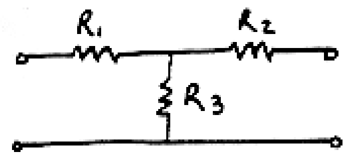
Chapter 17- Two-Port and Three Port Networks

Exercises

Ex. 17.4-1

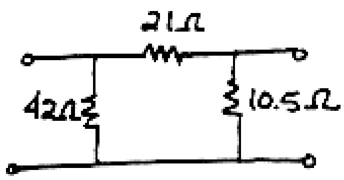


$$R_1 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{25(100)}{250} = 10\Omega$$

$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{(125)(125)}{250} = 12.5\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{100(125)}{250} = 50\Omega$$

Ex. 17.5-1



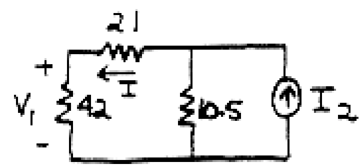
$$-Y_{12} = -Y_{21} = \frac{1}{21}$$

$$Y_{11} + Y_{12} = \frac{1}{42} \quad \text{so} \quad Y_{11} = \frac{1}{42} - \left(-\frac{1}{21}\right) = \frac{3}{42}$$

$$Y_{22} + Y_{21} = 10.5 \quad \text{so} \quad Y_{22} = \frac{1}{10.5} - \left(-\frac{1}{21}\right) = 1/7$$

$$Y = \begin{bmatrix} \frac{1}{41} & -\frac{1}{21} \\ -\frac{1}{21} & \frac{1}{7} \end{bmatrix}$$

To find the Z parameters, use the definitions



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \frac{42\ (21+10.5)}{42+31.5} = 18\Omega$$

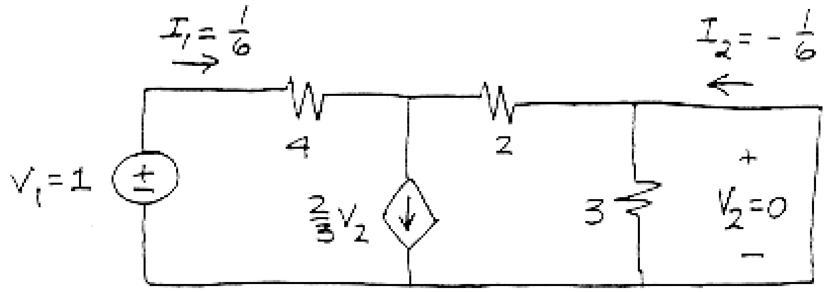
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = \frac{10.5(63)}{73.5} = 9\Omega$$

$$Z_{12} = Z_{21} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = 6\Omega$$

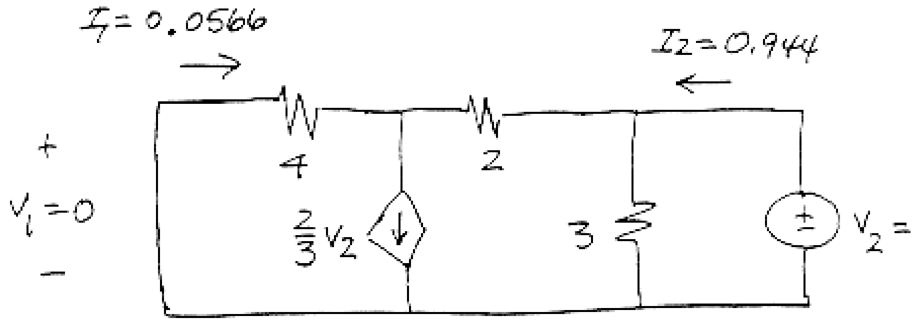
Since $I = \frac{10.5}{73.5} I_2$, then $V_1 = \frac{42(10.5)}{73.5} I_2 = 6I_2$

$$\therefore \quad Z = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}$$

Ex. 17.6-1

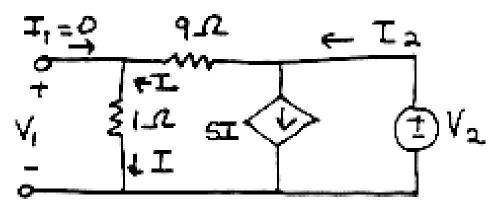


$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{6} \qquad Y_{21} = \frac{I_2}{V_1} = -\frac{1}{6} = -.167$$



$$Y_{12} = \frac{I_1}{V_2} = 0.0566 \qquad Y_{22} = \frac{I_2}{V_2} = 0.944$$

Ex. 17.7-1



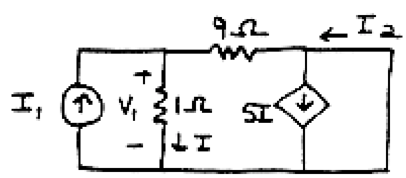
Set $I_1 = 0$
and connect V_2
and find I_2 and V_1

Note : $I_2 = 6I$, $\quad V_2 = (9+1)\,I = 10\,I$, $\quad V_1 = 1I$

Thus $h_{22} = \dfrac{I_2}{V_2} = \dfrac{6I}{10I} = 0.6$ S

$$h_{12} = \frac{V_1}{V_2} = \frac{I}{10I} = 0.1$$

Set $V_2 = 0$
and connect I_1
and find I_2 and V_1



$$V_1 = 1\,I$$

$$I_1 = I + \frac{V_1}{9} = \frac{10}{9}I$$

$$I_2 = 5I - \frac{V_1}{9} = \frac{44}{9}I$$

$$h_{11} = \frac{V_1}{I_1} = \frac{I}{\frac{10}{9}I} = 0.9\Omega$$

$$h_{21} = \frac{I_2}{I_1} = \frac{\frac{44}{9}I}{\frac{10}{9}I} = 4.4$$

Ex. 17.8-1 $Y = \begin{bmatrix} 2/15 & -1/5 \\ -1/10 & 2/5 \end{bmatrix}$ $\Delta Y = \dfrac{4}{75} - \dfrac{1}{50} = \dfrac{1}{30}$

then $Z = 30 \begin{bmatrix} 2/5 & 1/5 \\ 1/10 & 2/15 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 3 & 4 \end{bmatrix}$

Ex. 17.8-2 $T = \begin{bmatrix} -\dfrac{2/5}{(-1/10)} & -\dfrac{1}{(-1/10)} \\ -\dfrac{1/30}{(-1/10)} & -\dfrac{2/15}{(-1/10)} \end{bmatrix} = \underline{\begin{bmatrix} 4 & 10 \\ 1/3 & 4/3 \end{bmatrix}}$

Ex. 17.9-1 $T_a = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix}$ $T_b = \begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix}$ $T_c = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$$T_a T_b T_c = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix} T_c$$

$$= \begin{bmatrix} 3 & 12 \\ 1/6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \underline{\begin{bmatrix} 3 & 21 \\ 1/6 & 3/2 \end{bmatrix}}$$

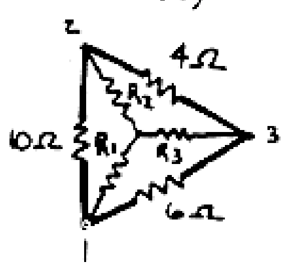
Problems

Section 17-4: T-to-T1 Transformations

17.4-1

(a)

Determine equivalent resistance, R_{ab} of Fig.(a)
Now the 10Ω resistance is part of two 3-terminals circuits that cannot be simplified by series-parallel combination. First replace circuit with thicker lines by an equivalent Y (Fig.(b)).



