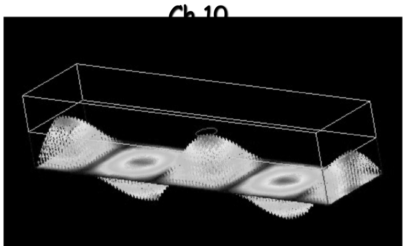
# EM Waves

INEL 4151
Ch 10



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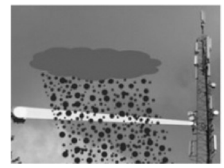
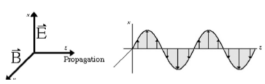
## Outline

- Plane Wave depending on media
  - general (lossy), lossless….
- Power in a Wave
- Applications and Concepts






10.1

## Plane waves

## The wave equation for E

- Start taking the curl of Faraday's law

$$\nabla\times\nabla\times E_s = -j\omega\mu\nabla\times H_s$$

- Then apply the vectorial identity

$$\nabla\times\nabla\times A = \nabla(\nabla\cdot A) - \nabla^2 A$$

- And you're left with

$$\nabla(\nabla\cdot E_s) - \nabla^2 E_s = -(j\omega\mu)(\sigma + j\omega\varepsilon)E_s$$
$$= -\gamma^2 E_s$$



## A Wave

$$\nabla^2 E - \gamma^2 E = 0$$

**Let's look at a special case for simplicity without loosing generality:**

- **•The electric field has only an *x*-component**
- **•The field travels in *z* direction**

**Then we have**

$$E(z,t)\hat{x}$$

*whose general solution is*

$$E(z) = E_o e^{-\gamma z} + E_o^{'} e^{+\gamma z}$$



## To change back to time domain

- From phasor

$$\gamma^2 = (j\omega\mu)(\sigma + j\omega\varepsilon)$$

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\varepsilon)} = \alpha + j\beta$$

$$E_{xs}(z) = E_o e^{-\gamma z} = E_o e^{-z(\alpha + j\beta)}$$

- …to time domain

$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\hat{x}$$

10.3-10.6

# Plane wave Propagation in Several Media

## Several Cases of Media

1. **Free space** $(\sigma = 0,\ \varepsilon = \varepsilon_o,\ \mu = \mu_o)$
2. **Lossless dielectric** $(\sigma = 0,\ \varepsilon = \varepsilon_r\varepsilon_o,\ \mu = \mu_r\mu_o \text{ or } \sigma << \omega\varepsilon)$
3. **Lossy dielectric (general)** $(\sigma \neq 0,\ \varepsilon = \varepsilon_r\varepsilon_o,\ \mu = \mu_r\mu_o)$
4. **Good Conductor** $(\sigma \cong \infty,\ \varepsilon = \varepsilon_o,\ \mu = \mu_r\mu_o \text{ or } \sigma >> \omega\varepsilon)$

Recall: Permittivity

$\varepsilon_o$=8.854 x $10^{-12}$ [F/m]

Permeability

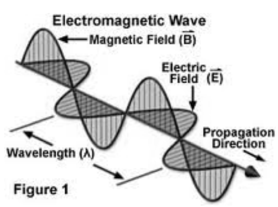
$\mu_o$= 4$\pi$ x $10^{-7}$ [H/m]



## 1. Free space

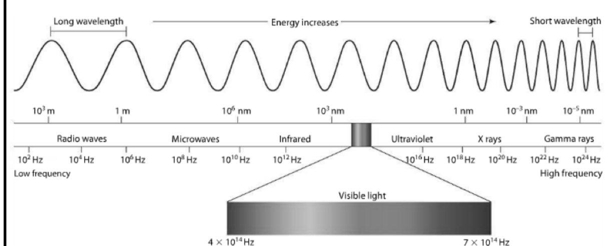
There are no losses, e.g. $E(z,t) = A\cos(\omega t - \beta z)\hat{x}$

Let's define
- The phase of the wave
- The angular frequency
- Phase constant
- The phase velocity of the wave
- The period T and wavelength $\lambda$
- In what direction does it moves?

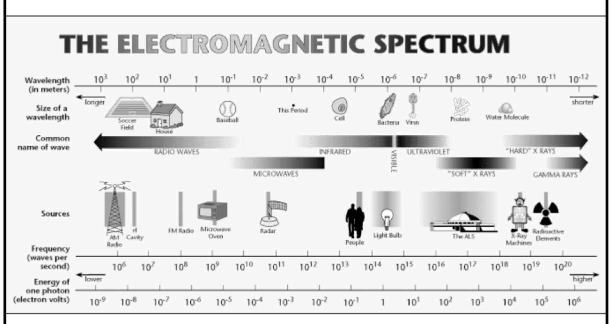





## EM wave Spectrum





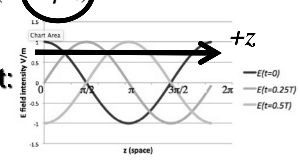






## Where does it moves?

$E(z,t) = A\cos(\omega t - \beta z)\hat{x}$

Choose 3 times: Plot:




$t = 0 \quad (\omega t = 0)$ $= \cos(-\beta z) = \cos\beta z$

$t = T/4 \quad (\omega t = 90^o)$ $= \cos(90 - \beta z) = \sin\beta z$

$t = T/2 \quad (\omega t = 180^o)$ $= \cos(180 - \beta z) = -\cos\beta z$

## So…

- Wave like this… Moves towards:

| Wave | Moves towards |
|---|---|
| $H(z,t) = A\cos(\omega t - \beta z)\hat{x}$ | $+\hat{z}$ |
| $E(z,t) = A\cos(\omega t + \beta z)\hat{y}$ | $-\hat{z}$ |
| $E(y,t) = A\cos(\omega t + \beta y)\hat{x}$ | $-\hat{y}$ |
| $H(x,t) = A\cos(\omega t - \beta x)\hat{z}$ | $+\hat{x}$ |

