# Chapter 7 Energy Storage Elements

## Exercises

**Ex. 7.3-1**

$$i_C(t)=1\frac{d}{dt}v_s(t)=\begin{cases}2 & 2<t<4\\ -1 & 4<t<8\\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad i_R(t)=1\,v_s(t)=\begin{cases}2t-4 & 2<t<4\\ 8-t & 4<t<8\\ 0 & \text{otherwise}\end{cases}$$

so $i(t)=i_C(t)+i_R(t)=\begin{cases}2t-2 & 2<t<4\\ 7-t & 4<t<8\\ 0 & \text{otherwise}\end{cases}$

**Ex. 7.3-2**

$$v(t)=\frac{1}{C}\int_{t_0}^{t} i_s(\tau)\,d\tau+v(t_0)=\frac{1}{\frac{1}{3}}\int_0^t i_s(\tau)\,d\tau-12$$

$$v(t)=3\int_0^t 4\,d\tau-12=12\,t-12 \quad \text{for} \quad 0<t<4$$

In particular, $v(4) = 36$ V.

$$v(t)=3\int_4^t -2\,d\tau+36=60-6t \quad \text{for} \quad 4<t<10$$

In particular, $v(10) = 0$ V.

$$v(t)=3\int_{10}^t 0\,d\tau+0=0 \quad \text{for} \quad 10<t$$

**Ex. 7.3-3** Break up into intervals

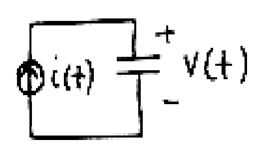
$$i=\begin{cases}0 & t<1\\ 1 & 1<t<2\\ 0 & t>2\end{cases} \qquad \text{now } v(t)=v(0)+\frac{1}{c}\int_0^t i\,dt=\frac{1}{10^{-4}}\int_0^t i\,dt$$

$\therefore$ $0<t<1 \Rightarrow v(t) = 10^4\int_0^t 0\,dt = \underline{0}$

$1<t<2 \Rightarrow v(t) = v(1)+10^4\int_1^t 1\,dt = 0+10^4\,\underline{t = 10^4\,t\text{ V}}$

$t>2 \Rightarrow v(t)=v(2)+10^4\int_2^t 0\,dt = 10^4(2) \;\underline{= 2\times10^4\text{ V}}$

**Ex. 7.3-4**




*because* $i(t) = 0$ when $t<0$

$$v(t)=\frac{1}{c}\int_{-\infty}^{t} i(t)dt=\frac{1}{5}\int_0^t(2+2\cos 5t)\,dt+v(0)=\underline{\frac{2}{5}t+\frac{2}{25}\sin 5t\,\frac{1}{5}\,V}$$

**Ex. 7.4-1** $W = \frac{1}{2}Cv^2 = \frac{1}{2}(2\times10^{-4})(100)^2 = \underline{1\text{ J}}$

$v_c(0^+) = v_c(0^-) = \underline{100\text{ V}}$

**Ex. 7.4-2** $W(t) = W(0) + \int_0^t vi\,dt \quad \Rightarrow W(0) = 0 \text{ since } v(0) = 0$

First find $\quad v(t) = v(0) + \frac{1}{C}\int_0^t i\,dt = 10^4\int_0^t 2\,dt = \underline{2\times10^4 t}$

$\therefore\ W(t) = \int_0^t (2\times10^4)(2)dt = 2\times10^4 t^2$

$W(1s) = 2\times10^4\text{ J} = \underline{20\text{ kJ}}$

b) $W(100s) = 2\times10^4(100)^2 = 2\times10^8\text{ J} = \underline{200\text{ mJ}}$

**Ex. 7.4-3**

We have $v(0^+) = v(0^-) = 3V$

$V_c(t) = \frac{1}{C}\int_{-\infty}^t i(t)dt = 5\int_0^t 3e^{5t}\,dt + 3\text{ V} = 3(e^{5t}-1)+3 = 3e^{5t}\text{ V}$

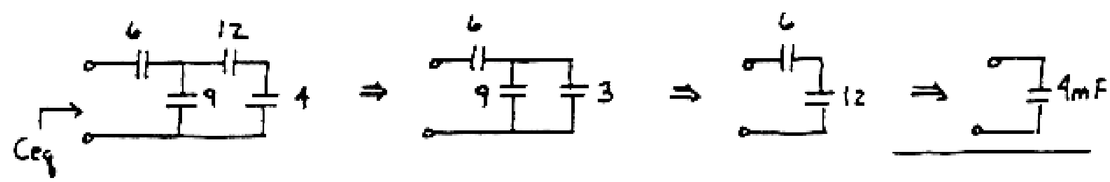
a) Now $v(t) = v_R(t) + v_c(t) = 5i(t) + v_c(t) = 15e^{5t} + 3e^{5t} = \underline{18e^{5t}\text{ V}},\ 0 \le t < 1$

b) $W_c(t) = \frac{1}{2}Cv_c^2(t) = \frac{1}{2}\times2(3e^{5t})^2 = 0.9e^{10t}\text{ J}$