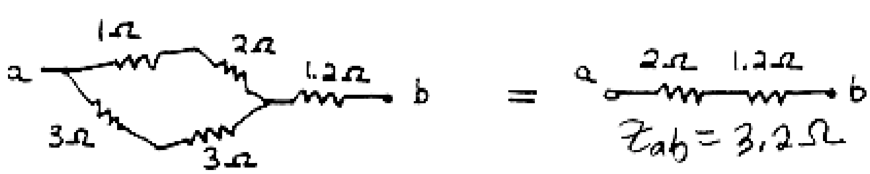
(b)

$$Z_1 = \frac{Z_{12}Z_{31}}{Z_{12}+Z_{23}+Z_{31}} = \frac{(10)\,(6)}{10+6+4} = 3\Omega$$

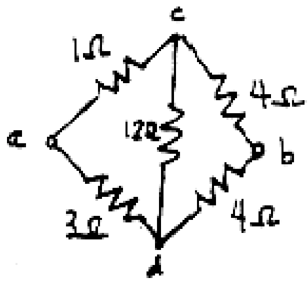
$$Z_2 = \frac{Z_{23}Z_{12}}{Z_{12}+Z_{23}+Z_{31}} = \frac{(4)\,(10)}{20} = 2\Omega$$

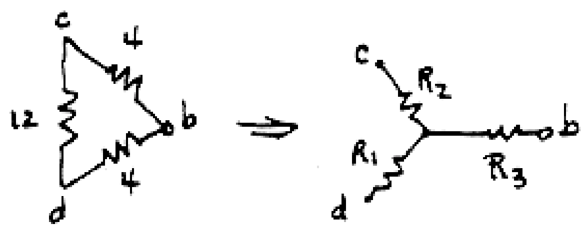
$$Z_3 = \frac{Z_{31}Z_{23}}{Z_{12}+Z_{23}+Z_{31}} = \frac{(6)\,(4)}{20} = 1.2\Omega$$

Thus have



17.4-2



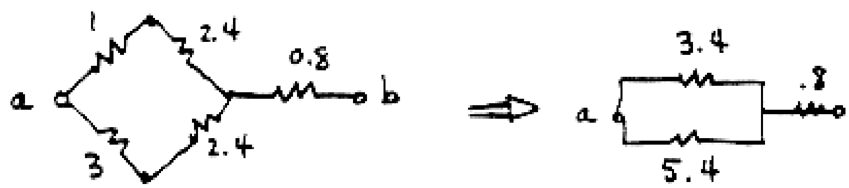


$$R_1 = \frac{12\times4}{12+4+4} = \frac{48}{20} = 2.4$$

$$R_2 = \frac{12\times4}{20} = 2.4$$

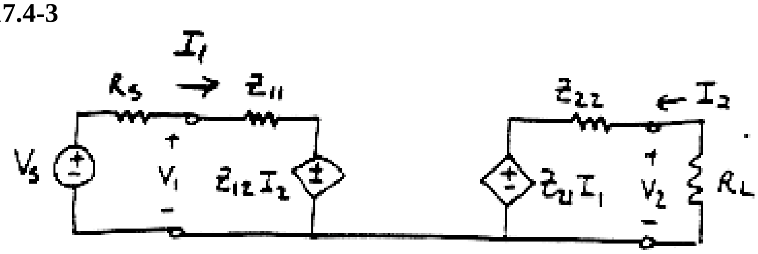
$$R_3 = \frac{4\times4}{20} = 0.8$$

Then



$$R_p = \frac{3.4(5.4)}{3.4+5.4} = 2.086 \quad \text{so } R_{ab} = 2.886\Omega$$

17.4-3



$$I_2 = \frac{-Z_{21}\, I_1}{Z_{22} + R_L}$$

$$A_i = \frac{-I_2}{I_1} = \frac{Z_{21}}{Z_{22} + R_L}$$

(forward current gain)

$$R_{in} = \frac{V_1}{I_1} = \frac{Z_{11}I_1 + Z_{12}I_2}{I_1}$$

$$= Z_{11} - \frac{Z_{12}\;A_i\;I_1}{I_1} \qquad \text{where } I_2 = A_i\, I_1 = \frac{Z_{21}\, I_1}{Z_{22} + R_L}$$

$$= Z_{11} - \frac{Z_{12}\;Z_{21}}{(Z_{22} + R_L)}$$

Forward voltage gain $A_v = \frac{V_2}{V_1}$ where $V_2 = -I_2 R_L = A_i R_L I_1$

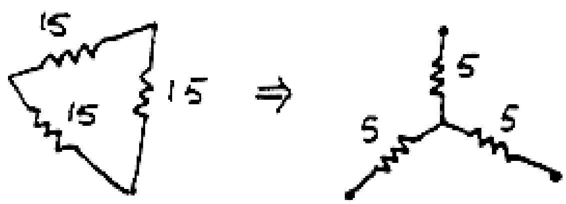
also note : $V = R_{in} I_1$

$$\text{So } A_v = \frac{V_2}{V_1} = \frac{A_i R_L}{R_{in}}$$

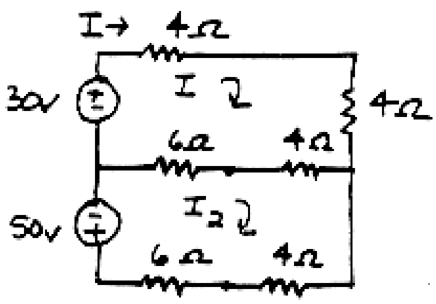
$\therefore A_p = A_i A_v = A_i^2 \frac{R_L}{R_{in}}$

17.4-4

Use Δ to Y transformation to transform R_1 Δ nework



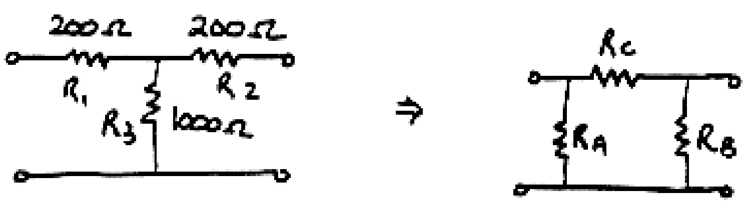
Have $5 \| R = 5 \| 20 = 4\Omega$



$$\begin{cases} 30 = 18I - 10I_2 \\ 50 = 10I - 20I_2 \end{cases}$$

$$I = \frac{\begin{vmatrix} 30 & -10 \\ 50 & -20 \end{vmatrix}}{18(-20) - (-10)10} = \frac{-100}{-260} = \underline{0.385A}$$

17.4-5

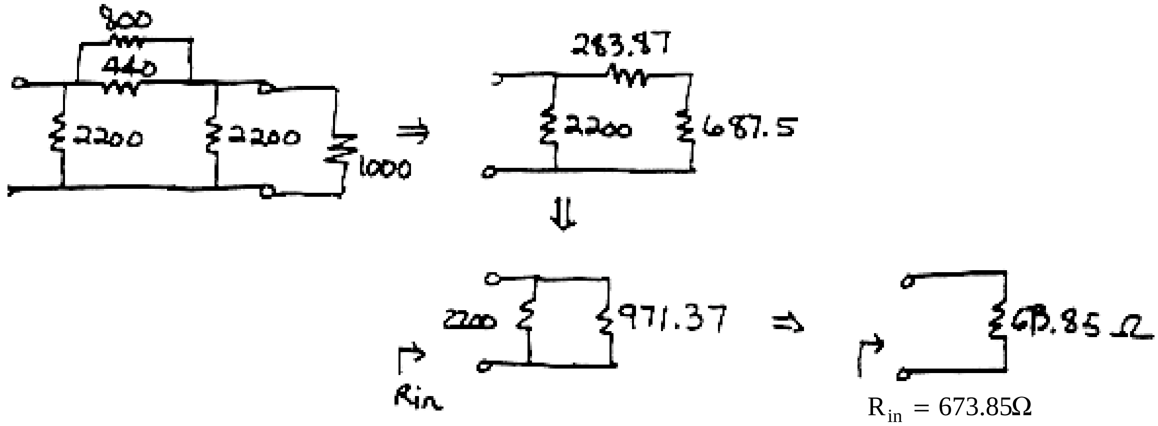


See Fig. 18-2.

$$R_A = R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \quad \text{Since } R_1 = R_2$$

$$\text{then } R_A = R_B = \frac{200(200) + 200(1000) + 1000(200)}{200} = 2200\Omega$$

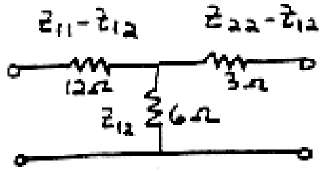
$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{44 \times 10^4}{10^3} = 440\Omega$$



$\underline{R_{in} = 673.85\Omega}$

<u>Section 17-5:</u> Equations of Two-Port Networks

17-5-1



T-network $\Rightarrow$ use Fig.17.5-1 to get parameters

Note $= Z_{12} = 6 \qquad Z_{22} = 9$

$Z_{11} = 18$

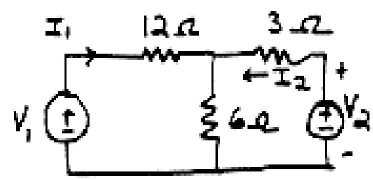
$$\text{so } Z = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}$$

Y parameters:

