Logic Circuits
INEL4205

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Office hours

- MWF 9:00- 10:30am
- S413

Digital Systems and Binary Numbers

Chapter 1

Digital Systems

- Can you give me an example of an electronic device that is not digital?
- What advantages do digital systems have?
- Why the name "digital"?
- Signals in most present day digital systems are binary: 0 1
- Binary digIT → BIT
- Groups of bits can be made to represent discrete symbols

Digital Systems

- Digital quantities emerge from:
  - Discrete measurements
  - Quantized from a continuous process
- Digital systems are composed of digital modules
- Need to understand digital circuits and their logical function

Number Systems and Binary Numbers

Decimal Number (Base 10) → 7392

$$7\times10^3 + 3\times10^2 + 9\times10^1 + 2\times10^0$$

Number Systems and Binary Numbers

Decimal Number (Base 10) → 7392.45

$$7\times10^{3}+3\times10^{2}+9\times10^{1}+2\times10^{0}+4\times10^{-1}+5\times10^{-2}$$

Number Systems and Binary Numbers

$$a_n\ a_{n-1}\ a_{n-2} \ldots\ a_0\ .a_{-1}\ a_{-2} \ldots\ a_{-m}$$

Decimal Number
(Base 10)

$$a_n \mathrm{x} 10^n + a_{n-1} \mathrm{x} 10^{n-1} + \ldots a_0 \mathrm{x} 10^0 + \ldots a_{-m} \mathrm{x} 10^{-m}$$

Number Systems and Binary Numbers

Binary Number (Base 2) → 11001.11

$$1x2^4+1x2^3+0x2^2+0x2^1+1x2^0+1x2^{-1}+1x2^{-2}$$

$$1x16+1x8+0x4+0x2+1x1+1x2^{-1}+1x2^{-2}$$

25.75

Number Systems and Binary Numbers

$$a_n \; a_{n-1} \; a_{n-2} \ldots \; a_0 \; . a_{-1} \; a_{-2} \ldots \; a_{-m}$$

Number Base *r*

$$A_n \text{ x } r^n + a_{n-1} \text{ x } r^{n-1} + \ldots a_0 \text{ x } r^0 + \ldots a_{-m} \text{ x } r^{-m}$$

Number Systems and Binary Numbers

Number Base 8 → $(732.45)_8$

$$7 x 8^2 + 3 x 8^1 + 2 x 8^0 + 4 x 8^{-1} + 5 x 8^{-2}$$

$$7 x 64 + 3 x 8 + 2 x 1 + 4 x 8^{-1} + 5 x 8^{-2}$$

474.578125

Number Systems and Binary Numbers

Table 1.1
Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Number Systems and Binary Numbers

$$a_n\ a_{n-1}\ a_{n-2} \ldots\ a_0\ .a_{-1}\ a_{-2} \ldots\ a_{-m}$$

Number Base *r*

$$A_n \times r^n + a_{n-1} \times r^{n-1} + \ldots a_0 \times r^0 + \ldots a_{-m} \times r^{-m}$$

Exercises:

Convert the following numbers to decimal:

1)$(10111.001)_2$

2)$(7532.42)_8$

Number Systems and Binary Numbers

Arithmetic operations with numbers in base r follow the same rules as for decimal numbers. Examples:

Augend	101101	Minuend	101101	Multiplicand	1011
Addend	+100111	Subtrahend	-100111	Multiplier	X101
Sum:	1010100	Difference	000110		1011
					0000
					1011
				Product	110111

Exercises:

Carry out the following operations:

1)$(1011.11+1100)_2$

2)$(5732-723)_8$

Number-Base Conversions

Conversion to binary

	Integer Quotient		Remainder	Coefficient
41/2=	20	+	½	$a_0=1$
20/2=	10	+	0	$a_1=0$
10/2=	5	+	0	$a_2=0$
5/2=	2	+	½	$a_3=1$
2/2=	1	+	0	$a_4=0$
1/2=	0	+	½	$a_5=1$

Number-Base Conversions

Conversion to binary

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

$(101001)_2$

Number-Base Conversions

Conversion to octal

Integer	Remainder
153	
19	1
2	3
0	2

$(231)^8$

Number-Base Conversions

Exercises

Convert $(1024)_{10}$ to binary

Convert $(1024)_{10}$ to octal

Number-Base Conversions

Conversion to binary

	Integer		Fraction	Coefficient
0.6875x2=	1	+	0.3750	a_{-1}=1
0.3750x2=	0	+	0.7500	a_{-2}=0
0.7500x2=	1	+	0.5000	a_{-3}=1
0.5000x2=	1	+	0.0000	a_{-4}=1

