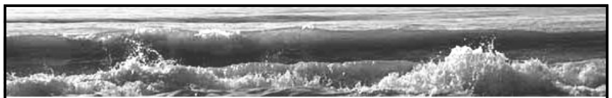


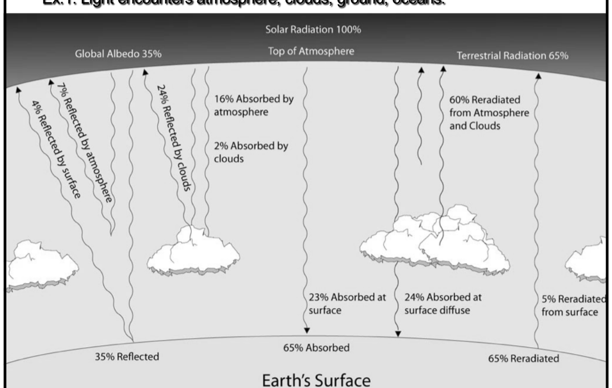
Wave Incidence

[Chapter 10 cont, Sadiku]

Dr. Sandra Cruz-Pol
Electrical and Computer Engineering Dept.
UPR-Mayagüez

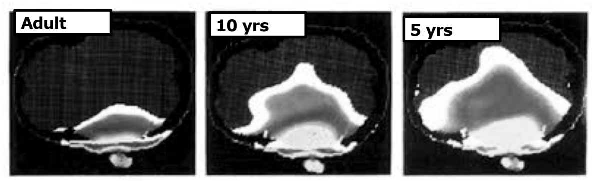
Ex.1. Light traveling in air encounters the water; another medium.

Ex.1. Light encounters atmosphere, clouds, ground, oceans.



Ex. 3. Wave encounters skin tissue, muscle mass and bones

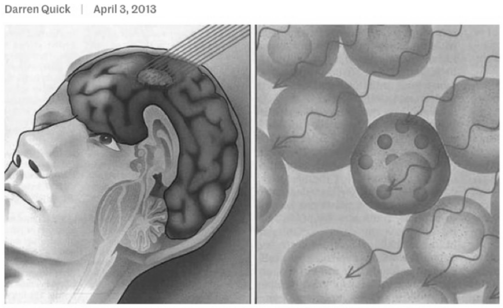
- Cellphone radiation



*Kid's skull is not yet developed

Ex. 4. Cancer treatment

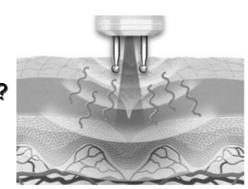
Darren Quick | April 3, 2013



Boron neutron capture therapy can kill tumors without harming healthy neighboring tissue.

Wave incidence

- For many applications, [such as fiber optics, line power transmission], it's necessary to know what happens to a wave when it meets a different medium.
  – How much is *transmitted*?
  – How much is *reflected* back?



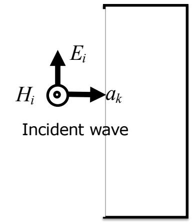
We will look at…

I. Normal incidence
Wave arrives at 0º from normal
– Standing waves

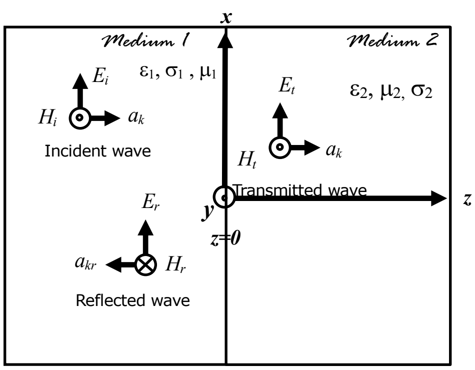
II. Oblique incidence
Wave arrives at another angle
– Snell's Law and Critical angle
– Parallel or Perpendicular polarization
– Brewster angle

Vital Definition

- Plane between media-interface
- Plane of incidence- (what you draw)

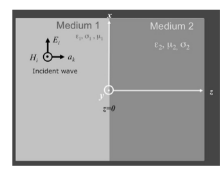


Reflection at Normal Incidence



Now in terms of equations

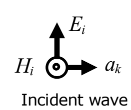
…



- Incident wave

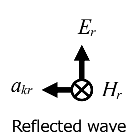
$$\vec{E}_{is}(z) = E_{io} e^{-\gamma_1 z}\hat{x}$$

$$\vec{H}_{is}(z) = H_{io} e^{-\gamma_1 z}\hat{y} = \frac{E_{io}}{\eta_1} e^{-\gamma_1 z}\hat{y}$$



