt{2} \angle-135^{\circ}}=1000 \angle 90^{\circ} \\
& \therefore\left|V_{o}\right|=1000 V
\end{aligned}
$$

(b) Do a source transformation to obtain


So have an RLC resonant circuit with $\omega_{0}=1 / \sqrt{\mathrm{LC}}=400$
Therefore the circuit is operated at resonance. This means that the $\mathrm{L}-\mathrm{C}$ parallel combination has an overall $\mathrm{Z}=\infty$ and hence $I_{s}=I_{o}$. When $R_{L}$ is suddenly changed from $100 \Omega$ to $1 \mathrm{k} \Omega$, due to the resonance condition, $V_{o}=R_{L} I_{o}$ suddenly increases by a factor of 10 . This in turn causes very large (equal \& opposite) currents in the $\mathrm{L} \& \mathrm{C}$ as the capacitor voltage is forced to abruptly change. Thus very large currents radiate electromagnetic radition and thus see sparks. Clearly we need a variable capacitor that varies as $\mathrm{R}_{\mathrm{L}}$ varies such that when $\mathrm{R}_{\mathrm{L}} \neq 100 \Omega$, $\Omega \neq \Omega_{\mathrm{o}}$ and thus have $\mathrm{V}_{\mathrm{o}}=\mathrm{R}_{\mathrm{L}} \mathrm{I}_{\mathrm{o}}$ constant while both $\mathrm{R}_{\mathrm{L}}$ and $\mathrm{I}_{\mathrm{o}}$ change abruptly.

## P13.5-9



$$
\begin{aligned}
\underset{\sim}{Z} & =R_{1}+j \omega L+\frac{1}{G_{2}+j \omega C} ; G_{2}=1 / R_{2} \\
& =\frac{\left(R_{1} G_{2}+1-\omega^{2} L C\right)+j\left(\omega L G_{2}+\omega C R_{1}\right)}{G_{2}+j \omega C}
\end{aligned}
$$

at resonance $\underset{\sim}{Z}=Z \angle 0^{\circ}$

$$
\begin{aligned}
& \text { or } \tan ^{-1} \frac{\omega \mathrm{LG}_{2}+\omega C R_{1}}{\left(\mathrm{R}_{1} \mathrm{G}_{2}+1-\omega^{2} \mathrm{LC}\right)}=\tan ^{-1} \frac{\omega \mathrm{C}}{\mathrm{G}_{2}} \\
& \text { thus } \frac{\omega \mathrm{LG}_{2}+\omega C R_{1}}{\left(\mathrm{R}_{1} \mathrm{G}_{2}+1-\omega^{2} L C\right)}=\frac{\omega C}{G_{2}} \Rightarrow \omega^{2}=\frac{\mathrm{C}-\mathrm{LG}_{2}^{2}}{L C^{2}} \& C>\mathrm{G}_{2}^{2} L
\end{aligned}
$$

with $\mathrm{R}_{1}=\mathrm{R}_{2}=1 \Omega$ and $\omega_{0}=100 \mathrm{rad} / \mathrm{s}$

$$
\omega_{0}^{2}=10^{4}=\frac{\mathrm{C}-\mathrm{L}}{\mathrm{LC}^{2}}, \text { if } \underline{\mathrm{C}=10 \mathrm{mF}} \Rightarrow \underline{\mathrm{~L}=5 \mathrm{mH}}
$$

and $\mathrm{C}>\mathrm{G}_{2}^{2} \mathrm{~L} \quad$ checks

P13.5-10

(a) $\quad Z_{\text {in }}=j \omega L+\frac{R / j \omega C}{R+1 / j \omega C}$

$$
Z_{i n}=\frac{\left(R-\omega^{2} R L C\right)+j \omega L}{1+j \omega R C}
$$

$$
\therefore\left|Z_{\text {in }}\right|=\sqrt{\frac{\left(\mathrm{R}-\omega^{2} \mathrm{RLC}\right)^{2}+(\omega \mathrm{L})^{2}}{1+(\omega R C)^{2}}}
$$

(b)

