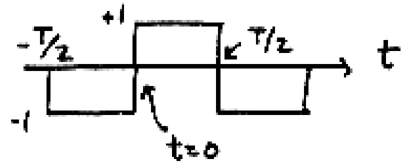
Chapter 15: – Fourier Series

Ex. 15.3-1 f(t) = K is a Fourier Series. The coefficients are a = K; a = b = 0 for n ≥ 1.

Ex. 15.3-2 f(t) = $A\cos\omega_0 t$ is a Fourier Series. $a_1 = A$ and all other coefficients are zero.

Ex. 15.4-1 Set origin at t = 0, so have an odd function; then $a_n = 0$ for n = 0,1, . . .



Note: $T = 4\left(\frac{\pi}{8}\right) = \pi/2$

Also, f(t) is half wave symmetric, then $b_n = 0$ for n = even

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin 2\pi n f_0 t\,dt = -\frac{2}{T}\int_{-T/2}^{0}\sin 2\pi n f_0 t\,dt + \frac{2}{T}\int_{0}^{T/2}\sin 2\pi n f_0 t\,dt$$

$$= \frac{4}{T}\int_0^{T/2}\sin 2\pi n f_0 t\,dt \qquad \text{where } 2\pi f_0 = \omega_0 \Rightarrow \omega_0 = \frac{2\pi}{T} = 4\ \text{rad/s}$$

$$= \frac{4}{2\pi n f_0 T}(1-\cos\pi n f_0 T) = \frac{4}{n\pi} \qquad n = 1, 3, 5, \ldots$$

so $f(t) = \frac{4}{\pi}\sum_n^N \frac{1}{n}\sin n\omega_0 t$; n odd and $\omega_0 = 4$ rad/s

Ex. 15.4-2 $T = \pi$, $\omega_0 = \frac{2\pi}{T} = 2$

odd function ⇒ quarter wave symmetric ⇒ $\begin{cases} a_0 = 0, & a_n = 0 \quad \text{for all n} \\ b_n = 0 & n = \text{even} \end{cases}$

$$b_n = \frac{8}{\pi}\int_0^{\pi/4} f(t)\sin n\omega_0 t\,dt \qquad \text{where } f(t) = \begin{cases} \dfrac{-2t}{\pi/6} & 0 < t < \pi/6 \\ -2 & \pi/6 \le t < \pi/4 \end{cases}$$

Thus $b_n = \frac{-24}{\pi^2}\frac{1}{n^2}\sin\left(\frac{n\pi}{3}\right)$

so $f(t) = \frac{-24}{\pi^2}\sum_n^N \frac{1}{n^2}\sin\left(\frac{n\pi}{3}\right)\sin(2nt)$ n odd

Ex. 15.4-3

a) f(t) is neither even nor odd. f(t) will contain both sine and cosine terms

b) $\frac{1}{4}$ wave symmetry ⇒ no even harmonics

c) average value of f(t) = 0 ⇒ $a_0 = 0$

Ex. 15.5-1 Odd function, $\omega_0 = \frac{2\pi}{T} = \pi$

$$C_n = \frac{1}{2}\int_0^2 f(t)e^{-jn\pi t}\,dt = \frac{1}{2}\int_0^1 e^{-jn\pi t}\,dt - \frac{1}{2}\int_1^2 e^{-jn\pi t}\,dt$$

$$= \frac{1}{2jn\pi}\left[-e^{-jn\pi}+1+e^{-j2n\pi}-e^{-jn\pi}\right] = \frac{1}{jn\pi}(1-e^{-jn\pi})$$

$$C_n = \begin{cases} \frac{2}{jn\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$$\text{so } f(t) = \frac{2}{j\pi}\left[e^{j\pi t}+\frac{1}{3}e^{j3\pi t}+\frac{1}{5}e^{j5\pi t}+ \dots -e^{-j\pi t}-\frac{1}{3}e^{-j3\pi t}-\frac{1}{5}e^{-j5\pi t}-\dots\right]$$

Ex. 15.5-2 Even function: $a_0 = \frac{1}{2}$, $a_n = 0$ for n even

$$T = 2\pi,\ \omega_0 = \frac{2\pi}{T} = 1 \text{ rad/s}$$

$$a_n = \frac{2}{\pi}\int_0^\pi \frac{t}{\pi}\cos nt\,dt = -\frac{4}{n^2\pi^2} \quad n \text{ odd}$$

$$\text{so } f(t) = \frac{1}{2} - \frac{4}{\pi^2}\left[\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + \dots\right]$$

Ex. 15.5-3 $$C_n = \frac{1}{T}\int_{-T/4}^{T/4} e^{-j\omega_0 nt}\,dt = \frac{1}{T}\left(\frac{T}{-j2\pi n}\right)e^{-j2\pi nt/T}\Big|_{-T/4}^{T/4} = \frac{1}{-j2\pi n}\left[e^{-j\pi n/2}-e^{j\pi n/2}\right]$$

$$C_n = \begin{cases} \frac{(-1)^{\frac{(n-1)}{2}}}{\pi n} & n \text{ odd} \\ 0 & n \text{ even},\ n\neq 0 \\ 1/2 & n=0 \end{cases}$$

Ex. 15.6-1
MathCad Spreadsheet

Setting up the index: n:=1,2. .30

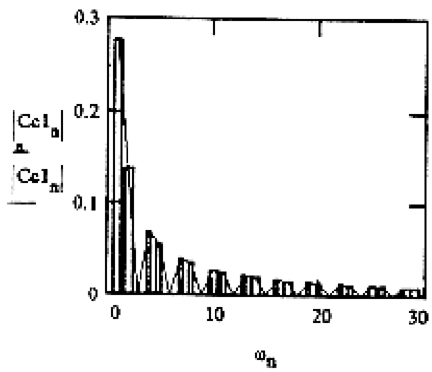
Setting the constraints: $A := K \quad T := 2\cdot\pi \quad \omega0 := \frac{2\cdot\pi}{T}$

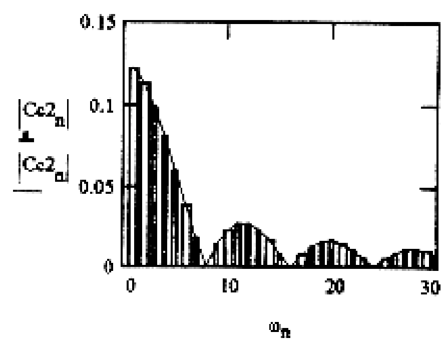
The various values of delta: $d1 : = \frac{T}{3}$ and $d2 : = \frac{T}{8}$

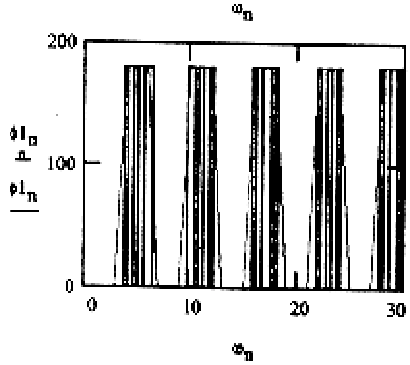
The coefficients: $Cc1_n := \frac{A\cdot d1}{T}\cdot\frac{\sin(x1_n)}{x1_n}$ where $x1_n := \frac{n\cdot\omega0\cdot d1}{2}$ also $\phi1_n := \arg(Cc1_n)\cdot\frac{180}{\pi}$

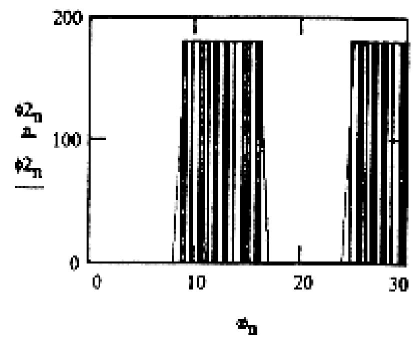
Similarily, $Cc2_n := \frac{A\cdot d2}{T}\cdot\frac{\sin(x1_n)}{x1_n}$ with $x1_n := \frac{n\cdot\omega0\cdot d2}{2}$ $\phi2_n := \arg(Cc2_n)\cdot\frac{180}{\pi}$

Now plotting, using $\omega_n := n\cdot\omega0$



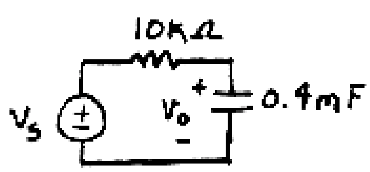






Ex. 15.8-1

$v_s(t) = 3.24\sum_{n=1}^{N}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin n\omega_0 t$



$\underline{n = 1}$ $\quad V_1 = \sin\frac{\pi}{2} \Rightarrow \sin 4t$

RC = 4s , $\omega_0 = 4\,\text{rad/s}$

$\underline{n = 3}$ $\quad V_3 = \frac{1}{9}\sin\frac{3\pi}{2} \Rightarrow -\frac{1}{9}\sin 12t$

$\underline{n = 5}$ $\quad V_5 = \frac{1}{25}\sin\frac{5\pi}{2} \Rightarrow \frac{1}{25}\sin 20t$

use n=1 also $\left|\frac{V_0}{V_s}\right| = \left[\frac{1}{1+(Rc\omega)^2}\right]^{1/2}$

at ω_0 $\left|\frac{V_{01}}{V_s}\right| = \frac{1}{\left[1+(4\omega_0)^2\right]^{1/2}} = \frac{1}{\sqrt{257}} = 0.062$ & $\phi = -\tan^{-1}(16) = -86^\circ$

$\left|\frac{V_{03}}{V_s}\right| = \frac{1/9}{\left[1+(4.12)^2\right]^{1/2}} = 0.0023$ & $\phi = -\tan^{-1}(4.12) = -88.8^\circ$

(or 3.7% of fundamental)

So use only n = 1 and n = 3 terms

$v_0 = 3.24 \times (.062) \sin(4t - 86^\circ) - 3.24(.0023) \sin(12t - 89^\circ) = \underline{0.20 \sin(4t-86^\circ) - 0.0075 \sin(12t-89^\circ)}$

Ex. 15.9-1

$$f(t) = e^{-at}u(t)$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{j\omega t} dt = \int_0^\infty e^{-at} e^{j\omega t} dt = \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_0^\infty = \frac{1}{a+j\omega}$$

Ex 15.10-1

$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at) e^{-j\omega t} dt$$

Let $\tau = at \quad \Rightarrow \quad t = \frac{\tau}{a}$

$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(\tau) e^{-j\omega\tau/a} d\frac{\tau}{a} = \frac{1}{a}\int_{-\infty}^{\infty} f(\tau) e^{-j(\omega/a)\tau} d\tau = \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

Ex 15.10-2

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(2\pi\delta(\omega)A\right) e^{j\omega t} dt = \frac{1}{2\pi}\int_{0^-}^{0^+}\left(2\pi\delta(\omega)A\right) dt = A$$