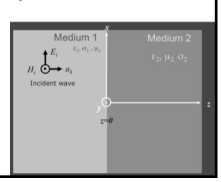
Reflected wave

- It's traveling along $-z$ axis



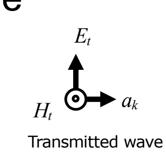
$$\vec{E}_{rs}(z) = E_{ro} e^{\gamma_1 z}\hat{x}$$

$$\vec{H}_{rs}(z) = H_{ro} e^{\gamma_1 z}(-\hat{y}) = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z}\hat{y}$$

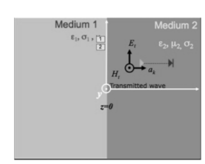


Transmitted wave

$$\vec{E}_{ts}(z) = E_{to} e^{-\gamma_2 z}\hat{x}$$



$$\vec{H}_{ts}(z) = H_{to} e^{-\gamma_2 z}\hat{y} = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z}\hat{y}$$



Sum to get TOTAL field

- At medium 1 and medium 2
- Tangential components must be continuous at the interface

$$\vec{E}_i(0)+\vec{E}_r(0)=\vec{E}_t(0)$$
$$\vec{H}_i(0)+\vec{H}_r(0)=\vec{H}_t(0)$$

Look at E fields

$$\vec{E}_i(0)+\vec{E}_r(0)=\vec{E}_t(0)$$
$$E_{io}e^{-\gamma_1 z}\hat{x}+E_{ro}e^{\gamma 1 z}\hat{x}=E_{to}e^{-\gamma_2 z}\hat{x}$$
$$z=0$$
$$E_{io}+E_{ro}=E_{to}$$
$$\frac{E_{io}}{E_{io}}+\frac{E_{ro}}{E_{io}}=\frac{E_{to}}{E_{io}}$$

Define

- Reflection coefficient, Γ

$$\Gamma=\frac{E_{ro}}{E_{io}}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}$$

- Transmission coefficient, τ

$$\tau=\frac{E_{to}}{E_{io}}=\frac{2\eta_2}{\eta_2+\eta_1}$$

Note:
- $1+\Gamma=\tau$
- Both are dimensionless and may be complex
- $0\le|\Gamma|\le 1$

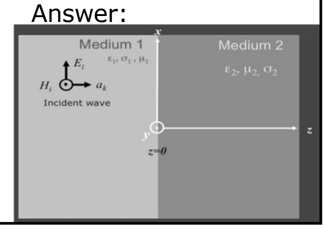
Example: PE 10.8

A 5GHz uniform plane wave $E_{is}=10e^{-j\beta z}\,a_x$ in free space is incident normally on a large plane, lossless dielectric slab ($z>0$) having $\varepsilon=4\varepsilon_o$ and $\mu=\mu_o$.

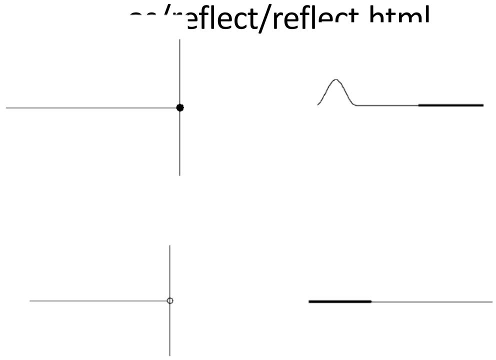
Find:
- the reflected wave E_{rs} and
- the transmitted wave E_{ts}.