Number-Base Conversions

Conversion to octal

	Product	Integer	Coefficient
0.513x8=	4.104	4	a_{-1}=4
0.104x8=	0.832	0	a_{-2}=0
0.832x8=	6.656	6	a_{-3}=6
0.656x8=	5.248	5	a_{-4}=5
0.248x8=	1.984	1	a_{-4}=1
0.984x8=	7.872	7	a_{-4}=7
0.872			

Number-Base Conversions

Exercises

Convert $(3.2154)_{10}$ to binary

Convert $(9.113)_{10}$ to octal

Octal and Hexadecimal Numbers

Table 1.2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Complements

- Simplify subtraction and logical manipulation
- Simpler, less expensive circuits
- Two types of complements
  - Radix Complement (r's complement)
  - Diminish Radix Complement ((r-1)'s complement)

Diminished Radix Complement

9's complement of	546700 is	999999-546700	=453299
9's complement of	012398 is	999999-012398	=987601
1's complement of	1011000 is	1111111-1011000	=0100111
1's complement of	0101101 is	1111111-0101101	=1010010

Radix Complement

10's complement of	546700 is	1000000-546700	=453300
10's complement of	012398 is	1000000-012398	=987602
2's complement of	1011000 is	10000000-1011000	=0101000
2's complement of	0101101 is	10000000-0101101	=1010011

Radix Complement

- 10’s complement
  - Leave all least significant zeros unchanged
  - Subtract the least significant non-zero digit from 10
  - Subtract all other digits from 9

- 2’s complement
  - Leave all least significant 0’s and the first 1 unchanged
  - Replace all other 1’s with 0’s, and 0’s with 1’s in all other significant digits

Diminished Radix Exercises

- Compute 9's complement of
  - 0912367
  - 82917364
  - 9999999
- Compute 1's complement of
  - 11001101
  - 00000000

Radix Exercises

- Compute 10’s complement of
  - 0912367
  - 82917364
  - 9999999
- Compute 2’s complement of
  - 11001101
  - 00000000
- Compare results with diminished radix comp.

Subtraction with Complements

- Subtraction of two n-digit unsigned numbers M-N in base r is as follows
  1. Add the minuend M to the r's complement of the subtrahend N: $M + (r^n - N) = M – N + r^n$
  2. If $M \geq N$ the sum will produce an end carry r^n which can be discarded
  3. If $M \leq N$ the sum does not produce and end carry and is equal to $r^n – (N – M)$, which is the r's complement of $(N – M)$. Take the r's complement and place a negative sign in front.

Example

Using 10's complement, subtract 72532 - 3250

M	=	72532
10's complement of *N*	=	96750
Sum	=	169282
Discard end carry	=	-100000
Answer	=	69282

Example

Using 10's complement, subtract 3250 - 72532

M	=	03250
10's complement of *N*	=	27468
Sum	=	30718
10's complement of Sum	=	69282
Answer	=	69282

Example

Using 2's complement, subtract 1010100 - 1000011

M	=	1010100
2's complement of N	=	+0111101
Sum	=	10010001
Discard end carry 2^7	=	-10000000
Answer	=	0010001

Example

Using 2's complement, subtract 1000011 - 1010100

M	=	1000011
2's complement of *N*	=	+0101100
Sum	=	1101111
(No carry) 2's complement of Sum	=	-0010001
Answer	=	-0010001

Example

Using 1's complement, subtract 1010100 - 1000011

M	=	1010100
1's complement of *N*	=	+0111100
Sum	=	10010000
End around carry	=	+1
Answer	=	0010001

Example

Using 2's complement, subtract 1000011 - 1010100

M	=	1000011
1's complement of *N*	=	0101011
Sum	=	1101110
(No carry) 1's complement of Sum	=	-0010001
Answer	=	-0010001

Exercise

- Subtract 28934 – 3456
- Subtract 3456 – 28934
- Subtract 110010 – 110100
- Subtract 0110011 - 0100001

Signed Binary Numbers

- Unsigned and Signed numbers are bit strings
- User determines when a number is signed or unsigned
- When signed, leftmost bit is the sign: 0 positive; 1 negative
- Convenient to use *signed-complement* system for negative numbers