Ex. 15.11-1

$$\mathcal{F}^{-1}\{\delta(\omega-\omega_0)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega t} dt = \frac{1}{2\pi}\, e^{j\omega_0 t}$$

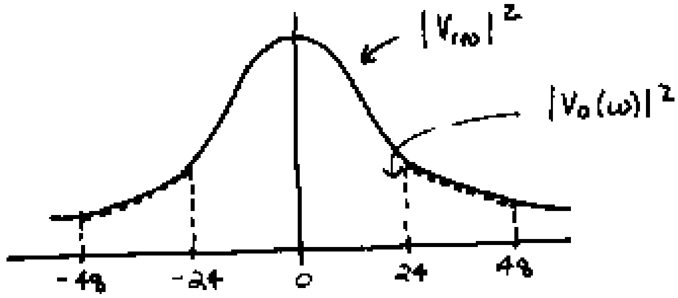
Take the Fourier Transform of both sides to get: $\mathcal{F}\left(e^{j\omega_0 t}\right) = 2\pi\delta(\omega-\omega_0)$

$$\mathcal{F}\{A\cos\omega_0 t\} = \mathcal{F}\left\{A\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right)\right\} = \frac{A}{2}\left(\mathcal{F}\left(e^{j\omega_0 t}\right) + \mathcal{F}\left(e^{-j\omega_0 t}\right)\right) = \frac{A}{2}\left(2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right)$$

$$= A\pi\delta(\omega-\omega_0) + A\pi\delta(\omega+\omega_0)$$

Ex 15.12-1

a.) $$V_{in}(\omega)=\frac{120}{24+j\omega} \quad \Rightarrow \quad V_{in}(\omega)=\frac{120^2}{24^2+\omega^2}=\frac{14400}{576+\omega^2}$$



b.)

$$W_{in}=\frac{1}{\pi}\int_0^{\infty}\frac{14400}{576+\omega^2}\,d\omega=\frac{14000}{\pi}\left(\frac{1}{24}\tan^{-1}\left(\frac{\omega}{24}\right)\right)\Bigg|_0^{\infty}=300 \text{ J}$$

$$W_{out}=\frac{1}{\pi}\int_{24}^{48}\frac{14400}{576+\omega^2}\,d\omega=\frac{14000}{\pi}\left(\frac{1}{24}\tan^{-1}\left(\frac{\omega}{24}\right)\right)\Bigg|_{24}^{48}=61.3 \text{ J}$$

$$\therefore \eta=\frac{W_{out}}{W_{in}}\times 100\%=\frac{61.3}{300}\times 100\%=20.5\%$$

Ex 15.13-1

$$f^+(t)=te^{-at}$$

$$f^-(t)=te^{at} \quad \Rightarrow \quad f^-(-t)=-te^{-at}$$

$$\therefore F^+(s)=\frac{1}{(s+a)^2} \quad \text{and} \quad F^-(s)=\frac{-1}{(s+a)^2}$$

Then $$F(\omega)=F^+(s)\big|_{s=j\omega}+F^-(s)\big|_{s=-j\omega}=\frac{1}{(s+a)^2}\Bigg|_{s=j\omega}+\frac{-1}{(s+a)^2}\Bigg|_{s=-j\omega}$$

$$=\frac{1}{(a+j\omega)^2}-\frac{1}{(a-j\omega)^2}=\frac{-j4a\omega}{\left(a^2+\omega^2\right)^2}$$

Problems

Section 15.3: The Fourier Series

P15.3-1 $f(t) = t^2$ for $0 \le t \le 2$

$$\omega_0 = \frac{2\pi}{2} = \pi$$

$$a_0 = \frac{1}{2}\int_0^2 t^2 dt = 4/3$$

$$a_n = \frac{2}{2}\int_0^2 t^2 \cos n\pi t \, dt = \frac{4}{(n\pi)^2}$$

$$b_n = \frac{2}{2}\int_0^2 t^2 \sin n\pi \, t \, dt = \frac{-4}{n\pi}$$

$$\therefore \quad \underline{f(t) = \frac{4}{3} + \frac{4}{\pi^2}\sum_{n=1}^{N}\frac{1}{n^2}\cos n\pi t - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi t}$$

P15.3-2 Even function: $T = 0.4, \omega_0 = \dfrac{2\pi}{T} = 5\pi \ \ \text{rad/s}$

$$f(t) = \begin{cases} A\cos\omega_0 t & 0 \le t \le .1 \\ 0 & .1 \le t < .3 \\ A\cos\omega_0 t & .3 \le t \le .4 \end{cases}$$

Choose period $-.1 \le t \le .3$ for integral

$$a_0 = \frac{1}{T}\int_{-.1}^{.1} A\cos\omega_0 t = A/\pi$$

$$a_n = \frac{2}{T}\int_{-.1}^{.1} A\cos\omega_0 t \ \cos n\omega_0 t \, dt$$

so $a_1 = 5A\int_{-.1}^{.1}\cos^2\omega_0 t \, dt = \dfrac{A}{2}$

$$a_n = 5A\int_{-.1}^{.1}\cos\omega_0 t \cos n\omega_0 t \, dt = 5A\int_{-.1}^{.1}\frac{1}{2}\left[\cos 5\pi(1+n)t + \cos 5\pi(1-n)t\right] dt$$

$$= \underline{\frac{2A}{\pi}\ \frac{\cos(n\pi/2)}{1-n^2} \quad n \ne 1}$$

and $\underline{b_n = 0}$

P15.3-3

$$a_n = \frac{2}{T}\left[\int_0^{\frac{T}{4}} \cos\left(n\frac{2\pi}{T}t\right)dt + \int_{\frac{T}{4}}^{\frac{T}{2}} 2\cos\left(n\frac{2\pi}{T}t\right)dt\right] = \frac{1}{n\pi}\left[\sin\left(n\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{4}} + 2\sin\left(n\frac{2\pi}{T}t\right)\Bigg|_{\frac{T}{4}}^{\frac{T}{2}}\right]$$

$$= \frac{1}{n\pi}\left[\left(\sin\left(\frac{n\pi}{2}\right)-0\right)+2\left((\sin n\pi)-\sin\left(\frac{n\pi}{2}\right)\right)\right]$$

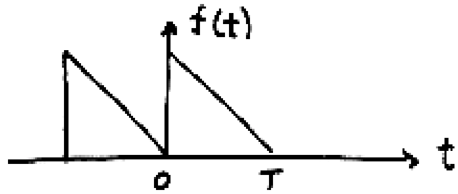
$$= -\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) = \begin{cases}\dfrac{(-1)^{\frac{n+1}{2}}}{n\pi} & \text{odd } n\\ 0 & \text{even } n\end{cases}$$

$$b_n = \frac{2}{T}\left[\int_0^{\frac{T}{4}} \sin\left(n\frac{2\pi}{T}t\right)dt + \int_{\frac{T}{4}}^{\frac{T}{2}} 2\sin\left(n\frac{2\pi}{T}t\right)dt\right] = -\frac{1}{n\pi}\left[\cos\left(n\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{4}} + 2\cos\left(n\frac{2\pi}{T}t\right)\Bigg|_{\frac{T}{4}}^{\frac{T}{2}}\right]$$

$$= -\frac{1}{n\pi}\left[(2\cos(n\pi)-1)-\cos\frac{n\pi}{2}\right]$$

$$= \begin{cases}\dfrac{3}{n\pi} & n \text{ is odd}\\ -\dfrac{2}{n\pi} & n = 2,6,10,\ldots\\ 0 & n = 4,8,12,\ldots\end{cases}$$

P15.3-4



$$\omega_0 = 2\pi/T$$

$$f(t) = A(1-t/T) \qquad 0 \le t \le T$$

$$a_0 = A/2$$

$$a_n = \frac{2}{T}\int_0^T A(1-t/T)\cos n\,\omega_0 t\,dt = 0 \qquad n > 0$$

$$b_n = \frac{2}{T}\int_0^T A(1-t/T)\sin n\omega_0 t\,dt = A/n\pi$$

$$\therefore\ f(t) = \frac{A}{2} + \sum_{n=1}^{\infty}\frac{A}{n\pi}\sin n\,\omega_0 t$$

P15.3-5 Since $A\sin\omega t = A\cos\left(\omega t-\frac{\pi}{2}\right) = A\cos\left(\omega\left(t-\frac{\pi}{2\omega}\right)\right)$

$$f_e(t)=f_d\left(t-\frac{\pi}{2\omega}\right) = \frac{2A}{\pi} + \sum_{n=1}^{\infty}\frac{4A(-1)^{n+1}}{\pi(4n^2-1)}\cos\left(2n\omega\left(t-\frac{\pi}{2\omega}\right)\right)$$

$$= \frac{2A}{\pi} + \sum_{n=1}^{\infty}\frac{4A(-1)^{n+1}}{\pi(4n^2-1)}\cos(2n\omega t - n\pi)$$

$$= \frac{2A}{\pi} + \sum_{n=1}^{\infty}\frac{4A(-1)^{n+1}}{\pi(4n^2-1)}(-1)^n\cos(2n\omega t) = \frac{2A}{\pi} - \sum_{n=1}^{\infty}\frac{4A}{\pi(4n^2-1)}\cos 2n\omega t$$

This is the Fourier Series of an even function (Why?) Indeed, $f_e(t)$ is even.

