Chapter 16: Filter Circuits

Exercises

Ex. 16.3-1

$$T_n(s) = \frac{1}{s+1}$$

$$T(s) = T_n\left(\frac{s}{1250}\right) = \frac{1}{\frac{s}{1250}+1} = \frac{1250}{s+1250}$$

Problems

<u>Section 16.3:</u> Filters

P16.3-1 Equation 16-3.2 and Table 16-3.2 provide a third-order Butterworth low-pass filter having a cutoff frequency equal to 1 rad/s.

$$H_n(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Frequency scaling so that $\omega_c = 2\pi 100 = 628$ rad/s:

$$H_L(s) = \frac{1}{\left(\frac{s}{628}+1\right)\left(\left(\frac{s}{628}\right)^2+\frac{s}{628}+1\right)}$$

$$= \frac{628^3}{(s+628)(s^2+628s+628^2)}$$

$$= \frac{247673152}{(s+628)(s^2+628s+394384)}$$

P16.3-2 Equation 16-3.2 and Table 16-3.2 provide a third-order Butterworth low-pass filter having a cutoff frequency equal to 1 rad/s and a dc gain equal to 1.

$$H_n(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Multiplying by 5 to change the dc gain to 5 and frequency scaling to change the cutoff frequency to $\omega_c = 100$ rad/s:

$$H_L(s) = \frac{5}{\left(\frac{s}{100}+1\right)\left(\left(\frac{s}{100}\right)^2+\frac{s}{100}+1\right)} = \frac{5 \cdot 100^3}{(s+100)(s^2+100s+100^2)}$$

$$= \frac{5000000}{(s+100)(s^2+100s+10000)}$$

P16.3-3 Use Table 16-3.2 to obtain the transfer function of a third-order Butterworth high-pass filter having a cutoff frequency equal to 1 rad/s and a dc gain equal to 5.

$$H_n(s) = \frac{5s^3}{(s+1)(s^2+s+1)}$$

Frequency scaling to change the cutoff frequency to $\omega_c = 100$ rad / s:

$$H_H(s) = \frac{5\left(\frac{s}{100}\right)^3}{\left(\frac{s}{100}+1\right)\left(\left(\frac{s}{100}\right)^2+\frac{s}{100}+1\right)}$$

$$= \frac{5 \cdot s^3}{(s+100)(s^2+100s+100^2)}$$

$$= \frac{5 \cdot s^3}{(s+100)(s^2+100s+10000)}$$

P16.3-4 Use Table 16-3.2 to obtain the transfer function of a fourth-order Butterworth high-pass filter having a cutoff frequency equal to 1 rad/s and a dc gain equal to 5.

$$H_n(s) = \frac{5 \cdot s^4}{\left(s^2+0.765s+1\right)\left(s^2+1.848s+1\right)}$$

Frequency scaling can be used to adjust the cutoff frequency 500 hertz = 3142 rad/s:

$$H_H(s) = \frac{5 \cdot \left(\frac{s}{3142}\right)^4}{\left(\left(\frac{s}{3142}\right)^2+0.765\left(\frac{s}{3142}\right)+1\right)\left(\left(\frac{s}{3142}\right)^2+1.848\left(\frac{s}{3142}\right)+1\right)}$$

$$= \frac{5 \cdot s^4}{\left(s^2+2403.6s+3142^2\right)\left(s^2+5806.4s+3142^2\right)}$$

P16.3-5 First, obtain the transfer function of a second-order Butterworth low-pass filter having a dc gain equal to 2 and a cutoff frequency equal to 2000 rad/s:

$$H_L(s) = \frac{2}{\left(\frac{s}{2000}\right)^2+1.414\left(\frac{s}{2000}\right)+1} = \frac{8000000}{s^2+2828s+4000000}$$

Next, obtain the transfer function of a second-order Butterworth high-pass filter having a passband gain equal to 2 and a cutoff frequency equal to 100 rad/s:

$$H_H(s) = \frac{2\cdot\left(\frac{s}{100}\right)^2}{\left(\frac{s}{100}\right)^2+1.414\left(\frac{s}{100}\right)+1} = \frac{2\cdot s^2}{s^2+141.4s+10000}$$

Finally, the transfer function of the bandpass filter is

$$H_B(s) = H_L(s)\cdot H_H(s) = \frac{16000000\cdot s^2}{\left(s^2+141.4s+10000\right)\left(s^2+2828s+4000000\right)}$$

P16.3-6

$$H_B(s) = 4\left(\frac{\frac{250}{1}s}{s^2+\frac{250}{1}s+250^2}\right)^2 = \frac{250000s^2}{\left(s^2+250s+62500\right)^2}$$

P16.3-7 First, obtain the transfer function of a second-order Butterworth high-pass filter having a dc gain equal to 2 and a cutoff frequency equal to 2000 rad/s:

$$H_L(s) = \frac{2\left(\frac{s}{2000}\right)^2}{\left(\frac{s}{2000}\right)^2+1.414\left(\frac{s}{2000}\right)+1} = \frac{2s^2}{s^2+2828s+4000000}$$

Next, obtain the transfer function of a second-order Butterworth low-pass filter having a pass-band gain equal to 2 and a cutoff frequency equal to 100 rad/s:

$$H_H(s) = \frac{2}{\left(\frac{s}{100}\right)^2+1.414\left(\frac{s}{100}\right)+1} = \frac{20000}{s^2+141.4s+10000}$$

Finally, the transfer function of the band-stop filter is

$$H_N(s) = H_L(s) + H_H(s) = \frac{2s^2\left(s^2+141.4s+10000\right)+20000\left(s^2+2828s+4000000\right)}{\left(s^2+141.4s+10000\right)\left(s^2+2828s+4000000\right)}$$

$$= \frac{2s^4+282.8s^3+40000s^2+56560000s+8\cdot10^{10}}{\left(s^2+141.4s+10000\right)\left(s^2+2828s+4000000\right)}$$

P16.3-8

$$H_N(s) = 4-4\left(\frac{\frac{250}{1}s}{s^2+\frac{250}{1}s+250^2}\right)^2 = \frac{4\left(s^2+62500\right)^2}{\left(s^2+250s+62500\right)^2}$$

P16.3-9

$$H_L(s) = 4\left(\frac{250^2}{s^2+\frac{250}{1}s+250^2}\right)^2 = \frac{4\cdot 250^4}{\left(s^2+250s+62500\right)^2}$$

P16.3-10

$$H_H(s) = 4\left(\frac{s^2}{s^2+\frac{250}{1}s+250^2}\right)^2 = \frac{4\cdot s^4}{\left(s^2+250s+62500\right)^2}$$

Section 16.4: Second-Order Filters

P16.4-1

$$T(s) = \frac{V_0(s)}{V_S(s)} = \frac{\frac{s}{RC}}{s^2+\frac{s}{RC}+\frac{1}{LC}}$$

So $K = 1\ ,\ \omega_0^2 = \frac{1}{LC}$ and $\frac{1}{RC} = \frac{\omega_0}{Q} \Rightarrow Q = RC\omega_0 = R\sqrt{\frac{C}{L}}$

Pick $\underline{C=1\mu F}$. Then $\underline{L} = \frac{1}{C\omega_0^2} = \underline{1\ H}$ and

$\underline{R} = Q\sqrt{\frac{L}{C}} = \underline{1000\ \Omega}$

P 16.4-2

$$T(s) = \frac{I_0(s)}{I_s(s)} = \frac{\frac{1}{LC}}{s^2+\frac{s}{RC}+\frac{1}{LC}}$$

So $K=1\ ,\ \omega_0^2=\frac{1}{LC}$ and $\frac{1}{RC}=\frac{\omega_0}{Q}\Rightarrow Q=RC\omega_0=R\sqrt{\frac{C}{L}}$

Pick $C = 1\mu F$ then $L=\frac{1}{C\omega_0^2}=25H$ and

$R = Q\sqrt{\frac{L}{C}} = 3535\Omega$

P16.4-3

$$T(s) = \frac{-\frac{1}{R_1RC^2}}{s^2+\frac{1}{RC}\left(2+\frac{R}{R_1}\right)s+\frac{1}{R^2C^2}}$$

Pick $C = 0.01\ \mu F$

$$\frac{1}{RC} = \omega_0 = 2000 \Rightarrow R = 50000 = 50k\Omega$$

$$\frac{\omega_0}{Q} = \frac{1}{RC}\left(2+\frac{R}{R_1}\right) \Rightarrow R_1 = \frac{R}{Q-2} = 8333 = 8.33\ k\Omega$$

P16.4-4 Pick $C = 0.02\mu F$. Then $R_1 = 40k\Omega,\ R_2 = 400k\Omega$ and $R_3 = 3.252k\Omega$.

