Wave Incidence

Ex. 1. Light traveling in air encounters the water; another medium.

Ex. 3. Wave encounters skin tissue, muscle mass and bones

- Cellphone radiation

*Kid’s skull is not yet developed

Ex. 4. Cancer treatment

Wave incidence

- For many applications, [such as fiber optics, line power transmission], it’s necessary to know what happens to a wave when it meets a different medium.

- How much is transmitted?
- How much is reflected back?
We will look at...

I. Normal incidence
   Wave arrives at $0^\circ$ from normal
   - Standing waves

II. Oblique incidence
    Wave arrives at another angle
    - Snell’s Law and Critical angle
    - Parallel or Perpendicular polarization
    - Brewster angle

Vital Definition

- Plane between media-interface
- Plane of incidence- (what you draw)

Reflection at Normal Incidence

- Incident wave
- Reflected wave
- Transmitted wave

Now in terms of equations

- Incident wave
  \[ \vec{E}_i(z) = E_{io} e^{-\gamma z} \hat{x} \]
  \[ \vec{H}_i(z) = H_{io} e^{-\gamma z} \hat{y} = \frac{E_{io}}{\eta_1} e^{-\gamma z} \hat{y} \]

Reflected wave

- It’s traveling along $-z$ axis
  \[ \vec{E}_r(z) = E_{ro} e^{\gamma z} \hat{x} \]
  \[ \vec{H}_r(z) = H_{ro} e^{\gamma z} (-\hat{y}) = -\frac{E_{ro}}{\eta_1} e^{\gamma z} \hat{y} \]

Transmitted wave

- \[ \vec{E}_{ts}(z) = E_{to} e^{-\gamma z} \hat{x} \]
- \[ \vec{H}_{ts}(z) = H_{to} e^{-\gamma z} \hat{y} = \frac{E_{to}}{\eta_2} e^{-\gamma z} \hat{y} \]
Sum to get TOTAL field

- At medium 1 and medium 2

\[ E_x(0) + E_x(0) = E_x(0) \]
\[ H_y(0) + H_y(0) = H_y(0) \]

- Tangential components must be continuous at the interface

\[ E_x(0) + E_x(0) = E_x(0) \]
\[ H_y(0) + H_y(0) = H_y(0) \]

Define

- Reflection coefficient, \( \Gamma \)

\[ \Gamma = \frac{E_\text{ref}}{E_\text{inc}} = \frac{n_2 - n_1}{n_2 + n_1} \]

- Transmission coefficient, \( \tau \)

\[ \tau = \frac{E_\text{trans}}{E_\text{inc}} = \frac{2n_2}{n_2 + n_1} \]

Note:

- \( 1 + \Gamma = \tau \)
- Both are dimensionless and may be complex
- \( 0 \leq |\Gamma| \leq 1 \)

Look at \( E \) fields

\[ E_{\text{inc}} e^{-j\omega t} \hat{x} + E_{\text{trans}} e^{j\omega t} \hat{x} = E_{\text{inc}} e^{-j\omega t} \hat{x} \]

\[ z = 0 \]
\[ E_{\text{inc}} + E_{\text{trans}} = E_{\text{inc}} \]
\[ E_{\text{trans}} + E_{\text{inc}} = E_{\text{inc}} \]

Example: PE 10.8

A 5GHz uniform plane wave \( E_\text{inc} = 10e^{j\omega t} \) in free space is incident normally on a large plane, lossless dielectric slab (\( z > 0 \)) having \( \varepsilon = 4\varepsilon_0 \) and \( \mu = \mu_0 \).

Find:

- the reflected wave \( E_\text{ref} \) and
- the transmitted wave \( E_\text{trans} \).

Answer:

\[ -3.33 e^{j\omega t} \hat{x} \]
\[ 6.67 e^{-j\omega t} \hat{x} \]

\( b_2 = 2b_1 = 200 \) pF/3

SEE http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html

Today we’ll see 3 Cases and the Standing waves form on each case

1. Medium 2 is perfect Conductor
2. Medium 2 is perfect Dielectric
   - \( \eta_2 > \eta_1 \)
3. Medium 2 is perfect Dielectric
   - \( \eta_1 > \eta_2 \)

http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html
Case 1: Conductor(2)

- Medium 1: perfect dielectric, $\sigma_1 = 0$
- Medium 2: perfect conductor, $\sigma_2 = \infty$

Find intrinsic impedance

Coef. Of reflex & transmission & E1 field

$$z_0 = 0, \quad \Gamma = -1, \tau = 0$$

$$E_0 = 2E_\infty \sin \beta_1 z \sin \omega t$$

$$\beta_1(z) = \beta_1 (\text{constant})$$

http://www.phy.ntnu.edu.tw/java/waveSuperposition/waveSuperposition.html

Case 2: Dielectric $\eta_2 > \eta_1$

- Medium 1: perfect dielectric $\sigma_1 = 0$
- Medium 2: perfect dielectric $\sigma_2 = 0$, $\eta_2 > \eta_1$

If $\eta_1 > \eta_2$, $\Gamma > 0$,

$\tau$ and $\Gamma$ are real.