P15.3-6

MathCad Spreadsheet

Index of summation, n: $N := 25$ $\quad n := 1,2..N$

Define parameters: $T := 2$ $\quad \omega := \frac{2\pi}{T}$

Define increment of time. Set up index to run over T=0 to T=2 of the signal.

$$T := \frac{2\cdot\pi}{\omega} \qquad dt := \frac{T}{200} \qquad i := 1,2..200 \quad t_i := dt\cdot i$$

Enter the formulas for the Fourier Series:

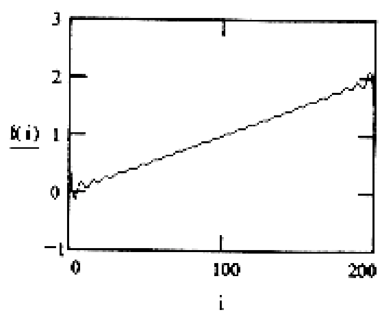
$$a_0 := \frac{1}{T}\cdot\int_0^T t\,dt \quad a_n := \frac{2}{T}\cdot\int_0^T t\cdot\cos(n\cdot\omega\cdot t)\,dt \quad b_n := \frac{2}{T}\cdot\int_0^T t\cdot\sin(n\cdot\omega\cdot t)\,dt$$

$$a_0 = 1 \qquad a_n = 0 \qquad b_n := \frac{2}{T}\cdot\frac{-(-\sin(n\cdot\omega\cdot T)+n\cdot\omega\cdot\cos(n\cdot\omega\cdot T)\cdot T)}{n^2\cdot\omega^2}$$

Enter the Fourier Series :

$$f(i) := a_0 + \sum_{n=1}^{N} b_n\cdot\sin(n\cdot\omega\cdot t_i)$$

Plot the periodic signal :



Section 15-4: Symmetry of the Function f(t)

P15.4-1

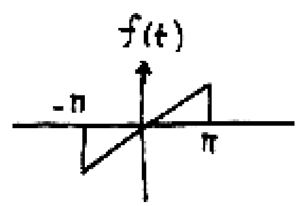
Choose $t_0 = -\pi$

$T = 2\pi,\ \omega_0 = \frac{2\pi}{2\pi} = 1$

average $a_0 = 0$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$$

$a_n = 0$ since have odd function



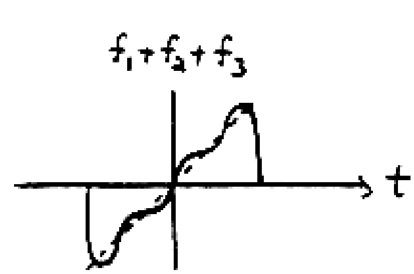
$f(t) = t \qquad -\pi < t < \pi$

$$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t\sin nt\,dt = \frac{1}{\pi}\left[\frac{\sin nt}{n^2} - \frac{t\cos nt}{n}\right]_{-\pi}^{\pi}$$

$$b_1 = \frac{1}{\pi}\left[\frac{\pi}{1}+\frac{\pi}{1}\right] = 2$$

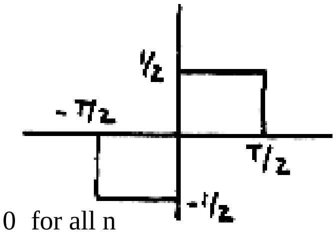
$b_2 = -1$

$b_3 = 2/3$



P15.4-2

Odd function, half-wave symmetry, from Table 15-1



$a_0 = 0$

$a_n = 0$ for even n

$b_n = 0$ for even n

$$a_n = \frac{4}{T}\int_0^{T/2}\left(\tfrac{1}{2}\right)\cos n\,\omega_0\,t\,dt = 0 \qquad n=\text{ odd } \therefore a_n = 0 \text{ for all n}$$

$\omega_0 = 2\pi/T$

$$b_n = \frac{4}{T}\int_0^{T/2}\left(\tfrac{1}{2}\right)\sin n\,\omega_0\,t\,dt$$

$$= \frac{4}{T}(1/2)\left[\frac{-\cos n\,\omega_0 t}{n\,\omega_0}\right]_0^{T/2} = \frac{2}{n\pi} \qquad n=1,3,5,...$$

$$\therefore f(t) = \frac{2}{\pi}\sum_{n=1}^{N}\frac{1}{n}\sin n\,\omega_0\,t;\quad n \text{ odd}$$

P15.4-3

$T = 8\text{s},\ \omega_0 = \pi/4$

even function $\Rightarrow b_n = 0$

$$a_0 = \text{average} = \frac{(2\times 2) - 2\times 1}{8} = 1/4$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\,\omega_0\,t\,dt = \frac{4}{8}\left[\int_0^1 2\cos n\frac{\pi}{4}t\,dt - \int_1^2 \cos n\frac{\pi}{4}t\,dt\right] = \frac{2}{n\pi}\left[3\sin\frac{n\pi}{4} - \sin\frac{n\pi}{2}\right]$$

$a_1 = .714,\ a_2 = .955,\ a_3 = .662$

P15.4-4

$\omega_0 = 2\omega,\ T = \dfrac{\pi}{\omega}$

$$a_0 = \frac{\omega}{\pi}\int_{\frac{-\pi}{2\omega}}^{\frac{\pi}{2\omega}} A\cos\omega t\,dt = \frac{2A}{\pi}$$

$$a_n = \frac{2\omega}{\pi}\int_{\frac{-\pi}{2\omega}}^{\frac{\pi}{2\omega}} A\cos\omega t\cos 2n\omega t\,dt$$

$$= \frac{2\omega A}{\pi}\left[\frac{\sin(2n-1)\omega t}{2(2n-1)\omega} + \frac{\sin(2n+1)\omega t}{2(2n+1)\omega}\right]_{-\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}}$$

$$= \frac{2A}{\pi}\left[\frac{\sin(2n-1)\frac{\pi}{2}}{2n-1} + \frac{\sin(2n+1)\frac{\pi}{2}}{2n+1}\right]$$

$$= \frac{2A}{\pi(4n^2-1)}\left[(2n+1)\sin(2n-1)\frac{\pi}{2} - (2n-1)\sin(2n-1)\frac{\pi}{2}\right] = -\frac{4A}{\pi(4n^2-1)}\cos(n\pi) = -\frac{4A(-1)^n}{\pi(4n^2-1)}$$

$b_n = 0$ due to symmetry

P15.4-5 $a_0 = 0$ because the average value is zero

$a_n = 0$ because the function is odd

$b_n = 0$ for even due to $\frac{1}{4}$ wave symmetry

Next

$$b_n = \int_{-\frac{T}{4}}^{\frac{T}{4}} t \sin\left(n\omega_0 t\right) dt = \frac{8\sin\left(\frac{n\pi}{2}\right) - 4n\pi\cos\left(\frac{n\pi}{2}\right)}{n^2\pi^2} = \begin{cases} \frac{8}{n^2\pi^2} & \text{for } n = 1,5,9,\ \ldots \\ -\frac{8}{n^2\pi^2} & \text{for } n = 3,7,11,\ \ldots \end{cases}$$

P15.4-6

Refer to Table 15.4-1. Take $A = \frac{1}{2}$ and $a_0 =$ average value of $f(t) = \frac{1}{2}$ Also $T = 2\pi$ so $\omega_0 = 1$ rad/s

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}\right)\sin\left((2n-1)t\right) = \frac{1}{2} + \frac{2}{\pi}\left(\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \ldots\right)$$

Section 15.5: Exponential Form of the Fourier Series

P15.5-1 MathCad Spredsheet:

Index of summation, n: $N := 15$ $n := 1,2..N$ $m := 1,2..N$

Define parameters : $A := \pi$ $T := 2\cdot\pi$ $\omega := \frac{2\cdot\pi}{T}$ $\omega := 1$

Define increment of time. Set up index to run over two periods of the signal.

$T := \frac{2\cdot\pi}{\omega}$ $dt := \frac{T}{200}$ $i := 1,2..400$ $t_i := dt\cdot i$

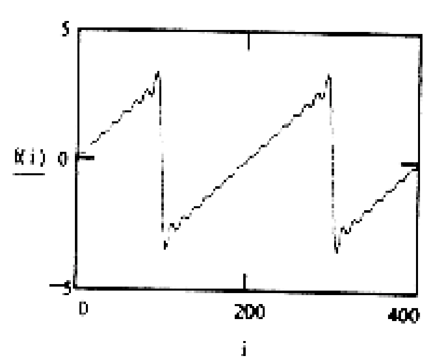
Enter the formulas for the Fourier Series:

$$C_n = i\cdot A\cdot\frac{\left(\cos(n\cdot\pi)\cdot\pi\cdot n - \sin(n\cdot\pi)\right)}{\left(\pi^2\cdot n^2\right)} \qquad C_m = i\cdot A\cdot\frac{\left(\cos(-m\cdot\pi)\cdot\pi\cdot -m - \sin(-m\cdot\pi)\right)}{\left[\pi^2\cdot(-m)^2\right]}$$

Enter the Fourier Series:

$$f(i) := \sum_{n=1}^{N} C_n\cdot\exp\left(-j\cdot n\cdot\omega\cdot t_i\right) + \sum_{m=1}^{N} C_m\cdot\exp\left(j\cdot -m\cdot\omega\cdot t_i\right)$$

Plot the periodic signal:



P15.5-2 MathCad Spreadsheet:

Index of summation, n: $N := 15$ $n := 1,2..N$ $m = 1,2..N$

Define parameters: $A := 1$ $T := 8$ $\omega := \frac{2\cdot\pi}{T}$ $\omega = 0.785$

Define increment of time. Set up index to run over two periods of the signal.

$T := \frac{2\cdot\pi}{\omega}$ $dt := \frac{T}{200}$ $i := 1, 2..100$ $t_i := dt\cdot i$

Enter the formulas for the Fourier Series:

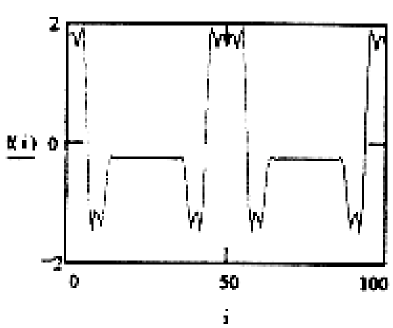
$$C_n := \frac{1}{T}\cdot\int_{-2}^{-1} -1\cdot\exp(-j\cdot n\cdot\omega\cdot t)\,dt + \frac{1}{T}\cdot\int_{-1}^{1} 2\cdot\exp(-j\cdot n\cdot\omega\cdot t)\,dt + \frac{1}{T}\cdot\left(\int_{1}^{2} -1\cdot\exp(j\cdot n\cdot\omega\cdot t)dt\right)$$

$$C_m := \frac{1}{T}\cdot\int_{-2}^{-1} -1\cdot\exp(-j\cdot-m\cdot\omega\cdot t)\,dt + \frac{1}{T}\cdot\int_{-1}^{1} 2\cdot\exp(-j\cdot-m\cdot\omega\cdot t)\,dt + \frac{1}{T}\cdot\left(\int_{1}^{2} -1\cdot\exp(-j\cdot-m\cdot\omega\cdot t)dt\right)$$

Enter the Fourier Series:

$$f(i) := \sum_{n=1}^{N} C_n\cdot\exp(j\cdot n\cdot\pi\cdot t_i) + \sum_{m=1}^{N} C_m\cdot\exp(j\cdot-m\cdot\pi\cdot t_i)$$

Plot the periodic signal:



P15.5-3

$$T = 2 \Rightarrow \omega_0 = \frac{2\pi}{2} = \pi;\ C_0 = \text{ average value } = -\frac{1}{2}$$

$$C_n = \frac{2}{2}\int_{-1}^{1} f(t)\,e^{-jn\pi t}\,dt$$

$$= \int_{-1}^{-\frac{4}{5}} (0.5)e^{-jn\pi t}\,dt + \int_{-\frac{4}{5}}^{\frac{1}{5}} (-1.5)e^{-jn\pi t}\,dt + \int_{\frac{1}{5}}^{1} (0.5)e^{-jn\pi t}\,dt$$

$$= \frac{.25}{-jn\pi}\left[\left(e^{jn\pi\frac{4}{5}} - e^{jn\pi}\right) - 3\left(e^{-j\frac{n\pi}{5}} - e^{jn\pi\frac{4}{5}}\right) + \left(e^{-jn\pi} - e^{-j\frac{n\pi}{5}}\right)\right]$$

P15.5-4 MathCad Spreadsheet:

Index of summation: $N := 15$ $n := 1, 2..N$ $m := 1, 2..N$

Define parameters: $A := 6$ $T := 4$

Fundamental frequency: $\omega := \frac{2\cdot\pi}{T}$ $\omega = 1.571$

Define increment of time. Set up index to run over two periods of the signal.