(c) at $\omega=1 / \sqrt{L C},\left|Z_{\text {in }}\right|=\frac{1}{\sqrt{(C / L)\left(1+R^{2} \mathrm{C} / \mathrm{L}\right)}}$

P13.5-11

$$
\begin{aligned}
& \text { Let } \mathrm{V}(\omega)=\mathrm{A} \angle \theta^{\circ}=\mathrm{A} \text { and } \mathrm{V}_{2}(\omega)=\mathrm{B} \angle \theta \\
& \begin{aligned}
& \mathrm{Y}(\omega)=\frac{\mathrm{I}(\omega)}{\mathrm{V}(\omega)}=\frac{\frac{\mathrm{V}(\omega)-\mathrm{V}_{2}(\omega)}{\mathrm{R}}}{\mathrm{~V}(\omega)}=\frac{\mathrm{A}-\mathrm{B} \angle \theta}{\mathrm{AR}} \\
&=\frac{\mathrm{A}-\mathrm{B} \cos \theta-j \mathrm{~B} \sin \theta}{A R} \\
&|\mathrm{Y}(\omega)|=\frac{\sqrt{(\mathrm{A}-\mathrm{B} \cos \theta)^{2}+(\mathrm{B} \sin \theta)^{2}}}{A R}
\end{aligned}
\end{aligned}
$$

PSpice Problems
SP 13-1


This simulation shows that the gain is

$$
g=600 \times 10^{-3}=0.6
$$

at the frequency

$$
\mathrm{w}=2 \pi \times 3.1823=20 \mathrm{rad} / \mathrm{sec}
$$



The phase shift is -45 at the frequency
$\omega=2 \pi \times 3.1831=20 \mathrm{rad} / \mathrm{sec}$
as required.

## SP 13-3



This simulation shows that the gain of the circuit is 0.1 at 10 Hz and 0.995 at $10,000 \mathrm{~Hz}$.

To satisfy the specifications on the corner frequencies, the gain must be $\leq 1.414$ at 200 Hz and $\geq 7.07$ at 2000 Hz . Both conditions are met.

The circuit satisfies the specifications.

SP 13-4


This simulation shows

- The high-frequency gain is $33.928 \cong 34 \mathrm{~dB}$
- The slope of the low-frequency asymptote is

$$
\frac{16 \mathrm{~dB}-(-4 \mathrm{~dB})}{\text { decade }}=20 \mathrm{~dB} / \text { decade }
$$

## SP 13-5



The peak of the frequency response is $72 \mathrm{~dB}=4000$ at $2.25 \mathrm{~Hz}=14130 \mathrm{rad} / \mathrm{sec}$. So $\mathrm{k}=4000$ and $\omega_{0}=14130$ $\omega_{1}$ and $\omega_{2}$ are identified as the frequencies where the gain is $72-3=69 \mathrm{~dB}$
$\mathrm{Q}=\frac{\omega_{0}}{\omega_{2}-\omega_{1}}=\frac{\mathrm{f}_{0}}{\mathrm{f}_{2}-\mathrm{f}_{1}}=\frac{2250}{2332.4-2172.1}=14$

$R 3$ and the dependent source $E 1$ model an ideal op amp. $V(R 4: 1)$ is $v_{s}(t) . V(R 7: 1)$ is $v_{0}(t) . V(V 6: t)$ is the answer given in this solution manual. After the transient part dies out, $\mathrm{V}(\mathrm{R} 7: 1)$ is identical to $\mathrm{V}(\mathrm{V} 6: \mathrm{t})$. The answer is correct.

SP 13-7


SP 13-8


|  |  |  |  |  |  | (A) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V1 | 1 | 0 | ac | 1. ${ }^{1}$ |  |  |
| R2 | 1 | 2 |  | 1.35 k | 24 にー |  |
| L3 | 2 | 3 |  | 500 m | ' |  |
| R4 | 3 | 0 |  | 47k | , |  |
| E5 | 4 | 0 | 3 | 010 | , |  |
| R6 | 4 | 6 |  | 1 | , |  |
| R7 | 4 | 5 |  | 213m | ' |  |
| C8 | 5 | 6 |  | 353u | ; |  |
| C9 | 6 | 0 |  | 3.53 m | ; |  |
| . ac dec 502020 k |  |  |  |  | ' |  |
| . probe |  |  |  |  | -25 |  |
| . End |  |  |  |  | $\begin{aligned} & 2 \mathrm{EHz} \\ & \square \mathrm{Udb}(6) \end{aligned}$ | $\text { 1. } 6 \mathrm{KHz}$ |
|  |  |  |  |  |  | Frequency |