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## 3. General Case (Lossy Dielectrics)

$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\hat{x}$$

- In general, we had $\gamma^2 = (j\omega\mu)(\sigma + j\omega\varepsilon)$ and $\gamma = \alpha + j\beta$

$$-\operatorname{Re}\gamma^2 = \beta^2 - \alpha^2 = \omega^2\mu\varepsilon$$
$$\left|\gamma^2\right| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$$

- From these we obtain

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \qquad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

- So , for a known material and frequency, we can find $\gamma = \alpha + j\beta$

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## Intrinsic Impedance, $\eta$

- If we divide *E* by *H*, we get units of ohms and the definition of the intrinsic impedance of a medium at a given frequency.

given $E = E_o e^{-\gamma z}\hat{x}$
Find $H$ (use Maxwell)

$$\eta = \frac{|E|}{|H|} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta|\angle\theta_\eta \quad [\Omega]$$

$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\hat{x}$$
$$H(z,t) = \frac{E_o}{|\eta|} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\hat{y}$$

*Not in-phase for a lossy medium

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## Note…

in time domain :
$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\hat{x}$$
$$H(z,t) = \frac{E_o}{|\eta|} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\hat{y}$$

phasor :
$$E(z) = E_o e^{-\alpha z} e^{-j(\beta z)}\hat{x}$$
$$H(z) = \frac{E_o}{|\eta|} e^{-\alpha z} e^{-j(\beta z - \theta_\eta)}\hat{y}$$

- *E* and *H* are perpendicular to one another
- Travel is perpendicular to the direction of propagation
- The amplitude is related by the impedance
- And so is the phase

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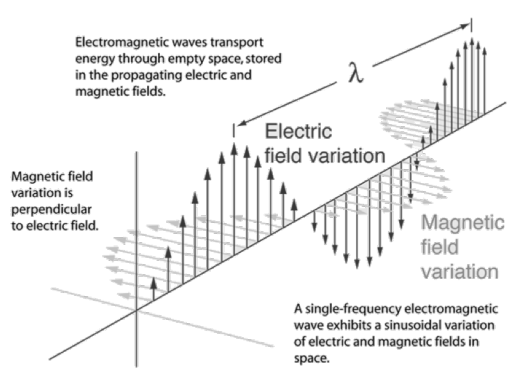


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## Example: Wave Propagation in Lossless materials

- A wave in a nonmagnetic material is given by

$$\vec{H} = \hat{z}\,50\cos(10^9 t - 5y)\ \text{[mA/m]}$$

Find:

(a) direction of wave propagation, Answer: +y,
(b) wavelength in the material $\lambda$=1.26m,
(c) phase velocity $u_p$= 2x$10^8$ m/s,

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## Loss Tangent

- If we divide the conduction current by the displacement current

$$\frac{|J_{cs}|}{|J_{ds}|} =$$



## Relation between $\tan\theta$ and $\varepsilon_c$

$$\nabla \times H = \sigma E + j\omega\varepsilon\, E = j\omega\varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]E$$

$$= j\omega\varepsilon_c E$$

The complex permittivity is

$$\varepsilon_c = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right] = \varepsilon' - j\varepsilon''$$

The loss tangent can be defined also as $\tan\theta = \dfrac{\varepsilon''}{\varepsilon'} = \dfrac{\sigma}{\omega\varepsilon}$



## Summary

| | Any medium | Lossless medium ($\sigma$=0) | Low-loss medium ($\varepsilon''/\varepsilon'$ <1/100) | Good conductor ($\varepsilon''/\varepsilon'$ >100) | Units |
|---|---|---|---|---|---|
| $\alpha$ | $\frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$ | $0$ | $\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ | $\sqrt{\pi f\mu\sigma}$ | [Np/m] |
| $\beta$ | $\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$ | $\omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_r\varepsilon_r}/c$ | $\omega\sqrt{\mu\varepsilon}$ | $\sqrt{\pi f\mu\sigma}$ | [rad/m] |
| $\eta$ | $\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ | $\sqrt{\frac{\mu}{\varepsilon}}$ | $\sqrt{\frac{\mu}{\varepsilon}}$ | $(1+j)\frac{\alpha}{\sigma} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$ | $[\Omega]$ |
| $u_c$ | $\omega/\beta$ | $\frac{1}{\sqrt{\mu\varepsilon}}$ | $\frac{1}{\sqrt{\mu\varepsilon}}$ | $\sqrt{\frac{4\pi f}{\mu\sigma}}$ | [m/s] |
| $\lambda$ | $\frac{2\pi}{\beta} = \frac{u_p}{f}$ | $\frac{u_p}{f}$ | $\frac{u_p}{f}$ | $\frac{u_p}{f}$ | [m] |

**In free space; $\varepsilon_o$ =8.85 x $10^{-12}$ F/m $\mu_o$=4π x $10^{-7}$ H/m

## $\beta$ can also be written as:

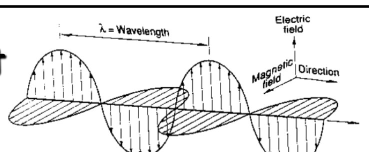
$$\} \; 8 \geq \sqrt{\frac{H\Diamond}{2}\left[\sqrt{1 \ni \left(\frac{®\leftarrow™}{©\geq\Diamond\sum}\right)^2} \ni 1\right]} \qquad \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