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{14}$$

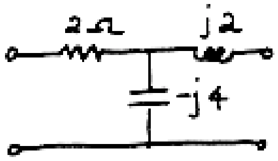
$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-6\,I_2}{(6+12)V_2} = -\frac{1}{21} = Y_{21}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{V_2/7}{V_2} = \frac{1}{7}$$



$$\text{So} \quad Y = \begin{bmatrix} 1/14 & -1/21 \\ -1/21 & 1/7 \end{bmatrix}$$

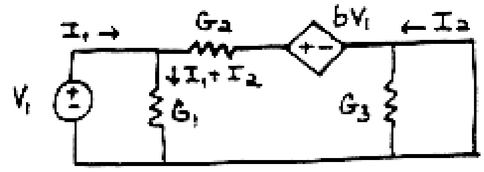
17.5-2



$$Z = \begin{bmatrix} 2-j4 & -j4 \\ -j4 & +j2 \end{bmatrix}$$

17.5-3

If $V_2 = 0 \rightarrow$ short circuit the output



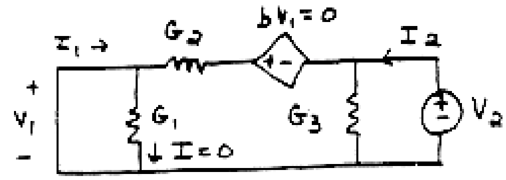
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}, \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$$

Now $V_1 = \dfrac{I_1 + I_2}{G_1}$ and $\dfrac{I_1 + I_2}{G_1} + \dfrac{I_2}{G_2} = bV_1$

So $I_1 = (G_1 - (b-1)G_2)V_1 = -1V_1$ & $I_2 = (b-1)G_2 V_1 = 3V_1$

Then $Y_{11} = -1S$ and $Y_{21} = 3S$

If $V_1 = 0 \rightarrow$ short circuit the input

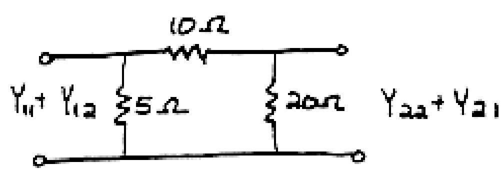


$$V_2 = \frac{I_1 + I_2}{G_3} = V_2$$

also $V_2 = \dfrac{-I_2}{G_2}$ $\quad \therefore Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0} = -G_2 = -1S$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = G_2 + G_3 = 4S$$

17.5-4



Using Fig. 17.5-2 as shown:

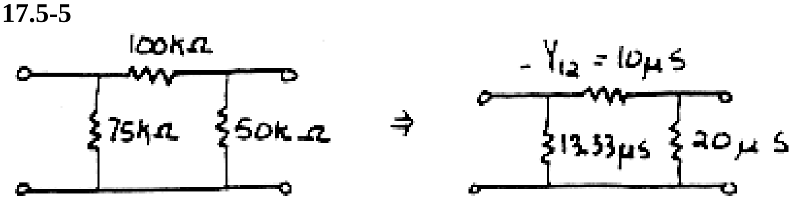
$Y_{11} + Y_{12} = 0.2S$

$-Y_{12} = -Y_{21} = 0.1$ or $Y_{12} = Y_{21} = -0.1S$

$Y_{11} = 0.2 - Y_{12} = 0.3$ S

$Y_{22} = 0.05 - Y_{21} = 0.15$ S

17.5-5



$Y_{12} = -10\mu S = \underline{Y_{21}}$

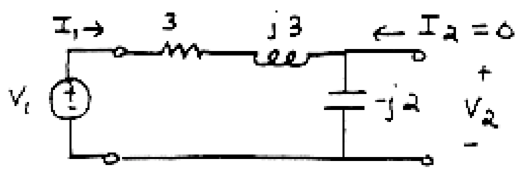
$Y_{11} + Y_{12} = 13.33\mu S$

$Y_{11} = \underline{23.33\mu S}$

$Y_{22} + Y_{21} = 20\mu S$ thus $Y_{22} = \underline{30\mu S}$

17.5-6

$\omega = 3$ rad/s



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = 3 + j3 - j2 = \underline{(3+j)\Omega}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \frac{-j2I_1}{I_1} = \underline{-j2\Omega}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \underline{-j2\Omega}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \underline{-j2\Omega}$$

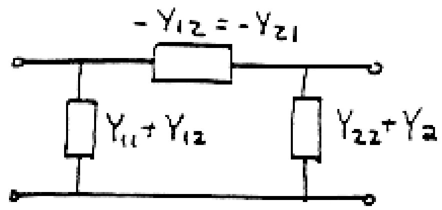
17.5-7

$$\left.\begin{aligned} \text{Now } Z_{11} - Z_{21} &= 4 \\ Z_{21} - Z_{12} &= 1/s \end{aligned}\right\} \quad \text{so } Z_{11} = 4 + 1/s = \frac{4s+1}{s}$$

$$Z_{22} - Z_{21} = 2s \qquad \text{so } Z_{22} = 2s + 1/s = \frac{2s^2+1}{s}$$

17.5-8

$$Y = \begin{bmatrix} \frac{s+1}{s} & -1 \\ -1 & s+1 \end{bmatrix} \quad \text{Use } \pi \text{ network}$$

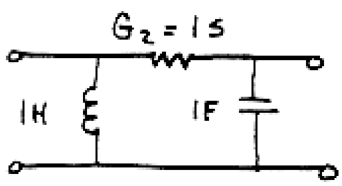


$$Y_{12} = -1s$$

$$Y_{11} = \frac{s+1}{s} \quad \text{so } Y_{11} + Y_{12} = \frac{s+1}{s} - 1 = \frac{1}{s}$$

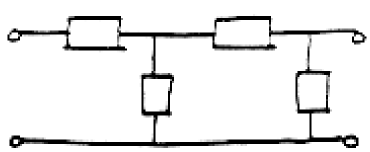
$$Y_{22} + Y_{21} = (s+1) - 1 = s$$

So have



17.5-9

$$Z = \begin{bmatrix} \frac{s^2+2s+2}{s^2+s+1} & \frac{1}{s^2+s+1} \\ \frac{1}{s^2+s+1} & \frac{s^2+1}{s^2+s+1} \end{bmatrix} \text{has 4 elements} \Rightarrow$$

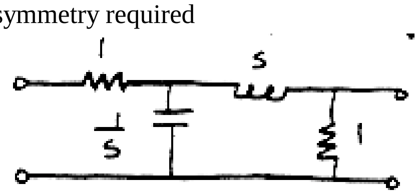


$(s^2 + s + 1)$ = char. eqn. $\Rightarrow$ leads to $R_1 = R_2 = 1$

$$\left.\begin{aligned} &\text{capacitance : } Z = \frac{1}{Cs} \qquad \text{use } C = 1 \\ &\text{inductor : } Z = Ls \qquad \text{so use } L = 1 \end{aligned}\right\} \text{to get } s^2+s+1$$

$Z_{12} = Z_{21}$ so some of symmetry required

Use a trial :

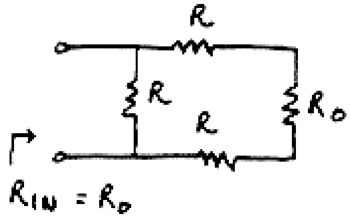


Try $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1(s+1/s)}{1+s+1/s} = \frac{(s^2+1)}{s^2+s+1}$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 1+\frac{1/s\,(s+1)}{1/s+s+1} = \frac{s^2+2s+2}{s^2+s+1}$

checks

17.5-10



$$R_o = \frac{R(2R+R_o)}{3R+R_o}$$

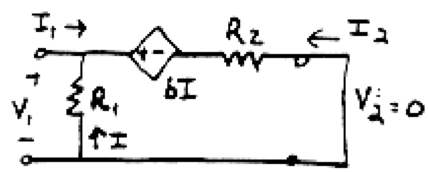
Solving for $R_o \Rightarrow R_o = R \pm \sqrt{4R^2+4(2R^2)}$

$= R \pm \sqrt{3}R = \underline{(\sqrt{3}-1)R}$

Section 17-6: Z and Y Parameters . . .