Signed Binary Numbers

- Number
  - Signed-magnitude
    - If positive ,0 followed by magnitude
    - If negative ,1 followed by magnitude
  - Signed-complement
    - If positive , leave number unchanged
    - If negative ,complement number

Signed Binary Numbers

Number of bits	Number	System	Positive	Negative
8	9	Signed-magnitude	00001001	10001001
8	9	Signed-Complement 1's	00001001	11110110
8	9	Signed-Complement 2's	00001001	11110111

Signed Binary Numbers

Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Arithmetic addition

+6	00000110	-6	11111010
+13	00001101	+13	00001101
+19	00010011	+7	00000111

+6	00000110	-6	11111010
-13	11110011	-13	11110011
-7	11111001	-19	11101101

Arithmetic subtraction

$$(\pm A) \quad - \quad (+B) \quad = \quad (\pm A) \quad + \quad (-B)$$

$$(\pm A) \quad - \quad (-B) \quad = \quad (\pm A) \quad + \quad (+B)$$

To convert a positive number into a negative number take the complement

Binary Codes

Table 1.4
Binary-Coded Decimal (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

BCD addition

- Since each digit must no exceed 9, worst case is 9+9+carry digit=9+9+1=19
- Result in the range 0 to 19, *i.e.*, 0000 to 1 1001
- If result less than or equal 9, no problem
- If result greater or equal 10, *i.e.*, 1010, add 6 (0110) to convert result to correct BCD and carry, as required

BCD addition

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

Other decimal Codes

Table 1.5
Four Different Binary Codes for the Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, −2, −1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
	1010	0101	0000	0001
Unused	1011	0110	0001	0010
bit	1100	0111	0010	0011
combi-	1101	1000	1101	1100
nations	1110	1001	1110	1101
	1111	1010	1111	1110

Gray code

Table 1.6
Gray Code

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

ASCII Code

Table 1.7
American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	—	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	—	o	DEL

ASCII Code

Control characters

NUL	Null	DLE	Data-link escape
SOH	Start of heading	DC1	Device control 1
STX	Start of text	DC2	Device control 2
ETX	End of text	DC3	Device control 3
EOT	End of transmission	DC4	Device control 4
ENQ	Enquiry	NAK	Negative acknowledge
ACK	Acknowledge	SYN	Synchronous idle
BEL	Bell	ETB	End-of-transmission block
BS	Backspace	CAN	Cancel
HT	Horizontal tab	EM	End of medium
LF	Line feed	SUB	Substitute
VT	Vertical tab	ESC	Escape
FF	Form feed	FS	File separator
CR	Carriage return	GS	Group separator
SO	Shift out	RS	Record separator
SI	Shift in	US	Unit separator
SP	Space	DEL	Delete

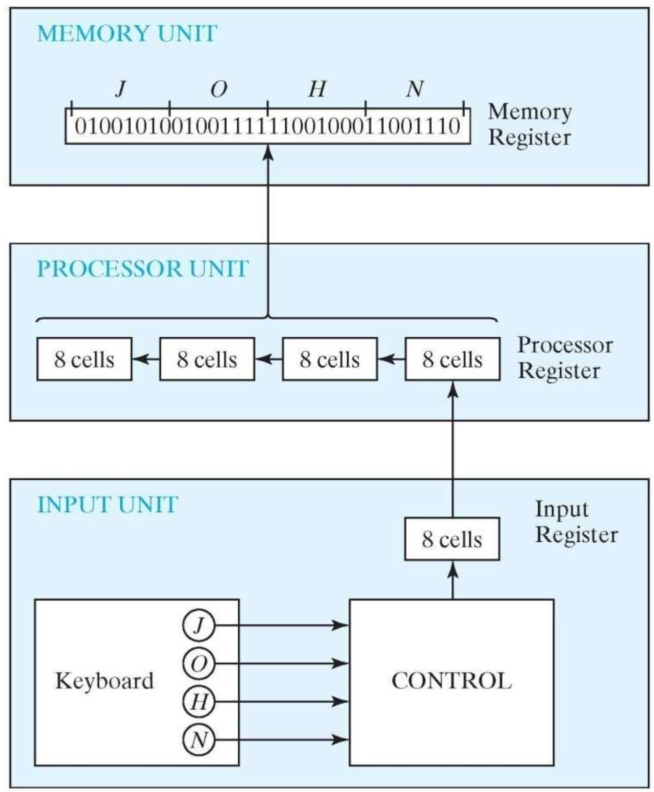
Error-detecting code

			With Even Parity	With Odd Parity
ASCII A	=	1000001	01000001	11000001
ASCII T	=	1010100	11010100	01010100