**If we substitute:**

$$\sqrt{\frac{\mu\varepsilon}{2}} = \sqrt{\frac{\mu_r\mu_o\varepsilon_r\varepsilon_r}{2}} = \frac{1}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}}$$



## Applet




**Harmonic oscillator damping**

http://www.cabrillo.edu/~jmccullough/Applets/Flash/Fluids,%20Oscillations%20and%20Waves/DampedSHM.swf

http://www.cabrillo.edu/~jmccullough/Applets/Applets_by_Topic/Oscillations.html



## Review: 1. Free Space

$$(\sigma = 0,\ \varepsilon = \varepsilon_o,\ \mu = \mu_o)$$

- Substituting in the general equations:

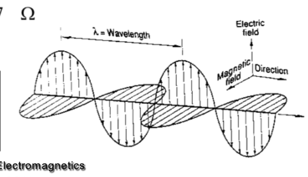
$$\alpha = 0, \quad \beta = \omega\sqrt{\mu\varepsilon} = \omega / c$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c \qquad \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\mu_o}{\varepsilon_o}}\angle 0^o = 120\pi\ \Omega = 377\ \ \Omega$$

$$E(z,t) = E_o\cos(\omega t - \beta z)\hat{x} \quad V/m$$

$$H(z,t) = \frac{E_o}{\eta_o}\cos(\omega t - \beta z)\hat{y} \quad A/m$$



## 2. Lossless dielectric

$(\sigma = 0,\ \varepsilon = \varepsilon_r\varepsilon_o,\ \mu = \mu_r\mu_o \text{ or } \sigma << \omega\varepsilon)$

- Substituting in the general equations:

$$\alpha = 0,\ \beta = \omega\sqrt{\mu\varepsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle 0^o$$



## 4. Good Conductors

$(\sigma \cong \infty,\ \varepsilon = \varepsilon_o,\ \mu = \mu_r\mu_o)$

- Substituting in the general equations:

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \qquad \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$$

Is water a good conductor???

$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\hat{x} \quad [V/m]$$

$$H(z,t) = \frac{E_o}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z}\cos(\omega t - \beta z - 45^o)\hat{y} \quad [A/m]$$

## Summary

| | Any medium | Lossless medium (σ=0) | Low-loss medium (ε"/ε' <1/100) | Good conductor $\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} > 100$ | Units |
|---|---|---|---|---|---|
| $\alpha$ | $\frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$ | 0 | $\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ | $\sqrt{\pi f\mu\sigma}$ | [Np/m] |
| $\beta$ | $\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$ | $\omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_r\varepsilon_r}/c$ | $\omega\sqrt{\mu\varepsilon}$ | $\sqrt{\pi f\mu\sigma}$ | [rad/m] |
| $\eta$ | $\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ | $\sqrt{\frac{\mu}{\varepsilon}}$ | $\sqrt{\frac{\mu}{\varepsilon}}$ | $(1+j)\frac{\alpha}{\sigma} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$ | [Ω] |
| $u_c$ | $\omega/\beta$ | $\frac{1}{\sqrt{\mu\varepsilon}}$ | $\frac{1}{\sqrt{\mu\varepsilon}}$ | $\sqrt{\frac{4\pi f}{\mu\sigma}}$ | [m/s] |
| $\lambda$ | $\frac{2\pi}{\beta} = \frac{u_p}{f}$ | $\frac{u_p}{f}$ | $\frac{u_p}{f}$ | $\frac{u_p}{f}$ | [m] |

**In free space; $\varepsilon_o$ =8.85 x $10^{-12}$ F/m $\mu_o$=4π x $10^{-7}$ H/m

## Example: Wave Propagation in Lossless materials

- A wave in a nonmagnetic material is given by

$$\vec{H} = \hat{z}50\cos(10^9 t - 5y)\ [\text{mA/m}]$$

Find:

d) Relative permittivity of material **2.25,**

e) Electric field phasor

$$\vec{E} = -\hat{x}12.57e^{-j5y}\ [\text{V/m}]$$



## Example

- In a free space,

$$\vec{H} = 10\cos(10^8 t - \beta x)\hat{z} \quad A/m$$

- Find $J_d$ and *E (use phasors)*

$$E = \frac{10\beta}{\omega\varepsilon}\cos(10^8 t - \beta x)\hat{y} \quad V/m$$

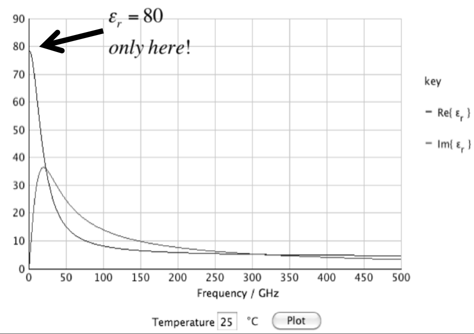
$$\vec{J}_d = \frac{\partial\vec{D}}{\partial t} = \frac{\varepsilon\partial\vec{E}}{\partial t} = j\omega\varepsilon\vec{E}$$



## Example: Water

## Permittivity of water vs. freq.

http://www.random-science-tools.com/electronics/water_dielectric.htm




## Example Water @ 60Hz, (low freq) Find $\alpha$, $\beta$ and $\eta$

- Attenuation const.