P16.4-5

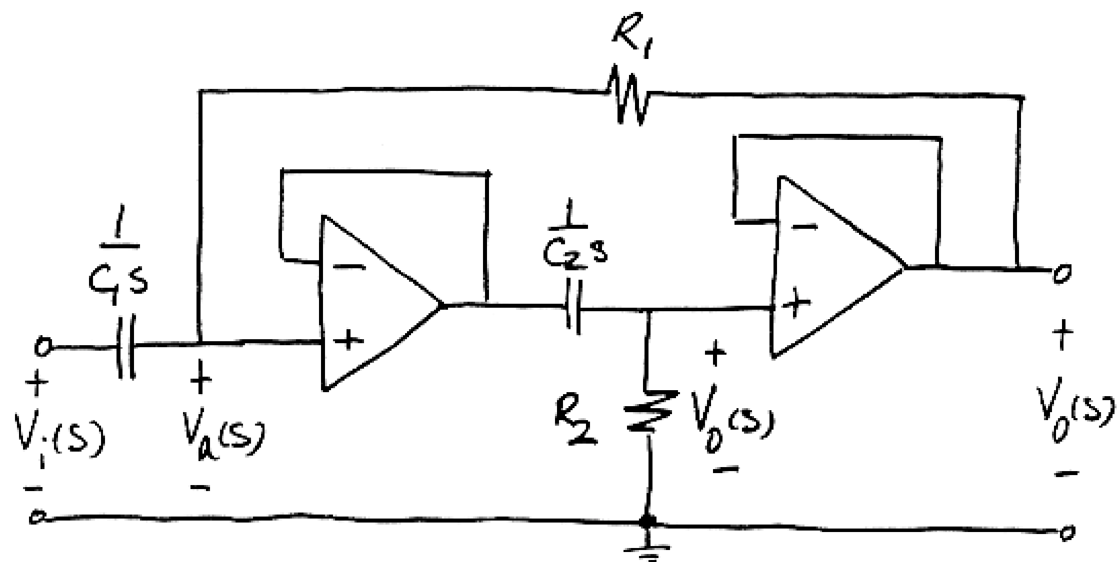
Pick $C_1 = C_2 = C = 1\mu F$

$$\frac{10^6}{\sqrt{R_1R_2}} = \omega_0$$

$$\frac{1}{R_1C} = \frac{\omega_0}{Q} \Rightarrow Q = \sqrt{\frac{R_1}{R_2}} \Rightarrow R_2 = \frac{R_1}{Q^2}$$

In this case $R_2 = R_1$ and $R_1 = \frac{10^6}{1000} = 1000 = 1k\Omega$

P 16.4-6



$$V_0(s) = \frac{R_2}{R_2 + \frac{1}{C_2 s}} V_a(s)$$

$$\frac{V_0(s) - V_a(s)}{R_1} - C_1 s\left(V_a(s) - V_i(s)\right) = 0$$

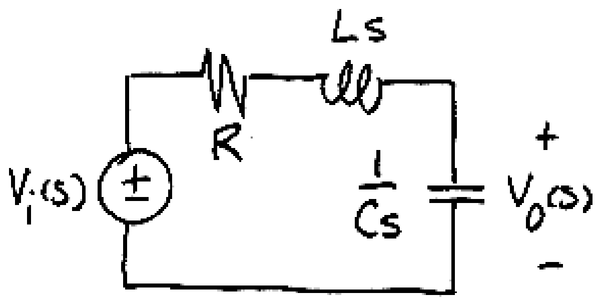
$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{s^2}{s^2 + \frac{s}{R_2 C_2} + \frac{1}{R_1 R_2 C_1 C_2}}$$

Pick $C_1 = C_2 = C = 1\mu F$. Then $\frac{1}{C\sqrt{R_1 R_2}} = \omega_0$

Also $\frac{1}{R_2 C} = \frac{\omega_0}{Q} \Rightarrow Q = \sqrt{\frac{R_2}{R_1}} \Rightarrow R_1 Q^2 = R_2$

In this case $R_1 = R_2 = R$ and $\frac{1}{CR} = \omega_0 \Rightarrow R = 1000\Omega$

P16.4-7



$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

When $R = 25$, $L = 10^{-2}$ and $C = 4 \times 10^{-6}$, then

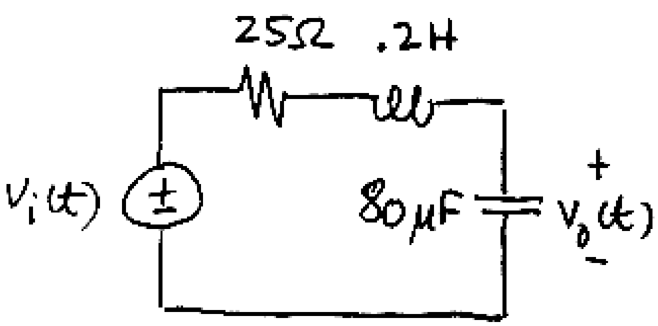
$$T(s) = \frac{25\times10^6}{s^2 + 2500s + 25\times10^6}$$

So

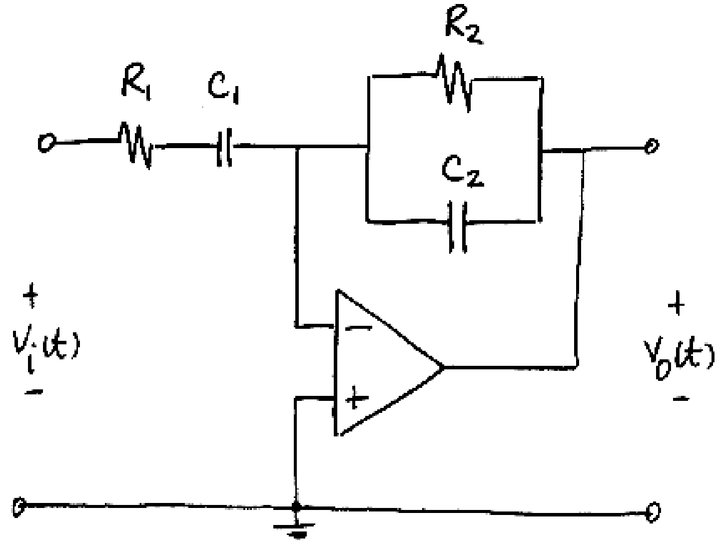
$$\omega_{old} = \sqrt{25\times10^6} = 5000$$

$$K_f = \frac{\omega_{new}}{\omega_{old}} = \frac{250}{5000} = 0.05$$

The scaled circuit is



P16.4-8



$$T(s) = \frac{V_0(s)}{V_i(s)} = -\frac{\dfrac{R_2}{1+R_2C_2s}}{R_1+\dfrac{1}{C_1s}} = -\frac{\dfrac{1}{R_1C_2}s}{s^2+\left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_2}\right)s+\dfrac{1}{R_1R_2C_1C_2}}$$

Pick $K_m = 1000$ so that the scaled capacitances will be $\dfrac{100\mu F}{1000} = 0.1\mu F$ and $\dfrac{500\mu F}{1000} = 0.5\mu F$

Before scaling $\left(R_1=20,\ C_1=100\mu F,\ R_2 = 10 \text{ and } C_2 = 500\mu F\right)$

$$T(s) = \frac{-100s}{s^2+700s+10^5}$$

After scaling ($R_1=20000,\ C_1= 0.1\mu F,\ R_2=10000,\ C_2= 0.5\mu F$)

$$T(s) = \frac{-100s}{s^2+700s+10^5}$$

P16.4-9 This is the frequency response of a bandpass filter, so $T(s) = \dfrac{K\dfrac{\omega_0}{Q}s}{s^2+\dfrac{\omega_0}{Q}s+\omega_0{}^2}$