$T := \frac{2\cdot\pi}{\omega}$ $dt := \frac{T}{200}$ $i := 1,2..400$ $t_i := dt \cdot i$

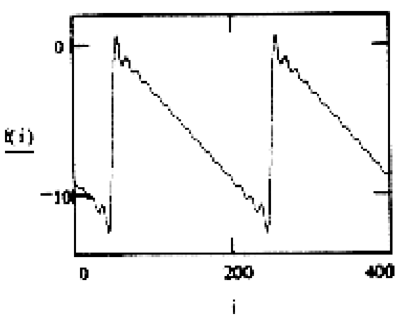
Enter the formulas for the Fourier Series:

$C_n := \frac{A\cdot j\cdot(-1)^{n}}{n\cdot\pi}$ $C_m := \frac{A\cdot j\cdot(-1)^{-m}}{-m\cdot\pi}$ <– The Fourier coefficients found using Table 15.5-1

Enter the Fourier Series:

$$f(i) := -6 + \sum_{n=1}^{N} C_n \cdot \exp\left[j\cdot n\cdot\omega\cdot(t_i+1)\right] + \sum_{m=1}^{N} C_n \cdot \exp\left[j\cdot m\cdot\omega\cdot(t_i+1)\right]$$

Plot the periodic signal:



The –6 shifts the plot vertically, while the (t+1) shifts the plot horizontally.

P15.5-5

$$C_n = \frac{1}{T}\int_0^1 (1-t)\ \exp(-jn\,\omega_0 t)\,dt$$

$$C_n = \frac{(-\exp(-jn2\pi) - jn2\pi + 1)}{n^2\,(2\pi)^2}$$

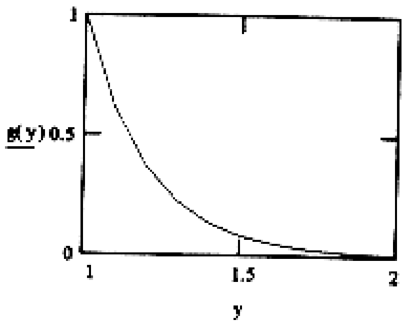
$$= \frac{1-\cos(2\pi n) - j\sin(2\pi n) - j2\pi n}{(2\pi n)^2}$$

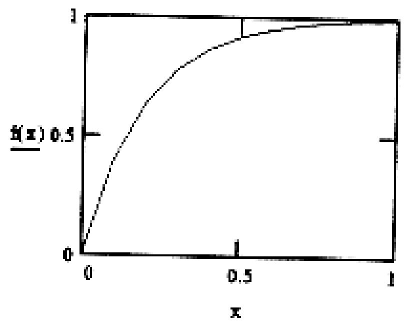
If n = integer, then $C_n = \frac{-jn2\pi}{(2\pi n)^2} = \underline{\frac{1}{j2\pi n}}$

P15.5-6 MathCad Spreadsheet:

Creating the signal: $x := 0, 0.1 .. 1$ $y := 1, 1.1 .. 2$

$$f(x) := \left(1 - \exp\left(\frac{-x}{0.2}\right)\right) \qquad g(y) := 150 \cdot \exp\left(-\frac{y}{0.2}\right)$$





Index of summation, n: $N := 15$ $n := 1, 2 .. N$ $m := 1, 2 .. N$

Define parameters: $A := 1$ $T := 2$

Fundamental frequency: $\omega := \frac{2 \cdot \pi}{T}$ $\omega := 3.142$

Define increment of time. Set up index to run over two periods of the signal.

$$T := \frac{2 \cdot \pi}{\omega} \qquad dt := \frac{T}{200} \qquad i := 1, 2 .. 400 \qquad t_i := dt \cdot i$$

Enter the formulas for the Fourier Series:

$$C_n := \frac{1}{2} \cdot \int_0^1 \left(1 - \exp\left(\frac{-t}{0.2}\right)\right) \cdot \exp(-j \cdot n \cdot \omega \cdot t)\, dt + \frac{1}{2} \cdot \int_1^2 150 \cdot \exp\left(-\frac{t}{0.2}\right) \cdot \exp(-j \cdot n \cdot \omega \cdot t)\, dt \quad *$$

$$C_m := \frac{1}{2} \cdot \int_0^1 \left(1 - \exp\left(\frac{-t}{0.2}\right)\right) \cdot \exp(-j \cdot - m \cdot \omega \cdot t)\, dt + \frac{1}{2} \cdot \int_1^2 150 \cdot \exp\left(-\frac{t}{0.2}\right) \cdot \exp(-j \cdot - m \cdot \omega \cdot t)\, dt$$

Enter the Fourier Series:

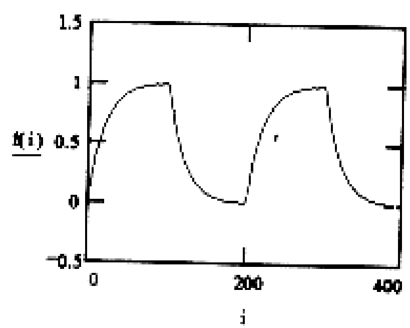
$$f(i) := \sum_{n=1}^{N} C_n \cdot \exp(-j \cdot n \cdot \omega \cdot t_i) + \sum_{m=1}^{N} C_m \cdot \exp(j \cdot - m \cdot \omega \cdot t_i) + 0.5$$

* Solving yields $C_n = \frac{5}{(j\pi n)(5 + j\pi n)}$

for $n = \text{odd}$

$C_n = 0, \ n = \text{even}$

Plot the periodic signal:



Section 15-6: The Fourier Spectrum

P15.6-1 Average value = 0 $\Rightarrow a_0 = 0$

half−wave symmetry $\Rightarrow$

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} \underbrace{\left(-\frac{4A}{T}t\right)}_{f(t)} \cos\left(n\frac{2\pi}{T}t\right)dt = -\frac{4A}{n^2\pi^2}(\cos(n\pi)-1)$$

and

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} \left(-\frac{4A}{T}t\right) \sin\left(n\frac{2\pi}{T}t\right)dt = -\frac{2A}{n\pi}\left(1-\cos(n\pi)\right)$$

n	$C_n = \sqrt{a_n^2 + b_n^2}$	$\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$
1	$1.509 \cdot A$	-57.5°
2	0	0
3	$0.434 \cdot A$	-78.0°
4	0	0
5	$0.257 \cdot A$	-82.7°
6	0	0
7	$0.183 \cdot A$	-84.8°

P15.6-2 MathCad Spreadsheet:

Determining the Fourier Coefficients:

Setting the index $N := 15$ $\quad n := 1, 2..N$ $\quad m := 1, 2..N$

Parameters: $T := 2\cdot\pi$ $\quad wo := \frac{2\cdot\pi}{T}$ $\quad j := \sqrt{-1}$

Finding the coefficients of the exponential Fourier Series. Split the function into four regions.

$$C1_n := \frac{4}{T}\cdot\int_{\frac{3\cdot T}{16}}^{\frac{T}{4}} \left(\frac{16}{T}\cdot t - 3\right)\cdot \exp(-j\cdot n\cdot wo\cdot t)\,dt \qquad C2_n := \frac{4}{T}\cdot\int_{\frac{T}{4}}^{\frac{T}{2}} \sin\left(2\cdot\frac{\pi}{T}\cdot t\right)\cdot\exp(-j\cdot n\cdot wo\cdot t)\,dt$$

$$C3_n := \frac{4}{T}\cdot\int_{\left(\frac{3}{16}+\frac{1}{2}\right)\cdot T}^{3\cdot\frac{T}{4}} \left(11 - \frac{16}{T}\cdot t\right)\cdot\exp(-j\cdot n\cdot\omega_0\cdot t)\,dt \qquad C4_n := \frac{4}{T}\cdot\int_{\frac{3\cdot T}{4}}^{T} \sin\left(2\cdot\frac{\pi}{T}\cdot t\right)\cdot\exp(-j\cdot n\cdot wo\cdot t)\,dt$$

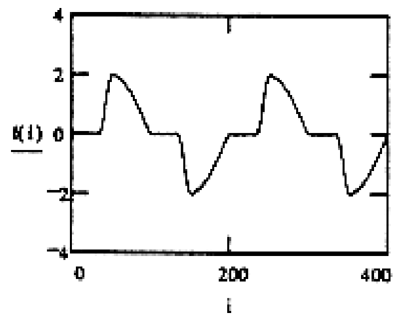
$$C_n := C1_n + C2_n + C3_n + C4_n$$

Verify that these coefficients are indeed correct by using them to plot the function:
Define increment of time. Set up index to run over two periods of the signal.

$$dt := \frac{T}{200} \qquad i := 1, 2..400 \qquad t_i := dt \cdot i$$

Enter the Fourier Series: $f(i) := \sum_{n=1}^{N} |C_n| \cdot \cos\left(n \cdot wo \cdot t_i + \arg(C_n)\right)$

Plot the periodic signal:



Here are the coefficients:

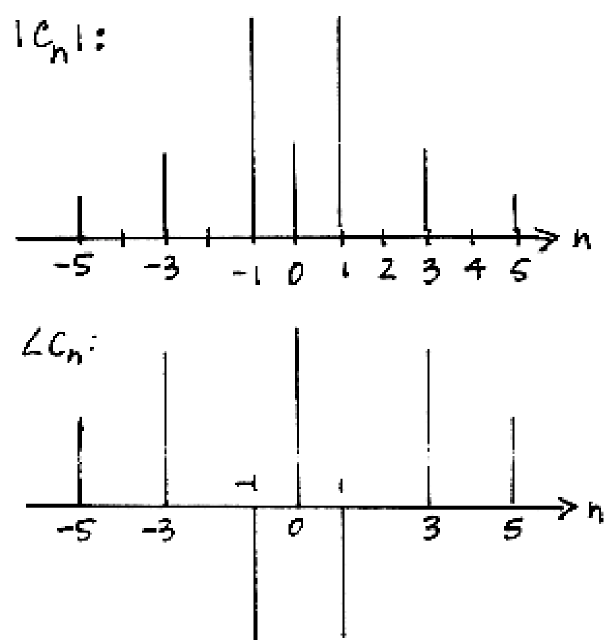
C_n

–0.604 – 1.247i
$-1.388i10^{-15}$
0.545 + 0.222i
0
–0.077 – 0.179i
0
0.056 + 0.127i
0
0.029 – 0.0077i
0
–0.013 + 0.037i
0
0.025 – 0.012i
0
$5.485.10^{-4}$ + 0.001i