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

- Phase constant

$$\beta = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

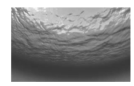
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

- For distilled water

$\mu_r = 1$
$\varepsilon_r = 81$
$\sigma = 0.0001$

$\frac{\sigma}{\omega\varepsilon} = 370$
$\alpha = 1.5\times10^{-4}$
$\beta = 1.5\times10^{-4}$
$\eta =$

- For sea water



$\mu_r = 1$
$\varepsilon_r = 80$
$\sigma = 4$

$\frac{\sigma}{\omega\varepsilon} = 14{,}986{,}341$
$\alpha = 0.03$
$\beta = 0.03$
$\eta =$

## Example Water @ 10GHz, (microwaves) Find $\alpha$, $\beta$ and $\eta$

- Attenuation const.

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

- Phase constant

$$\beta = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

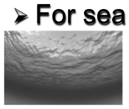
- For distilled water

$\mu_r = 1$
$\varepsilon_r = 65$
$\sigma = 0.0001$

$\frac{\sigma}{\omega\varepsilon} = 2.8\times10^{-6}$
$\alpha = 0.0023$
$\beta = 1{,}688$
$\eta =$

$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ $\omega\sqrt{\mu\varepsilon}$

- For sea water



$\mu_r = 1$
$\varepsilon_r = 62$
$\sigma = 4$

$\frac{\sigma}{\omega\varepsilon} = 0.116$
$\alpha = 95$
$\beta = 1651$
$\eta =$

## Exercise: Lossy media propagation

For each of the following determine if the material is low-loss dielectric, good conductor, etc.

(a) Glass with $\mu_r$=1, $\varepsilon_r$=5 and $\sigma$=$10^{-12}$ S/m at 10 GHZ $\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$
(b) Animal tissue with $\mu_r$=1, $\varepsilon_r$=12 and $\sigma$=0.3 S/m at 100 MHZ
(c) Wood with $\mu_r$=1, $\varepsilon_r$=3 and $\sigma$=$10^{-4}$ S/m at 1 kHZ

Answer:
(a) low-loss, $\alpha$= 8.4x$10^{-11}$ Np/m, $\beta$= 468 r/m, $\lambda$= 1.34 cm, $u_p$=1.34x$10^8$, $\eta_c$=168 Ω
(b) general, $\alpha$= 9.75, $\beta$=12, $\lambda$=52 cm, $u_p$=0.5x$10^8$ m/s, $\eta_c$=39.5+j31.7 Ω
(c) Good conductor, $\alpha$= 6.3x$10^{-4}$, $\beta$= 6.3x$10^{-4}$, $\lambda$= 10km, $u_p$=0.1x$10^8$, $\eta_c$=6.28(1+j) Ω

$\omega\sqrt{\mu\varepsilon}$
$= c/\sqrt{\mu_r\varepsilon_r}$

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## Today's class

- Skin depth
- Short cut! ☺
- Power

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## Skin depth, $\delta$

**We know that a wave attenuates in a lossy medium until it vanishes, but how deep does it go?**

- Is defined as the depth at which the electric amplitude is decreased to 37%

$$e^{-1} = 0.37 = (37\%)$$

$$e^{-\alpha z} = e^{-1} \text{ at } z = 1/\alpha = \delta$$

Skin depth
$\delta = 1/\alpha$ [m]

$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\hat{x} \quad [V/m]$$

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## Microwave Oven

Most food are lossy media at microwave frequencies, therefore EM power is lost in the food as heat.

- Find depth of penetration for potato which at 2.45 GHz has the complex permittivity given. $\varepsilon_c = \varepsilon_o (30 - j1)$

$$\gamma = \sqrt{(j\omega\mu_o)(j\omega\varepsilon_c)}$$
$$= \frac{2\pi f}{c}\sqrt{(j-30)}$$
$$= 202 - j195 \ \ [/\text{m}]$$

The power reaches the inside as soon as the oven in turned on!

$$\delta = 1/\alpha = 5\text{mm}$$

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## Shortcut!

- You can use Maxwell's or use

$$\vec{H} = \frac{1}{\eta}\hat{k} \times \vec{E}$$
$$\vec{E} = -\eta\,\hat{k} \times \vec{H}$$

where *k* is the direction of propagation of the wave, i.e., the direction in which the EM wave is traveling (a unitary vector).

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## Waves - Summary

- Static charges > static electric field, *E*
- Steady current > static magnetic field, *H*
- Static magnet > static magnetic field, *H*
- Time-varying current > time varying *E(t)* & *H(t)* that are interdependent > **electromagnetic wave**
- Time-varying magnet > time varying *E(t)* & *H(t)* that are interdependent > **electromagnetic wave**

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## EM waves don't need a medium to propagate

- Sound waves need a medium like air or water to propagate
- EM waves don't. They can travel in free space in the complete absence of matter.

Look at a "wind wave"; the energy moves, the plants stay at the same place.