$$z_{\min} = \frac{-\pi n}{\beta_1} = \frac{-n \lambda_1}{2} \quad n = 1, 2, 3$$

$$z_{\max} = \frac{(2n+1)\pi}{2\beta_1} = \frac{(2n+1)\lambda_1}{4} \quad n = 1, 2, 3$$

Case 3: Dielectric $\eta_1 > \eta_2$

- Medium 1 = perfect dielectric $\sigma_1 = 0$
- Medium 2 = perfect dielectric $\sigma_2 = 0$, $\eta_1 > \eta_2$

If $\eta_2 < \eta_1$, $\Gamma < 0$, $\tau$ and $\Gamma$ are real.

$$z_{\min} = \frac{n \pi}{\beta_1} = \frac{n \lambda_1}{2} \quad n = 1, 2, 3$$

$$z_{\max} = \frac{(2n+1)\pi}{2\beta_1} = \frac{(2n+1)\lambda_1}{4} \quad n = 1, 2, 3$$

Standing waves due to reflection

$$E_z = E_1 + E_2 = E_\infty (e^{-j\beta_2 z} + \Gamma e^{+j\beta_2 z}) = E_\infty e^{-j\beta_2 z} (1 + \Gamma e^{+j\beta_2 z})$$

***At every half-wavelength, everything repeats!***

Standing waves due to reflection

$$E_z = E_1 + E_2 = E_\infty (e^{-j\beta_2 z} + \Gamma e^{+j\beta_2 z}) = E_\infty e^{-j\beta_2 z} (1 + \Gamma e^{+j\beta_2 z})$$

At every half-wavelength, all electromagnetic properties repeat.
Normal Incidence

Standing Wave Ratio, \( s \)

- Measures the amount of reflections, the more reflections, the larger the standing wave that is formed.
- The ratio of \( |E|_{\text{max}} \) to \( |E|_{\text{min}} \)

\[
\frac{s}{|E|_{\text{min}}} = \frac{|E|_{\text{max}}}{|E|_{\text{min}}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}
\]

Ex. Given a wave travelling in F.S. and hitting dielectric

- Lossless dielectric at 10 MHz has \( \varepsilon_r = 16 \), nonmagnetic
- Find SWR on air

Power Flow in Medium 1

- The net average power density flowing in lossless medium 1

\[
P_{\text{ave1}}(z) = \frac{1}{2} \text{Re} \left[ \bar{E}_1 \times H_1^* \right] = \frac{1}{2} \text{Re} \left[ \bar{E}_0 \left( e^{-j\beta z} + \Gamma e^{j\beta z} \right) \times \bar{H}_0^* \left( e^{j\beta z} + \bar{\Gamma} e^{-j\beta z} \right) \right]
\]

\[
P_{\text{ave1}}(z) = \frac{1}{2} \text{Re} \left[ \bar{E}_0 \left( 1 - |\Gamma|^2 \right) \frac{\bar{E}_0^*}{2\eta_1} \right] = P_{\text{ave1}}^+ + P_{\text{ave1}}^-.
\]

Power Flow in Transmitted wave

- The net average power density flowing in lossless medium 2

\[
P_{\text{ave2}}(z) = P_t = \frac{1}{2} \text{Re} \left[ E_t \times H_t^* \right] = \frac{1}{2} \text{Re} \left[ \bar{E}_0 \left( e^{-j\beta z} + \bar{\Gamma} e^{j\beta z} \right) \times \bar{H}_0^* \left( e^{j\beta z} + \bar{\Gamma} e^{-j\beta z} \right) \right]
\]

\[
P_{\text{ave2}}(z) = \frac{1}{2} \text{Re} \left[ \bar{E}_0 \left( 1 - |\Gamma|^2 \right) \frac{\bar{E}_0^*}{2\eta_2} \right] = P_{\text{ave2}}^+ + P_{\text{ave2}}^-.
\]

Power in Lossy Media

\[
P_{\text{ave1}}(z) = \frac{1}{2} \left| E_0 \right|^2 \left( e^{-2\alpha z} - |\Gamma|^2 e^{2\alpha z} \right)
\]

\[
P_{\text{ave2}}(z) = \frac{1}{2} \left| E_0 \right|^2 \left( 1 - e^{-2\alpha z} \text{Re} \left( \frac{1}{\eta_z} \right) \right)
\]

where

\[
\Gamma = \frac{\sqrt{\varepsilon_1 - \sqrt{\varepsilon_2}}}{\sqrt{\varepsilon_1 + \sqrt{\varepsilon_2}}} \quad \text{and} \quad \varepsilon_{\text{eff}} = \varepsilon_2 - j \frac{\sigma_2}{\omega_2}
\]

Ex. Antenna Radome

A 10GHz aircraft radar uses a narrow-beam scanning antenna mounted on a gimbal behind a dielectric radome.

- Even though the radome shape is far from planar, it is approximately planar over the narrow extent of the radar beam.
- If the radome material is a lossless dielectric with \( \mu_r = 1 \) and \( \varepsilon_r = 9 \), choose its thickness \( d \) such that the radome appears transparent to the radar beam.
- Mechanical integrity requires \( d \) to be greater than 2.3 cm.

Answer:

\( \lambda/2 = 0.5 \text{cm}, \quad d = 2.5 \text{cm} \)