17.6-1

Set $V_2 = 0$

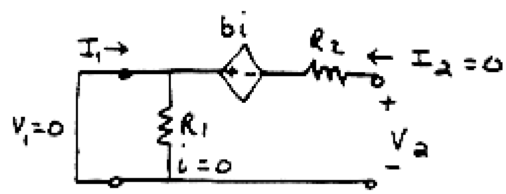


$I = -V_1/R_1$, $I_2 = -\frac{(b+R_1)}{R_1R_2}V_1$

$I_1 = -I_2 - I = \left(\frac{b+R_1+R_2}{R_1R_2}\right)V_1$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{b+R_1+R_2}{R_1R_2} \quad \text{and} \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-(b+R_1)}{R_1R_2}$$

Set $V_1 = 0$



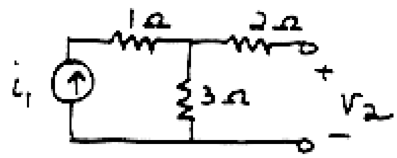
$I_2 = -I_1$

KVL : $V_2 = R_2I_2 \Rightarrow I_2 = V_2/R_2$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{R_2} \quad \text{and} \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{1}{R_2}$$

17.6-2

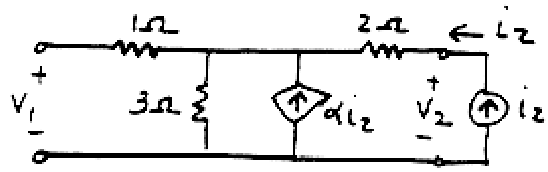
(1) Place source i_1 and open circuit output



$\boxed{Z_{11} = 4\,\Omega}$

$v_2 = 3i,$ so $\boxed{Z_{21} = 3\Omega}$

(2) Set $i_1 = 0$



$i_{3\Omega} = \alpha i_2 + i_2 = (1+\alpha)i_2$

$v_1 = 3(1+\alpha)i_2$ so $\boxed{Z_{12} = 3(1+\alpha)}$

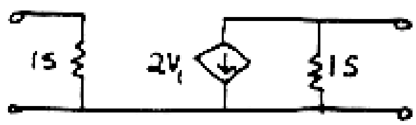
$v_2 = 2i_2 + 3(1+\alpha)i_2$ so $\boxed{Z_{22} = (5+3\alpha)}$

So $Z = \begin{bmatrix} 4 & 3(1+\alpha) \\ 3 & (5+3\alpha) \end{bmatrix}$

17.6-3

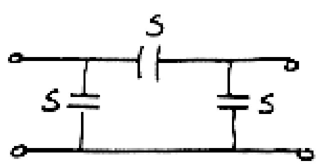
Use two Parallel networks

$Y_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$



network 1

$Y_2 = \begin{bmatrix} 2s & -s \\ -s & 2s \end{bmatrix}$



network 2
(admittance form)

Since $Y_{12} = Y_{21} = -s \quad \Rightarrow Y_{11} = 2s$ and $Y_{22} = 2s$

$Y_{11} = Y_{12} = s$

$Y_T = Y_1 + Y_2 = \begin{bmatrix} 1+2s & -s \\ 2-s & 1+2s \end{bmatrix} \qquad Y_T \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} 1/s \\ 0 \end{bmatrix}$

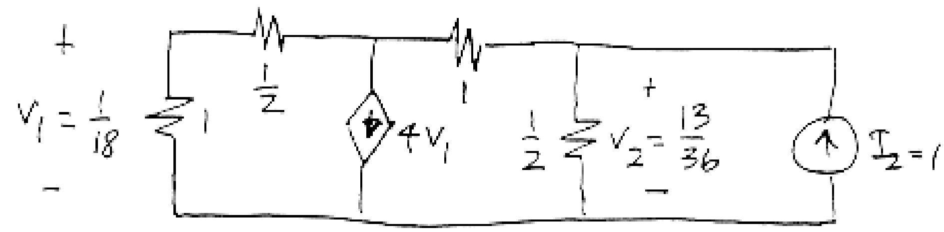
So $\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = Y_T^{-1} \begin{bmatrix} 1/s \\ 0 \end{bmatrix} = \dfrac{1}{q} \begin{bmatrix} 2s+1 & s \\ s-2 & 2s+1 \end{bmatrix} \begin{bmatrix} 1/s \\ 0 \end{bmatrix}$

where $q = 3s^2 + 6s + 1$

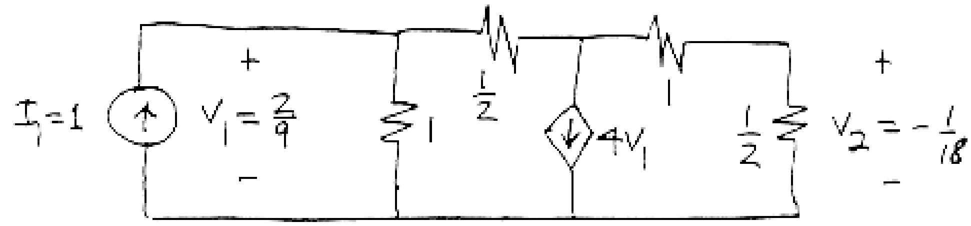
So $V_2(s) = \dfrac{(s-2)}{s(3s^2+6s+1)} = \dfrac{1}{3}\left[\dfrac{-6}{s} + \dfrac{-1.25}{s+1.82} + \dfrac{7.25}{s+0.184}\right]$

Thus $v_2(t) = \dfrac{1}{3}\left[-6 - 1.25e^{-0.182t} + 7.25e^{-0.184t}\right] \qquad t \ge 0$

17.6-4

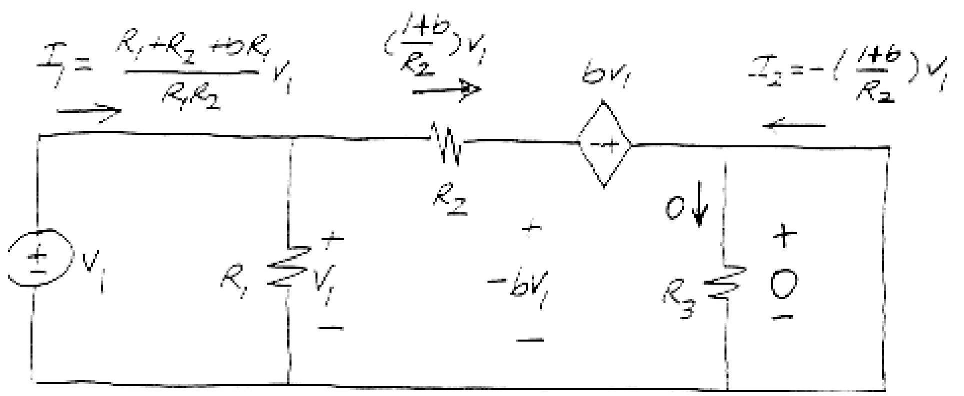


$$Z_{12} = \frac{V_1}{I_2} = \frac{1}{18} \qquad Z_{22} = \frac{V_2}{I_2} = \frac{13}{36} = .361$$



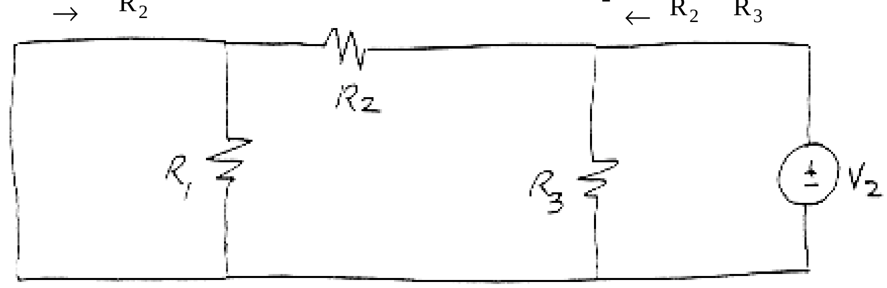
$$Z_{11} = \frac{V_1}{I_1} = \frac{2}{9} \qquad Z_{21} = \frac{V_2}{I_1} = -\frac{1}{8}$$

17.6-5



$$Y_{11} = \frac{I_1}{V_1} = \frac{R_1 + R_2 + bR_1}{R_1 R_2} \qquad Y_{21} = \frac{I_2}{V_1} = -\left(\frac{1+b}{R_2}\right)$$

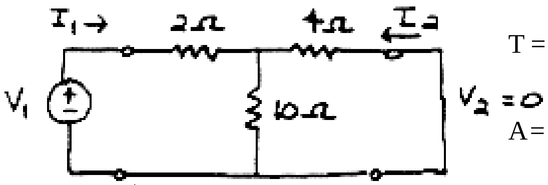
$$\underset{\rightarrow}{I_1} = -\frac{V_2}{R_2} \qquad \underset{\leftarrow}{I_2} = \frac{V_2}{R_2} + \frac{V_2}{R_3}$$



$$Y_{12} = \frac{I_1}{V_1} = -\frac{1}{R_2} \qquad Y_{22} = \frac{I_2}{V_2} = \frac{R_2 + R_3}{R_2 R_3}$$