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10.7-10.8

## Power and Poynting Vector

## Power in a wave

- A wave **carries power** and **transmits** it wherever it goes

The power density per area carried by a wave is given by the *Poynting vector.*

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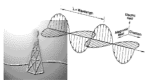
## Power in a wave

- To get Power density in W/m² carried by an electromagnetic wave:



$$\vec{E}\,\vec{H} \Rightarrow \left[\frac{V}{m}\cdot\frac{A}{m}\right] = \left[\frac{W}{m^2}\right]$$

**But we need to have a vector that shows where is the power going**
**That's the Poynting Vector.**



## Poynting Vector Derivation

- Start with ***E*** dot Ampere's

$$E\cdot\left[\nabla\times H = \sigma E + \varepsilon\frac{\partial E}{\partial t}\right]$$

$$E\cdot(\nabla\times H) = E\cdot E\sigma + E\cdot\varepsilon\frac{\partial E}{\partial t}$$

- Apply vector identity

$$\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)\text{ or in this case :}$$
$$\nabla\cdot(H\times E) = E\cdot(\nabla\times H) - H\cdot(\nabla\times E)$$

- And end up with:

$$H\cdot(\nabla\times E) + \nabla\cdot(H\times E) = \sigma E^2 + \frac{1}{2}\varepsilon\frac{\partial E^2}{\partial t}$$



## Poynting Vector Derivation…

- Substitute Faraday in 1rst term

$$H\cdot\left(-\mu\frac{\partial H}{\partial t}\right) + \nabla\cdot(H\times E) = \sigma E^2 + \frac{1}{2}\varepsilon\frac{\partial E^2}{\partial t}$$

$$\text{As in derivative of square function : } H\cdot\left(-\mu\frac{\partial H}{\partial t}\right) = \frac{\mu}{2}\frac{\partial(H\cdot H)}{\partial t}$$

and if invert the order, it's (-)
$$\nabla\cdot(H\times E) = -\nabla\cdot(E\times H)$$

$$-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla\cdot(E\times H) = \sigma E^2 + \frac{\varepsilon}{2}\frac{\partial E^2}{\partial t}$$

Rearrange

$$\nabla\cdot(E\times H) = -\left(\frac{\varepsilon}{2}\frac{\partial E^2}{\partial t} + \frac{\mu}{2}\frac{\partial H^2}{\partial t}\right) - \sigma E^2$$

## Poynting Vector Derivation…

- Taking the integral *wrt* volume

$$\int_v \nabla\cdot(\vec{E}\times\vec{H})dv = -\frac{\partial}{\partial t}\int_v\left(\frac{\varepsilon}{2}E^2 + \frac{\mu}{2}H^2\right)dv - \int_v \sigma E^2 dv$$

- Applying Theorem of Divergence

$$\oint_S(\vec{E}\times\vec{H})\cdot d\vec{S} = -\frac{\partial}{\partial t}\int_v\left(\frac{\varepsilon}{2}E^2 + \frac{\mu}{2}H^2\right)dv - \int_v \sigma E^2 dv$$

Total power across surface of volume | Rate of change of stored energy in E or H | Ohmic losses due to conduction current

- Which means that the total power coming out of a volume is either due to the electric or magnetic field energy variations or is lost in ohmic losses.

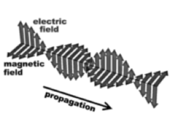


## Power: Poynting Vector

- Waves carry energy and information
- Poynting says that the net power flowing out of a given volume is = to the decrease in time in energy stored minus the conduction losses.

$$\vec{\mathcal{P}}(t) = \vec{E}\times\vec{H} \quad [W/m^2]$$

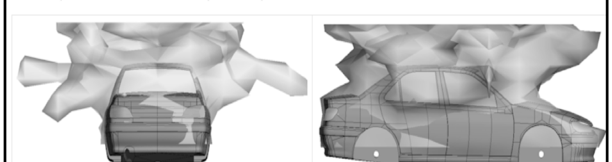
***Represents the instantaneous power density vector associated to the electromagnetic wave.***






## Bluetooth antenna in car

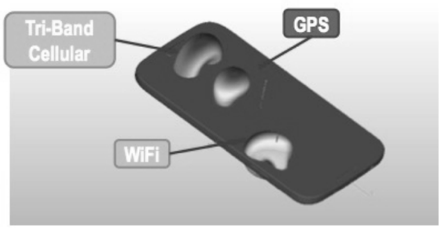
Bluetooth may also be considered as a means of communicating with devices outside the vehicle, e.g. a mobile phone. With such applications in mind, the power density outside the vehicle was also investigated. The question to answer is how far away from the vehicle can a -70 dBm (-40 dBW) signal level strength still be expected. This power density boundary was visualized with isosurfaces, which clearly indicate where the field strength is equal to -70 dBm. Within this boundary bluetooth devices are thus very likely to connect with the car's bluetooth system, but outside of this boundary connectivity will be limited.



(a) Side view (b) Rear view

Poynting vector (power density) -70dBm isosurface outside the vehicle
(click to enlarge images)

## Cellphone power radiation




Source: AnSys= Fred German

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## Time Average Power Density

- The Poynting vector *averaged in time* is

$$\vec{\mathcal{P}}_{ave} = \frac{1}{T}\int_0^T \vec{\mathcal{P}}\,dt = \frac{1}{T}\int_0^T \left(\vec{E}\times\vec{H}\right)dt = \frac{1}{2}\mathrm{Re}\left\{\vec{E}_s \times \vec{H}_s^*\right\}$$

- For the general case wave:

$$\vec{E}_s = E_o e^{-\alpha z} e^{-j\beta z}\hat{x} \quad [V/m]$$

$$\vec{H}_s = \frac{E_o}{\eta} e^{-\alpha z} e^{-j\beta z}\hat{y} \quad [A/m]$$

$$\vec{\mathcal{P}}_{ave} = \frac{E_o^2}{2|\eta|} e^{-2\alpha z}\cos\theta_\eta \hat{z} \quad [\mathrm{W/m^2}]$$