From peak of the frequency response

$\omega_0 = 2p\times10\times10^6 = 62.8\times10^6$

$k = 10dB = 3.16$

next

$$\frac{\omega_0}{Q} = BW = (10.1{\times}10^6-9.9{\times}10^6)\ 2\pi = (0.2{\times}10^6)2\pi = 1.26{\times}10^6$$

so

$$T(s) = \frac{3.16(1.26)10^6 s}{s^2+(1.26)10^6 s+62.8^2.10^{12}} = \frac{(3.98)10^6 s}{s^2+(1.26)10^6 s+3.944.10^{15}}$$

P16.4-10

(a) Assume ideal op amp (inverting), then $H = \dfrac{V_0}{V_s} = -\dfrac{Z_2}{Z_1}$

where $Z_1 = R_1 - j/\omega C_1$ and $Z_2 = \dfrac{R_2 \; 1/j\omega C_2}{R_2 + 1/j\omega C_2}$

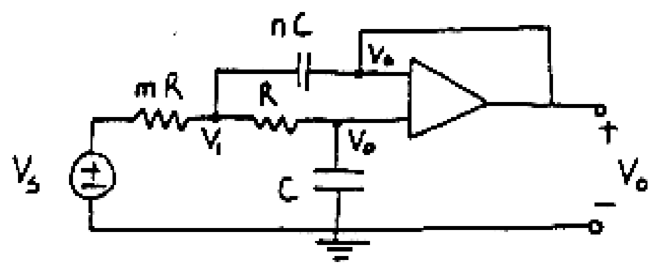
$$\therefore \; H = -\frac{j\omega R_2 C_1}{\left(1+j\omega/\omega_2\right)\left(1+j\omega/\omega_1\right)} \quad \text{where } \omega_1 = 1/R_1C_1, \; \omega_2 = 1/R_2C_2$$

(b) $\omega_1 = 1/R_1C_1$ and $\omega_2 = 1/R_2C_2$

(c) at $\omega_1 << \omega << \omega_2$

$$|H| \approx \frac{\omega R_2 C_1}{\omega/\omega_1} = \omega_1 R_2 C_1 = R_2/R_1.$$

P16.4-11



Assume $V_i \simeq 0$, then $V+ = V- = V_0$

Use $s = j\omega$

Voltage divider yields: $V_0 = V_1 \dfrac{1/sC}{R + 1/sC}$, $\Rightarrow V_1 = (1+sRC)V_0$

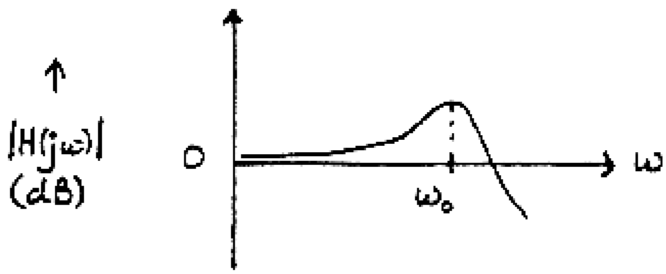
KCL at V_1: $(V_1 - V_s)/mR + (V_1 - V_0)/R + (V_1 - V_0)\, snC = 0$

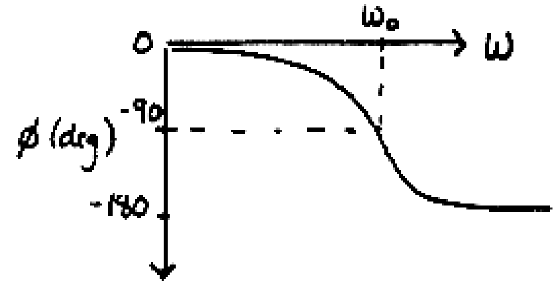
Plugging V_1 into above yields

$$V_0\left[\frac{1}{mR} + sC + \frac{sC}{m} + s^2 nRC^2\right] = \frac{V_s}{mR}$$

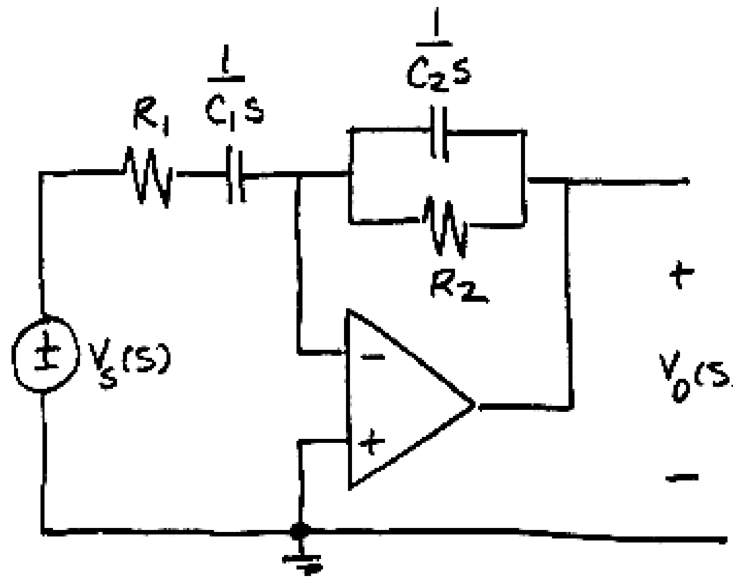
$$\therefore \frac{V_0}{V_s} = \frac{1}{1+s(m+1)RC + nm R^2C^2s^2}$$

$$\Rightarrow H(j\omega) = \frac{V_0}{V_s} = \frac{1}{1-\left(\omega/\omega_0\right)^2 + j\left(\omega/Q\omega_0\right)} \quad \text{where } \omega_0 = \frac{1}{\sqrt{mn}\,RC} \text{ and } Q = \frac{\sqrt{mn}}{m+1}$$





P16.4-12

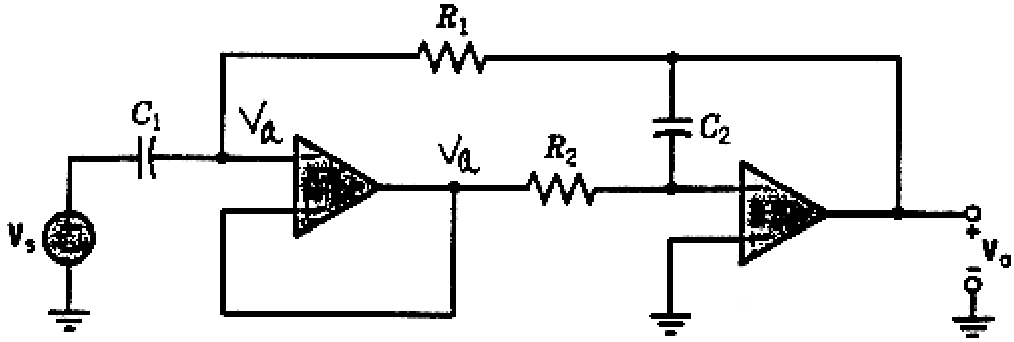


$$H(s) = \frac{V_0(s)}{V_S(s)} = -\frac{R_2 \left\| \frac{1}{C_2 s} \right.}{R_1 + \frac{1}{C_1 s}} = \frac{\frac{R_2}{R_2 C_2 s+1}}{\frac{R_1 C_1 s+1}{C_1 s}} = \frac{-\frac{1}{R_1 C_2} s}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = 70.7 \text{ k rad/sec} = 2\pi(11.25 \text{ kHz})$$

$$BW = \frac{\omega_0}{Q} = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} = 150 \text{ k rad/s} = 2\pi(23.9 \text{ kHz})$$