P15.6-3

From P15.5-3

n	C_n
1	$0.64 \angle -126^\circ$
2	0
3	$0.21 \angle 162^\circ$
4	0
5	$0.127 \angle 90^\circ$
6	0
7	$0.09 \angle 18^\circ$



P15.6-4 MathCad Spreadsheet:

Index. of summation: $N:=20$ $n:=1,2..N$ $m:=1,2..N$ n indexes pos, m indexes neg, as the summations should be run from –N to +N

Define parameters: $A:=1$ $T:=1$

Define increment of time. Set up index to run over two periods of the signal.

$$\omega_0 := \frac{2\cdot\pi}{T} \qquad dt := \frac{T}{200} \qquad i := 1,2..400 \qquad t_i := dt\cdot i$$

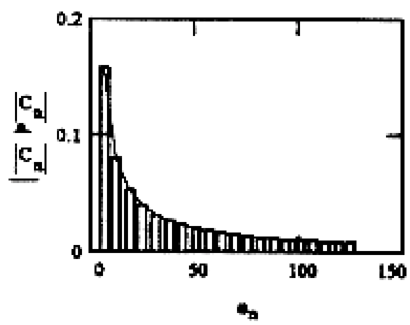
Enter the formulas for the Fourier Series:

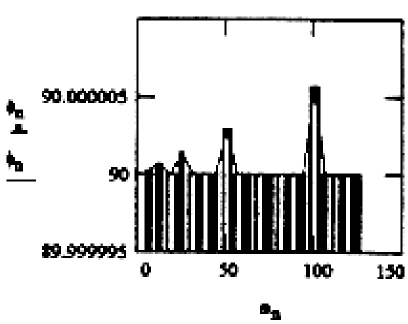
$$C_n := \frac{1}{T}\cdot\left[\int_0^1 (1-t)\cdot\exp(-j\cdot n\cdot\omega_0\cdot t)\,dt\right] \qquad C_m := \frac{1}{T}\cdot\left[\int_0^1 (1-t)\cdot\exp(-j\cdot -m\cdot\omega_0\cdot t)\,dt\right]$$

Enter the Fourier Series:

$$f(i) := \sum_{n=1}^{N} C_n\cdot\exp(-j\cdot n\cdot\omega_0\cdot t_i) + \sum_{m=1}^{N} C_m\cdot\exp(-j\cdot m\cdot\omega_0\cdot t_i) + 0.5$$

Now to get the Fourier Spectrum: $\omega_n := n\cdot\omega_0$ $\phi_n := \arg(C_n)\cdot\frac{180}{\pi}$





$\lvert C_n \rvert$	ϕ_n
0.159	90
0.08	90
0.053	90
0.04	90
0.032	90
0.027	90
0.023	90
0.02	90
0.018	90
0.016	90
0.014	90
0.013	90
0.012	90
0.011	90
0.011	90
0.01	90
0.009	90
0.009	90
0.008	90
0.008	90

P15.6-5

MathCad Spreadsheet

Index. of summation: N :=15 n:= 1,2,..N m:=1,2..N

n indexes pos, m indexes neg, as the summations should be run from –N to +N

Define parameters: A :=1 T:=2

Define increment of time. Set up index to run over two periods of the signal.

$$\omega0 := \frac{2\cdot\pi}{T} \quad dt := \frac{T}{200} \quad i := 1,2..400 \quad t_i := dt\cdot i$$

Enter the formulas for the Fourier Series:

Continued

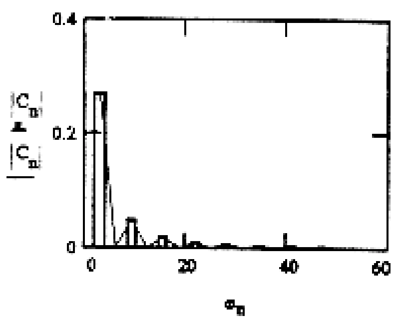
$$C_n := \frac{1}{2}\cdot\int_0^1\left(1-\exp\left(\frac{-t}{0.2}\right)\right)\cdot\exp(-j\cdot n\cdot\omega0\cdot t)dt+\frac{1}{2}\cdot\int_1^2 150\cdot\exp\left(-\frac{t}{0.2}\right)\cdot\exp(-j\cdot n\cdot\omega0\cdot t)dt$$

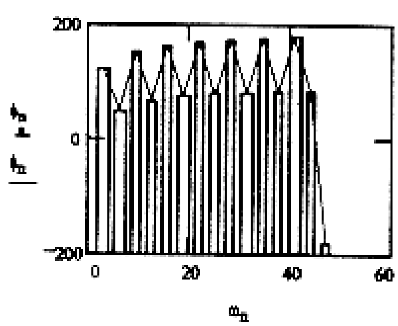
$$C_m := \frac{1}{2}\cdot\int_0^1\left(1-\exp\left(\frac{-t}{0.2}\right)\right)\cdot\exp(-j\cdot -m\cdot\omega0\cdot t)dt+\frac{1}{2}\cdot\int_1^2 150\cdot\exp\left(-\frac{t}{0.2}\right)\cdot\exp(-j\cdot -m\cdot\omega0\cdot t)dt$$

Enter the Fourier Series:

$$f(i) := \sum_{n=1}^{N} C_n\cdot\exp(-j\cdot n\cdot\omega0\cdot t_i)+\sum_{m=1}^{N} C_m\cdot\exp(j\cdot -m\cdot\omega0\cdot t_i)+0.5$$

Now to get the Fourier SP ectrum: $\omega_n := n\cdot\omega0$ $\phi_n := \arg(C_n)\cdot\frac{180}{\pi}$





C_n
–0.145 + 0.227I
$4.118\cdot 10^{-4}+5.174i\cdot 10^{-4}$
–0.044 + 0.022i
$1.451\cdot 10^{-4}+3.648i\cdot 10^{-4}$
–0.019 + 0.005i
$6.981\cdot 10^{-5}+2.632i\cdot 10^{-4}$
0.01 + 0.002i
$4.043\cdot 10^{-5}+2.032i\cdot 10^{-4}$
$-0.006+6.568i\cdot 10^{-4}$
$2.624\cdot 10^{-5}+1.648i\cdot 10^{-4}$
$-0.004+2.498i\cdot 10^{-4}$
$1.836\cdot 10^{-5}+1.384i\cdot 10^{-5}$
$-0.003+6.92i\cdot 10^{-5}$
$1.355\cdot 10^{-5}+1.192i\cdot 10^{-4}$
$-0.002-1.809i\cdot 10^{-5}$

ϕ_n
122.578
51.488
153.362
68.303
164.524
75.144
170.242
78.749
173.892
80.957
176.556
82.445
178.674
83.514
–179.54

Section 15.8: Circuits and Fourier Series

P15.8-1 Refer to Table 15.4-1. Take A = 5,

$T = \pi \Rightarrow \omega_o = 2$ and $a_o = 5$. Then

$$v_s(t) = 5+\frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\sin\left(2\left(2k-1\right)t\right)$$

Let $n = 2k-1$

$$v_s(t) = 5+\frac{20}{\pi}\sum_{\substack{n=1\\n=odd}}^{\infty}\frac{1}{n}\sin 2nt = 5+\frac{20}{\pi}\left(\sin 2t+\frac{1}{3}\sin 6t+\frac{1}{5}\sin 10t+\ldots\right)$$

The transfer function of the circuit is

$$T(\omega) = \frac{I(\omega)}{V_s\left(\omega\right)} = \frac{1}{4+j2\omega}$$

$$T\left(n\omega_0\right) = T\left(2n\right) = \frac{1}{4+j4n} = \frac{1}{4\sqrt{1+n^2}}e^{-\tan^{-1}(n)}$$

$$T(0) = \frac{1}{4}$$

so

$$i(t) = \frac{5}{4}+\frac{5}{\pi}\sum_{\substack{n=1\\n=odd}}^{\infty}\frac{1}{n\sqrt{1+n^2}}\sin\left(2nt-\tan^{-1}(n)\right)$$

P15.8-2

$$\frac{V_0(s)}{V_s(s)} = \frac{Z_p\ (s)}{Z_p(s)+Z_s(s)}$$ where $$z_p(s) = \frac{RL}{1+j\omega R_L C}$$ and $Z_s(s) = R + j\omega L$. After some algebra

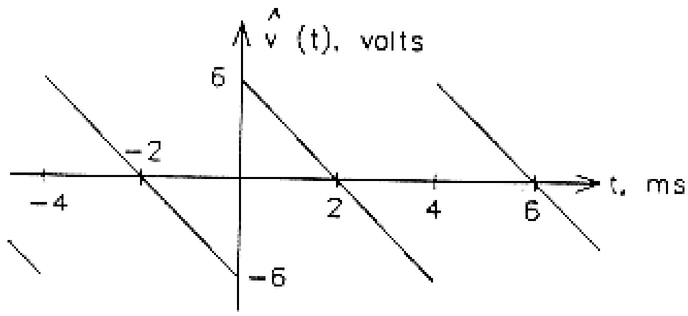
$$\frac{V_0(s)}{V_s(s)} = \frac{\frac{1}{LC}}{s^2+\left(\frac{R}{L}+\frac{1}{R_L C}\right)s+\left(\frac{1}{LC}+\frac{R}{R_L LC}\right)}$$

We require

$$2\pi\times 8000 = \omega_0 = \sqrt{\frac{1}{LC}+\frac{R}{R_L LC}}$$

or $$L = \frac{1}{{\omega_0}^2 C}\left(1+\frac{R}{R_L}\right) = \frac{1}{(2\pi\times 8000)^2\,(.004)}\left(1+\frac{2}{100}\right) = .1\,\mu\text{H}$$

P15.8-3



Rather than find the Fourier Series of v(t) directly, consider the signal $\hat{v}(t)$ shown above. These two signals are related by $v(t) = \hat{v}(t-1)-6$

since v(t) is <u>delayed</u> by 1 ms and shifted down by 6 V. For example, at t = 2ms

$v\,(2ms) = -3\text{ V}$

$\hat{v}(2-1ms)-6=3-6=-3\text{V}$

The Fourier series of $\hat{v}(t)$ is obtained as follows

$$T = 4ms$$

$$\omega_0 = \frac{2\pi \text{ radians}}{4ms} = \frac{\pi}{2}\text{rad/ms}$$

$$\hat{\omega}_0 = \text{average value of } \hat{v}(t) = 0$$

$$\hat{a}_n = 0$$

because $\hat{v}(t)$ is an odd function.