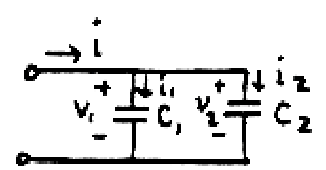
Now when $t = 0.2s \Rightarrow W_c(t)\big|_{t=.2s} = \underline{6.65\text{ J}}$

When $t = .8s \Rightarrow W_c(t)\big|_{t=.8s} = \underline{2.68\text{k J}}$

**Ex. 7.5-1**



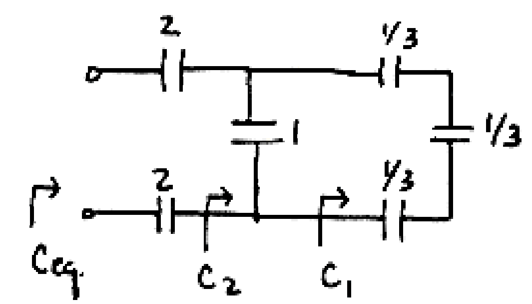

**Ex. 7.5-2**




$v_1 = v_2 \Rightarrow \frac{dv_1}{dt} = \frac{dv_2}{dt} \Rightarrow \frac{i_1}{C_1} = \frac{i_2}{C_2} \Rightarrow \underline{i_1 = \frac{C_1}{C_2}i_2}$

from KCL : $i = i_1 + i_2 = \left(\frac{C_1}{C_2}+1\right)i_2 \Rightarrow \underline{i_2 = \frac{C_2}{C_1+C_2}i}$

**Ex. 7.5-3**




$\frac{1}{C_1} = \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}} \Rightarrow C_1 = 1/9\text{ mF}$

$C_2 = 1 + C_1 = 1 + 1/9 = 10/9\text{ mF}$

So $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{10/9}$, $1/C_{eq} = 19/10$

then $\underline{C_{eq} = \frac{10}{19}\text{ mF}}$

**Ex. 7.6-1**

$$v_L(t)=1\frac{d}{dt}i_s(t)=\begin{cases}2 & 2<t<4\\ -1 & 4<t<8\\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad v_R(t)=1\,i_s(t)=\begin{cases}2t-4 & 2<t<4\\ 8-t & 4<t<8\\ 0 & \text{otherwise}\end{cases}$$

so $v(t)=v_L(t)+v_R(t)=\begin{cases}2t-2 & 2<t<4\\ 7-t & 4<t<8\\ 0 & \text{otherwise}\end{cases}$

**Ex. 7.6-2**

$$i(t)=\frac{1}{L}\int_{t_0}^{t}v_s(\tau)\,d\tau+i(t_0)=\frac{1}{\frac{1}{3}}\int_0^t v_s(\tau)\,d\tau-12$$

$i(t)=3\int_0^t 4\,d\tau-12=12\,t-12 \quad \text{for} \quad 0<t<4$ In particular, $i(4)$ = 36 A.

$i(t)=3\int_4^t -2\,d\tau+36=60-6t \quad \text{for} \quad 4<t<10$ In particular, $i(10)$ = 0 A.

$$i(t)=3\int_{10}^t 0\,d\tau+0=0 \quad \text{for} \quad 10<t$$

**Ex. 7.6-3**

$$i(t) = i(0)+\frac{1}{L}\int_0^t v(t) = 0+\frac{1}{1}\int_0^t 9t^2dt = \underline{3t^3\text{ A}}$$

**Ex. 7.7-1**

$$v = L\,{}^{di}\!/_{dt} = \left(\tfrac{1}{4}\right)\,{}^{d}\!/_{dt}\left(4te^{-t}\right) = \underline{(1-t)e^{-t}\text{ V}}$$

$$P = vi = \left[\,(1-t)e^{-t}\,\right]\left(4te^{-t}\right) = \underline{4t(1-t)e^{-2t}\text{ W}}$$

$$W = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\left(\tfrac{1}{4}\right)\left(4te^{-t}\right)^2 = \underline{\;2t^2\,e^{-2t}\text{ J}}$$

**Ex. 7.7-2**

$$v(t) = L\,{}^{di}\!/_{dt} = {}^{1}\!/_{2}\,{}^{di}\!/_{dt}$$

$$i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ -2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases} \qquad \therefore\ v(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t<1 \\ -1 & 1<t<2 \\ 0 & t>2 \end{cases}$$

$$\text{now } p(t) = v(t)\,i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ 2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases}$$

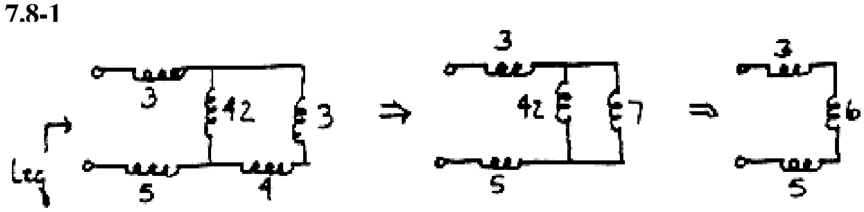
$$W(t) = W(t_0) + \int_{t_0}^{t} p(t)dt \quad \Rightarrow \quad t<0:\ W(t) = 0 \text{ since } p(t) = 0$$