P16.4-13



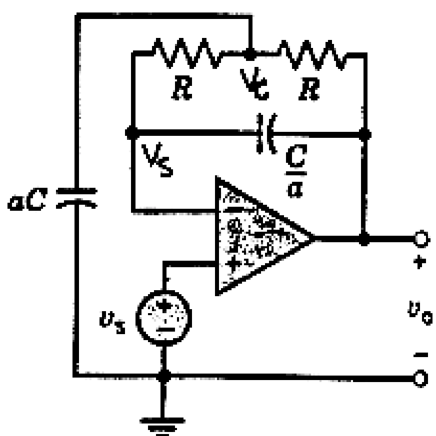
$$\left.\begin{array}{l} C_1 s(V_a - V_s) + \dfrac{V_a - V_0}{R_1} = 0 \\ -\dfrac{V_a}{R_2} - C_2 s V_0 = 0 \end{array}\right\} \Rightarrow H(s) = \frac{V_0(s)}{V_s(s)} = \frac{-\dfrac{1}{R_2C_2}s}{s^2 + \dfrac{1}{R_1C_1}s + \dfrac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} = 10^4 \text{ rad/sec}$$

$$BW = \frac{1}{R_1C_1} = 10^3 \text{rad/sec}$$

$$Q = \frac{\omega_0}{BW} = 10$$

P16.4-14



$$aCs\, V_c(s) + \frac{V_c(s) - V_s(s)}{R} + \frac{V_c(s) - V_0(s)}{R} = 0$$

$$\frac{C}{a} s \left(V_s(s) - V_0(s)\right) + \frac{V_s(s) - V_c(s)}{R} = 0$$

$$H(s) = \frac{V_0(s)}{V_s(s)} = \frac{s^2 + \left(\dfrac{2}{a} + a\right)\dfrac{1}{RC}s + \dfrac{1}{(RC)^2}}{s^2 + \left(\dfrac{2}{a}\right)\dfrac{1}{RC}s + \dfrac{1}{(RC)^2}}$$

We require

$$10^5 = \frac{1}{RC}$$

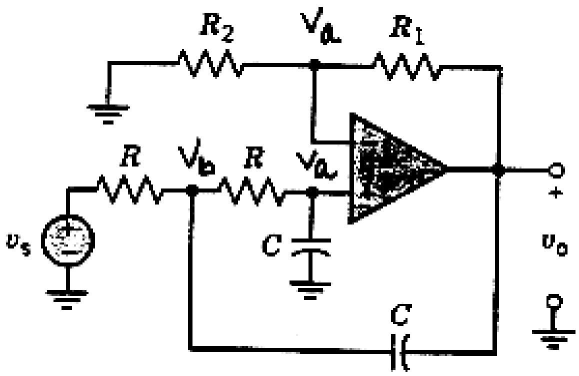
Pick C = 0.01μF then R = 1000Ω

$$\text{Next } \left|H(\omega_0)\right| = \frac{\dfrac{2}{a} + a}{\dfrac{2}{a}} = 1 + \frac{a^2}{2}$$

We require

$$201 = 1 + \frac{a^2}{2} \Rightarrow a = 20$$

P16.4-15



$$\frac{V_a}{R_2}+\frac{V_a-V_0}{R_1}=0$$

$$Cs\,V_a+\frac{V_a-V_b}{R}=0$$

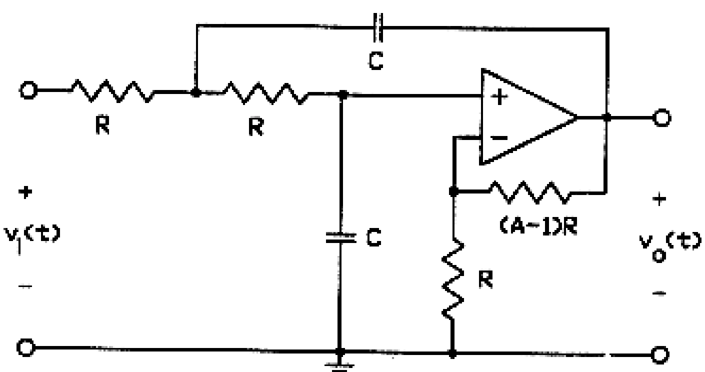
$$C_s\,(V_b-V_0)+\frac{V_b-V_s}{R}=0$$

$$\frac{V_0(s)}{V_s(s)}=\frac{\left(1+\frac{R_1}{R_2}\right)\frac{1}{R^2C^2}}{s^2+\left(2-\frac{R_1}{R_2}\right)\frac{1}{RC}s+\frac{1}{R^2C^2}}$$

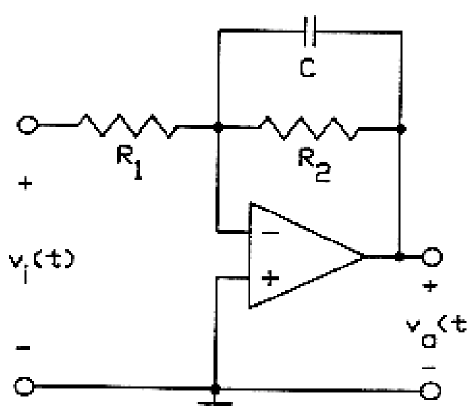
$$\omega_0=\frac{1}{RC}=\frac{1}{\left(1.2\times10^3\right)\left(20\times10^{-9}\right)}=41.67\text{ k rad/sec}$$

Section 16.5: High-Order Filters

P16.5-1 This filter is designed as a cascade connection of a Sallen-key low-pass filter designed as described in Table 16.4-2 and a first-order low-pass filter designed as described in Table 16.5-2.



Sallen-Key Low-Pass Filter



First-Order Low-Pass Filter

MathCad Speadsheet

The transfer function is of the form $T(s)=\frac{c}{s\hat{}2+bs+a}$

Enter the transfer function coefficients: $a := 628^2$ $b: = 628$

Determine the filter specifications: $\omega_0 := \sqrt{a}$ $Q := \frac{\omega_0}{b}$ $\omega_0 = 628$ $Q = 1$

Pick a convenient value for the capacitance: $C := 0.1\cdot10^{-6}$

Calculate resistance values: $R := \frac{1}{C\cdot\omega_0}$ $A := 3-\frac{1}{Q}$ $R = 1.592\cdot10^4$ $R\cdot(A-1) = 1.592\cdot10^4$

Calculate the dc gain: A=2.

The dc gain of the Sallen-key filter is 2. Therefore, the dc gain of the first-order filter must be ½ so that the dc gain of the whole filter is 1.

MathCad Spreadsheet

The transfer function is of the form $T(s) = \frac{-k}{s+p}$

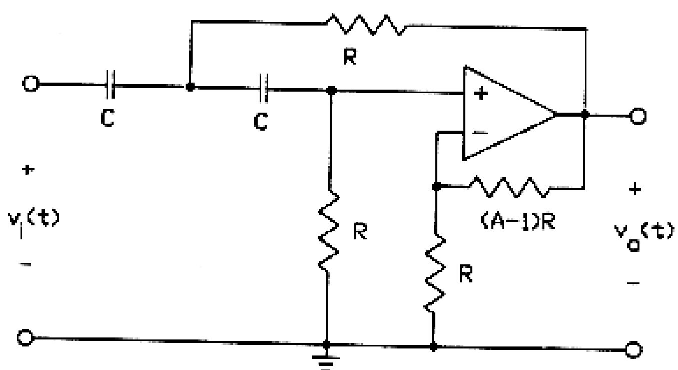
Enter the transfer function coefficients: $p := 628 \quad k := 0.5 \cdot p$

Pick a convenient value for the capacitance: $C : 0.1 \cdot 10^{-6}$

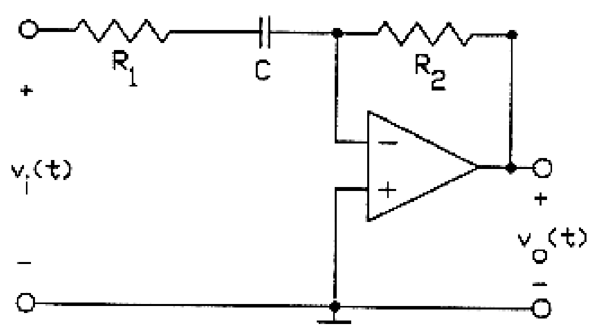
Calculate resistance values: $R2 := \frac{1}{C \cdot p} \quad R1 := \frac{1}{C \cdot k} \quad R1 = 3.185 \cdot 10^{4} \quad R2 = 1.592 \cdot 10^{4}$

P16.5-2 This filter is designed as a cascade connection of a Sallen-key high-pass filter, designed as described in Table 16.4-2, and a first-order high-pass filter, designed as described in Table 16.5-2.