$$\hat{b}_n = \frac{1}{2}\int_0^4 (6-3t)\sin n\frac{\pi}{2}t\,dt$$

$$= 3\int_0^4 \sin n\frac{\pi}{2}t\;dt - \frac{3}{2}\int_0^4 t\sin n\frac{\pi}{2}tdt$$

$$= 3\frac{-\cos n\frac{\pi}{2}t}{n\frac{\pi}{2}}\Bigg|_0^4 - \frac{3}{2}\left[\frac{1}{\frac{n^2\pi^2}{4}}\sin n\frac{\pi}{2}t - \frac{n\pi}{2}t\cos n\frac{\pi}{2}t\right]_0^4$$

$$= \frac{6}{n\pi}(-1+\cos 2n\pi) - \frac{6}{n^2\pi^2}\left((\sin 2n\pi - 0)-(2n\pi - \cos 2\pi - 0)\right) = \frac{12}{n\pi}$$

so

$$\hat{v}(t) = \sum_{n=1}^{\infty}\frac{12}{n\pi}\sin n\frac{\pi}{2}t$$

Then

$$v(t) = -6+\sum_{n=1}^{\infty}\frac{12}{n\pi}\sin n\frac{\pi}{2}(t-1) = -6+\sum_{n=1}^{\infty}\frac{12}{n\pi}\sin\left(n\frac{\pi}{2}t - n\frac{\pi}{2}\right)$$

where t is in ms. Equivalently

$$v(t) = -6 + \frac{12}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(n\frac{\pi}{2}10^3 t - n\frac{\pi}{2}\right)$$

where t is in s

Next, the transfer function of the circuit is

$$T(s) = \frac{R}{\frac{1}{Cs}+Ls+R} = \frac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

so

$$T(\omega) = \frac{j\omega\frac{R}{L}}{\left(\frac{1}{LC}-\omega^2\right)+j\omega\frac{R}{L}} = \frac{10^4\, j\omega}{\left(10^8-\omega^2\right)+10^4\, j\omega}$$

Then

$$T(0) = 0$$

and

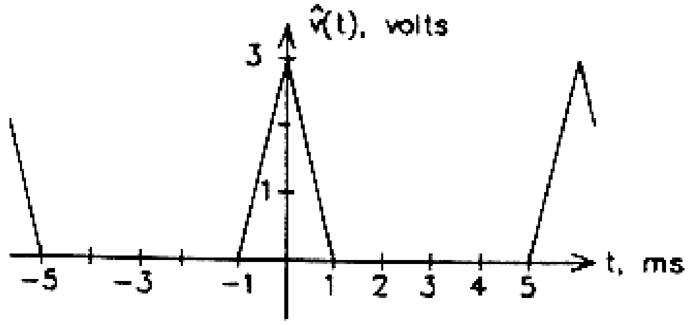
$$T\left(n\,\omega_0\right) = T\left(n\frac{\pi}{2}10^3\right) = \frac{j20n\pi}{\left(400-n^2\pi^2\right)+j20n\pi}$$

$$= \frac{1}{\sqrt{\left(400-n^2\pi^2\right)^2+400n^2\pi^2}}\, e^{j\left(90-\tan^{-1}\frac{20n\pi}{400-n^2\pi^2}\right)}$$

$$\text{Finally, } v_0(t) = \frac{12}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(n\frac{\pi}{2}10^3 t-n\frac{\pi}{2}+90^\circ-\tan^{-1}\left(\frac{20n\pi}{400-n^2\pi^2}\right)\right)}{n\sqrt{\left(400-n^2\pi^2\right)^2+400n^2\pi^2}}$$

P15.8-4 Rather than find the Fourier Series of v(t) directly, consider the signal $\hat{v}(t)$ shown below.

These two signals are related by $v(t) = \hat{v}(t-2)-1$



Let's calculate the Fourier Series of $\hat{v}(t)$, taking advantage of its symmetry.

$T = 6\text{ms}$ $\qquad \omega_O = \frac{2\pi\,\text{rad}}{6\text{ms}} = \frac{\pi}{3}\text{rad/ms}$

$$a_o = \text{average value of } \hat{v}(t) = \frac{\frac{3.2}{2}}{6} = \frac{1}{2}\text{ V}$$

$b_n = 0$ because $\hat{v}(t)$ is an even function

$$a_n = 2\left(\frac{2}{6}\int_0^1\left(3-3t\right)\cos n\frac{\pi}{3}t\,dt\right)$$

$$a_n = 2\int_0^1 \cos n\frac{\pi}{3}t\,dt - 2\int_0^1 t\cos n\frac{\pi}{3}t\,dt$$

$$= 2\left(\frac{\sin n\frac{\pi}{3}}{n\frac{\pi}{3}} - \frac{1}{\frac{n^2\pi^2}{9}}\left(\cos n\frac{\pi}{3}t + n\frac{\pi}{3}t\sin n\frac{\pi}{3}t\right)\right)\Bigg|_0^1$$

$$= \frac{6}{n\pi}\sin n\frac{\pi}{3} - \left(\frac{18}{n^2\pi^2}\left(\cos n\frac{\pi}{3} - 1\right) + \frac{6}{n\pi}\sin n\frac{\pi}{3}\right) = -\frac{18}{n^2\pi^2}\left(\cos n\frac{\pi}{3} - 1\right)$$

So

$$\hat{v}(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1 - \cos n\frac{\pi}{3}\right)\cos n\frac{\pi}{3}t$$

$$v(t) = \hat{v}(t-2) - 1 = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1 - \cos\frac{n\pi}{3}\right)\cos\left(n\frac{\pi}{3}t - n\frac{2\pi}{3}\right)$$

where t is in ms. Equivalently

$$v(t) = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1 - \cos\frac{n\pi}{3}\right)\cos\left(n\frac{\pi}{3}10^3 t - n\frac{2\pi}{3}\right)$$

where t is in seconds.

Next we calculate the transfer function of the circuit:

$$T(\omega) = \frac{-\frac{R_2}{1 + j\omega C_2 R_2}}{R_1 + \frac{1}{j\omega C_1}} = \frac{j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

$$T(n\omega_0) = T\left(n\frac{\pi}{3}10^3\right) = \frac{-jn\frac{\pi}{3}}{\left(1 + jn\frac{\pi}{3}\right)\left(1 + jn\frac{2\pi}{3}\right)}$$

$$|T(n\omega_0)| = \frac{n\frac{\pi}{3}}{\sqrt{\left(1 + \frac{n^2\pi^2}{9}\right)\left(1 + \frac{4n^2\pi^2}{9}\right)}} = \frac{n\pi}{\sqrt{(9 + n^2\pi^2)(9 + 4n^2\pi^2)}}$$

$$\angle T(n\omega_0) = -90^\circ - \left(\tan^{-1} n\frac{\pi}{3} + \tan^{-1} n\frac{2\pi}{3}\right)$$

The output voltage is

$$v_0(t) = \sum_{n=1}^{\infty} \frac{18\left(1 - \cos\frac{n\pi}{3}\right)\cos\left(n\frac{\pi}{3}10^3 t - n\frac{2\pi}{3} - 90^\circ - \tan^{-1} n\frac{\pi}{3} - \tan^{-1} n\frac{2\pi}{3}\right)}{n^2\pi^2\sqrt{(9 + n^2\pi^2)(9 + 4n^2\pi^2)}}$$

At t= 4ms

$$v_0(.004) = \sum_{n=1}^{\infty} \frac{18\left(1 - \cos\frac{n\pi}{3}\right)\cos\left(n\frac{4\pi}{3} - n\frac{2\pi}{3} - 90^\circ - \tan^{-1} n\frac{\pi}{3} - \tan^{-1} n\frac{2\pi}{3}\right)}{n^2\pi^2\sqrt{(9 + n^2\pi^2)(9 + 4n^2\pi^2)}}$$

P 15.9-1 Let $g(t) = e^{-at}u(t) - e^{at}u(-t)$. Notice that $f(t) = \lim_{a \to 0} g(t)$.

Next

$$G(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt - \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt = \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_0^{\infty} - \left. \frac{e^{(a-j\omega)t}}{(a-j\omega)} \right|_{-\infty}^{0}$$

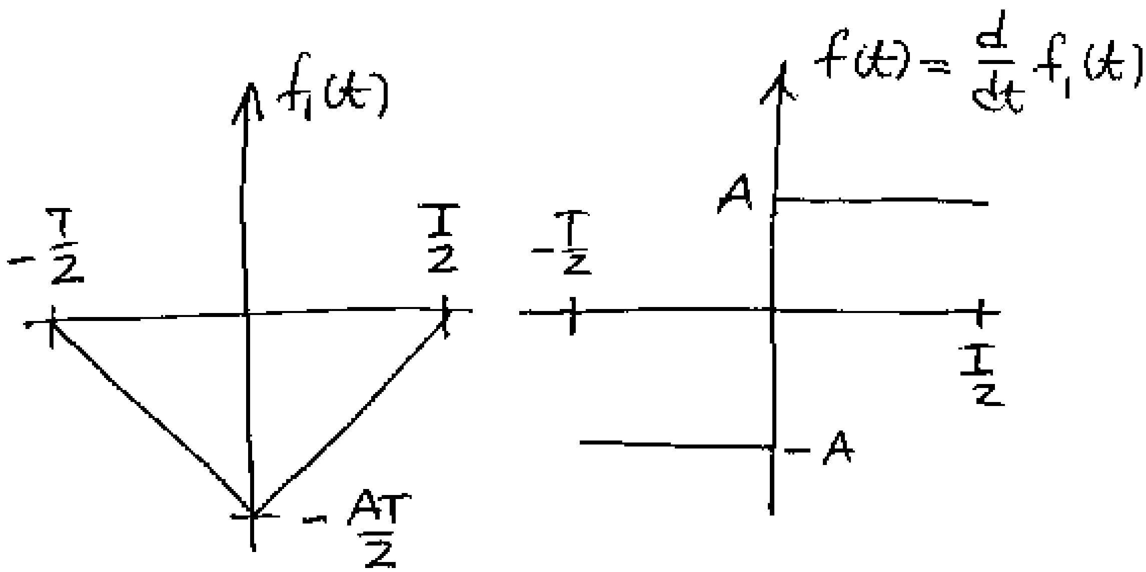
$$= \left(0 - \frac{1}{-(a+j\omega)} \right) - \left(\frac{1}{(a-j\omega)} - 0 \right) = \frac{-2j\omega}{a^2 + \omega^2}$$

Finally $F(\omega) = \lim_{a \to 0} G(\omega) = \lim_{a \to 0} \frac{-2j\omega}{a^2 + \omega^2} = \frac{2}{j\omega}$

P 15.9-2

$$F(\omega) = \int_{-\infty}^{\infty} Ae^{-at}u(t)\, e^{-j\omega t} dt = \int_0^{\infty} Ae^{-at} e^{-j\omega t} dt = \left. \frac{Ae^{-(a+j\omega)t}}{-(a+j\omega)} \right|_0^{\infty} = 0 - \frac{A}{-(a+j\omega)} = \frac{A}{a+j\omega}$$