## SP 13-9





## Verification Problems

## VP 13-1

When $\omega<630 \mathrm{rad} / \mathrm{sec}, \mathrm{T}(\omega) \simeq \frac{1}{10}$ which agrees with the tabulated values of $|\mathrm{T}(\omega)|$ corresponding to $\omega=200$ and $400 \mathrm{rad} / \mathrm{sec}$.

When $\omega>6300 \mathrm{rad} / \mathrm{sec}, \mathrm{T}(\omega) \simeq 1$ which agrees with the tabulated values of $|\mathrm{T}(\omega)|$ corresponding to $\omega=12600,25000,50000$ and $100000 \mathrm{rad} / \mathrm{sec}$.

At $\omega=630$ we expect $|\mathrm{T}(\omega)|=-3 \mathrm{~dB}=0.707$. This agrees with the tabulated value of $|\mathrm{T}(\omega)|$ corresponding to $\omega=6310$.

At $\omega=630$ we expect $|T(\omega)|=-20+3=-17 \mathrm{~dB}=0.14$ which agrees with the tabulated values of $|\mathrm{T}(\omega)|$ corresponding to $\omega=400$ and $795 \mathrm{rad} / \mathrm{s}$.

This data does seem reasonable.

VP 13-2

$$
\mathrm{BW}=\frac{\omega_{0}}{\mathrm{Q}}=\frac{10,000}{70}=143 \neq 71.4 \mathrm{rad} / \mathrm{s}
$$

This report is not correct.

VP 13-3

$$
\begin{aligned}
\omega_{0} & =\frac{1}{\sqrt{\mathrm{LC}}}=10 \mathrm{k} \mathrm{rad} / \mathrm{s}=1.59 \mathrm{kHz} \\
\mathrm{Q} & =\frac{1}{\mathrm{R}} \sqrt{\frac{\mathrm{~L}}{\mathrm{C}}}=20 \\
\mathrm{BW} & =\frac{\mathrm{R}}{\mathrm{~L}}=500 \mathrm{rad} / \mathrm{s}=79.6 \mathrm{~Hz}
\end{aligned}
$$

The reported results are correct.

## VP 13-4

The network function indicates a zero at $200 \mathrm{rad} / \mathrm{s}$ and a pole at $800 \mathrm{rad} / \mathrm{s}$. In contrast, the Bode plot indicates a pole at $200 \mathrm{rad} / \mathrm{s}$ and a zero at $800 \mathrm{rad} / \mathrm{s}$.

The Bode plot and network function don't correspond to each other.
where

$$
\begin{aligned}
\mathrm{k} & =\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \\
\mathrm{Z} & =\frac{1}{\mathrm{C}_{1} \mathrm{R}_{1}} \\
\mathrm{p} & =\frac{1}{\mathrm{C}_{2} \mathrm{R}_{2}}
\end{aligned}
$$

## Design Problems

DP 13-1 Pick an appropriate circuit from Table 13.4-2.


The specifications indicate that

$$
\begin{aligned}
& 2=\mathrm{k}=\frac{R_{2}}{R_{1}}, 5=\mathrm{k} \frac{p}{z}=\frac{C_{1}}{C_{2}}, \\
& 2 \pi \cdot 1000<z=\frac{1}{C_{1} R_{1}} \text { and } 2 \pi \cdot 10,000>p=\frac{1}{C_{2} R_{2}}
\end{aligned}
$$

Try $z=2 \pi \cdot 2000 \mathrm{rad} / s$. Pick $C_{1}=0.05 \mu F$. Then

$$
R_{1}=\frac{1}{C_{1} z}=1.592 \mathrm{k} \Omega, \quad R_{2}=2 R_{1}=3.183 \mathrm{k} \Omega, \quad C_{2}=\frac{C_{1}}{k \frac{p}{k}}=0.01 \mu \mathrm{~F}
$$