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## Total AVE Power in W

The total power through a surface *S* is

$$P_{ave} = \int_S \vec{\mathcal{P}}_{ave}\cdot d\vec{S} \quad [W]$$

- Note that the units now are in Watts
- Note that power nomenclature, *P* is not cursive.
- Note that the dot product indicates that the **surface area needs to be perpendicular** to the Poynting vector so that all the power will go thru. (give example of receiver antenna)

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## Exercise

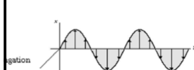
$$\vec{\mathcal{P}}_{ave} = \frac{E_o^2}{2|\eta|} e^{-2\alpha z}\cos\theta_\eta \hat{z} \quad [\mathrm{W/m^2}]$$

$$P_{ave} = \int_S \vec{\mathcal{P}}_{ave}\cdot d\vec{S} \quad [W]$$

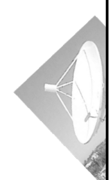
- Find average power received by antenna given
- S=3m² $\mathbf{a_z}$

$$\vec{E} = \hat{x}\sqrt{120\pi}\,e^{-j5z}$$

$$\vec{\mathcal{P}}_{ave} = \frac{(\sqrt{120\pi})^2}{2(120\pi)} e^0 \cos(0)\hat{z} = 0.5\,\hat{z}$$



$$P_{ave} = \int_S \vec{\mathcal{P}}_{ave}\cdot d\vec{S} = 0.5\cdot 3 = 1.5W$$



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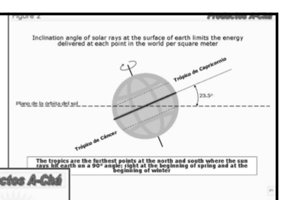
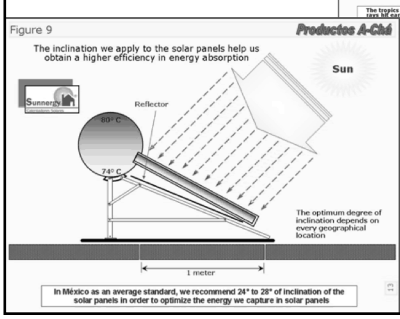
## Solar Panels

Optimum inclination of solar panels for maximum power transmission depends on geographical location.




http://www.altenergymag.com/emagazine.php?issue_number=05.08.01&article=gonzalez

## PE 10.7

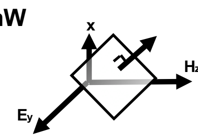
$$P_s = \frac{|E_o|^2}{2\eta} = \frac{|\eta H_o|^2}{2\eta} = \eta\frac{|H_o|^2}{2} = 60\pi(.04)$$

In free space, solar TEM wave at some latitude & season can be measured to be *H=0.2 cos (ωt-βx)* **z** A/m. Find the total power passing through a solar panel of side 10cm on plane $\hat{x}+\hat{z}=1$

Answer; $P_{tot}$ = $2.4\pi\frac{(.10)^2}{\sqrt{2}}$ =53mW

- square plate at $\hat{z}=3$

Answer; $P_{tot}$ = 0mW!

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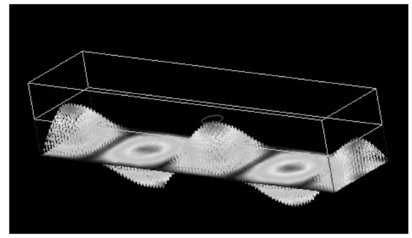
## Exercises: Power

2. A 5GHz wave traveling In a nonmagnetic medium with $\varepsilon_r$=9 is characterized by $\vec{E} = \hat{y}3\cos(\omega t + \beta x) - \hat{z}2\cos(\omega t + \beta x)$[V/m] Determine the direction of wave travel and the average power density carried by the wave

- Answer:

$$\vec{\mathcal{P}}_{ave} = \frac{E_o^2}{2|\eta_1|}e^{-2\alpha z}\cos\theta_\eta \hat{a}_k = -\hat{x}\frac{\left(3^2+2^2\right)}{2(40\pi)}$$

$$= -\hat{x}0.05 \ [W/m^2]$$



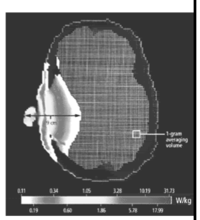
EM Wave propagation

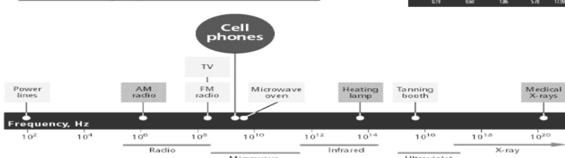
## Applications and Concepts

## Cell phone & brain

- Computer model for Cell phone Radiation inside the Human Brain
- SAR=*Specific Absorption Rate* [W/Kg] FCC limit 1.6W/kg, (depends on frequency)

http://www.ewg.org/cellphoneradiation/Get-a-Safer-Phone/Samsung/Impression+%28SGH-a877%29/




## Human absorption

- ***30-300 MHz is where the human body absorbs RF energy most efficiently***
- **** The FCC limit in the US for public exposure from cellular telephones at the ear level is a SAR level of 1.6 watts per kilogram (1.6 W/kg) as averaged over one gram of tissue.***

- http://handheld-safety.com/SAR.aspx
- http://www.fcc.gov/Bureaus/Engineering_Technology/Documents/bulletins/oet56/oet56e4.pdf
- http://www.rfcafe.com/references/electrical/fcc-maximum-permissible-exposure.htm