P 15.9-3 First notice that



Then, from line 6 of Table 15.10-2: $\mathcal{F}\{f_1(t)\} = \left(-\frac{AT}{2}\right)\left(\frac{T}{2}\right) Sa^2\left(\frac{\omega T}{4}\right) = \left(\frac{-AT^2}{4}\right) Sa^2\left(\frac{\omega T}{4}\right)$

Also, from line 7 of Table 15.10-2: $\mathcal{F}\{f(t)\} = \mathcal{F}\left\{\frac{d}{dt} f_1(t)\right\} = j\omega\, \mathcal{F}\{f_1(t)\} = -j\omega \frac{AT^2}{4} Sa^2\left(\frac{\omega T}{4}\right)$

This can be written as: $\mathcal{F}\{f(t)\} = -j\omega \frac{AT^2}{4} \frac{\sin^2\left(\frac{\omega T}{4}\right)}{\left(\frac{\omega T}{4}\right)^2} = \frac{4A}{j\omega} \sin^2\left(\frac{\omega T}{4}\right)$

P 15.9-4 First notice that: $\mathcal{F}^{-1}\left\{\delta\left(\omega-\omega_0\right)\right\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\delta\left(\omega-\omega_0\right)e^{-j\omega t}d\omega=\frac{1}{2\pi}e^{-j\omega_0 t}$

Therefore $\mathcal{F}\left\{e^{-j\omega_0 t}\right\}=2\pi\delta\left(\omega-\omega_0\right)$. Next, $10\cos 50t=5\,e^{j50t}+5\,e^{-j50t}$.

Therefore $\mathcal{F}\left\{10\cos 50t\right\}=\mathcal{F}\left\{5\,e^{j50t}\right\}+\mathcal{F}\left\{5\,e^{-j50t}\right\}=10\pi\delta(\omega-50)+10\pi\delta(\omega+50)$.

P15.9-5

$$F\left(\omega\right)=-2\int_1^2 e^{-j\omega t}dt=\left.\frac{-2e^{-j\omega t}}{-j\omega}\right|_1^2=\frac{2}{j\omega}\left(e^{-j2\omega}-e^{-j\omega}\right)=\frac{2}{j\omega}\left(\left(\cos 2\omega-j\sin 2\omega\right)-\left(\cos\omega-j\sin\omega\right)\right)$$

$$=\frac{2j}{\omega}\left(\cos\omega-\cos 2\omega\right)+\frac{2}{\omega}\left(\sin\omega-\sin 2\omega\right)$$

P 15.9-6

$$F\left(\omega\right)=\int_0^B\frac{A}{B}t\,e^{-j\omega t}dt=\frac{A}{B}\left[\frac{e^{-j\omega t}}{\left(-j\omega\right)^2}\left(-j\omega t-1\right)\right]_0^B=\frac{A}{B}\left[\frac{e^{-j\omega B}}{-\omega^2}\left(j\omega B-1\right)-\frac{1}{\omega^2}\right]$$

$$=\frac{A}{B}\left[\frac{-Be^{-j\omega B}}{j\omega}+\frac{e^{-j\omega B}}{\omega^2}-\frac{1}{\omega^2}\right]$$

P 15.9-7

$$F\left(\omega\right)=\int_{-2}^2 e^{-j\omega t}dt-\int_{-1}^1 e^{-j\omega t}dt=\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-2}^2-\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^1=\frac{1}{j\omega}\left(e^{j2\omega}-e^{-j2\omega}\right)-\frac{1}{j\omega}\left(e^{j\omega}-e^{-j\omega}\right)$$

$$=\frac{2}{\omega}\left(\sin 2\omega-\sin\omega\right)$$

P15.12-1

$$i_s\left(t\right)=40\,\text{signum}\left(t\right)$$

$$I_s\left(\omega\right)=40\left(\frac{2}{j\omega}\right)=\frac{80}{j\omega}$$

$$H\left(\omega\right)=\frac{I\left(\omega\right)}{I_s\left(\omega\right)}=\frac{1}{4+j\omega}$$

$$I\left(\omega\right)=H\left(\omega\right)I_s\left(\omega\right)=\frac{1}{4+j\omega}\times\frac{80}{j\omega}=\frac{20}{j\omega}-\frac{20}{4+j\omega}$$

$$\therefore i\left(t\right)=10\,\text{signum}\left(t\right)-20\,e^{-4t}u\left(t\right)$$

P15.12-2 $i_s(t) = 100\cos 3t \quad \text{A}$

$$I_s(\omega) = 100\pi\left[\delta(\omega-3)+\delta(\omega+3)\right]$$

$$H(\omega) = \frac{I(\omega)}{I_s(\omega)} = \frac{1}{4+j\omega}$$

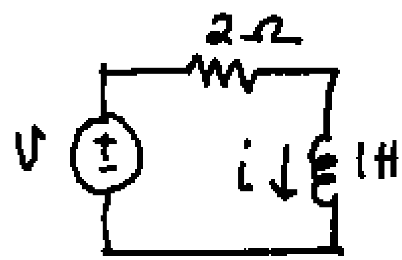
$$I(\omega) = 100\pi\left[\frac{\delta(\omega-3)+\delta(\omega+3)}{4+j\omega}\right]$$

$$i(t) = \frac{100\pi}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\delta(\omega-3)+\delta(\omega+3)}{4+j\omega}\right]e^{j\omega t}d\omega = 50\left[\frac{e^{-j3t}}{4-j3}+\frac{e^{j3t}}{4+j3}\right]$$

$$= 10\left[e^{-j(3t-36.9)}+e^{j(3t-36.9)}\right]$$

$$= 10\cos(3t-36.9)$$

P 15.12-3 $v(t) = 10\cos 2t$



$$V(\omega) = 10\pi\left[\delta(\omega+2)+\delta(\omega-2)\right]$$

$$Y(\omega) = \frac{1}{2+j\omega}$$

$$I(\omega) = Y(\omega)V(\omega) = \frac{10\pi\left[\delta(\omega+2)+\delta(\omega-2)\right]}{2+j\omega}$$

$$i(t) = \frac{10\pi}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\delta(\omega+2)+\delta(\omega-2)}{2+j\omega}\right]e^{j\omega t}d\omega = 5\left[\frac{e^{-j2t}}{2-j2}+\frac{e^{j2t}}{2+j2}\right]$$

$$= 5\left[e^{-j(2t-45)}+e^{j(2t-45)}\right] = 5\cos(2t-45) \quad \text{A}$$

P15.12-4

$$v(t)=e^t u(-t)+u(t)$$

$$\mathcal{F}\left\{e^t u(-t)\right\}=\int_{-\infty}^{\infty} e^t u(-t)e^{-j\omega t}dt=\int_{-\infty}^{0} e^t e^{-j\omega t}dt=\left.\frac{e^{(1-j\omega)t}}{1-j\omega}\right|_{-\infty}^{0}=\frac{1}{1-j\omega}$$

$$\mathcal{F}\left\{u(t)\right\}=\pi\delta(\omega)+\frac{1}{j\omega}$$

$$\therefore V(\omega)=\frac{1}{1-j\omega}+\pi\delta(\omega)+\frac{1}{j\omega}$$

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{j\omega}\right)}{\frac{1}{2}+\frac{1}{j\omega}}=\frac{1}{2+j\omega},\quad H(\omega)=\frac{\frac{1}{2+j\omega}}{1+\frac{1}{2+j\omega}}=\frac{1}{3+j\omega}$$

$$V_o(\omega)=\frac{1}{3+j\omega}\left[\frac{1}{1-j\omega}+\pi\delta(\omega)+\frac{1}{j\omega}\right]=\frac{-\frac{1}{12}}{3+j\omega}+\frac{\frac{1}{4}}{1-j\omega}+\frac{\frac{1}{3}}{j\omega}+\frac{\pi\delta(\omega)}{3+j\omega}$$

$$\mathcal{F}^{-1}\left\{\frac{\pi\delta(\omega)}{3+j\omega}\right\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega)}{3+j\omega}e^{-j\omega t}d\omega=\frac{1}{6}$$

$$\therefore\quad v_o(t)=-\frac{1}{12}e^{-3t}u(t)+\frac{1}{4}e^t u(-t)+\frac{1}{3}\,\text{signum}(t)+\frac{1}{6}$$

P15.12-5

$$v_s(t)=15e^{-5t}u(t)\quad \text{V}\quad\Rightarrow\quad V(\omega)=\frac{15}{5+j\omega}$$

$$W_s=\int_{-\infty}^{\infty}\left(15e^{-5t}u(t)\right)^2dt=\int_0^{\infty}\left(15e^{-5t}\right)^2dt=22.5\quad \text{J}$$

$$H(\omega)=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=\frac{\frac{1}{RC}}{\frac{1}{RC}+j\omega}$$

C=10 μF. Try R=10 kΩ. Then

$$V_o(\omega)=\frac{10}{10+j\omega}\times\frac{15}{5+j\omega}$$

$$W_o=\frac{1}{\pi}\int_0^{\infty}\left(\frac{10}{10+j\omega}\times\frac{15}{5+j\omega}\right)^2d\omega=\frac{1}{\pi}\int_0^{\infty}\left(\frac{300}{25+\omega^2}-\frac{300}{100+\omega^2}\right)^2d\omega=15\quad \text{J}$$

P 15.12-6

$$H(\omega)=\frac{4}{4+j\omega}$$

$$V_s(\omega)=\mathcal{F}\left\{8u(t)-8u(t-1)\right\}=\left(8\pi\delta(\omega)+\frac{8}{j\omega}\right)-\left(8\pi\delta(\omega)+\frac{8}{j\omega}\right)e^{-j\omega}$$

$$V_s(\omega)=\frac{8}{j\omega}\left(1-e^{-j\omega}\right)\quad \text{since}\quad \delta(\omega)e^{-j\omega}=\delta(\omega)$$

$$V_o(\omega)=\frac{4}{4+j\omega}\times\frac{8}{j\omega}\left(1-e^{-j\omega}\right)=\left(\frac{8}{j\omega}-\frac{8}{4+j\omega}\right)-\left(\frac{8}{j\omega}-\frac{8}{4+j\omega}\right)e^{-j\omega}$$