Section 17-7: Hybrid Transmission Parameters

17.7-1



$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

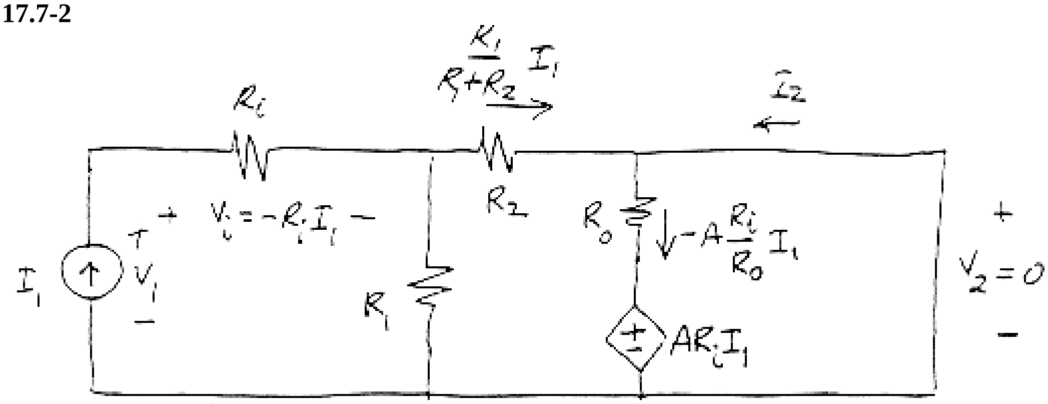
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{12}{10} = 1.2 \text{ since by voltage divider } V_2 \text{ across } 10\Omega \text{ is } V_2 = \frac{10}{12}V_1$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{34}{5} = 6.8\Omega \quad \text{since } -I_2 = \frac{V_1}{2+4\|10} = \frac{5}{34}V_1$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{10} = 0.1\text{S}$$

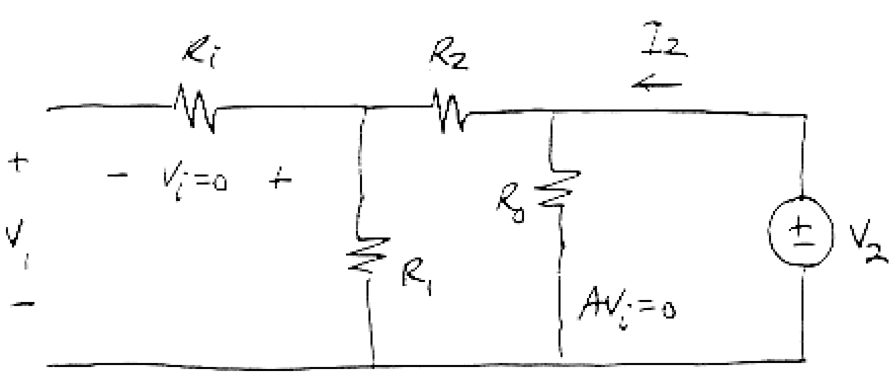
$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{1}{10/(10+4)} = 1.4 \quad \text{since } I_2 = \frac{10}{10+4}I_1 \text{(current divider)}$$

17.7-2



$$V_1 = (R_i + R_1\|R_2)I_1 \Rightarrow h_{11} = R_i + R_1\|R_2 = 600\text{k}\Omega$$

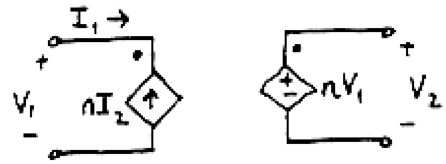
$$I_2 = -A\frac{R_i}{R_o}I_1 - \frac{R_1}{R_1+R_2}I_1 \Rightarrow h_{21} = -(A\frac{R_i}{R_o} + \frac{R_1}{R_1+R_2}) = -10^6$$



$$I_2 = \frac{V_2}{R_o\|(R_1+R_2)} \Rightarrow h_{22} = \frac{R_o+R_1+R_2}{R_o(R_1+R_2)} = 10^{-3}$$

$$V_1 = \frac{R_1}{R_1+R_2}V_2 \quad \Rightarrow h_{12} = \frac{R_1}{R_1+R_2} = \frac{1}{2}$$

17.7-3

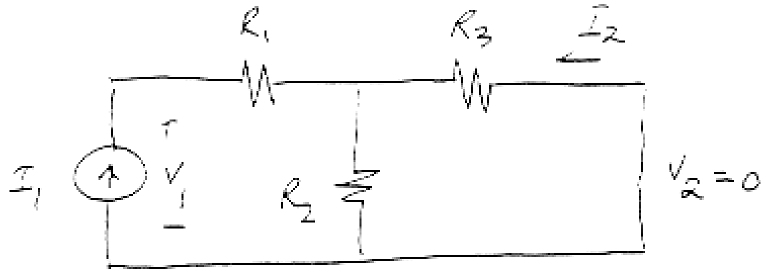


$$V_2 = nV_1 \qquad \begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{cases}$$
$$I_1 = -nI_2$$

Then $h_{11} = 0$ and $h_{22} = 0$

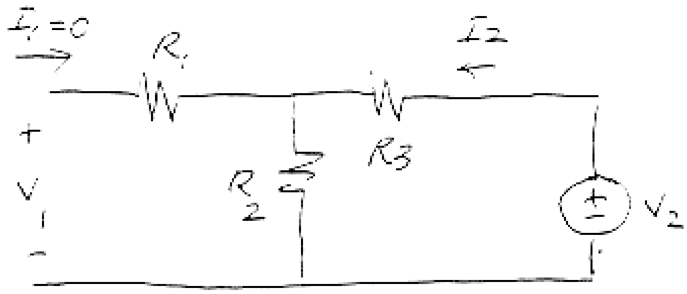
$h_{12} = 1/n$ and $h_{21} = 1/-n$

17.7-4



$$V_1 = (R_1 + R_2 // R_3)\, I_1 \Rightarrow h_{11} = R_1 + \frac{R_2R_3}{R_2+R_3}$$

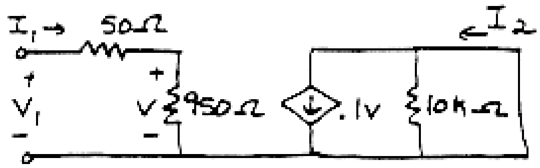
$$I_2 = -\frac{R_2}{R_2+R_3}\, I_1 \Rightarrow h_{21} = -\frac{R_2}{R_2+R_3}$$



$$I_2 = \frac{V_2}{R_2+R_3} \Rightarrow h_{22} = \frac{1}{R_2+R_3}$$

$$V_1 = \frac{R_2}{R_2+R_3} V_2 \Rightarrow h_{12} = \frac{R_2}{R_2+R_3}$$

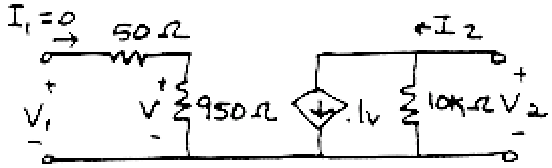
17.7-5



$I_2 = .1V,\ V = 950\, I_1$

so $I_2 = .1\,(950)\, I_1$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 50 + 950 = \underline{1000\Omega} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = 95$$



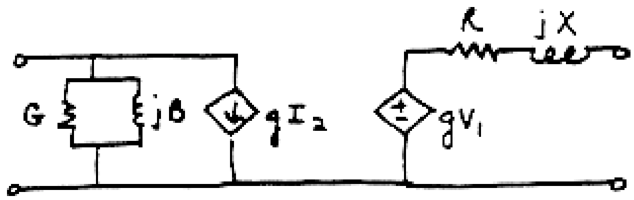
Continued

$V = 0$ since $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 0 \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = 10^{-4}$$

17.7-6

$Y = G + jB$
$Z = R + jX$



$$\text{open ckt: } G = \frac{P}{V^2} = \frac{100}{(10^3)^2} = 100\mu S, \qquad Y = \frac{I}{V} = \frac{0.42}{1000} = 420\mu S$$

$$B = -\sqrt{Y^2 - G^2} = -408\mu S \qquad g = \frac{V_2}{V_1} = \frac{500}{1000} = 1/2$$

$$\text{shortckt: } R = \frac{g^2 P}{I_1^2} = \frac{400}{4\times10^2} = 1\Omega, \qquad Z = g^2\frac{V_1}{I_1} = \frac{126}{4\times10} = 3.15\Omega$$

$$X = \sqrt{Z^2 - R^2} = 3\Omega$$

Section 17-8: Relationships between Two-Port Parameters

17.8-1 $\tilde{I} = \tilde{Y}\,\tilde{V}$

$$\text{Y parameters } \begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{cases} \quad (1) \qquad \text{h parameters } \begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{cases} \quad (2)$$