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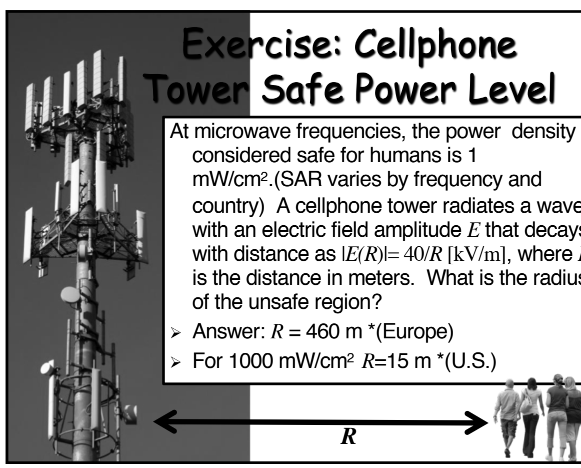
**ICNIRP= Intl Commission of Non-ionizing radiation protection**

## ICNIRP (1998:511) reference levels for occupational & general public exposure- table7

| Frequency range | Electric field strength (V/m) | | Equivalent plane wave power density $S_{eq}$(W/m²) | |
|---|---|---|---|---|
| | general public | occupational | general public | Occupational |
| 1-25 Hz | 10,000 | 20,000 | - | |
| 0.025- 0.82 KHz | 250/f(KHz) | 500/f(KHz) | | |
| 0.82 -3 KHz | 250/f(KHz) | 610 | - | |
| 3-1000 KHz | 87 | 610 | - | |
| 1-10 MHz | $87/f^{1/2}$ (MHz) | 610/f (MHz) | - | |
| 10-400 MHz | 28 | 61 | 2 | 10 |
| 400-2000 MHz | $1.375f^{1/2}$ (MHz) | $3f^{1/2}$ (MHz) | f/200 | f/40 |
| 2-300 GHz | 61 | 137 | 10 | 50 |

## Exercise: Cellphone Tower Safe Power Level




At microwave frequencies, the power density considered safe for humans is 1 mW/cm². (SAR varies by frequency and country) A cellphone tower radiates a wave with an electric field amplitude $E$ that decays with distance as $|E(R)| = 40/R$ [kV/m], where $R$ is the distance in meters. What is the radius of the unsafe region?

- Answer: $R$ = 460 m *(Europe)
- For 1000 mW/cm² $R$=15 m *(U.S.)

## Some internationally allowed standard levels of the of cell tower radiation

- Australian standard limits the radiation level to 200 mW/cm$^2$
- Russia, Italy and Canada allows only 10 mW/cm$^2$
- China, 6 mW/cm$^2$
- New Zealand 0.02 mW/cm$^2$ .
- United States allows 580 to 1,000 mW/cm$^2$

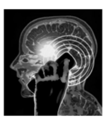
Radiofrequency Radiation Health Studies, Wireless Antenna Site Consumer Information Package, Sage Associates, Montecito, CA, 2000, www. sageassocciates.net. (b) Tower concerns should be health, not aesthetics, Burlington Free Press, January 12, 2001.



## Antennas hiding

- http://www.rense.com/general56/rad.htm


## Radiación en Teléfonos móviles



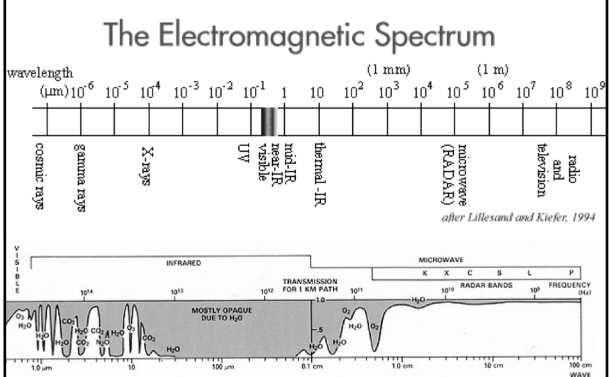
En 2011, la Organización Mundial de la Salud de la ONU clasificó a los teléfonos celulares en Categoría de Peligro de Cáncer, a pesar de que producen radiación no-ionizante

### 6 Tips para Protegerte:

1. Mantén las llamadas cortas.
2. Mantén distancia de 1“ (2.5 cm) del cuerpo)
3. Usa el cable auricular
4. Usa bocina altoparlante,
5. Apaga el bluetooth y WiFi cuando no lo estés usando
6. Envía texto en vez
7. *Coteja el nivel de radiación en tu modelo. Depende de la región. Los mismos modelos en Europa emiten menos radiación debido a exigencias de sus leyes. Busca en sarvalues.com SAR levels