The passband gain of the Sallen key stage is 2 and the passband gain of the first-order stage is 2.5 So the overall passband gain is $2 \times 2.5 = 5$



Sallen-Key High-Pass Filter



First-Order High-Pass Filter

MathCad Spreadsheet:

The transfer function is of the form $T(s) = \frac{cs\hat{}2}{s\hat{}2+bs+a}$

Enter the transfer function coefficients: $a := 10000 \quad b : 100$

Determine the filter specifications: $\omega_0 := \sqrt{a} \quad Q := \frac{\omega_0}{b} \quad \omega_0 = 100 \quad Q = 1$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \frac{1}{C \cdot \omega_0} \quad A := 3 - \frac{1}{Q} \quad R = 1.10^{5} \quad R \cdot (A-1) = 1 \cdot 10^{5}$

Calculate the passband gain: $A = 2$

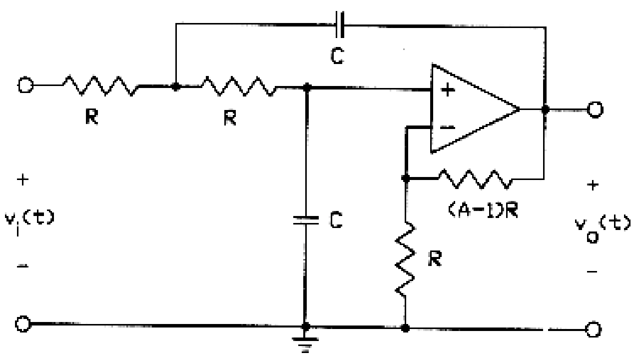
The transfer function is of the form $T(s) = \dfrac{-ks}{s + p}$

Enter the transfer function coefficients: $p := 100$ $k := 2.5$

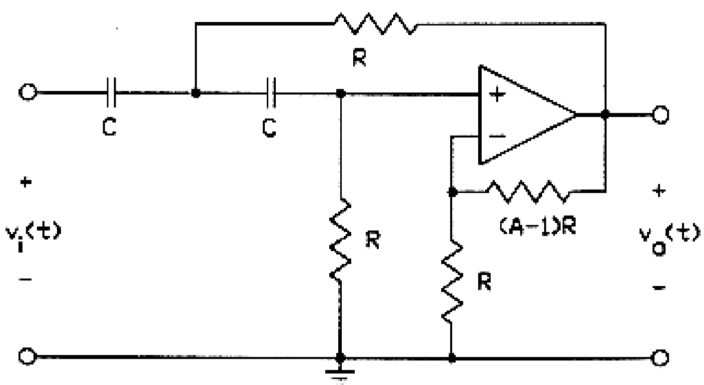
Pick a convenient value for the capactiance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R1 := \dfrac{1}{C \cdot p}$ $R2 := k \cdot R1$ $R1 = 1 \cdot 10^{5}$ $R2 = 2.5 \cdot 10^{5}$

P16.5-3 This filter is designed as a cascade connection of a Sallen-key low-pass filter, a Sallen-key high-pass filter and an inverting amplifier.



Sallen-key Low-Pass Filter



Sallen-Key High-Pass Filter

MathCad Spreadsheet:

The transfer function is of the form $T(s) = \dfrac{c}{s\hat{}2+bs+a}$.

Enter the transfer function coefficients: $a := 4000000$ $b := 2828$

Determine the filter specifications: $\omega_0 := \sqrt{a}$ $Q := \dfrac{\omega_0}{b}$ $\omega_0 = 2 \cdot 10^{3}$ $Q = 0.707$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values : $R := \dfrac{1}{C \cdot \omega_0}$ $A := 3 - \dfrac{1}{Q}$ $R = 5 \cdot 10^{3}$ $R \cdot (A-1) = 2.93 \cdot 10^{3}$

Calculate the dc gain: $A = 1.586$

The transfer function is of the form $T(s) = \dfrac{cs\hat{}2}{s\hat{}2+bs\ +a}$.

Enter the transfer function coefficients: $a: = 10000$ $b: = 141.4$

Determine the filter specifications: $\omega_0 : = \sqrt{a}$ $Q : = \dfrac{\omega_0}{b}$ $\omega_0 = 100$ $Q = 0.707$

Pick a convenient value for the capacitance: $C = 0.1 \cdot 10^{-6}$

Calculate resistance values: $R: = \dfrac{1}{C \cdot \omega_0}$ $A: = 3 - \dfrac{1}{Q}$ $R = 1 \cdot 10^5$ $R \cdot (A-1) = 5.86 \cdot 10^4$

Calculate the pass band gain: $A = 1.586$

The passband gain of these two stages is

$1.586 \times 1.586 = 2.515$

The required passband gain is

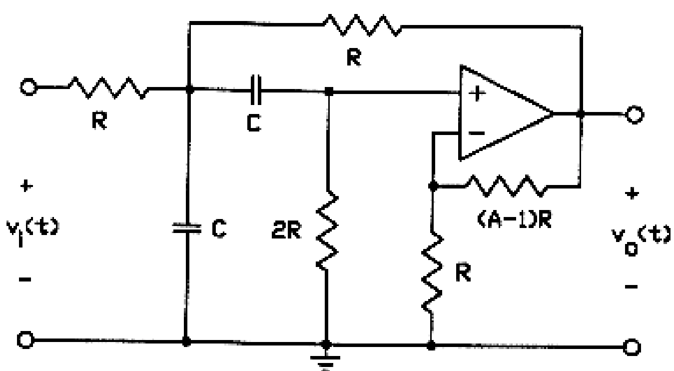
$$\frac{1.6 \times 10^6}{141.4 \times 2828} = 4.00$$

An amplifier with a gain equal to $\dfrac{4.0}{2.515} = 1.59$ is needed to achieve the specified gain.

P16.5-4

This filter is designed as the cascade connection of two identical Sallen-key bandpass filters

Sallen-Key BandPass Filter



MathC

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Enter the transfer function coefficients: $a : = 62500$ $b: = 250$

Determine the filter specifications: $\omega_0 : = \sqrt{a}$ $Q: = \dfrac{\omega_0}{b}$ $\omega_0 = 250$ $Q = 1$

Pick a convenient value for the capacitance: $C: = 0.1 \cdot 10^{-6}$

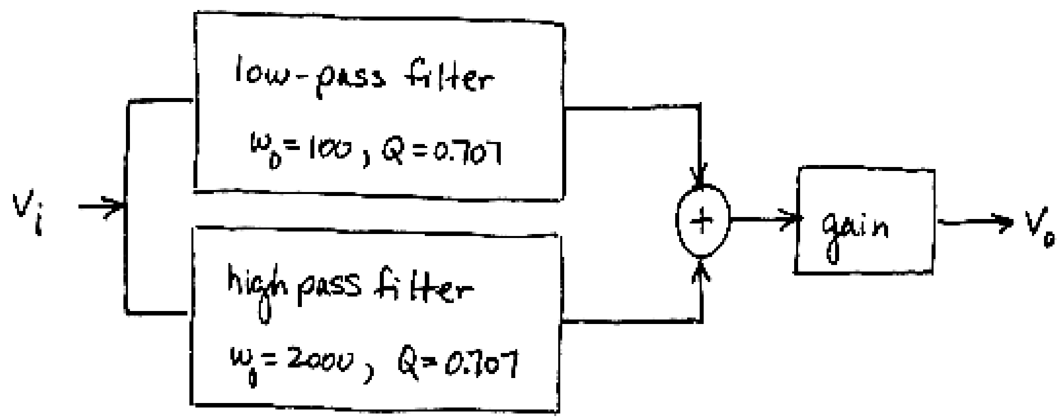
Calculate resistance values: $R := \dfrac{1}{C \cdot \omega_0}$ $A := 3 - \dfrac{1}{Q}$

