Table 4.1 Euclidean Algorithm Example

Dividend	Divisor	Quotient	Remainder
a = 1160718174	b = 316258250	q_1 = 3	r_1 = 211943424
b = 316258250	r_1 = 211943424	q_2 = 1	r_2 = 104314826
r_1 = 211943424	r_2 = 104314826	q_3 = 2	r_3 = 3313772
r_2 = 104314826	r_3 = 3313772	q_4 = 31	r_4 = 1587894
r_3 = 3313772	r_4 = 1587894	q_5 = 2	r_5 = 137984
r_4 = 1587894	r_5 = 137984	q_6 = 11	r_6 = 70070
r_5 = 137984	r_6 = 70070	q_7 = 1	r_7 = 67914
r_6 = 70070	r_7 = 67914	q_8 = 1	r_8 = 2156
r_7 = 67914	r_8 = 2156	q_9 = 31	r_9 = 1078
r_8 = 2156	r_9 = 1078	q_{10} = 2	r_{10} = 0

Table 4.2 Arithmetic Modulo 8

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

(a) Addition modulo 8

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

(b) Multiplication modulo 8

w	$-w$	w^{-1}
0	0	—
1	7	1
2	6	—
3	5	3
4	4	—
5	3	5
6	2	—
7	1	7

(c) Additive and multiplicative inverses modulo 8

Table 4.3 Properties of Modular Arithmetic for Integers in Z_n

Property	Expression
Commutative Laws	$(w + x) \bmod n = (x + w) \bmod n$ $(w \times x) \bmod n = (x \times w) \bmod n$
Associative Laws	$[(w + x) + y] \bmod n = [w + (x + y)] \bmod n$ $[(w \times x) \times y] \bmod n = [w \times (x \times y)] \bmod n$
Distributive Law	$[w \times (x + y)] \bmod n = [(w \times x) + (w \times y)] \bmod n$
Identities	$(0 + w) \bmod n = w \bmod n$ $(1 \times w) \bmod n = w \bmod n$
Additive Inverse $(-w)$	For each $w \in Z_n$, there exists a z such that $w + z \equiv 0 \bmod n$

Table 4.4 Extended Euclidean Algorithm Example

i	r_i	q_i	x_i	y_i
–1	1759		1	0
0	550		0	1
1	109	3	1	–3
2	5	5	–5	16
3	4	21	106	–339
4	1	1	–111	355
5	0	4		

Result: $d = 1$; $x = -111$; $y = 355$

Table 4.5 Arithmetic in GF(7)

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

(a) Addition modulo 7

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

(b) Multiplication modulo 7

w	$-w$	w^{-1}
0	0	—
1	6	1
2	5	4
3	4	5
4	3	2
5	2	3
6	1	6

(c) Additive and multiplicative inverses modulo 7

Table 4.6 Arithmetic in GF(2^3)

		000	001	010	011	100	101	110	111
	+	0	1	2	3	4	5	6	7
000	0	0	1	2	3	4	5	6	7
001	1	1	0	3	2	5	4	7	6
010	2	2	3	0	1	6	7	4	5
011	3	3	2	1	0	7	6	5	4
100	4	4	5	6	7	0	1	2	3
101	5	5	4	7	6	1	0	3	2
110	6	6	7	4	5	2	3	0	1
111	7	7	6	5	4	3	2	1	0

(a) Addition

		000	001	010	011	100	101	110	111
	×	0	1	2	3	4	5	6	7
000	0	0	0	0	0	0	0	0	0
001	1	0	1	2	3	4	5	6	7
010	2	0	2	4	6	3	1	7	5
011	3	0	3	6	5	7	4	1	2
100	4	0	4	3	7	6	2	5	1
101	5	0	5	1	4	2	7	3	6
110	6	0	6	7	1	5	3	2	4
111	7	0	7	5	2	1	6	4	3

(b) Multiplication

w	$-w$	w^{-1}
0	0	—
1	1	1
2	2	5
3	3	6
4	4	7
5	5	2
6	6	3
7	7	4

(c) Additive and multiplicative inverses

Table 4.8 Extended Euclid [($x^8 + x^4 + x^3 + x + 1$), ($x^7 + x + 1$)]

Initialization	$a(x) = x^8 + x^4 + x^3 + x + 1$; $v_{-1}(x) = 1$; $w_{-1}(x) = 0$ $b(x) = x^7 + x + 1$; $v_0(x) = 0$; $w_0(x) = 1$
Iteration 1	$q_1(x) = x$; $r_1(x) = x^4 + x^3 + x^2 + 1$ $v_1(x) = 1$; $w_1(x) = x$
Iteration 2	$q_2(x) = x^3 + x^2 + 1$; $r_2(x) = x$ $v_2(x) = x^3 + x^2 + 1$; $w_2(x) = x^4 + x^3 + x + 1$
Iteration 3	$q_3(x) = x^3 + x^2 + x$; $r_3(x) = 1$ $v_3(x) = x^6 + x^2 + x + 1$; $w_3(x) = x^7$
Iteration 4	$q_4(x) = \mathrm{x}$; $r_4(x) = 0$ $v_4(x) = x^7 + x + 1$; $w_4(x) = x^8 + x^4 + x^3 + x + 1$
Result	$d(x) = r_3(x) = \mathrm{gcd}(a(x), b(x)) = 1$ $\mathrm{w}(x) = w_3(x) = (x^7 + x + 1)^{-1} \bmod (x^8 + x^4 + x^3 + x + 1) = x^7$

Table 4.9 Generator for $GF(2^3)$ using $x^3 + x + 1$

Power Representation	Polynomial Representation	Binary Representation	Decimal (Hex) Representation
0	0	000	0
g^0 (= g^7)	1	001	1
g^1	g	010	2
g^2	g^2	100	4
g^3	$g + 1$	011	3
g^4	$g^2 + g$	110	6
g^5	$g^2 + g + 1$	111	7
g^6	$g^2 + 1$	101	5