Check: $\mathrm{p}=\frac{1}{C_{2} R_{2}}=31.42 \mathrm{krad} / \mathrm{s}<2 \pi \cdot 10,000$

DP 13-2


$$
\begin{aligned}
\mathrm{H}(\omega) & =\frac{\mathrm{V}_{\mathrm{o}}(\omega)}{\mathrm{V}_{\mathrm{s}}(\omega)} \\
& =\frac{1}{\mathrm{j} \omega \mathrm{C}} \| \mathrm{R}
\end{aligned}
$$

$$
H(\omega)=\frac{V_{o}(\omega)}{V_{s}(\omega)}=\frac{\frac{1}{j \omega C} \| R}{j \omega L+\left(\frac{1}{j \omega C} \| R\right)}=\frac{\frac{R}{1+j \omega C R}}{j \omega L+\frac{R}{1+j \omega C R}}=\frac{\frac{1}{L C}}{-\omega^{2}+j \omega \frac{1}{R C}+\frac{1}{L C}}
$$

Pick $\frac{1}{\sqrt{\mathrm{LC}}}=\omega_{0}=2 \pi\left(100 \cdot 10^{3}\right) \mathrm{rad} / \mathrm{s}$. When $\omega=\omega_{0}$

$$
\mathrm{H}_{0}(\omega)=\frac{\frac{1}{\mathrm{LC}}}{-\frac{1}{\mathrm{LC}}+\mathrm{j} \frac{1}{\sqrt{\text { LC }} \frac{1}{\mathrm{RC}}+\frac{1}{\mathrm{LC}}}}
$$

So

$$
\left.\mid \mathrm{H}_{( } \omega_{0}\right) \left\lvert\,=\mathrm{R} \sqrt{\frac{\mathrm{C}}{\mathrm{~L}}}\right.
$$

We require

$$
-3 \mathrm{~dB}=0.707=\left|\mathrm{H}_{\left(\omega_{0}\right)}\right|=\mathrm{R} \sqrt{\frac{\mathrm{C}}{\mathrm{~L}}}=1000 \sqrt{\frac{\mathrm{C}}{\mathrm{~L}}}
$$

Finally

$$
\left.\begin{array}{l}
\frac{1}{\sqrt{\mathrm{LC}}}=2 \pi\left(100 \cdot 10^{3}\right) \\
0.707=1000 \sqrt{\frac{\mathrm{C}}{\mathrm{~L}}}
\end{array}\right\} \Rightarrow \begin{gathered}
\mathrm{C}=1.13 \mathrm{nF} \\
\mathrm{~L}=2.26 \mathrm{mH}
\end{gathered}
$$

DP 13-3

$\mathrm{R}_{1}=10 \mathrm{k} \Omega$
$\mathrm{R}_{2}=866 \mathrm{k} \Omega$
$\mathrm{R}_{3}=8.06 \mathrm{k} \Omega$
$\mathrm{R}_{4}=1 \mathrm{M} \Omega$
$\mathrm{R}_{5}=2.37 \mathrm{M} \Omega$
$\mathrm{R}_{6}=499 \mathrm{k} \Omega$
$\mathrm{C}_{1}=0.47 \mu \mathrm{~F}$
$\mathrm{C}_{2}=0.1 \mu \mathrm{~F}$
ckt A is inverting summer

$$
\begin{equation*}
\Rightarrow \mathrm{V}_{\mathrm{s}}=-\mathrm{R}_{3}\left\lfloor\frac{\mathrm{~V}_{\mathrm{in}}}{\mathrm{R}_{1}}+\frac{\mathrm{V}_{\mathrm{f}}}{\mathrm{R}_{\mathrm{z}}}\right\rfloor \tag{1}
\end{equation*}
$$