SEE http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html

Answer:



SEE http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html



Today we'll see 3 Cases

and the Standing waves form on each case

1. Medium 2 is perfect Conductor
2. Medium 2 is perfect Dielectric
   - $\eta_2>\eta_1$
3. Medium 2 is perfect Dielectric
   - $\eta_1>\eta_2$

Case 1: Conductor(2)

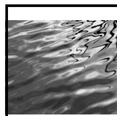
- Medium 1: perfect dielectric, $\sigma_1=0$
- Medium 2: perfect conductor, $\sigma_2=\infty$

Find intrinsic impedance
Coef. Of reflex & transmission
& E1 field

$$(1+j)\frac{\alpha}{\sigma}$$

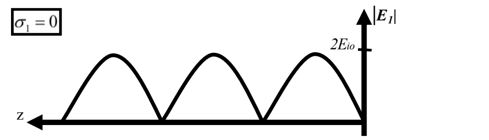
$$\eta_2=0,\quad \Gamma=-1, \tau=0$$
$$E_{1s}=-2jE_{io}\sin\beta_1 z\,\hat{x}\,(phasor)$$
$$E_1(z,t)=2E_{io}\sin\beta_1 z\sin\omega t\,\hat{x}$$

http://www.phy.ntnu.edu.tw/java/waveSuperposition/waveSuperposition.html



The EM field forms a Standing Wave on medium 1

$$E_1=2E_{io}\sin\beta_1 z\sin\omega t\,\hat{x}$$



$$Minima\ @\ -\beta_1 z=0,\pi,2\pi$$
$$Maxima\ @\ -\beta_1 z=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}$$
$$z_{max}=-\frac{n\pi}{\beta_1}=-\frac{n\lambda_1}{4}\qquad n=1,3,5$$

Conducting material $\sigma_2=\infty$

Case 2: Dielectric 2 $\eta_2>\eta_1$

- Medium 1: perfect dielectric $\sigma_1=0$
- Medium 2: perfect dielectric $\sigma_2=0$, $\eta_2>\eta_1$

$$\text{If } \eta_2>\eta_1,\quad \Gamma>0,\quad \tau \text{ and } \Gamma \text{ are real.}$$

$$E_{1s}=E_{is}+E_{rs}=E_{oi}(e^{-j\beta_1 z}+\Gamma e^{+j\beta_1 z})=E_{oi}e^{-j\beta_1 z}(1+\Gamma e^{+2j\beta_1 z})$$

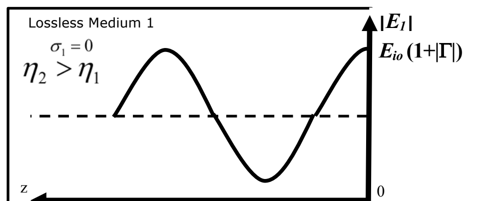
$$-2\beta_1 z_{max}=0,2\pi,4\pi,6\pi\ldots$$
$$\text{or } -\beta_1 z_{max}=0,\pi,2\pi,3\pi,\ldots$$

$$z_{max}=-\frac{n\pi}{\beta_1}=-\frac{n\lambda_1}{2}\qquad n=0,1,2,3$$

$$z_{min}=-\frac{(2n+1)\pi}{2\beta_1}=-\frac{(2n+1)\lambda_1}{4}\qquad n=0,1,2,3$$

Standing waves due to reflection

$$E_1=E_i+E_r=E_{oi}(e^{-j\beta_1 z}+\Gamma e^{+j\beta_1 z})=E_{oi}e^{-j\beta_1 z}(1+\Gamma e^{+2j\beta_1 z})$$



$$z_{max}=-\frac{n\pi}{\beta_1}=-\frac{n\lambda_1}{2}\qquad n=0,1,2,3$$

***At every **half-wavelength**, everything repeats! ***

Case 3: Dielectric 2 $\eta_1>\eta_2$

- Medium 1 = perfect dielectric $\sigma_1=0$
  - Medium 2 = perfect dielectric $\sigma_2=0$ $\eta_1>\eta_2$

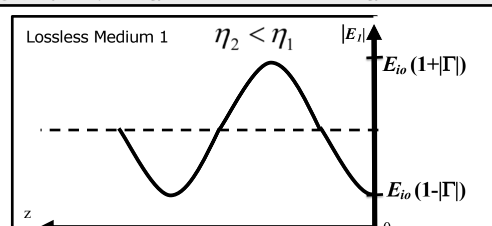
$$\text{If } \eta_2<\eta_1,\quad \Gamma<0, \tau \text{ and } \Gamma \text{ are real.}$$

