Pick the appropriate circuit from Table 13.4-2.



We require

$$200 = p_1 = \frac{1}{C_1 R_1}$$
  

$$500 = p_2 = \frac{1}{C_2 R_2}$$
  

$$34dB = 50 = k = C_1 R_2$$
  
Pick  $C_1 = 1\mu$ F. Then  $R_1 = 5k\Omega$ ,  $R_2 = 50k\Omega$  and  $C_2 = 0.04\mu$ F.





## P13.4-18

$$H_{1,2} = -\frac{R_2/R_1}{1+j\omega R_2 C} \Rightarrow H_{Total} = H_{1,2}^2 = (R_2/R_1)^2 \left(\frac{1}{1+j\omega R_2 C}\right)^2$$

(a) At low frequency,  

$$H_T = (R_2/R_1)^2$$
  $\therefore$  for  $H_T = 1$ , need  $R_1 = R_2$   
Now also  $\omega_1 = 1000 = 1/R_2C$ , so let  $C = 1\mu F$   
 $\therefore R_1 = R_2 = 1/(1000) (10^{-6}) = 1k\Omega$ 

(b) At 
$$\omega = 10,000$$
  
 $|\mathbf{H}| = \frac{1}{1 + (\omega R_2 C)^2} = \frac{1}{1 + [(10^4)(10^3)(10^{-6})]^2} = 10^{-2}$   
 $\Rightarrow 20 \log |\mathbf{H}| = 20 \log 10^{-2} = -40 \mathrm{dB}$ 



(a) 
$$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = -\frac{R_2/R_1}{1+j\omega R_2 C} = -\frac{10}{1+j\frac{\omega}{10,000}}$$
  
(b) 10=20dB  
(c) 10,000 rad/sec

P13.4-20



Assume  $V_i \simeq 0$ , then  $V_+ = V_- = V_o$ Use  $s = j\omega$ 

Voltage divider yields :  $V_o = V_1 \frac{1/sC}{R+1/sC} \Rightarrow V_1 = (1+sRC)V_0$ KCL at  $V_1$ :  $(V_1-V_s)/mR + (V_1-V_o)/R + (V_1-V_o)snC = 0$ 

Plugging V<sub>1</sub> into above yields

$$V_{o}\left[\frac{1}{mR} + sC + \frac{sC}{m} + s^{2}nRC^{2}\right] = \frac{V_{s}}{mR}$$
  

$$\therefore \frac{V_{o}}{V_{s}} = \frac{1}{1 + s(m+1)RC + nmR^{2}C^{2}s^{2}}$$
  

$$\Rightarrow H(j\omega) = \frac{V_{o}}{V_{s}} = \frac{1}{1 - (\omega/\omega_{o})^{2} + j(\omega/Q\omega_{o})} \text{ where } \omega_{0} = \frac{1}{\sqrt{mn}RC}$$
  

$$Q = \frac{\sqrt{mn}}{2}$$



P13.4-21  $R_1$   $R_2$   $V_0$  +  $V_{S}(\omega)$   $W_{S}(\omega)$   $W_{S}(\omega)$  $W_{S$ 

$$\left. \begin{array}{l} V_{o}(\omega) = \displaystyle \frac{1}{j\omega C_{2}} \\ R + \displaystyle \frac{1}{j\omega C_{2}} \\ V_{a}(\omega) \\ 0 = \displaystyle \frac{V_{a}(\omega) - V_{s}(\omega)}{R_{1}} + j\omega C_{1}(V_{a}(\omega) - V_{o}(\omega)) \end{array} \right\} \Rightarrow V_{o}(1 + j\omega C_{1}R_{1})(1 + j\omega C_{2}R_{2}) = j\omega C_{1}R_{1}V_{o} + V_{s}$$

$$T(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{1}{1 + C_2 R_2 j \omega - \omega^2 C_1 C_2 R_1 R_2} = \frac{1}{-\omega^2 + .8 j \omega + 1}$$
  