Next use $\frac{1}{j\omega}=\frac{1}{j\omega}+\pi\delta(\omega)-\pi\delta(\omega)$ to write

$$V_o(\omega)=\left(8\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-\pi\delta(\omega)-\frac{8}{4+j\omega}\right)-\left(8\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-\pi\delta(\omega)-\frac{8}{4+j\omega}\right)e^{-j\omega}$$

$$=\left(8\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-\frac{8}{4+j\omega}\right)-\left(8\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-\frac{8}{4+j\omega}\right)e^{-j\omega}$$

$$v_o(t)=8u(t)-8e^{-4t}u(t)-\left(8u(t-1)-8e^{-4(t-1)}u(t-1)\right)$$

$$=8(1-e^{-4t})u(t)-8(1-e^{-4(t-1)})u(t-1)\quad \text{V}$$

SP 15-1

```
Iin    1     0     pulse (3.1415  -3.1415   -3.1415   6.2830  0  0  6.2830)
R1     1     0     1


.tran   0.1   6.2832
.four   0.15915   v(1)
.probe
.end
```

FOURIER COMPONENTS OF TRANSIENT RESPONSE V (1)

DC COMPONENT = 1.883515E-04

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.592E-01	2.000E+00	1.000E+00	-1.078E-04	0.000E+00
2	3.183E-01	1.001E+00	5.003E-01	-1.800E+02	-1.800E+02
3	4.775E-01	6.676E-01	3.338E-01	-3.232E-04	-2.153E-04
4	6.366E-01	5.013E-01	2.506E-01	-1.800E+02	-1.800E+02
5	7.958E-01	4.016E-01	2.008E-01	-5.370E-04	-4.291E-04
6	9.549E-01	3.353E-01	1.676E-01	-1.800E+02	-1.800E+02
7	1.114E+00	2.880E-01	1.440E-01	-7.493E-04	-6.415E-04
8	1.273E+00	2.526E-01	1.263E-01	-1.800E+02	-1.800E+02
9	1.432E+00	2.251E-01	1.126E-01	-9.600E-04	-8.522E-04

SP 15-2

```
Iin     1    0   pulse     (1 0 0 1 0 0 1)
R1      1    0   1

.tran   0.11
.four   1   V(1)
.probe
.end
```

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = -5.080439E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000E+00	3.182E-01	1.000E+00	-1.771E+02	0.000E+00
2	2.000E+00	1.590E-01	4.996E-01	-1.742E+02	2.895E+00
3	3.000E+00	1.059E-01	3.327E-01	-1.713E+02	5.787E+00
4	4.000E+00	7.925E-02	2.490E-01	-1.684E+02	8.676E+00
5	5.000E+00	6.326E-02	1.988E-01	-1.655E+02	1.156E+01
6	6.000E+00	5.257E-02	1.652E-01	-1.627E+02	1.444E+01
7	7.000E+00	4.491E-02	1.411E-01	-1.598E+02	1.731E+01
8	8.000E+00	3.914E-02	1.230E-01	-1.569E+02	2.017E+01
9	9.000E+00	3.464E-02	1.088E-01	-1.541E+02	2.303E+01

SP 15-3

```
Vin     1   0    pulse     (40 –40 –0.25m 0 0 0.5m 1m)
R1      1   0    1

.tran   1u  1m
.four   1k  v(1)
.probe
.end
```

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = -7.999887E-02

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000E+03	5.093E+01	1.000E+00	-9.036E+01	0.000E+00
2	2.000E+03	1.600E-01	3.142E-03	8.928E+01	1.796E+02
3	3.000E+03	1.698E+01	3.333E-01	8.892E+01	1.793E+02
4	4.000E+03	1.600E-01	3.142E-03	-9.144E+01	-1.080E+00
5	5.000E+03	1.019E+01	2.000E-01	-9.180E+01	-1.440E+00
6	6.000E+03	1.600E-01	3.142E-03	8.784E+01	1.782E+02
7	7.000E+03	7.274E+00	1.428E-01	8.748E+01	1.778E+02
8	8.000E+03	1.600E-01	3.142E-03	-9.288E+01	-2.520E+00
9	9.000E+03	5.657E+01	1.111E-01	-9.324E+01	-2.880E+00

SP 15-4

```
Vin    1    0   pulse  (25  5  0  0  0  4  5)
R1     1    0   1

.tran   0.01 5
.four   0.2 v(1)
.probe
.end
```

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 8.960000E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE(DEG)
1	2.000E-01	7.419E+00	1.000E+00	1.253E+02	0.000E+00
2	4.000E-01	6.030E+00	8.127E-01	1.606E+02	3.528E+01
3	6.000E-01	4.061E+00	5.473E-01	-1.642E+02	-2.894E+02
4	8.000E-01	1.935E+00	2.609E-01	-1.289E+02	-2.542E+02
5	1.000E+00	8.000E-02	1.078E-02	-9.360E+01	-2.189E+02
6	1.200E+00	1.182E+00	1.593E-01	1.217E+02	-3.600E+00
7	1.400E+00	1.704E+00	2.297E-01	1.570E+02	3.168E+01
8	1.600E+00	1.537E+00	2.072E-01	-1.678E+02	-2.930E+02
9	1.800E+00	8.954E–01	1.207E-01	-1.325E+02	-2.578E+02

SP 15-5

```
Vin    1     0     pulse  (1 -1 -0.5 1 0 0 1)
R1     1     0     1

.tran    0.1   1
.four    1   v(1)
.probe
.end

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

 DC COMPONENT =     1.299437E-02

HARMONIC   FREQUENCY        FOURIER     NORMALIZED      PHASE       NORMALIZED
   NO         (HZ)        COMPONENT     COMPONENT      (DEG)      PHASE (DEG)

    1     1.000E+00       6.364E-01    1.000E+00    -1.777E+02    0.000E+00
    2     2.000E+00       3.180E-01    4.996E-01     4.679E+00    1.823E+02
    3     3.000E+00       2.117E-01    3.326E-01    -1.730E+02    4.682E+00
    4     4.000E+00       1.585E-01    2.490E-01     9.366E+00    1.870E+02
    5     5.000E+00       1.264E-01    1.987E-01    -1.683E+02    9.376E+00
    6     6.000E+00       1.051E-01    1.651E-01     1.407E+01    1.917E+02
    7     7.000E+00       8.972E-02    1.410E-01    -1.636E+02    1.409E+01
    8     8.000E+00       7.817E-02    1.228E-01     1.880E+01    1.965E+02
    9     9.000E+00       6.916E-02    1.087E-01    -1.588E+02    1.883E+01
```

Verification Problems

VP 15-1

$f(t) = 2 + \cos\frac{t}{2} \Rightarrow\ a_0 = 2\ ,\ a_1 = 1$ and all other coefficients are zero. The computer printout is correct.

VP 15-2 Table 15.4-2 shows that the average value of a full wave rectified sinewave is $\frac{2A}{\pi}$ where A is the amplitude of the sinewave. In this case $a_0 = \frac{2(400)}{\pi} = 255.$

Unfortunately the report says, "half-wave rectified." The report is not correct.

Design Problems

DP 15-1

For sinusoidal analysis, shift horizontal axis to average, which is 6V.

Have odd function and half-wave symmetry $\Rightarrow a_n = 0$

$T = \pi$, $\omega_0 = 2\pi/\pi = 2$

Need third harmonic : $b_n = \frac{2\times 2}{T}\int_0^{T/2} f(t)\sin n\omega_0 t\, dt$

$$b_3 = \frac{4}{T}\int_0^{\pi/2} \sin 6t\, dt = -\frac{4}{6\pi}\cos 6t\Big|_0^{\pi/2} = 0.424$$

so $v = 0.424\sin 6t = 0.424\cos(6t - 90^\circ)$

$V = 0.424\angle 0^\circ$ (assume sin input and output for ease), $Z_c = \frac{-j}{6c}$ for third harmonic

$\therefore$ transfer function $H(j\omega) = \frac{16}{16 - j/6C}$

$V_0 = VH = (0.424\angle 0^\circ)\,|H|\,\angle\theta = 1.36 \;\leftarrow$ choose

so $|H| = 3.2$ requires $C = \frac{1}{205}F$ so $H = \frac{16}{16 - j34} = 3.2\angle 64.9^\circ$

$\therefore$ third harmonic of $\underline{v_2 = 1.36\sin(6t + 64.9^\circ)}$ V

DP 15-2 Refer to Table 15.4-2.

$$v_s(t) = \frac{2A}{\pi} - \sum_{n=1}^{N}\frac{4A}{\pi}\left(\frac{1}{4n^2 - 1}\right)\cos(2n\omega_0 t)$$

So here $v_s(t) = \frac{360}{\pi} - \sum_{n=1}^{N}\frac{640}{\pi}\left(\frac{1}{4n^2 - 1}\right)\cos(2n377t)$

or $v_s(t) = v_{so} + \sum_{n=1}^{N} v_{sn}(t)$ and $v_0(t) = v_{o0} + \sum_{n=1}^{N} v_{on}(t)$

ripple $\le 0.04 \cdot$ dc output

$\max\left(\sum_{n=1}^{N} v_{on}(t)\right) \le 0.04 \cdot v_{o0}$

or $|v_{01}(t)| \le 0.04\, v_{o0}$

but $v_{o0} = v_{sO}$ when dc the L becomes a short

$V_{on} = \left(\frac{R}{R + j\omega_{0n}L}\right)V_{sn}$

$$V_{01} = \frac{R}{R + j\omega_0 L}\ V_{s1} = \frac{1}{1 + j377L}\ V_{s1}\ ,\ \text{but}\ |V_{s1}| = \frac{640}{\pi(3)}$$

$$\text{So}\ V_{01} = \frac{1}{1 + j377L}\left(\frac{640}{3\pi}\right)$$

$$\text{but}\ |V_{01}| \le 0.04\ v_{o0}\ \ \text{and}\ v_{o0} = v_{s0} = \frac{360}{\pi}$$

$$\text{then}\ \frac{1}{\sqrt{1+(377)^2 L^2}} \cdot \frac{640}{3\pi} \le 0.04\left(\frac{360}{\pi}\right)$$

Solving for L yields L > 1.54mH

DP 15-3 $V_{o0} = V_{s0} = 1/\pi \Leftarrow$ to dc

Transfer Function First harmonic:

$$= \frac{Z_p}{Z_L + Z_p} V_{s1}\ \text{where}\ Z_p = \frac{R}{1 + j\omega RC}$$

$$\text{So}\ \frac{V_{01}}{V_{s1}} = \frac{\dfrac{R}{1 + j\omega RC}}{j\omega L + \dfrac{R}{1 + j\omega RC}} = \frac{1/LC}{(j\omega)^2 + (j\omega)\dfrac{1}{RC} + \dfrac{1}{LC}}$$

$$\text{and}\ |V_{01}| = \frac{1}{20}|V_{o0}| = \frac{1}{20}|V_{s0}|$$

$$\text{or}\ \frac{1/LC}{\sqrt{\omega^4 + \left(\dfrac{\omega}{RC}\right)^2 + \left(\dfrac{1}{LC}\right)^2}} = \left(\frac{1}{20}\right)\left(\frac{1}{\pi}\right)$$

$\omega = 800\pi$

with R=75 kΩ

and choosing L=0.1 mH

yields C=0.1 F