$$0<t<1:\ W(t) = \int_0^t 4t\,dt = 2t^2$$

$$1<t<2:\ W(t) = W(1) + \int_1^t 4(t-2)dt = 2t^2 - 8t + 8$$

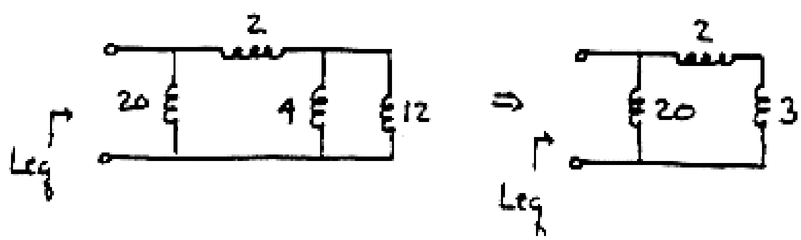
$$t>2:\ W(t) = W(2) = 0$$

**Ex. 7.8-1**




$L_{eq} = 3 + 6 + 5 = \underline{14\text{mH}}$

**Ex. 7.8-2**




$$L_{eq} = \frac{(20)(5)}{20+5} = \underline{4\text{mH}}$$

**Ex. 7.8-3**

$$i = i_1 + i_2$$

$$i_1 = \frac{1}{L_1}\int v\,dt + i_1(0), \quad i_2 = \frac{1}{L_2}\int v\,dt + i_2(0) \qquad \text{but } i_1(0)=0 \text{ and } i_2(0)=0$$

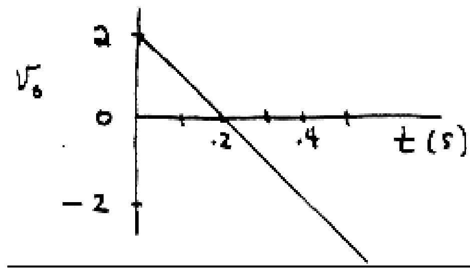
$$\text{so } i = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt, \text{ now also } i = \frac{1}{L_P}\int v\,dt + i(0) \qquad \text{where } i(0)=0$$

$$\therefore \frac{i_1}{i} = \frac{{}^{1}\!/_{L_1}\int v\,dt}{{}^{1}\!/_{L_P}\int v\,dt} = \frac{{}^{1}\!/_{L_1}}{{}^{1}\!/_{L_1} + {}^{1}\!/_{L_2}} = \underline{\frac{L_2}{L_1+L_2}}$$

**Ex. 7.10-1**

$$v_o = -\frac{1}{RC}\int_0^t v_s dt + v_o(0) = -2\int_0^t 5\,dt + 2$$

$$\frac{1}{RC} = \frac{1}{(1\times10^3)(500\times10^{-6})} = 2 \qquad \underline{v_o = (2-10\,t)\text{ V , } t\geq0}$$




**Ex. 7.10-2**

$$v_o = -RC\frac{dv_s}{dt} = -1\frac{d\,(0.2\sin 10t)}{dt} = -\underline{2\cos 10t\text{ V, } t\geq0}$$

$$RC = (20\times10^3)(50\times10^{-6}) = 1$$

## Problems

### Section 7-3: Capacitors

**P7.3-1**

$$v = v(0) + \frac{1}{C}\int i\,dt \quad \text{and} \quad q = Cv$$

$$\therefore Cv = Cv(0) + \int i\,dt = Cv(0) + it \leftarrow \text{ since i = constant}$$

$$\therefore \quad t = \frac{q - Cv(0)}{i} = \frac{150\mu C - (15\mu F)(5V)}{25mA} \underline{= 3ms}$$

**P7.3-2**

$$i(t) = C\frac{d}{dt}v(t) = \frac{1}{8}\frac{d}{dt}12\cos(2t+30^\circ) = \frac{1}{8}(12)(2)\cos(2t+30^\circ+90^\circ) = 3\cos(2t+120^\circ)\text{ A}$$

**P7.3-3**

$$(3\cdot10^{-3})\cos(500t+45^\circ) = C\frac{d}{dt}12\cos(500t-45^\circ) = C(12)(500)\cos(500t-45+90) = C(6000)\cos(500t+45^\circ)$$

$$\text{so} \quad C = \frac{3\cdot10^{-3}}{6\cdot10^3} = \frac{1}{2}\cdot10^{-6} = \frac{1}{2}\,\mu F$$

**P 7.3-6**

(a) $i(t) = C\dfrac{d}{dt}v(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$

(b) $v(t) = \dfrac{1}{C}\int_0^t i(\tau)\,d\tau + v(0) = 2\int_0^t i(\tau)\,d\tau$

For $0 < t < 2$, $i(t) = 0$ A so $v(t) = 2\int_0^t 0\,d\tau + 0 = 0$ V

For $2 < t < 6$, $i(t) = 0.2\,t - 0.4$ V so

$$v(t) = 2\int_0^t (0.2\tau - 0.4)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_1^t = 0.2t^2 - 0.8t + 0.8 \text{ V}$$

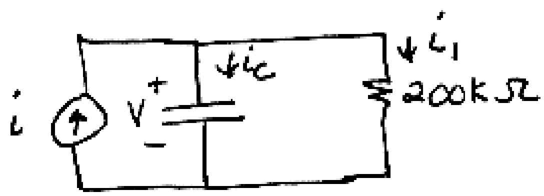
$v(6) = 0.2\left(6^2\right) - 0.8(6) + 0.8 = 3.2$ V . For $6 < t$, $i(t) = 0.8$ A so $v(t) = 2\int_0^t 0.8\,d\tau + 3.2 = 1.6t - 6.4$ V