$\Rightarrow \mathrm{V}_{\mathrm{o}}=-\mathrm{V}_{\mathrm{s}} \frac{\mathrm{Z}_{\mathrm{f}}}{\mathrm{Z}_{\mathrm{i}}}=-\mathrm{V}_{\mathrm{s}} \frac{\mathrm{R}_{5}}{\mathrm{R}_{4}}\left[\frac{1}{\mathrm{C}_{1} \mathrm{R}_{5} \mathrm{~s}+1}\right]$
$\Rightarrow \mathrm{V}_{\mathrm{f}}=-\frac{1}{\mathrm{C}_{\mathrm{s}} \mathrm{R}_{6} \mathrm{~s}} \mathrm{~V}_{\mathrm{o}}$
Continued

Solving (1) $\rightarrow$ (3) for $V_{o} / V_{\text {in }}$ yields

$$
V_{o} / V_{\text {in }}=\frac{\frac{R_{3}}{R_{1} R_{4} C_{1}} s}{s^{2}+\frac{1}{R_{5} C_{1}} s+\frac{R_{3}}{R_{2} R_{4} R_{6} C_{1} C_{2}}}
$$

plugging in the values for the resistors \& capacitors, can draw



$H_{1}(\omega)=-K_{1} \frac{j \omega}{1+j \frac{\omega}{p_{1}}}$
where

$$
\mathrm{K}_{1}=\mathrm{R}_{2} \mathrm{C}_{1}, \mathrm{p}_{1}=\frac{1}{\mathrm{C}_{1} \mathrm{R}_{1}}
$$

We require

$$
\begin{aligned}
& 10=-\mathrm{K}_{1} \mathrm{~K}_{2}=\mathrm{R}_{2} \mathrm{C}_{1} \frac{\mathrm{R}_{4}}{\mathrm{R}_{3}} \\
& 200=\mathrm{p}_{1}=\frac{1}{\mathrm{R}_{1} \mathrm{C}_{1}} \\
& 500=\mathrm{p}_{2}=\frac{1}{\mathrm{C}_{2} \mathrm{R}_{4}}
\end{aligned}
$$

Pick $\mathrm{C}_{1}=1 \mu \mathrm{~F}$. Then $\mathrm{R}_{1}=\frac{1}{\mathrm{p}_{1} \mathrm{C}_{1}}=5 \mathrm{k} \Omega$
Pick $\mathrm{C}_{2}=0.1 \mu \mathrm{~F}$. Then $\mathrm{R}_{4}=\frac{1}{\mathrm{p}_{2} \mathrm{C}_{2}}=20 \mathrm{k} \Omega$
Next $10=\frac{\mathbf{R}_{2}}{R_{3}}\left(10^{-6}\right)\left(20 \cdot 10^{3}\right) \Rightarrow \frac{R_{2}}{R_{3}}=500$
Let $R_{2}=500 \mathrm{k} \Omega$ and $\mathrm{R}_{3}=1 \mathrm{k} \Omega$

DP 13-5


$$
H_{1}(\omega)=-K_{1} \frac{j \omega}{1+j \frac{\omega}{p_{1}}}
$$

where

$$
\mathrm{K}_{1}=\mathrm{R}_{2} \mathrm{C}_{1}, \mathrm{p}_{1}=\frac{1}{\mathrm{C}_{1} \mathrm{R}_{1}}
$$

We require

$$
\begin{aligned}
& 20 \mathrm{~dB}=10=-\mathrm{K}_{1} \mathrm{~K}_{2}=\mathrm{R}_{2} \mathrm{C}_{1} \frac{\mathrm{R}_{4}}{\mathrm{R}_{3}} \\
& 0.1=\mathrm{p}_{1}=\frac{1}{\mathrm{R}_{1} \mathrm{C}_{1}} \\
& 100=\mathrm{p}_{2}=\frac{1}{\mathrm{R}_{4} \mathrm{C}_{2}}
\end{aligned}
$$

Pick $\mathrm{C}_{1}=20 \mu \mathrm{~F}$. Then $\mathrm{R}_{1}=\frac{1}{\mathrm{p}_{1} \mathrm{C}_{1}}=500 \mathrm{k} \Omega$
Pick $\mathrm{C}_{2}=1 \mu \mathrm{~F}$. Then $\mathrm{R}_{4}=\frac{1}{\mathrm{p}_{2} \mathrm{C}_{2}}=10 \mathrm{k} \Omega$
Next

$$
10=\frac{\mathrm{R}_{2}}{\mathrm{R}_{3}}\left(20 \cdot 10^{-6}\right)\left(10 \cdot 10^{3}\right) \Rightarrow \frac{\mathrm{R}_{2}}{\mathrm{R}_{3}}=50
$$