$$z_{max}=-\frac{(2n+1)\pi}{2\beta_1}=-\frac{(2n+1)\lambda_1}{4}\qquad n=1,2,3$$

$$z_{min}=-\frac{n\pi}{\beta_1}=-\frac{n\lambda_1}{2}\qquad n=0,1,2,3$$

Standing waves due to reflection

$$E_1=E_i+E_r=E_{oi}(e^{-j\beta_1 z}+\Gamma e^{+j\beta_1 z})=E_{oi}e^{-j\beta_1 z}(1+\Gamma e^{+2j\beta_1 z})$$



$$z_{max}=-\frac{(2n+1)\lambda_1}{4}\qquad n=0,1,2,3$$

At every half-wavelength, all *em* properties repeat

Standing Wave Ratio, *s*

◆Measures the amount of reflections, the more reflections, the larger the standing wave that is formed.

◆The ratio of $|E_1|_{max}$ to $|E_1|_{min}$

$$s = \frac{|E_1|_{\max}}{|E_1|_{\min}} = \frac{|H_1|_{\max}}{|H_1|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

or

$$|\Gamma| = \frac{s-1}{s+1}$$

Ideally
s=1 (0 dB)
No reflections

Ex. Given a wave travelling in F.S. and hitting dielectric

- Lossless dielectric at 10MHz has

ε_r=16, nonmagnetic

- Find SWR on air

Power Flow in Medium 1

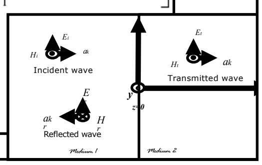
- The net average power density flowing in lossless medium 1

$$P_{ave1}(z) = \frac{1}{2}\text{Re}[E_1 \times H_1^*]$$

$$= \frac{1}{2}\text{Re}\left[\hat{x}E_{io}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right) \times \hat{y}\,\frac{E_{io}^*}{\eta_1}\left(e^{j\beta_1 z} + \Gamma^* e^{-j\beta_1 z}\right)\right]$$

$$= \hat{z}\frac{|E_{io}|^2}{2\eta_1}\left(1-|\Gamma|^2\right)$$

$$= P_{ave}^i + P_{ave}^r$$



Power Flow in Transmitted wave

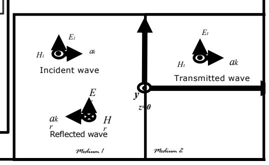
- The net average power density flowing in lossless medium 2

$$P_{ave2}(z) = P_t = \frac{1}{2}\text{Re}[E_2 \times H_2^*]$$

$$= \frac{1}{2}\text{Re}\left[\hat{x}\tau\, E_{io}e^{-j\beta_2 z} \times \hat{y}\tau^*\,\frac{E_{io}^*}{\eta_2}e^{j\beta_2 z}\right]$$

$$P_t = \hat{z}|\tau|^2\frac{|E_{io}|^2}{2\eta_2}$$

$$\textit{where } \hat{a}_k = \hat{z}$$



Power in Lossy Media

$$P_{ave1}(z) = \hat{z}\frac{|E_o^i|^2}{2\eta_{c1}}\left(e^{-2\alpha_1 z} - |\Gamma|^2 e^{2\alpha_1 z}\right)$$

$$P_{ave2}(z) = \hat{z}|\tau|^2\frac{|E_o^i|^2}{2}e^{-2\alpha_2 z}\,\text{Re}\left(\frac{1}{\eta_{c2}^*}\right)$$

where

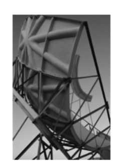
$$\Gamma = \frac{\sqrt{\varepsilon_{c1}} - \sqrt{\varepsilon_{c2}}}{\sqrt{\varepsilon_{c1}} + \sqrt{\varepsilon_{c2}}} \quad \text{and} \quad \varepsilon_{c2} = \varepsilon_2 - j\frac{\sigma_2}{\omega_2}$$

Ex. Antenna Radome


Antenna with radome

A 10GHz aircraft radar uses a narrow-beam scanning antenna mounted on a gimbal behind a dielectric radome.

- Even though the radome shape is far from planar, it is approximately planar over the narrow extent of the radar beam.
- If the radome material is a lossless dielectric with μ_r=1 and ε_r=9, choose its thickness *d* such that the radome appears transparent to the radar beam.
- Mechanical integrity requires *d* to be greater that 2.3 cm.


Antenna with no radome

Answer:
λ/2=.5cm, d=2.5cm