**P7.3-7**

$$v(t) = v(0) + \frac{1}{C}\int_0^t i\,dt = 25 + 2.5\times10^4\int_0^t \left(6\times10^{-3}\right)e^{-6t}\,dt = 25 + 150\int_0^t e^{-6t}\,dt = 25 + 150\left[-\left(\tfrac{1}{6}\right)e^{-6t}\right]_0^t$$

$$v(t) = \underline{50 - 25e^{-6t} \text{ V}}$$

**P7.3-8**




$$i_1 = \frac{v}{200} = \frac{1}{40}\left(1-2e^{-2t}\right)\text{mA} = 25\left(1-2e^{-2t}\right)\mu\text{A}$$

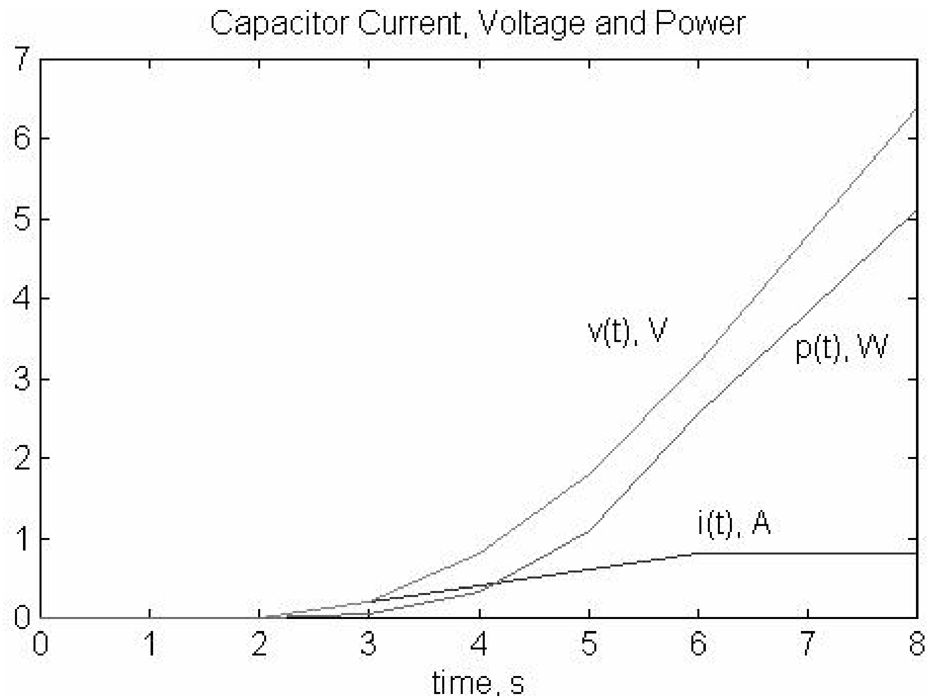
$$i_C = C\frac{dv}{dt} = 10(-2)\left(-10e^{-2t}\right) = 200e^{-2t}\mu\text{A}$$

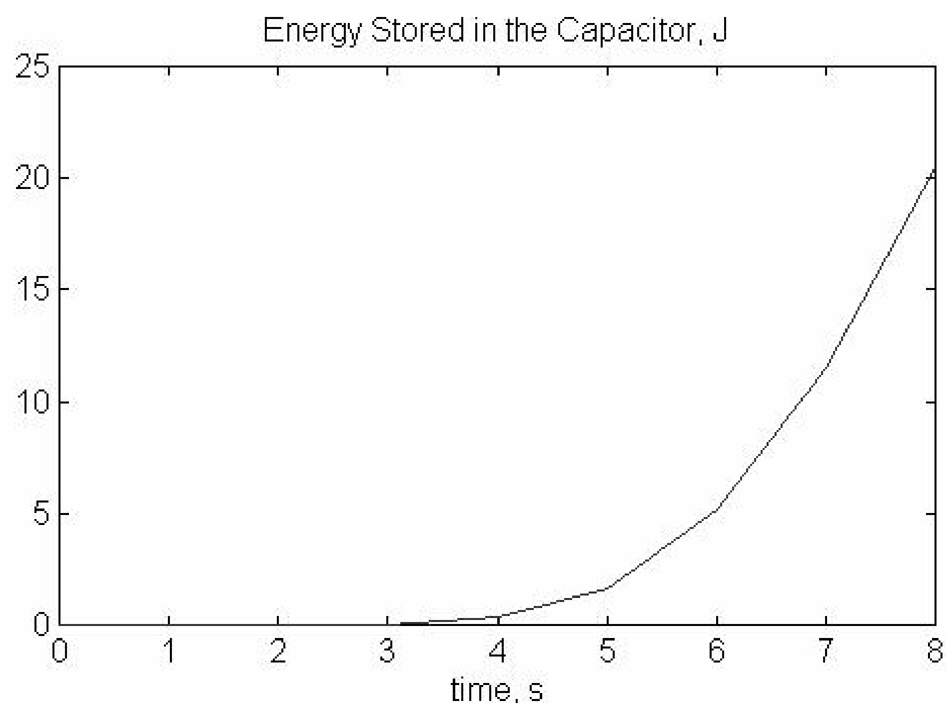
$$\therefore i = i_1 + i_c = 200e^{-2t} + 25 - 50e^{-2t}$$