Binary Storage and Registers

- Binary Cell: device with two stable states and capable to store one bit (0 or 1)
- Register: A group of binary cells
- Register Transfer: basic operation to transfer binary information from one set of registers to another

Binary Storage and Registers
Transfer of information among registers



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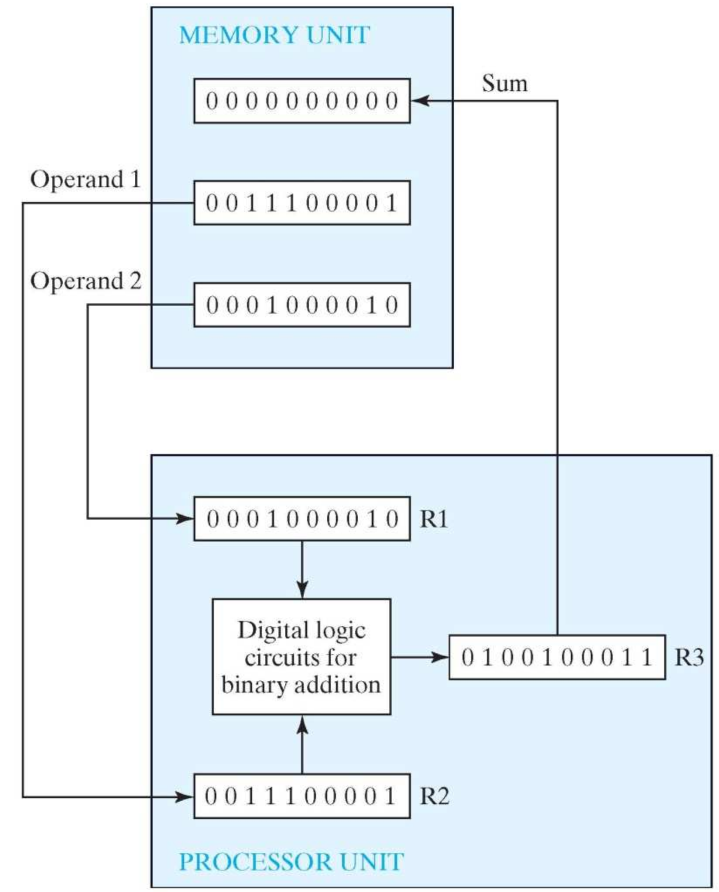
Binary Storage and Registers
Example of binary information processing