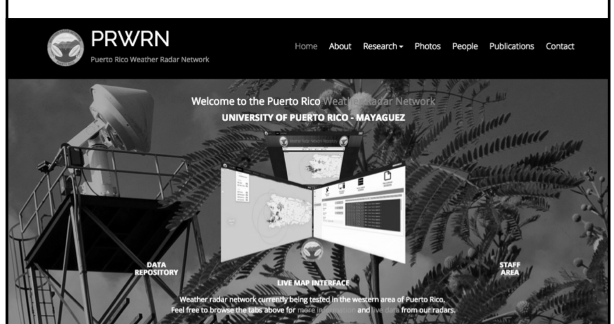
## Radar bands

| Band Name | Nominal Freq Range | Specific Bands | Application |
|---|---|---|---|
| HF, VHF, UHF | 3-30 MHz0, 30-300 MHz, 300-1000MHz | 138-144 MHz<br>216-225, 420-450 MHz<br>890-942 | TV, Radio, |
| L | 1-2 GHz (15-30 cm) | 1.215-1.4 GHz | Clear air, soil moist |
| S | 2-4 GHz (8-15 cm) | 2.3-2.5 GHz<br>2.7-3.7> | Weather observations<br>Cellular phones |
| C | 4-8 GHz (4-8 cm) | 5.25-5.925 GHz | TV stations, short range Weather |
| X | 8-12 GHz (2.5–4 cm) | 8.5-10.68 GHz | Cloud, light rain, airplane weather. Police radar. |
| Ku | 12-18 GHz | 13.4-14.0 GHz, 15.7-17.7 | Weather studies |
| K | 18-27 GHz | 24.05-24.25 GHz | Water vapor content |
| Ka | 27-40 GHz | 33.4-36.0 GHz | Cloud, rain |
| V | 40-75 GHz | 59-64 GHz | Intra-building comm. |
| W | 75-110 GHz | 76-81 GH, 92-100 GHz | Rain, tornadoes |
| millimeter | 110-300 GHz | | Tornado chasers |






## Radares UPRM



## Decibel Scale

- In many applications need comparison of two powers, a power ratio, e.g. reflected power, attenuated power, gain,…
- The decibel (dB) scale is logarithmic

$$G = \frac{P_1}{P_2}$$

$$G[dB] = 10\log\left(\frac{P_1}{P_2}\right)$$

- Note that for voltages, fields, and electric currents, the log is multiplied by 20 instead of 10.

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## Attenuation rate, A

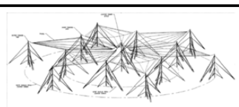
- Represents the rate of decrease of the magnitude of $P_{ave}(z)$ as a function of propagation distance`

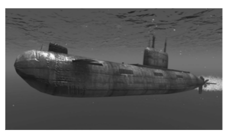
$$A = 10\log\left(\frac{P_{ave}(z)}{P_{ave}(0)}\right) = 10\log\left(e^{-2\alpha z}\right)$$

$$\alpha_{dB}[dB/m] = 8.68\alpha[Np/m]$$
$$\alpha_{dB}z[dB] = 20dB = 8.68\alpha z$$
$$20dB = 8.68\alpha z$$
$$z = 20/8.68\alpha = 23$$

$$\vec{\mathcal{P}}_{ave} = \frac{E_o^2}{2|\eta_1|}e^{-2\alpha z}\cos\theta_\eta$$

## Submarine antenna





A submarine at a depth of 200m receives signals at 1kHz from an antenna array on land.

a) Determine the power density incident upon the submarine antenna due to the EM wave with $|E_o|$= 10V/m.

- [At 1kHz, sea water has $\varepsilon_r$=81, $\sigma$=4].

$$\alpha = \sqrt{f\pi\mu\sigma} = .126$$

$$\eta = (1+j)\frac{\alpha}{\sigma} = 0.044\angle 45^o\ \Omega$$

$$\mathcal{P}_{ave} = \frac{E_o^2}{2|\eta|}e^{-2\alpha z}\cos\theta_\eta \hat{z} \quad [\text{W/m}^2]$$

$$\mathcal{P}_{ave} = 10^{-19}[W/m^2] = -190\,dBW/m^2$$

b) At what depth the amplitude of *the power* has decreased to 1% its initial value at $z$=0 (sea surface)? Answer= 18.3m

## Pure Water is blue?

- Yes, it's turquoise-blue due to its electromagnetic spectrum emission

http://ece.uprm.edu/~pol/OceanBlue

## Remember

- Perfect dielectric = lossless
- Nonmagnetic means $\eta_r$=1

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## Eyes are Antennas

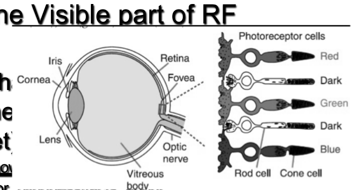
Designed to see the Visible part of RF Spectrum

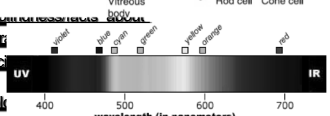
- Normally humans h
  eyes that allows the
  within 400nm(violet
- http://book.bionumbers.org/how
- https://nei.nih.gov/health/color_blindness/facts_about
- https://en.wikipedia.org/wiki/Tetra

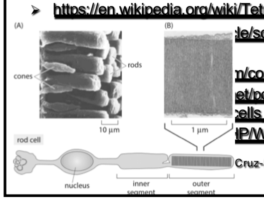
le/sci

n/colo

et/post/is_visible_light_wavelength_related_to_th
cells_of_our_eyes
P/WCL/color1.html

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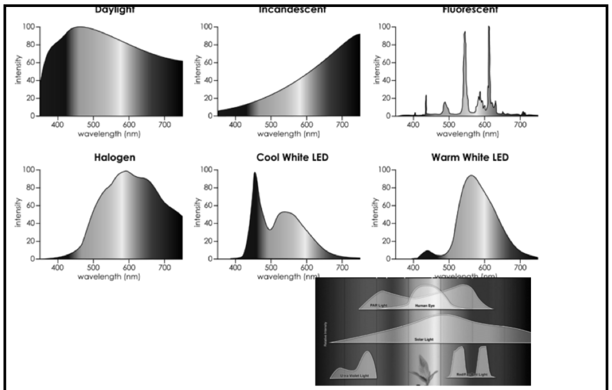


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