$$= \underline{25 + 150e^{-2t}\mu\text{A}}$$

Section 7-4: Energy Storage in a Capacitor

**P7.4-1**



These plots were produced using three MATLAB scripts:

| | |
|---|---|
| capvol.m | function v = CapVol(t)<br>  if t<2<br>    v = 0;<br>  elseif t<6<br>    v = 0.2*t*t - .8*t +.8;<br>  else<br>    v = 1.6*t - 6.4;<br>  end |
| capcur.m | function i = CapCur(t)<br>  if t<2<br>    i=0;<br>  elseif t<6<br>    i=.2*t - .4;<br>  else<br>    i =.8;<br>  end |
| c7s4p1.m | t=0:1:8;<br>for k=1:1:length(t)<br>  i(k)=CapCur(k-1);<br>  v(k)=CapVol(k-1);<br>  p(k)=i(k)*v(k);<br>  w(k)=0.5*v(k)*v(k);<br>end<br><br>plot(t,i,t,v,t,p)<br>text(5,3.6,'v(t), V')<br>text(6,1.2,'i(t), A')<br>text(6.9,3.4,'p(t), W')<br>title('Capacitor Current, Voltage and Power')<br>xlabel('time, s')<br><br>% plot(t,w)<br>% title('Energy Stored in the Capacitor, J')<br>% xlabel('time, s') |

**P7.4-2**

$$i_c = C\frac{dv}{dt} = \left(10\times10^{-6}\right)(-5)(-4000)e^{-4000t} = \underline{0.2e^{-4000t}\text{ A}}$$

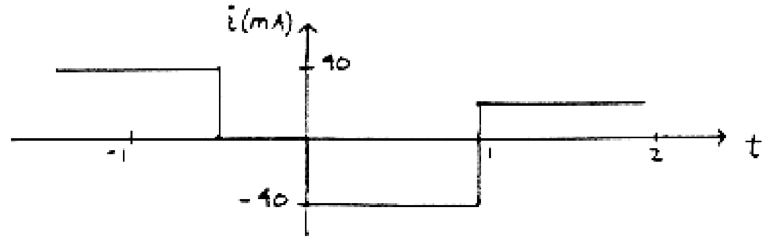
$$\underline{i_c(0) = 0.2\text{ A}} \qquad \underline{i_c(10\text{ms}) = 8.5\times10^{-19}\text{ A}}$$

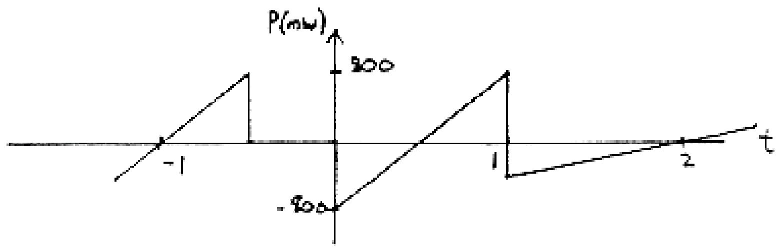
$$W(t) = \frac{1}{2}Cv^2(t) \qquad v(0) = 5-5e^0 = 0 \qquad \underline{W(0) = 0}$$

$$v(10\text{ms}) = 5-5e^{-40} = 5 - 4.24\times10^{-18} \cong 5 \qquad \underline{W(10) = 1.25\times10^{-4}\text{ J}}$$

**P7.4–3** $i(t) = C\,dv_c/dt$ so read off slope of $v_c(t)$ to get $i(t)$

$P(t) = v_c(t)i(t)$ so multiply $v_c(t)$ & $i(t)$ curves to get $p(t)$

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**P7.4-4** $v_c(t) = v_c(0) + \frac{1}{C}\int_0^t i\,dt = v_c(0) + \frac{1}{2}\int_0^t 50\cos\left(10t+\frac{\pi}{6}\right)dt \quad = \left[v_c(0) - \frac{5}{2}\sin\frac{\pi}{6}\right] + \frac{5}{2}\sin\left(10t+\frac{\pi}{6}\right)$

Now since $v_c(t)_{ave} = 0 \Rightarrow v_c(0) = \frac{5}{2}\sin\frac{\pi}{6}$

$\therefore\ v_c(t) = \frac{5}{2}\ \sin\left(10t+\frac{\pi}{6}\right)\ \text{V}$

$\therefore W_{max} = \frac{1}{2}Cv_{c_{max}}^2 = \frac{1}{2}(2)(2.5)^2 = \underline{6.25\mu\ \text{J}}$

First non-negative t for max energy occurs : $10t + \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow \underline{t = \frac{\pi}{30} = 0.1047\ \text{s}}$

**P7.4-5**

$$W = \frac{1}{2}Cv^2 \Rightarrow v = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2\left(4e^{-10t}\right)}{.001}} = 40\sqrt{5}\,e^{-5t}$$