Rewrite eqn.(1) into form of (2) to get I_1 and V_2 on right side of eqn.

$$-Y_{11}V_1 = I_1 + Y_{12}V_2$$

$$-Y_{21}V_1 + I_2 = Y_{22}V_2 \qquad \text{or} \qquad \hat{Y}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \overset{*}{Y}\begin{bmatrix} -I_1 \\ V_2 \end{bmatrix}$$

where $\hat{Y}$ and $\overset{*}{Y}$ are new matrices related to Y

and h requires $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = H\begin{bmatrix} -I_1 \\ V_2 \end{bmatrix}$

$$\therefore\; H = \hat{Y}^{-1}\,Y^* = \begin{bmatrix} -Y_{11} & 0 \\ -Y_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & Y_{12} \\ 0 & Y_{22} \end{bmatrix} = \begin{bmatrix} -1/Y_{11} & 0 \\ -Y_{21}/Y_{11} & 1 \end{bmatrix} \begin{bmatrix} -1 & Y_{12} \\ 0 & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_{11}} & -Y_{12}/Y_{11} \\ \frac{Y_{21}}{Y_{11}} & Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}} \end{bmatrix}$$

17.8-2 $\Delta Z = (3)(6) - (2)(2) = 14$

$$Y = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix} = \begin{bmatrix} \frac{6}{14} & -\frac{2}{14} \\ -\frac{2}{14} & \frac{3}{14} \end{bmatrix}$$

17.8-3 $\Delta Y = (0.1)(0.5) - (0.4)(0.1) = .01$

$$H = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 4 & 0.1 \end{bmatrix}$$

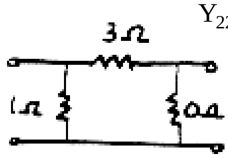
17.8-4 $\Delta Y = (0.5)(0.6) - (-0.4)(-0.4)$

$$H = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix} = \begin{bmatrix} 2 & 0.8 \\ -0.8 & 0.28 \end{bmatrix}$$

<u>Section17-9:</u> Interconnection of Two-Port Networks

17.9-1 $Y = Y_a + Y_b$ in parallel

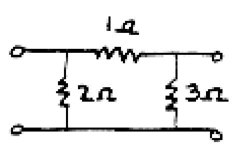
$$Y_a = \begin{bmatrix} 4/3 & -1/3 \\ -1/3 & 1/3 \end{bmatrix}$$



$Y_{22} = 0 - Y_{21} = 1/3$

$Y_{11} + Y_{12} = 1$ so $Y_{11} = 1 + 1/3 = 4/3$

$$Y_b = \begin{bmatrix} 3/2 & -1 \\ -1 & 4/3 \end{bmatrix}$$



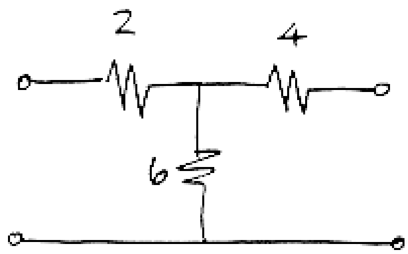
$Y_{11} + Y_{12} = 1/2$

$Y_{11} = 1/2 - (-1) = 3/2$

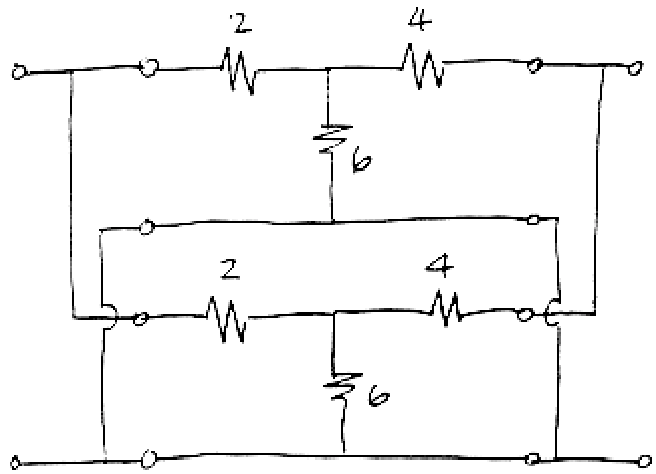
$Y_{21} + Y_{22} = 1/3$ so $Y_{22} = 1 + 1/3 = 4/3$

$$Y = \begin{bmatrix} (4/3 + 3/2) & -4/3 \\ -4/3 & 5/3 \end{bmatrix} = \underline{\begin{bmatrix} 17/6 & -4/3 \\ -4/3 & 5/3 \end{bmatrix}}$$

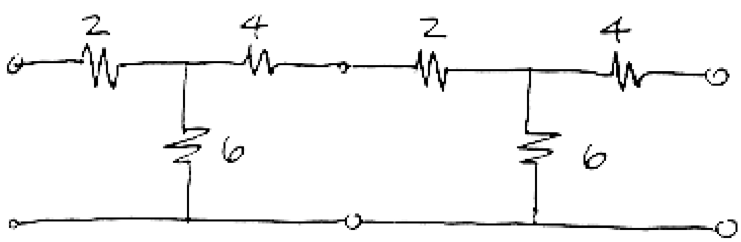
17.9-2



$$Y=\begin{bmatrix}\frac{10}{44} & \frac{-6}{44}\\ \frac{-6}{44} & \frac{8}{44}\end{bmatrix}\quad T=\begin{bmatrix}\frac{8}{6} & \frac{44}{6}\\ \frac{1}{6} & \frac{10}{6}\end{bmatrix}$$



$$Y_p = Y + Y = \begin{bmatrix}\frac{20}{44} & \frac{-12}{44}\\ \frac{-12}{44} & \frac{16}{44}\end{bmatrix}$$



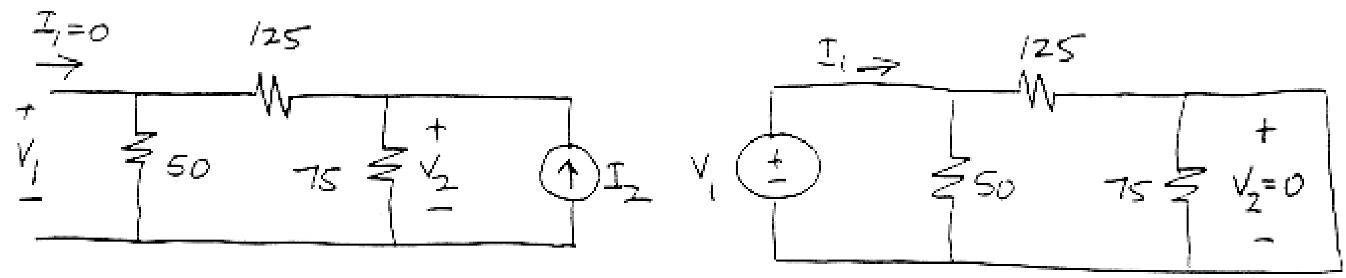
$$T_C = T \cdot T' = \begin{bmatrix}\frac{108}{36} & \frac{792}{36}\\ \frac{18}{36} & \frac{144}{36}\end{bmatrix}$$

17.9-3

$$Y = \begin{bmatrix}\frac{1}{s}+s & -s\\ -s & \frac{1}{s}+s\end{bmatrix} + \begin{bmatrix}G_1+G_2 & -G_2\\ -G_2 & G_2+G_3\end{bmatrix}$$

Verification Problems

VP 17-1

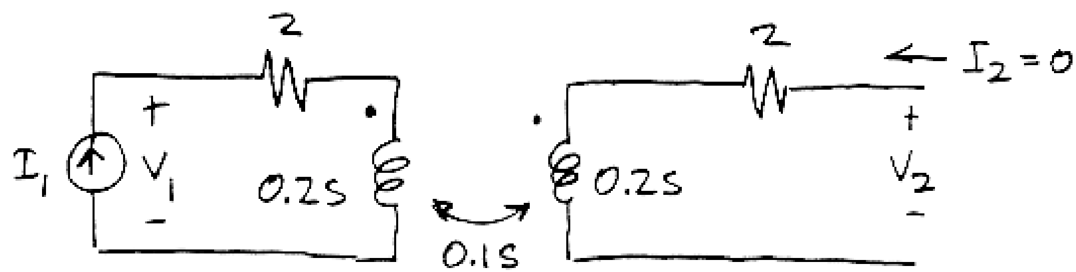


$$V_1 = 50\left(\frac{75}{175+75}\right) I_2 = 15\, I_2 \qquad I_1 = \left(\frac{1}{50}+\frac{1}{125}\right)V_1 = 0.028V_1$$