Second order poles with  $\omega_o = 0$  and  $\delta = .4$ 



Section 13-5: Resonant Circuits

P13.5-1

$$\omega_{0} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(\frac{1}{120}\right)\left(\frac{1}{30} \times 10^{-6}\right)}} = 60 \text{ k rad/sec}$$

$$Q = R\sqrt{\frac{C}{L}} = 10,000 \sqrt{\frac{\frac{1}{30} \times 10^{-6}}{\frac{1}{120}}} = 20$$

$$\omega_{1} = -\frac{\omega_{0}}{2Q} + \sqrt{\left(\frac{\omega_{0}}{2Q}\right)^{2} + \omega_{0}^{2}} = 58.52 \text{ k rad/s}$$

$$\omega_{2} = \frac{\omega_{0}}{2Q} + \sqrt{\left(\frac{\omega_{0}}{2Q}\right)^{2} + \omega_{0}^{2}} = 61.52 \text{ k rad/s}$$

$$BW = \frac{1}{RC} = \frac{1}{(10000)\left(\frac{1}{30} \times 10^{-6}\right)} = 3 \text{ k rad/s}$$
Notice that BW =  $\omega_{2} - \omega_{1} = \frac{\omega_{0}}{Q}$ 

## P13.5-2

$$R = k = |H(\omega_0)| = \frac{8}{20 \cdot 10^{-3}} = 400\Omega$$
  

$$\omega_0 = 1000 \text{ rad/s}$$
  

$$|H(\omega)| = \frac{k}{\sqrt{1 + Q^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$$
  
At  $\omega = 897.6 \text{ rad/s}, |H(\omega)| = \frac{4}{20.10^{-3}} = 200, \text{ so}$   

$$200 = \frac{400}{\sqrt{1 + Q^2 \left(\frac{897.6}{1000} - \frac{1000}{897.6}\right)^2}} \implies Q = 8$$

Now

$$\frac{1}{\sqrt{LC}} = \omega_0 = 1000$$
  
$$400\sqrt{\frac{C}{L}} = Q = 8$$
$$\Rightarrow \begin{array}{c} C = 20\mu F \\ L = 50 \text{ mH} \end{array}$$

P13.5-4

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4 \text{ rad/s}, Q = \frac{1}{R} \sqrt{\frac{L}{C}} = 10, BW = \frac{R}{L} = 10^3 \text{ rad/s}$$

P13.5-5 
$$R = Z(\omega_0) = 100\Omega$$
$$\frac{1}{100C} = BW = 500 \Rightarrow C = 20\mu F$$
$$\frac{1}{\sqrt{(20 \cdot 10^{-6})L}} = \omega_0 = 2500 \Rightarrow L = 8mH$$

P13.5-6

$$R = \frac{1}{Y(\omega_0)} = 100\Omega$$
$$\frac{100\Omega}{L} = BW = 500 \implies L = 0.2H$$
$$\frac{1}{\sqrt{(0.2)C}} = \omega_0 = 2500 \implies C = 0.8\mu F$$



 $\omega = \omega_0$  is the frequency at which the imaginary part of Y( $\omega$ ) is zero:

$$R_{1}(LCR_{1}R_{2}) - L(R_{1} + R_{2} - \omega_{0}^{2}CLR_{2}) = 0$$
  
$$\omega_{0} = \sqrt{\frac{LR_{2} - CR_{1}^{2}R_{2}}{CL^{2}R_{2}}} = 12.9 \text{ M rad/sec}$$

P13.5-8

$$1000 \underline{10^{\circ}} \oplus -j^{100} \oplus R_{L} \neq V_{0}$$

$$\omega = 400$$

(a) Using voltage divider

$$V_{o} = (1000 \angle 0^{\circ}) \frac{\frac{(100)(-j100)}{100-j100}}{\frac{(100)(-j100)}{100-j100} + j100} = (1000 \angle 0^{\circ}) \frac{100/\sqrt{2} \angle -135^{\circ}}{100/\sqrt{2} \angle -135^{\circ} + j100} = \frac{10^{5}/\sqrt{2} \angle -135^{\circ}}{50\sqrt{2} \angle -135^{\circ}} = 1000 \angle 90^{\circ}$$
  
$$\therefore |V_{o}| = 1000V$$

(b) Do a source transformation to obtain

$$I_{s} = 10 - \frac{1}{2} + L = C - R_{L} = V_{o}$$

т.

So have an RLC resonant circuit with  $\omega_0 = 1/\sqrt{LC} = 400$ 

Therefore the circuit is operated at resonance. This means that the L-C parallel combination has an overall  $Z = \infty$ and hence  $I_s = I_o$ . When  $R_L$  is suddenly changed from 100 $\Omega$  to 1 k $\Omega$ , due to the resonance condition,  $V_o = R_L I_o$ suddenly increases by a factor of 10. This in turn causes very large (equal & opposite) currents in the L&C as the capacitor voltage is forced to abruptly change. Thus very large currents radiate electromagnetic radiation and thus see sparks. Clearly we need a variable capacitor that varies as  $R_L$  varies such that when  $R_L \neq 100 \Omega$ ,  $\Omega \neq \Omega_o$  and thus have  $V_o = R_L I_o$  constant while both  $R_L$  and  $I_o$  change abruptly.