$\therefore\ \underline{v(0.1) = 40\sqrt{5}\,e^{-5(.1)} = 54.25\ \text{V}}$

$$i(t) = C\,\frac{dv_c}{dt} = \left(10^{-3}\right)(-5)\left(40\sqrt{5}\,e^{-5t}\right) = \frac{1}{\sqrt{5}}e^{-5t}$$

$\therefore\ \underline{i(0.1) = \frac{1}{\sqrt{5}}e^{-5(.1)} = -0.2712\ \text{A}}$

**P7.4-6** Max. charge on capacitor $= Cv = (10\mu\text{F})(6\text{V}) = 60\mu\text{C}$

$\Delta t = \frac{\Delta q}{i} = \frac{60\mu\text{C}}{10\mu\text{A}} = \underline{6\ \text{sec}}$ to charge

stored energy $= W = \frac{1}{2}Cv^2 = \frac{1}{2}(10\mu\text{F})(6\text{V})^2 = \underline{180\mu\ \text{J}}$

P7.4-7

$$P(\Delta t) = \left(1.5\times10^3 W\right)(1hr)\left(\frac{60min}{1hr}\right)\left(\frac{60sec}{1min}\right) = 5.4\times10^6 J$$

$$W = \frac{1}{2}Cv^2 \Rightarrow C = {2W}\big/{v^2}$$

$$C = \frac{2\left(5.4\times10^6\right)}{(500)^2} = 43.2\ F$$

$$\text{size of capacitor} = (43.2F)\left(\frac{10^6\mu F}{F}\right)\left(\frac{1cm^3}{100\mu F}\right)$$

$$\text{size} = 4.32\times10^5\ cm^3\left(\frac{1m}{100cm}\right)^3 = \underline{0.432\ m^3}$$

Capacitor may be practical, it depends on size of car! Would be 76 cm on a side- pretty large !

## Section 7-5: Series and Parallel capacitors

**P7.5-1**

$$2\mu F \parallel 4\mu F = 6\mu F$$

$$6\mu F \text{ in series with } 3\mu F = \frac{6\mu F\cdot 3\mu F}{6\mu F+3\mu F} = 2\mu F$$

$$i(t) = 2\mu F\,\frac{d}{dt}\,(6\cos 100t) = (2\cdot10^{-6})\,(6)\,(100)\,\,(-\sin 100t) = \underline{-1.2\ \sin 100t\ \ mA}$$

**P7.5-2**

$$4\mu F \text{ in series with } 4\mu F = \frac{4\mu F\cdot 4\mu F}{4\mu F+4\mu F} = 2\mu F$$

$$2\ \mu F \parallel 2\mu F = 4\mu F$$

$$4\mu F \text{ in series with } 4\mu F = 2\mu F$$

$$i(t) = 2\mu F\,\frac{d}{dt}(3+3e^{-250t}) = (2\cdot10^{-6})\,\,(0+3(-250)e^{-250t}) = -1.5\cdot10^{-3}\ e^{-250t}\ A = \underline{-1.5\ e^{-250t}\ \ mA}$$

**P7.5-3** $C \text{ in series with } C = \dfrac{C\cdot C}{C+C} = \dfrac{C}{2}$

$$C \parallel C \parallel \frac{C}{2} = \frac{5}{2}C$$

$$C \text{ in series with } \frac{5}{2}C = \frac{C\cdot\frac{5}{2}C}{C+\frac{5}{2}C} = \frac{5}{7}C$$

$$(25\cdot10^{-3})\cos 250t = \frac{5}{7}C\,\,\frac{d}{dt}(14\ \sin 250t\ ) = \left(\frac{5}{7}C\right)(14)\,\,(250)\cos 250t$$

$$\text{so } 25\cdot10^{-3} = 2500C \Rightarrow C = 10\cdot10^{-6} = 10\mu F$$

## Section 7-6: Inductors

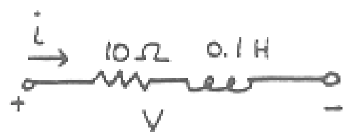
P7.6-1 Find max. voltage across coil:

$v(t) = L\, {}^{di}\!/_{dt} = 200\ \left[100(400)\cos 400t\right]\ \text{V}$

$\therefore\ v_{max} = 8\times10^6\,\text{V}$

thus have a field of $8\times10^6\,{}^{\text{V}}\!/_{2\text{m}} = 4\times10^6\ {}^{\text{V}}\!/_{\text{m}}$ which exceeds dielectric strength in air of $3\times10^6$ V/m $\therefore$ will get a discharge as the air is ionized.