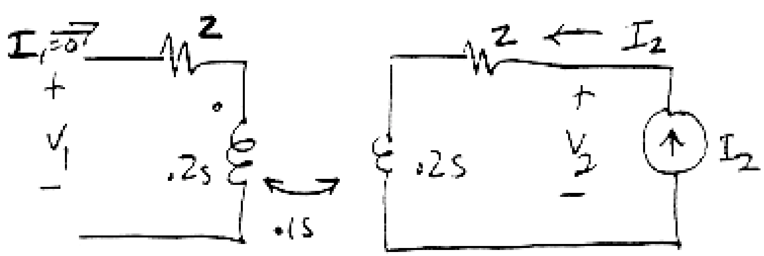
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 15 \qquad Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 28\text{mS}$$

$Y_{11} \neq 24\text{mS}$, so the report is n<u>o</u>t correct.

VP 17-2



$$\left.\begin{aligned} V_1 &= (2+0.2s)\ I_1 \\ V_2 &= (0.1s)I_1 \end{aligned}\right\} \Rightarrow \begin{aligned} Z_{11} &= 2+0.2s=0.2(s+10) \\ Z_{21} &= 0.1s \end{aligned}$$



$$Z_{22} = 2+.2s \qquad Z_{12} = .1s$$

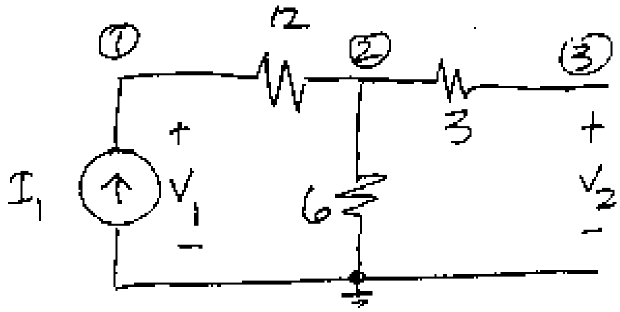
$$\Delta Z = (2+.2s)(2+.2s) - (.1s)(.1s) = .01(3s^2 + 80s + 40)$$

$$T = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix} = \begin{bmatrix} \frac{2(s+10)}{s} & \frac{.1(3s^2+80s+40)}{s} \\ .1s & \frac{2(s+10)}{s} \end{bmatrix} \leftarrow \text{Does not agree!}$$

PSpice Problems

SP 17-1

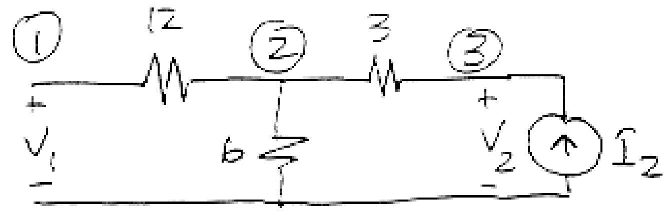
```
I1  0   1  1
R1  1   2  12
R2  2   3  3
R3  2   0  6
I2  0   3  0

.end
```




```
   NODE    VOLTAGE :    NODE   VOLTAGE     NODE    VOLTAGE
(   1)     18.0000    (   2)    6.0000   (   3)     6.0000
```

$V_1 = 18 \Rightarrow Z_{11} = 18$ $\quad\quad$ $V_2 = 6 \Rightarrow Z_{21} = 6$

```
I1  0   1  0
R1  1   2  12
R2  2   3  3
R3  2   0  6
I2  0   3  1

.end
```

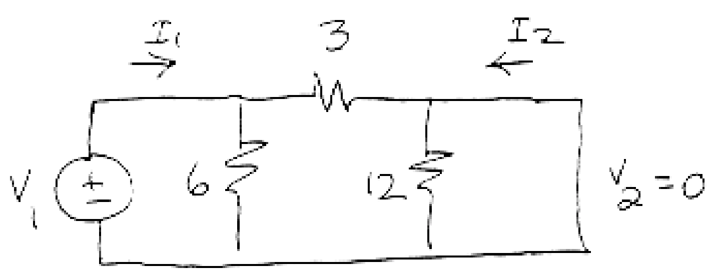



```
   NODE    VOLTAGE      NODE   VOLTAGE      NODE   VOLTAGE
(     1)     6.0000    (    2)   6.0000    (     3)   9.0000
```

$V_1 = 6 \Rightarrow Z_{12} = 6$ $\quad\quad$ $V_2 = 9 \Rightarrow Z_{22} = 9$

SP 17-2

```
V1  1   0  1
R1  1   0  6
R2  1   2  3
R3  2   0  12
V2  2   0  0


.end
```



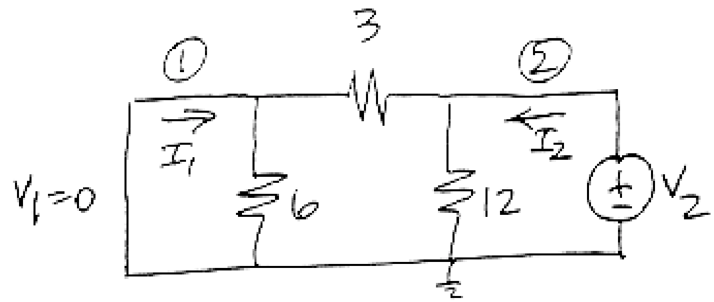

```
   NODE    VOLTAGE      NODE  VOLTAGE
(     1)    1.0000    (     2)  0.0000
```

```
VOLTAGE SOURCE CURRENTS
NAME          CURRENT
V1           -5.000E-01 = -I1
V2            3.000E-01 = -I2
```

$Y_{11} = .5$

$Y_{21} = -.33$

```
V1    1    0    0
R!    1    0    6
R2    1    2    3
R3    2    0    12
V2    2    0    1

.end
```

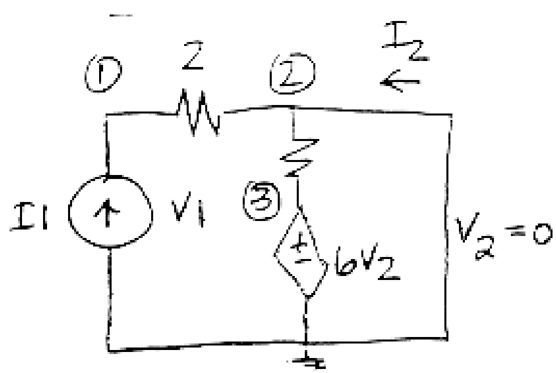



```
   NODE     VOLTAGE        NODE     VOLTAGE
(    1)      0.0000     (    2)      1.0000

VOLTAGE SOURCE CURRENTS
NAME          CURRENT
V1             3.333E-01 = -I1
V2            -4.1.67E-01 = I2
```

$Y_{12} = -.333,\ Y_{22} = -.4167$

SP 17-3

```
I1  0  1  1
R1  1  2  2
R2  2  3  3
E   3  0  2  0  6
V2  2  0  0

.end
```



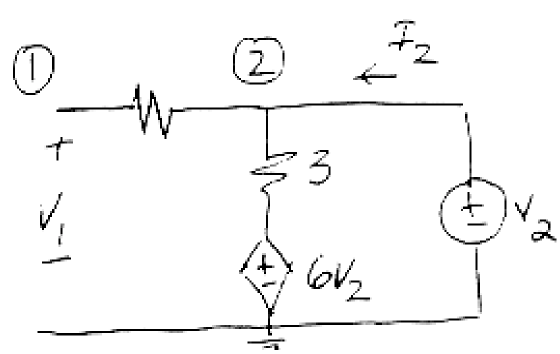

```
   NODE     VOLTAGE       NODE   VOLTAGE       NODE     VOLTAGE
(    1)      2.0000    (    2)   0.0000    (    3)      0.0000
```

$V_1=2 \Rightarrow h_{11}=2$

```
VOLTAGE SOURCE CURRENTS
NAME         CURRENT
V2             1.000E+00=–I2
```