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Binary Logic

Table 1.8
Truth Tables of Logical Operations

AND

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

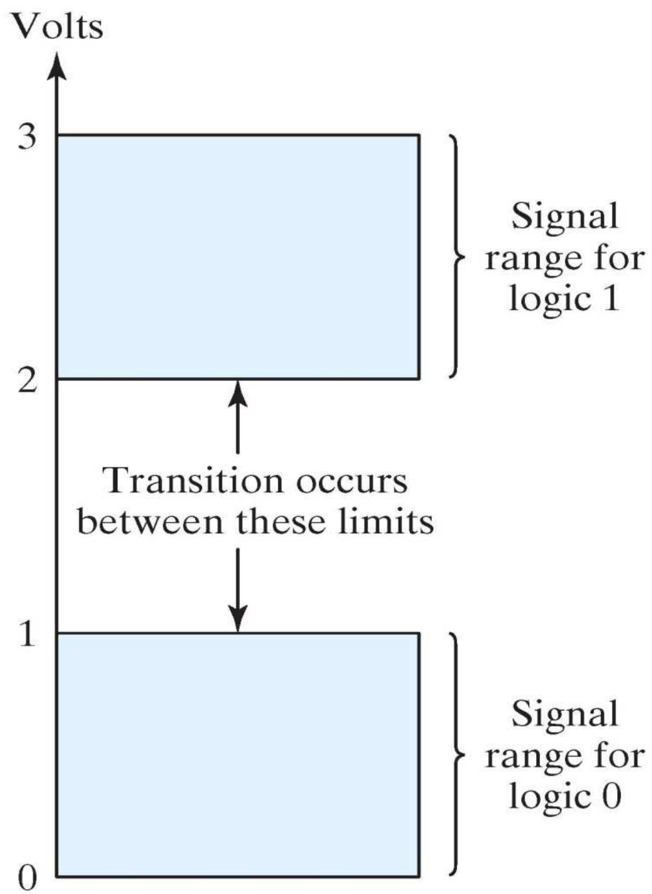
OR

x	y	$x + y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT

x	x'
0	1
1	0

Logic Gates



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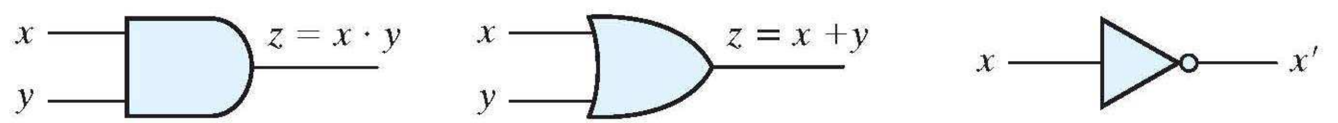
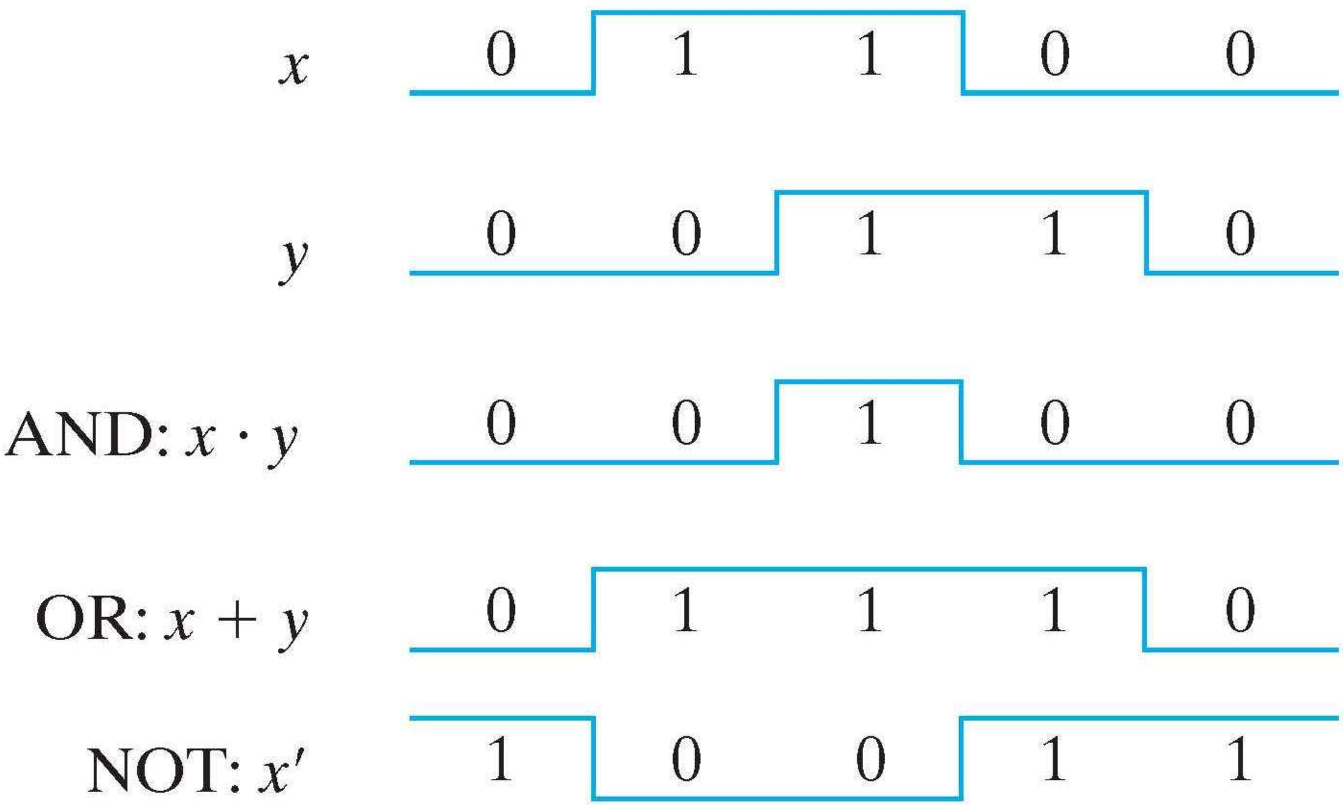
Logic Gates