**P7.6-2**




$v = L\,{}^{di}\!/_{dt} + 10\,i$

$= (.1)\,(4e^{-t} - 4te^{-t}) + 10(4te^{-t})$

$\underline{= 0.4\,e^{-t} + 39.6t\,e^{-t}\ \text{V}}$

**P 7.6-3**

(a) $$v(t) = L\frac{d}{dt}i(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \\ -4 & 2 < t < 3 \\ 0 & 3 < t \end{cases}$$

(b) $$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0) = \int_0^t v(\tau)\,d\tau$$

For $0 < t < 1$, $v(t) = 0$ V so $i(t) = \int_0^t 0\,d\tau + 0 = 0\ \text{A}$

For $1 < t < 2$, $v(t) = 4t - 4$ V so $i(t) = \int_0^t (4\tau - 4)\,d\tau + 0 = \left(2\tau^2 - 4\tau\right)\Big|_1^t = 2t^2 - 4t + 2\ \text{A}$

$i(2) = 4\left(2^2\right) - 4(2) + 2 = 2\ \text{A}$

For $2 < t < 3$, $v(t) = -4t + 12$ V so

$$i(t) = \int_0^t (-4\tau + 12)\,d\tau + 2 = \left(-2\tau^2 + 12\tau\right)\Big|_1^t + 2 = -2t^2 + 12t - 14\ \text{A}$$

$i(3) = -4\left(3^2\right) + 12(3) - 14 = 4\ \text{A}$

For $0 < t < 1$, $v(t) = 0$ V so $i(t) = \int_0^t 0\,d\tau + 4 = 4\ \text{A}$

**P7.6-4**

$$v(t) = (250\cdot10^{-3})\ \frac{d}{dt}\ (120\cdot10^{-3})\ \sin(500t-30^\circ) = (.25)(.12)(500)\ \sin(500t-30^\circ+90^\circ) = 15\sin(500t+60^\circ)$$

**P7.6-5**

$$i_L(t) = \frac{1}{5\,mH}\int_0^t v_s(\tau)\,d\tau - 2\mu A$$

for $0 \le t < 1\ \mu s$ $\quad v_s(t) = 4\,mV$

$$i_L(t) = \frac{1}{5mH}\int_0^t 4\,mV\,d\tau - 2\mu A = \left(\frac{4}{5}\ \frac{mV}{mH}\right)t - 2\mu A$$

$$i_L(1\mu s) = \left(\frac{4}{5}\ \frac{mV}{mH}\cdot 1\mu s\right) - 2\mu A = -\frac{6}{5}\ \mu A$$

for $1\mu s \le t < 3\mu s$ $\quad v_s(t) = -1\ mV$

$$i_L(t) = \frac{1}{5mH}\int_{1\mu s}^{t} -1\,mV\,d\tau - \frac{6}{5}\ \mu A = -\frac{1}{5}\ \frac{mV}{mH}\ (t-1\mu s) - \frac{6}{5}\ \mu A = \left(-\frac{1\ mV}{5\ mH}\right)t - 1\mu A$$

$$i_L(3\mu s) = \left(-\frac{1\ mV}{5\ mH}\cdot 3\mu s\right) - 1\mu A = -\frac{8}{5}\ \mu A$$

for $3\mu s \le t$ $\quad v_s(t) = 0$ so $i_L(t)$ remains $-\frac{8}{5}\ \mu A$

**P7.6-6**

$$v(t) = 2k\Omega\cdot i_s(t) + 4mH\cdot\frac{d}{dt}\ i_s(t)\ \text{(in general)}$$

for $0 < t < 1\ \mu s$ $\quad i_s(t) = 1\ \frac{mA}{\mu s}\cdot t \Rightarrow \frac{d}{dt} i_s(t) = 1\ \frac{mA}{\mu s}$

so $v(t) = (2k\Omega)\left(1\frac{mA}{\mu s}\right)t + 4\ mH\left(1\ \frac{mA}{\mu s}\right) = \left(\frac{2V}{\mu s}\right)t + 4\ V$

for $1\mu s < t < 3\mu s$ $\quad i_s(t) = 1mA \Rightarrow \frac{d}{dt}\ i_s(t) = 0$

so $v(t) = 2k\Omega\cdot 1mA + 4mH\cdot 0 = 2\ V$

for $3\mu s < t < 5\mu s$ $\quad i_s(t) = 4mA - \left(1\frac{mA}{\mu s}\right)t$

so $\frac{d}{dt} i_s(t) = -1\ \frac{mA}{\mu s}$ and $v(t) = 4V - \left(\frac{2V}{\mu s}\right)t$

when $5\mu s < t < 7\mu s$ $\quad i_s(t) = -1mA$ and $\frac{d}{dt} i_s(t) = 0$

so $v(t) = -2\ V$

when $7\mu s < t < 8\mu s$ $\quad i_s(t) = 1\ \frac{mA}{\mu s}\cdot t - 8mA$

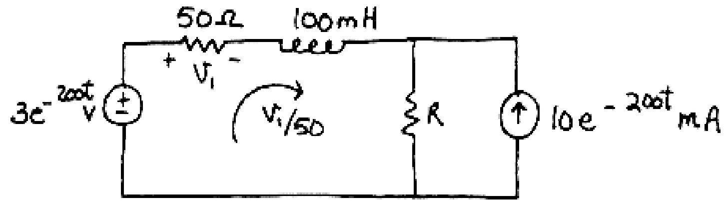
so $\frac{d}{dt}\ i_s(t) = 1\ \frac{mA}{\mu s}$

and $v(t) = -12\ V + \left(\frac{2\ V}{\mu s}\right)t$

when $8\mu s < t$, then $i_s(t) = 0 \Rightarrow \frac{d}{dt} i_s(t) = 0$

so $v(t) = 0$

**P7.6-7**




$v_1(t) = e^{-200t}\text{V}$

$$\text{KVL a: } -3e^{-200t} + v_1 + .1\frac{d}{dt}\left(\frac{v_1}{50}\right) + R\left[\frac{v_1}{50} + .01e^{-200t}\right] = 0$$

$$-3e^{-200t} + e^{-200t} - .4e^{-200t} + R\left[\frac{1}{50} + .01\right] e^{-200t} = 0$$

$$-2.4 + R(.03) = 0 \Rightarrow \underline{R = \frac{2.4}{.03} = 80\Omega}$$

**P 7.6-8**

(a) $$v(t) = L\frac{d}{dt}i(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$$