$\Rightarrow h_{21} = -1$

```
I1  0  1  0
R1  1  2  2
R2  2  3  3
E   3  0  2  0  6
V2  2  0  1

.end
```



NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE
(1)	1.0000	(2)	1.0000	(3)	6.0000

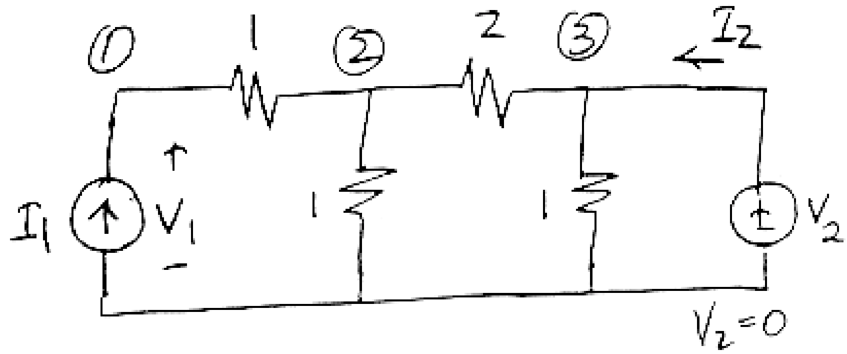
$V_1=1 \Rightarrow h_{12}=1$

VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V2	1.667E+00=$-I_2 \Rightarrow h_{22}$= 1.667

SP 17-4

```
I1   0  1   1
R1   1  3   1
R2   3  0   1
R3   2  3   2
R4   2  0   1
V2   2  0   0

.end
```



NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE
(1)	1.6667	(2)	0.0000	(3)	.6667

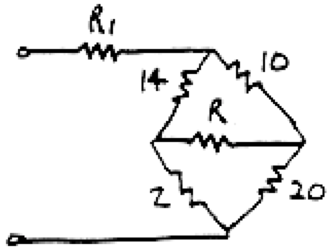
$V_1=1.667 \Rightarrow h_{11}=1.667$

VOLTAGE SOURCE CURRENTS

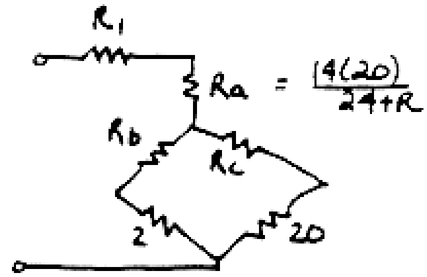
NAME	CURRENT
V2	3.333E-01==$-I_2 \Rightarrow h_{21}$= -.33

Design Problems

DP 17-1



$\Rightarrow$



$$R_a = \frac{140}{24+R}$$
$$R_b = \frac{14R}{24+R}$$
$$R_c = \frac{10R}{24+R}$$

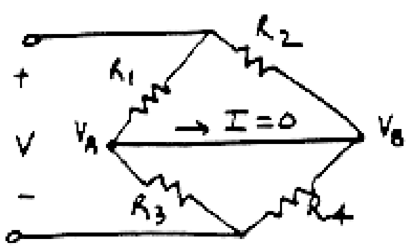
$$R_{in} = R_1 + R_a + \frac{(2+R_b)(20+R_c)}{22+R_b+R_c}$$

Need $R_1 < 10$, $R < 10 \Rightarrow$ try $R = 6\Omega$

$$R_{in} = R_1 + \frac{140}{30} + \frac{(4.8)(22)}{26.8} = R_1 + 4.667 + 3.94$$

Now $R_{in} = 16.6 = R_1 + 4.667 + 3.94$ $\quad$ $\underline{\text{thus } R_1 = 8\Omega}$

DP 17-2



Need $V_A + V_B$ for balance

$$\frac{R_1 V}{R_1+R_3} = \frac{R_2 V}{R_2+R_4} \qquad (1)$$

$$\frac{R_3 V}{R_1+R_3} = \frac{R_4 V}{R_2+R_4} \qquad (2)$$

Dividing (1) by (2) yields: $\underline{\frac{R_1}{R_3} = \frac{R_2}{R_4}}$ for balance

DP17-3

$$\begin{cases} V_1 = h_{11}I_1 + h_{12}V_2 & (1) \\ I_2 = h_{21}I_1 + h_{22}V_2 & (2) \end{cases}$$

also $V_2 = -I_2 R_L$ $\quad$ So $\quad I_2 = h_{21}I_1 - h_{22}R_L I_2$

Then $\frac{I_2}{I_1} = h_{21}\left(\frac{1}{1+h_{22}R_L}\right)$

$$A_i = \frac{I_L}{I_1} = -\frac{I_2}{I_1} = -h_{21}\left(\frac{1}{1+h_{22}R_L}\right)$$

require : $79 = 80\left(\frac{1}{1+h_{22}R_L}\right)$ $\quad$ or $\quad \frac{79}{80}\left(1+\frac{R_L}{80}\right) = 1$ $\qquad$ R_L in kΩ

$$\Rightarrow \underline{R_L = 1\text{k}\Omega}$$

For R_{in} , need $R_{in} = \dfrac{V_1}{I_1}$

$$I_2 = -\frac{V_2}{R_L} = h_{21}I_1 + h_{22}V_2 \qquad \text{from (2)}$$

$$\text{or} \quad V_2(h_{22} + 1/R_L) = -h_{21}I_1 \qquad (3)$$

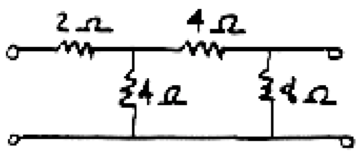
Substituting (3) into (1) $\Rightarrow V_1 = h_{11}I_1 + \dfrac{h_{12}(-h_{21})}{(h_{22} + 1/R_L)} I_1$

$$R_{in} \approx h_{11} - h_{12}R_L\, h_{21}$$

(since $h_{22} << 1/R_L$)

$R_{in} = 45 - (5\times10^{-4})(10^3)(80) = \underline{5\Omega}$ okay since $< 10\Omega$

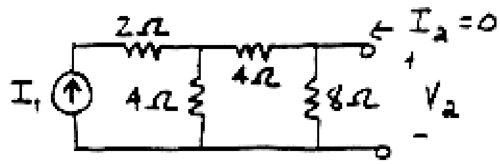
DP 17-4



$$Z_{11} = 2 + \frac{4(12)}{4+12} = 5\Omega$$

$$Z_{22} = \frac{8(8)}{8+8} = 4\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$



$$V_2 = 8\left[\frac{4}{4+12} I_1\right] = 2I_1 \quad \text{so} \quad Z_{21} = 2\Omega \text{ Similarly } Z_{12} = 2\Omega$$

Connect R_L and $V_s \Rightarrow V_2 = -I_2 RL$

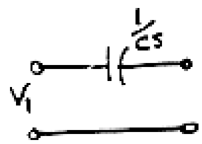
$$V_1 = V_s$$

Thèvenin: $Z_T = Z_{22} = 4\Omega$ so for max power transfer use $R_L = 4\Omega$

$$P_{RL} = \frac{\left(\frac{Vs}{2}\right)^2}{4} = 89.3\text{W} \Rightarrow \underline{V_s = 37.8\text{V}}$$

DP 17-5

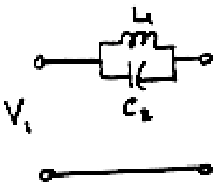
T_a



$$Z = \frac{1}{Cs}$$

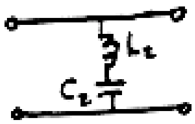
$$T = T_a T_b T_c T_d$$

T_b



$$Z = \frac{L_1 s}{L_1 C_2 s^2 + 1}$$ resonant frequency 5kHz

T_c

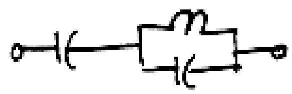


$$Y(s) = \frac{C_2 s}{L_2 C_2 s^2 + 1}$$ resonant frequency 10kHz

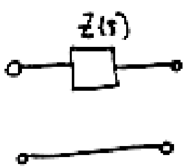
T_d

$$Y(s) = \frac{1}{R_L}$$

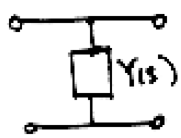
$C_1 L_1 C_2$



essentially a short circuit at 7.5kHz



$$T = \begin{bmatrix} 1 & Z(s) \\ 0 & 1 \end{bmatrix}$$



$$T = \begin{bmatrix} 1 & 0 \\ Y(s) & 1 \end{bmatrix}$$