$R = 4 \cdot 10^4$ $2 \cdot R = 8 \cdot 10^4$ $R \cdot (A-1) = 4 \cdot 10^4$

Calculate the passband gain: $A \cdot Q = 2$

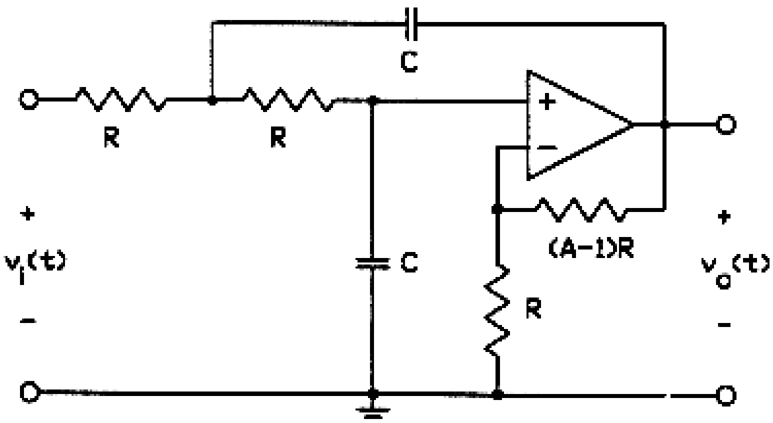
This filter is designed using this structure:

P16.5-5



Here's the low-pass filter design:

Sallen-Key Low-Pass Filter



MathCad Spreadsheet

The transfer function is of the form $T(s) = \dfrac{c}{s\hat{}2+bs+a}$

Enter the transfer function coefficients: $a := 10000$ $b := 141.4$

Determine the filter specifications: $\omega_0 := \sqrt{a}$ $Q := \dfrac{\omega_0}{b}$ $\omega_0 = 100$ $Q = 0.707$

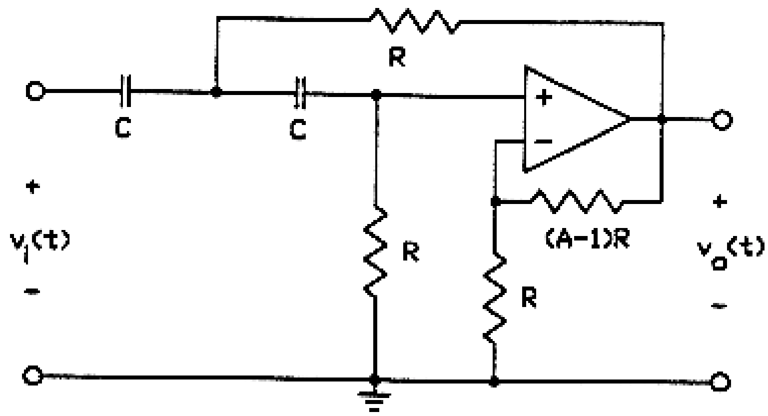
Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \dfrac{1}{C \cdot \omega_0}$ $A := 3 - \dfrac{1}{Q}$ $R = 1 \cdot 10^{5}$ $R \cdot (A-1) = 5.86 \cdot 10^{4}$

Calculate the dc gain: $A = 1.586$

Here's the high-pass filter design:

Sallen-Key Low-Pass Filter



MathCad Spreadsheet

The transfer function is of the form $T(s) = \dfrac{cs\hat{}2}{s\hat{}2+bs+a}$

Enter the transfer function coefficients: $a := 4000000$ $b := 2828$

Determine the filter specifications: $\omega_0 := \sqrt{a}$ $Q := \dfrac{\omega_0}{b}$ $\omega_0 = 2\cdot 10^3$ $Q = 0.707$

Pick a convenient value for the capacitance: $C := 0.1\cdot 10^{-6}$

Calculate resistance values: $R := \dfrac{1}{C\cdot\omega_0}$ $A := 3-\dfrac{1}{Q}$ $R = 5\cdot 10^3$ $R\cdot(A-1) = 2.93\cdot 10^3$

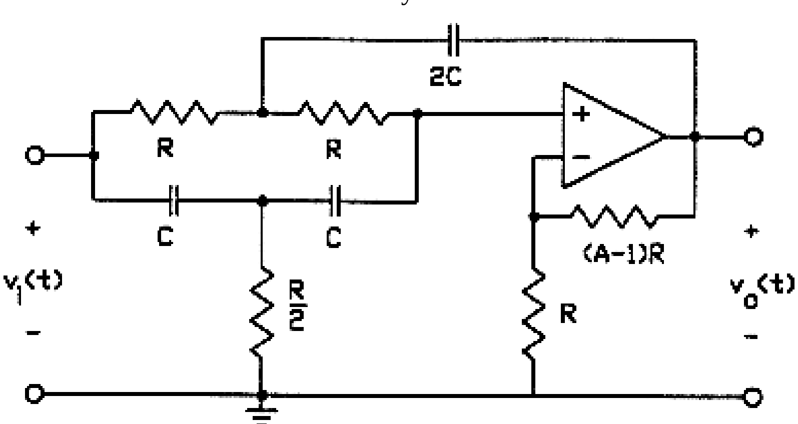
Calculate the dc gain: $A = 1.586$

The required passband gain is 2, but both Sallen-key filters have pass band gains equal to 1.586. The amplifier has a gain of $\dfrac{2}{1.586} = 1.26$, to make the passband gain of the entire filter 2.

P16.5-6

This filter is designed as the cascade connection of two identical Sallen-key notch filters.

Sallen-Key Notch Filter



The transfer function is of the form $T(s) = \frac{c(s\hat{}2+a)}{s\hat{}2+bs+a}$.

Enter the transfer function coefficients: $a := 62500$ $b := 250$

Determine the filter specifications : $\omega_0 := \sqrt{a}$ $Q := \frac{\omega_0}{b}$ $\omega_0 = 250$ $Q = 1$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$ $2 \cdot C = 2 \cdot 10^{-7}$

Calculate resistance values: $R := \frac{1}{C \cdot \omega_0}$ $A := 2 - \frac{1}{2 \cdot Q}$

$R = 4 \cdot 10^4$ $\frac{R}{2} = 2 \cdot 10^4$ $R \cdot (A-1) = 2 \cdot 10^4$

Calculate the passband gain: $A = 1.5$

The required passband gain is 4. An amplifier having a gain equal to $\frac{4}{(1.5)(1.5)} = 1.78$ is needed to achieve the required gain.

P16.5-7

(a) $H_1 = V_1 / V_s = \frac{R_1}{R_1 + 1/sC} = \frac{R_1 sC}{1 + R_1 sC}$

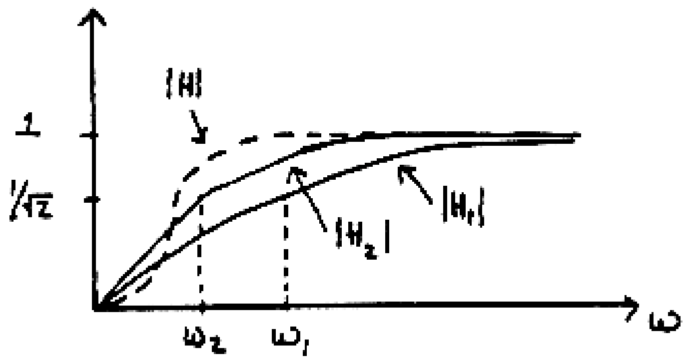
$$\Rightarrow |H_1(j\omega)| = \frac{\omega R_1 C}{\sqrt{1+(\omega R_1 C)^2}} = \frac{\omega / \omega_1}{\sqrt{1+(\omega / \omega_1)^2}}, \quad \omega = 1_1 / R_1 C$$

(b) $H_2 = V_2 / V_1 = \frac{sL}{R_2 + sL}$ $\Rightarrow |H_2(j\omega)| = \frac{\omega / \omega_2}{\sqrt{1+\left(\omega / \omega_2\right)^2}}$ $\omega_2 = R_2 / L$

(c) $$H(j\omega) = H_1(j\omega)\, H_2(j\omega) = \frac{(sR_1C)}{(1+sR_1C)} \frac{(s L/R_2)}{(1+s L/R_2)} = \frac{s^2 R_1 LC/R_2}{1+s(R_1 C + L/R_2) + s^2 (R_1 LC/R_2)}$$