(b) $$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0) = 2\int_0^t v(\tau)\,d\tau$$

For $0 < t < 2$, $v(t) = 0$ V so $i(t) = 2\int_0^t 0\,d\tau + 0 = 0$ A

For $2 < t < 6$, $v(t) = 0.2\,t - 0.4$ V so

$$i(t) = 2\int_0^t (0.2\tau - 0.4)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_1^t = 0.2t^2 - 0.8t + 0.8 \text{ A}$$

$i(6) = 0.2\left(6^2\right) - 0.8(6) + 0.8 = 3.2$ A .

For $6 < t$, $v(t) = 0.8$ V so $i(t) = 2\int_0^t 0.8\,d\tau + 3.2 = 1.6t - 6.4$ A

## Section 7-7: Energy Storage in an Inductor

**P7.7-1**

$$v(t) = 100\cdot10^{-3}\frac{d}{dt}i(t) = \begin{cases} 0 & t<0 \\ 0.4 & 0<t>1 \\ 0 & t>1 \end{cases}$$

$$p(t) = v(t)\,i(t) = \begin{cases} 0 & t<0 \\ 1.6t & 0<t<1 \\ 0 & t>1 \end{cases}$$

$$W(t) = \int_0^t p(\tau)\,d\tau = \begin{cases} 0 & t<0 \\ 0.8t^2 & 0<t<1 \\ 0.8 & t>1 \end{cases}$$

**P7.7-2**

$$p(t)=v(t)\,i(t) = \left[5\frac{d}{dt}(4\sin 2t)\right](\ 4\sin 2t\ ) = 5\ (8\cos 2t)\ (4\sin 2t)$$

$$= 80\ [2\cos 2t \sin 2t] = 80\ [\sin(2t+2t) + \sin(2t-2t)] = 80\sin 4t\ \text{W}$$

$$W(t)=\int_0^t p(\tau)\,d\tau\ = 80\int_0^t \sin 4\tau\ d\tau = -\ \frac{80}{4}\ [\cos 4\tau\ |_0^t] = 20\ (1-\cos 4t)$$

**P7.7-3**

$$i(t)= \frac{1}{25\cdot 10^{-3}}\int_0^t 6\cos 100t + 0 = \frac{6}{(25\cdot 10^{-3})(100)}\ [\sin 100\tau\ |_0^t] = 2.4\sin 100\ t$$

$$p(t) = v(t)\,i(t) = (6\cos 100t)(2.4\sin 100t) = 7.2\ \ [\ 2(\cos 100t)(\sin 100t)\ ] = 7.2\ [\ \sin 200t + \sin 0\ ] = 7.2\sin 200t$$

$$W(t) = \int_0^t p(\tau)\,d\tau\ = 7.2\int_0^t \sin\ 200\tau\, d\tau =\ -\ \frac{7.2}{200}\ \left[\cos\ 200\tau|_0^t\right] = 0.036[1-\cos 200t]\text{J} = 36\ [1-\cos 200t\ ]\ \ \text{mJ}$$

## Section 7-8: Series and Parallel Inductors

**P7.8-1**

$$6H \parallel 3H\ =\ \frac{6H\cdot 3H}{6H+3H}\ =\ 2H$$

$$2H\ +\ 2H = 4H$$

$$i(t) = \frac{1}{4}\int_0^t 6\cos 100\tau\ \ d\tau\ = \frac{6}{4\cdot 100}\left[\sin 100\tau|_0^t\right] =\ 15\sin 100t\ \text{mA}$$

**P7.8-2**

$$4mH + 4mH = 8mH$$

$$8mH \parallel 8mH\ \ = \frac{8mH\cdot 8mH}{8mH+8mH} = 4mH$$

$$4mH + 4mH\ =\ 8mH$$

$$v(t) = (8\cdot 10^{-3})\ \frac{d}{dt}\ (5+3e^{-250t})(10^{-3}) = (8\cdot 10^{-6})\,(0+3(-250)e^{-250t})\ = -6e^{-250t}\ \ \text{mV}$$

**P7.8-3**

$$L \parallel L\ =\ \frac{L\cdot L}{L+L}\ \ =\ \frac{L}{2}$$

$$L + L + \frac{L}{2} = \frac{5}{2}\,L$$

$$25\cos 250t = \frac{5}{2}\,L\ \frac{d}{dt}\ (14\cdot 10^{-3}\sin 250\ t) = \left(\frac{5}{2}L\right)(14\cdot 10^{-3})(250)\cos 250t$$

$$\text{so}\ \ L = \frac{25}{\frac{5}{2}(14\cdot 10^{-3})\,(250)} = 2.86\ \text{H}$$