P13.5-9

$$Z = R_1 + j\omega L + \frac{1}{G_2 + j\omega C}; G_2 = 1/R_2$$
$$= \frac{(R_1G_2 + 1 - \omega^2 LC) + j(\omega LG_2 + \omega CR_1)}{G_2 + j\omega C}$$

at resonance  $Z_{\sim} = Z \angle 0^{\circ}$ 

or 
$$\tan^{-1} \frac{\omega LG_2 + \omega CR_1}{\left(R_1G_2 + 1 - \omega^2 LC\right)} = \tan^{-1} \frac{\omega C}{G_2}$$
  
thus  $\frac{\omega LG_2 + \omega CR_1}{\left(R_1G_2 + 1 - \omega^2 LC\right)} = \frac{\omega C}{G_2} \Rightarrow \omega^2 = \frac{C - LG_2^2}{LC^2} \& C > G_2^2 LC^2$ 

with 
$$R_1 = R_2 = 1\Omega$$
 and  $\omega_0 = 100$  rad/s  
 $\omega_0^2 = 10^4 = \frac{C-L}{LC^2}$ , if  $\underline{C} = 10\text{mF} \Rightarrow \underline{L} = 5\text{mH}$   
and  $C > G_2^2 L$  checks





P13.5-11

Let 
$$V(\omega) = A \angle \theta^{\circ} = A$$
 and  $V_2(\omega) = B \angle \theta$   
 $Y(\omega) = \frac{I(\omega)}{V(\omega)} = \frac{\frac{V(\omega) - V_2(\omega)}{R}}{V(\omega)} = \frac{A - B \angle \theta}{AR}$   
 $= \frac{A - B \cos \theta - j B \sin \theta}{AR}$   
 $|Y(\omega)| = \frac{\sqrt{(A - B \cos \theta)^2 + (B \sin \theta)^2}}{AR}$ 

**PSpice** Problems





This simulation shows that the gain is

 $g = 600 \times 10^{-3} = 0.6$ 

at the frequency

 $w = 2\pi \times 3.1823 = 20 rad/sec$ 

SP 13-2



The phase shift is  $-45^{\circ}$  at the frequency

 $\omega = 2\pi \times 3.1831 = 20 rad/sec$ 

as required.



This simulation shows that the gain of the circuit is 0.1 at 10Hz and 0.995 at 10,000 Hz.

To satisfy the specifications on the corner frequencies, the gain must be  $\leq 1.414$  at 200 Hz and  $\geq 7.07$  at 2000 Hz. Both conditions are met.

The circuit satisfies the specifications.



This simulation shows

- The high-frequency gain is  $33.928 \cong 34$ dB
- The slope of the low-frequency asymptote is

$$\frac{16dB - (-4dB)}{decade} = 20 dB/decade$$

SP 13-5



The peak of the frequency response is 72dB = 4000 at 2.25Hz = 14130rad/sec. So k = 4000 and  $\omega_0$  = 14130  $\omega_1$  and  $\omega_2$  are identified as the frequencies where the gain is 72–3 = 69dB

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1} = \frac{2250}{2332.4 - 2172.1} = 14$$

SP 13-6



R3 and the dependent source E1 model an ideal op amp. V(R4:1) is  $v_s(t)$ . V(R7:1) is  $v_0(t)$ . V(V6:t) is the answer given in this solution manual. After the transient part dies out, V(R7:1) is identical to V(V6:t). The answer is correct.





SP 13-8





### Verification Problems

#### VP 13-1

When  $\omega < 630 \text{ rad} / \sec$ ,  $T(\omega) \approx \frac{1}{10}$  which agrees with the tabulated values of  $|T(\omega)|$  corresponding to  $\omega = 200$  and 400 rad/sec.

When  $\omega > 6300 \text{ rad} / \text{sec}$ ,  $T(\omega) \approx 1$  which agrees with the tabulated values of  $|T(\omega)|$  corresponding to  $\omega = 12600, 25000, 50000 \text{ and } 100000 \text{ rad/sec}$ .