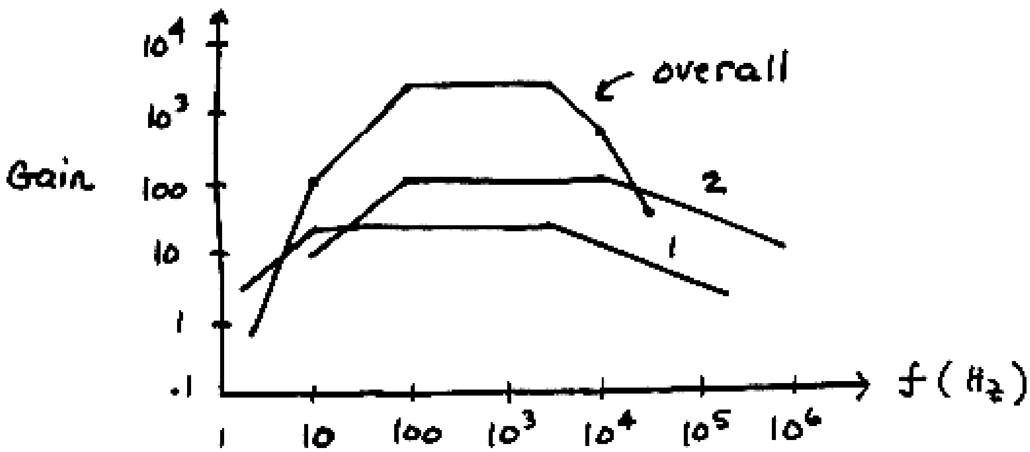
$$H(j\omega) = \frac{\left(s/\omega_0\right)^2}{1 + 1/Q (s/\omega_0)^2 + (s/\omega_0)^2}$$

where $\omega_0^2 = R_2 / R_1\, LC$, $\omega_0 Q = \frac{1}{R_1 C + L/R_2}$



(d) The circuit of Figure P13-32d is not the same as $H=H_1H_2$ since the first filter (looking from V_s) consists of C and $R_1 \parallel (R_2+sL)$. Thus effectively H_1 is altered, and we won't get the response as sketched in part (c). So we need to place a buffer to the right of R_1 to isolate each first-order filter, then $H=H_1H_2$ will be obtained.

P16.5-8



total midband gain = 2000

$F(H_z)$	$\lvert G_1 \rvert$	$\lvert 0_2 \rvert$	$\left\lvert V_0 / V_s \right\rvert$
1	2	1	2
10	20	10	200
100	20	100	2000
2000	20	100	2000
10,000	4	100	400
100,000	0.4	10	4

P16.5-9

$$H_{1,2} = -\frac{R_2/R_1}{1+j\omega R_2C} \quad \Rightarrow \quad H_{Total} = H_{1,2}^2 = \left(R_2/R_1\right)^2\left(\frac{1}{1+j\omega R_2C}\right)^2$$

(a) At low frequency,

$$H_T = \left(\frac{R_2}{R_1}\right)^2 \qquad \therefore \text{ for } H_T = 1 \text{ , need } R_1 = R_2$$

Now also $\omega_1 = 1000 = \frac{1}{R_2 C}$, so let $\underline{C = 1\mu F}$

$$\therefore R_1 = R_2 = \frac{1}{(1000)(10^{-6})} = 1k\Omega$$

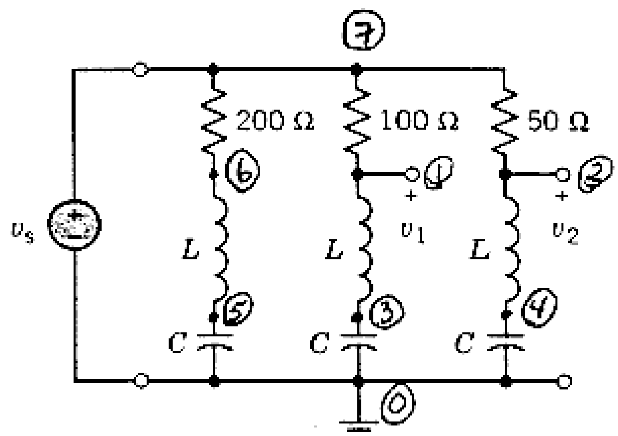
(b) At $\omega = 10{,}000$

$$|H| = \frac{1}{1+(\omega R_2 C)^2} = \frac{1}{1+\left[(10^4)(10^3)(10^{-6})\right]^2} = 10^{-2}$$

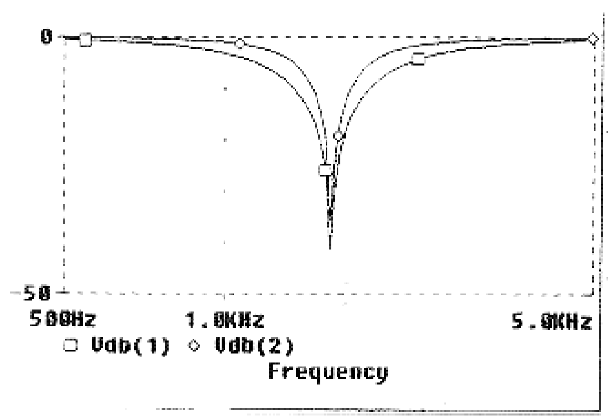
$$\Rightarrow \quad 20\log|H| = 20\log 10^{-2} = \underline{-40\text{dB}}$$

PSpice Problems

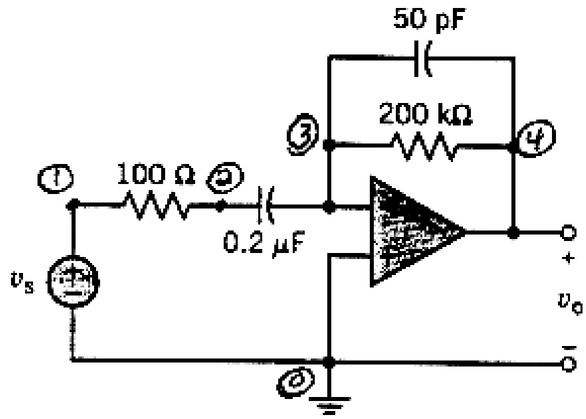
SP 16-7

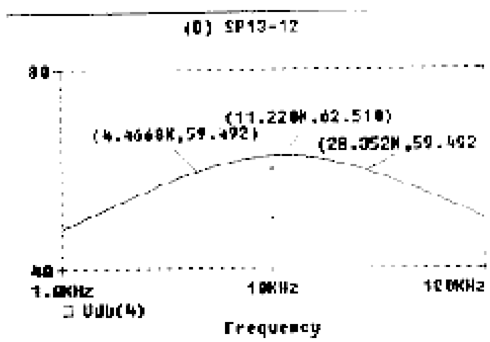



```
Vs    7   0    ac   1
R1    7   6    200
R2    7   1    100
R3    7   2    50
L1    6   5    10m
L2    1   3    10m
L3    2   4    10m
C1    5   0    1u
C2    3   0    1u
C3    4   0    1u

.ac dec 100 100 10k
.probe
.end
```



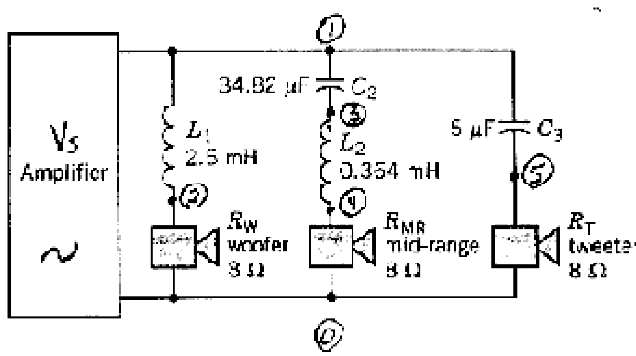
SP 16-8

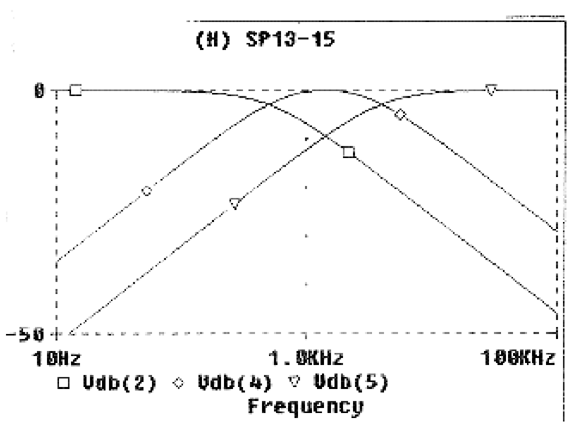





```
Vs   1  0  ac   1
R1   1  2  100
C1   2  3  0.2u
R2   3  4  200k
C2   3  4  50p

Xoa5 3    0   4    FGOA

.subckt FGOA 1 2 4
*nodes listed in order - + o
Ri   1   2   500k
E        3     0     1     2     100k
Ro   4   3   1k
.ends FGOA

.ac dec 100 1k 100k
.probe
.end
```