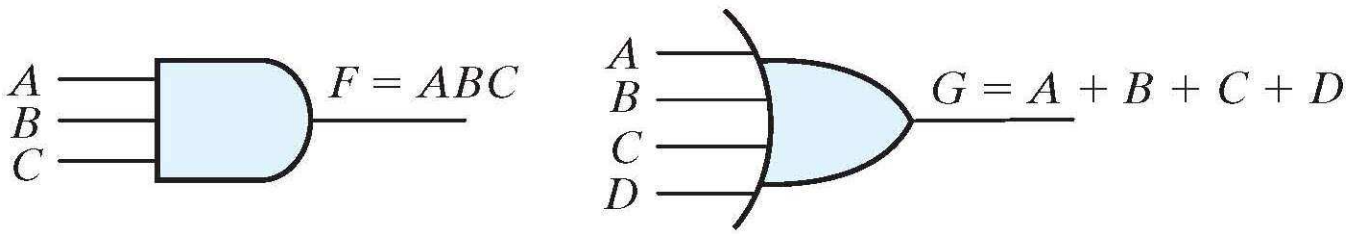


(a) Two-input AND gate (b) Two-input OR gate (c) NOT gate or inverter

Logic Gates

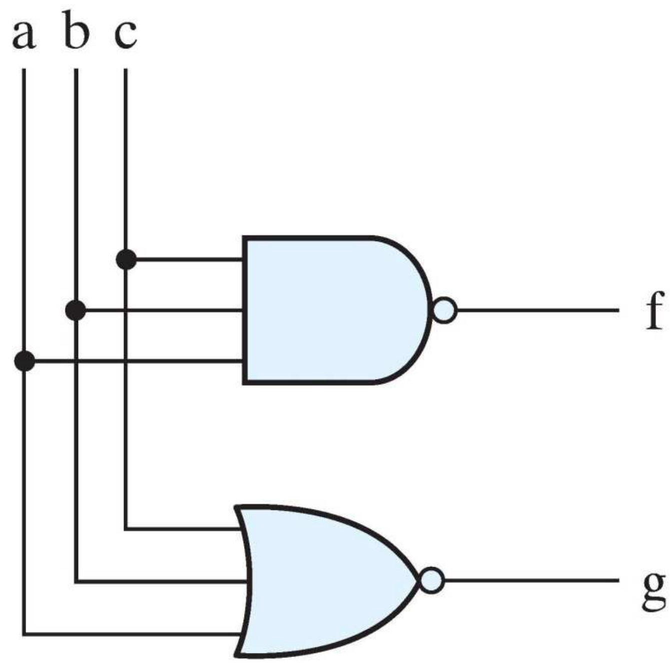


Logic Gates



(a) Three-input AND gate (b) Four-input OR gate

Logic Gates



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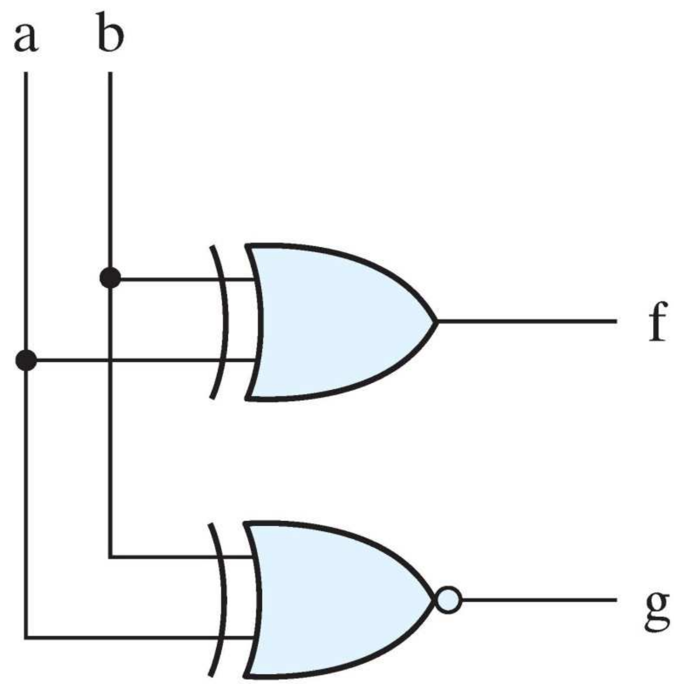

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Homework Assignment Chapter 1

- 1.1
- 1.3
- 1.9
- 1.10
- 1.14
- 1.15
- 1.18
- 1.21
- 1.25
- 1.28
- 1.34
- 1.35
- 1.36

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