At  $\omega = 630$  we expect  $|T(\omega)| = -3dB = 0.707$ . This agrees with the tabulated value of  $|T(\omega)|$  corresponding to  $\omega = 6310$ .

At  $\omega$ =630 we expect  $|T(\omega)| = -20 + 3 = -17$ dB = 0.14 which agrees with the tabulated values of  $|T(\omega)|$  corresponding to  $\omega$  =400 and 795 rad/s.

This data does seem reasonable.

**VP 13-2** BW=
$$\frac{\omega_0}{Q} = \frac{10,000}{70} = 143 \neq 71.4 \text{ rad/s}$$
 This report is not correct.

**VP 13-3** ω<sub>0</sub>

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10k \text{ rad/s} = 1.59 \text{ kHz}$$
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 20$$
$$BW = \frac{R}{L} = 500 \text{ rad/s} = 79.6 \text{ Hz}$$

The reported results are correct.

## **VP 13-4**

The network function indicates a zero at 200 rad/s and a pole at 800 rad/s. In contrast, the Bode plot indicates a pole at 200 rad/s and a zero at 800 rad/s.

The Bode plot and network function don't correspond to each other.

where

$$k = \frac{R_2}{R_1}$$
$$Z = \frac{1}{C_1 R_1}$$
$$p = \frac{1}{C_2 R_2}$$

р

# Design Problems

**DP 13-1** Pick an appropriate circuit from Table 13.4-2.



The specifications indicate that

$$2 = k = \frac{R_2}{R_1}, 5 = k\frac{p}{z} = \frac{C_1}{C_2},$$
  

$$2\pi \cdot 1000 < z = \frac{1}{C_1 R_1} \text{ and } 2\pi \cdot 10,000 > p = \frac{1}{C_2 R_2}$$
  
Try  $z = 2\pi \cdot 2000 \text{ rad/s. Pick } C_1 = 0.05 \mu F.$  Then  

$$R_1 = \frac{1}{C_1 z} = 1.592 \text{ k}\Omega, \quad R_2 = 2R_1 = 3.183 k\Omega, \quad C_2 = \frac{C_1}{k\frac{p}{k}} = 0.01 \mu F$$

Check :  $p = \frac{1}{C_2 R_2} = 31.42k \text{ rad/s} < 2\pi \cdot 10,000$ 



$$H(\omega) = \frac{V_{o}(\omega)}{V_{s}(\omega)} = \frac{\frac{1}{j\omega C} \|R}{j\omega L + \left(\frac{1}{j\omega C} \|R\right)} = \frac{\frac{R}{1 + j\omega CR}}{j\omega L + \frac{R}{1 + j\omega CR}} = \frac{\frac{1}{LC}}{-\omega^{2} + j\omega \frac{1}{RC} + \frac{1}{LC}}$$

Pick  $\frac{1}{\sqrt{LC}} = \omega_0 = 2\pi (100 \cdot 10^3)$  rad/s. When  $\omega = \omega_0$ 

$$H_{0}(\omega) = \frac{\frac{1}{LC}}{-\frac{1}{LC} + j\frac{1}{\sqrt{LC}}\frac{1}{RC} + \frac{1}{LC}}$$

So

$$\left| \mathbf{H}_{(\boldsymbol{\omega}_0)} \right| = \mathbf{R} \sqrt{\frac{\mathbf{C}}{\mathbf{L}}}$$

We require

$$-3dB = 0.707 = |H(\omega_0)| = R\sqrt{\frac{C}{L}} = 1000\sqrt{\frac{C}{L}}$$

Finally

$$\frac{1}{\sqrt{LC}} = 2\pi (100 \cdot 10^3)$$
$$\Rightarrow C = 1.13 nF$$
$$L = 2.26 mH$$



Solving (1)  $\rightarrow$  (3) for  $\frac{V_o}{V_{in}}$  yields  $V_{0} / V_{in} = \frac{\frac{R_{3}}{R_{1}R_{4}C_{1}}s}{s^{2} + \frac{1}{R_{5}C_{1}}s + \frac{R_{3}}{R_{2}R_{4}R_{6}C_{1}C_{2}}}$ 

plugging in the values for the resistors & capacitors, can draw



Continued



$$H_{1}(\omega) = -K_{1} \frac{j\omega}{1+j\frac{\omega}{p_{1}}}$$

where

$$K_1 = R_2 C_1, \ p_1 = \frac{1}{C_1 R_1}$$

We require

$$10 = -K_1 K_2 = R_2 C_1 \frac{R_4}{R_3}$$
$$200 = p_1 = \frac{1}{R_1 C_1}$$
$$500 = p_2 = \frac{1}{C_2 R_4}$$