SP 16-9






```
Vs  1   0  ac  1
L1  1   2  2.5m
Rw  2   0  8
C2  1   3  34.82u
L2  3   4  0.364m
Rmr 4   0  8
C3  1   5  5u
Rt  5   0  8

.ac dec 100 10 100k
.probe
.end
```

Bw=4.07k - 493 $H_Z \simeq$ 3600H_Z

Verification Problems

VP 16.1

$$\omega_0 = \sqrt{10000} = 100 \quad \text{rad/s}$$

$$\frac{\omega_0}{Q} = 25 \quad \Rightarrow Q = \frac{100}{25} = 4 \neq 5$$

This filter does <u>not</u> satisfy the specifications.

VP 16.2

$$\omega_0 = \sqrt{10000} = 100 \quad \text{rad/s}$$

$$\frac{\omega_0}{Q} = 25 \Rightarrow Q = \frac{100}{25} = 4$$

$$K = \frac{75}{25} = 3$$

This filter does satisfy the specifications.

VP16.3

$$\omega_0 = \sqrt{400} = 20 \quad \text{rad/s}$$

$$\frac{\omega_0}{Q} = 25 \Rightarrow Q = \frac{20}{25} = 0.8$$

$$K = \frac{600}{400} = 1.5$$

This filter does satisfy the specifications.

VP16.4

$$\omega_0 = \sqrt{625} = 25 \quad \text{rad/s}$$

$$\frac{\omega_0}{Q} = 62.5 \quad \Rightarrow Q = \frac{25}{62.5} = 0.4$$

$$K = \frac{750}{625} = 1.2$$

This filter does satisfy the specifications.

VP 16.5

$$\omega_0 = \sqrt{144} = 12 \text{ rad / s}$$

$$\frac{\omega_0}{Q} = 30 \quad \Rightarrow \quad Q = \frac{12}{30} = 0.4$$

This filter does <u>not</u> satisfy the specifications.

Design Problems

DP 16.1

$$\frac{V_0(s)}{V_1(s)} = \frac{-\frac{s}{RC}}{s+\frac{2}{R_3C}s+\frac{2}{RR_3C^2}}$$

$$2\pi(100.10^3) = \omega_0 = \sqrt{\frac{2}{RR_3C^2}}$$

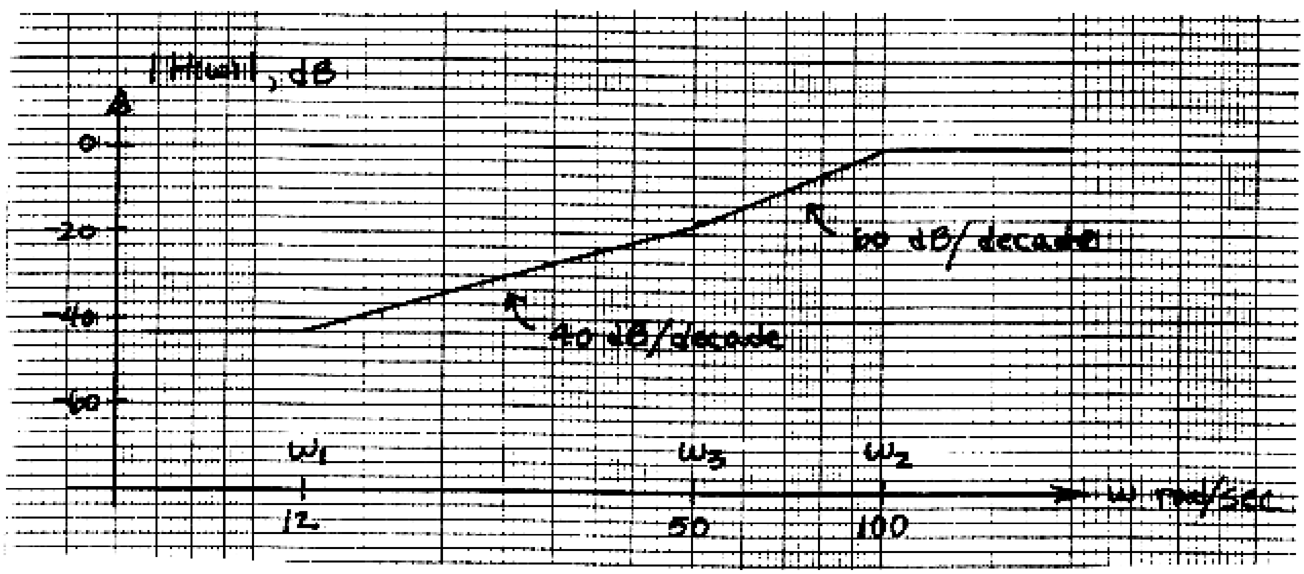
$$2\pi(10.10^3) = BW = \frac{\omega_0}{Q} = \frac{2}{R_3C}$$

$C = 100$ pF is specified so

$$R_3 = \frac{2}{(100\times10^{-12})\,(2\pi\times10\times10^3)} = 318\text{ k}\Omega$$

$$R = \frac{2}{R_3C^2\omega_0^2} = 1.6\text{ k}\Omega$$

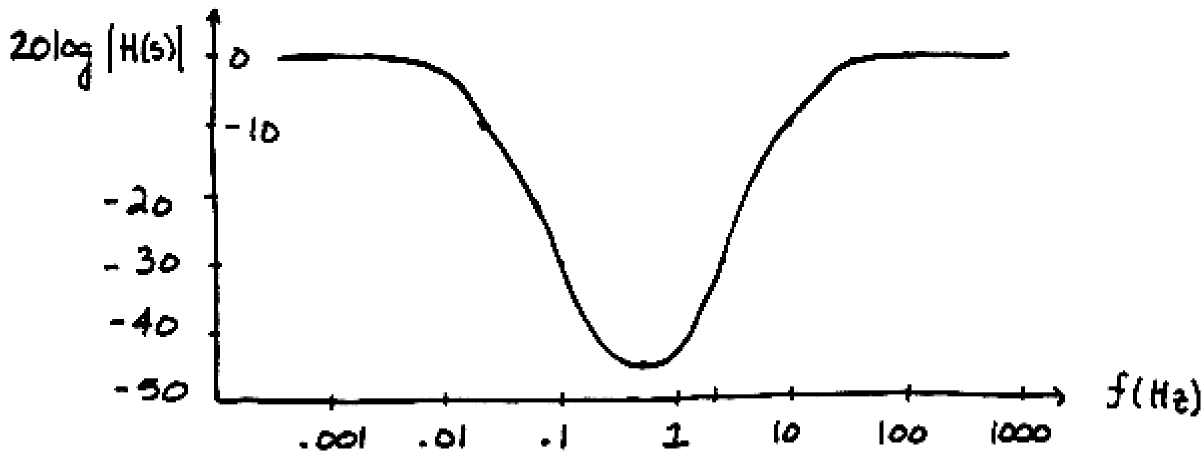
DP16.2



DP 16.3

Choose $\omega_1 = 0.1$, $\omega_2 = 2$, $\omega_3 = 5$, $\omega_4 = 100$ rad/s

The corresponding Bode magnitude plot is:



$$H(s) = \frac{\left(1+\frac{s}{\omega_1}\right)^2\left(1+\frac{s}{\omega_3}\right)^2}{\left(1+\frac{s}{\omega_2}\right)^2\left(1+\frac{s}{\omega_4}\right)^2}$$

Minimum is -46.2 db at $f_{min} = 0.505$ Hz