Let $\mathrm{R}_{2}=200 \mathrm{k} \Omega$ and $\mathrm{R}_{3}=4 \mathrm{k} \Omega$

## DP 13-6

The network function of this circuit is

$$
T(\omega)=\frac{1+\frac{R_{2}}{R_{3}}}{1+j \omega R_{1} C}
$$

The phase shift of this network function is $\quad \theta=-\tan ^{-1} \omega R_{1} C$

The gain of this network function is

$$
\mathrm{G}=\frac{1+\frac{\mathrm{R}_{3}}{\mathrm{R}_{2}}}{\sqrt{1+\left(\omega \mathrm{R}_{1} \mathrm{C}\right)^{2}}}=\frac{1+\frac{\mathrm{R}_{3}}{\mathrm{R}_{2}}}{\sqrt{1+(\tan \theta)^{2}}}
$$

Design of this circuit proceeds as follows. Since the frequency and capacitance are known, $\mathrm{R}_{1}$ is calculated from

$$
\mathrm{R}_{1}=\frac{\tan (-\theta)}{\omega \mathrm{C}}
$$

Next pick $\mathrm{R}_{2}=10 \mathrm{k} \Omega$ (a convenient value) and calculated $\mathrm{R}_{3}$ using

$$
\begin{aligned}
\mathrm{R}_{3} & =\left(\mathrm{G} \cdot \sqrt{1+(\tan \theta)^{2}}-1\right) \cdot \mathrm{R}_{2} \\
\theta=-45 \mathrm{deg}, \mathrm{G}=2, \omega=1000 \mathrm{rad} / \mathrm{s} \Rightarrow \mathrm{R}_{1} & =10 \mathrm{k} \Omega, \mathrm{R}_{2}=10 \mathrm{k} \Omega, \mathrm{R}_{3}=18.284 \mathrm{k} \Omega, \mathrm{C}=0.1 \mu \mathrm{~F}
\end{aligned}
$$

DP 13-7 From Table 13.4-2 and the Bode plot:

$$
\begin{aligned}
800 & =\mathrm{z}=\frac{1}{\mathrm{R}_{1}\left(0.5 \times 10^{-6}\right)} \Rightarrow \mathrm{R}_{1}=2.5 \mathrm{k} \Omega \\
32 \mathrm{~dB} & =40=\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \Rightarrow \mathrm{R}_{2}=100 \mathrm{k} \Omega \\
200 & =\mathrm{p}=\frac{1}{\mathrm{R}_{2} \mathrm{C}} \Rightarrow \mathrm{C}=\frac{1}{(200)(100 \mathrm{k} \Omega)}=0.05 \mu \mathrm{~F} \\
20 \mathrm{~dB} & =10=\mathrm{k} \frac{\mathrm{p}}{\mathrm{z}}=\frac{0.5 \mu \mathrm{~F}}{\mathrm{C}}=\frac{0.5 \mu \mathrm{~F}}{0.05 \mu \mathrm{~F}}
\end{aligned}
$$

## DP 13-8

$$
\begin{aligned}
H(\omega)= & \frac{-R_{2}}{1+\frac{1}{j \omega C}}=-\frac{j \omega \mathrm{CR}_{2}}{1+j \omega C R_{1}} \\
195^{\circ}= & 180+90-\tan ^{-1} \omega C R_{1} \\
& \Rightarrow R_{1}=\frac{\tan (270-195)}{(1000)\left(0.1 \times 10^{-6}\right)}=37.3 \mathrm{k} \Omega \\
10= & \lim _{w \rightarrow \infty}|H(\omega)|=\frac{R_{2}}{R_{1}} \Rightarrow R_{2}=10 R_{1}=373 \mathrm{k} \Omega
\end{aligned}
$$