Pick  $C_1 = 1\mu F$ . Then  $R_1 = \frac{1}{p_1 C_1} = 5k\Omega$ Pick  $C_2 = 0.1\mu F$ . Then  $R_4 = \frac{1}{p_2 C_2} = 20k\Omega$ Next  $10 = \frac{R_2}{R_3}(10^{-6})(20 \cdot 10^3) \Rightarrow \frac{R_2}{R_3} = 500$ Let  $R_2 = 500k\Omega$  and  $R_3 = 1k\Omega$ 

$$H_2(\omega) = \frac{K_2}{1 + j\frac{\omega}{P_2}}$$

where

$$K_2 = -\frac{R_4}{R_3}, p_2 = \frac{1}{C_2 R_4}$$



$$20dB = 10 = -K_1K_2 = R_2C_1\frac{K_4}{R_3}$$
  

$$0.1 = p_1 = \frac{1}{R_1C_1}$$
  

$$100 = p_2 = \frac{1}{R_4C_2}$$
  
Pick C<sub>1</sub> = 20µF. Then R<sub>1</sub> =  $\frac{1}{p_1C_1} = 500k\Omega$   
Pick C<sub>2</sub> = 1µF. Then R<sub>4</sub> =  $\frac{1}{p_2C_2} = 10k\Omega$ 

Next

$$10 = \frac{R_2}{R_3} (20 \cdot 10^{-6})(10 \cdot 10^3) \Rightarrow \frac{R_2}{R_3} = 50$$

Let  $R_2 = 200k\Omega$  and  $R_3 = 4k\Omega$ 

## **DP 13-6**

Design of this

The network function of this circuit is

$$T(\omega) = \frac{1 + \frac{R_2}{R_3}}{1 + j\omega R_1 C}$$

The phase shift of this network function is

$$\theta = -\tan^{-1} \omega R_1 C$$

The gain of this network function is

The gain of this network function is
$$G = \frac{1 + \frac{R_3}{R_2}}{\sqrt{1 + (\omega R_1 C)^2}} = \frac{1 + \frac{R_3}{R_2}}{\sqrt{1 + (\tan \theta)^2}}$$
Design of this circuit proceeds as follows. Since the frequency and capacitance are known, R<sub>1</sub> is calculated from

$$R_1 = \frac{\tan(-\theta)}{\omega C}$$

Next pick  $R_2 = 10k\Omega$  (a convenient value) and calculated  $R_3$  using

$$R_3 = (G \cdot \sqrt{1 + (\tan \theta)^2 - 1}) \cdot R_2$$
  
  $\theta = -45 \text{ deg}, G = 2, \omega = 1000 \text{ rad/s} \Rightarrow R_1 = 10 \text{k}\Omega, R_2 = 10 \text{k}\Omega, R_3 = 18.284 \text{ k}\Omega, C = 0.1 \mu\text{F}$ 

DP 13-7 From Table 13.4-2 and the Bode plot:

$$800 = z = \frac{1}{R_1(0.5 \times 10^{-6})} \Rightarrow R_1 = 2.5 \text{ k}\Omega$$
  

$$32dB = 40 = \frac{R_2}{R_1} \Rightarrow R_2 = 100 \text{ k}\Omega$$
  

$$200 = p = \frac{1}{R_2C} \Rightarrow C = \frac{1}{(200)(100\text{ k}\Omega)} = 0.05\mu\text{F}$$
  

$$20dB = 10 = \text{k}\frac{\text{p}}{\text{z}} = \frac{0.5\mu\text{F}}{\text{C}} = \frac{0.5\mu\text{F}}{0.05\mu\text{F}}$$

**DP 13-8** 

$$H(\omega) = \frac{-R_2}{1 + \frac{1}{j\omega C}} = -\frac{j\omega CR_2}{1 + j\omega CR_1}$$
  

$$195^{\circ} = 180 + 90 - \tan^{-1} \omega CR_1$$
  

$$\Rightarrow R_1 = \frac{\tan(270 - 195)}{(1000)(0.1 \times 10^{-6})} = 37.3 \text{ k}\Omega$$
  

$$10 = \lim_{w \to \infty} |H(\omega)| = \frac{R_2}{R_1} \Rightarrow R_2 = 10R_1 = 373 \text{ k}\Omega$$