Aperture Antennas

INEL 5305
Prof. Sandra Cruz-Pol
ECE, UPRM

Ref. Balanis Chpt. 12

Aperture Antennas

- Most common at microwave frequencies
- Can be flushed-mounted
- We will analyze radiation characteristics at **far field**
  - Rectangular aperture
  - Circular aperture

Far field is the $\mathfrak{F}$ of the near field

- Fourier Transform for 1-D

$$W(k_x) = \int_{-\infty}^{\infty} w(x) e^{jk_x x} dx$$

f *t*

$$w(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(k_x) e^{-jk_x x} dk_x$$

- For two-dimensions, x and y;

$$U(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y) e^{jk_x x + jk_y y} dx\, dy$$

$$u(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(k_x, k_y) e^{-jk_x x - jk_y y} dk_x dk_y$$

Properties of Fourier Transform

$$\mathscr{F}_t \frac{ds(t)}{dt} = j\omega \mathscr{F}_t s(t)$$

$$\mathscr{F}_x \frac{\partial u(x, y)}{\partial x} = -jk_x \mathscr{F}_x u(x, y)$$

$$\mathscr{F}_x \frac{\partial u^2(x, y)}{\partial x^2} = (-jk_x)^2 \mathscr{F}_x u(x, y)$$

$$\mathscr{F}_{yx} \frac{\partial u^2(x, y)}{\partial x^2} = -k_x^2 \mathscr{F}_{yx} u(x, y)$$

$$\nabla^2 \mathbf{E} + k_o^2 \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E} + k_o^2 \mathbf{E} = 0$$

$$\left(\frac{\partial E_x(x,y,z)}{\partial x} + \frac{\partial E_y(x,y,z)}{\partial y} + \frac{\partial E_z(x,y,z)}{\partial z}\right) = 0$$

Taking the Fourier transform of the 2 equations above:

$$\frac{\partial^2}{\partial z^2}\mathbf{E}(k_x,k_y,z) + (k_o^2 - k_x^2 - k_y^2)\mathbf{E}(k_x,k_y,z) = 0$$

$$\left(k_x E_x(k_x,k_y,z) + k_y E_y(k_x,k_y,z) + j\frac{\partial E_z(k_x,k_y,z)}{\partial z}\right) = 0$$

Now, we define, $k_z^2 = k_o^2 - k_x^2 - k_y^2$

And we obtain, $\frac{\partial^2 \mathbf{E}(k_x,k_y,z)}{\partial z^2} + k_z^2 \mathbf{E}(k_x,k_y,z) = 0$

Which has a solution of $\mathbf{E}(k_x,k_y,z) = \mathbf{f}(k_x,k_y)e^{-jk_z z}$

Then we take the inverse transform

$$\mathbf{E}(x,y,z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{f}(k_x,k_y)e^{-j\mathbf{k}\cdot\mathbf{r}}dk_x dk_y$$

If z=0, then, we are at the aperture

$$\mathbf{E}_a(x,y) = \mathbf{E}_{\tan}(x,y,0) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{f}(k_x,k_y)e^{-jk_x x - jk_y y}dk_x dk_y$$

Which looks like:

$$u(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(k_x,k_y)e^{-jk_x x - jk_y y}dk_x dk_y$$

Which is the inverse of F…

This is the Fourier transform for 2 dimensions, so:

$$U(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u(x,y)e^{jk_x x + jk_y y}dx\,dy$$

$$\mathbf{f}_t(k_x,k_y) = \iint_{S_a} E_a(x,y)e^{jk_x x + jk_y y}dxdy$$

It can be shown that,

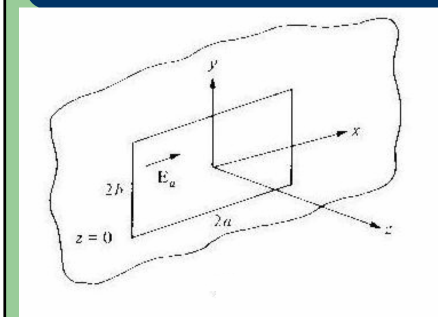
$$\mathbf{E}(r) \approx \frac{jk_o\cos\theta}{2\pi r}e^{-jk_o r}\mathbf{f}_t\left(k_o a\cos\theta\cos\phi, k_o b\sin\theta\sin\phi\right)$$

Therefore, if we know the field at the aperture, we can used these equations to find $\mathbf{E}(r)$.

=>First, we'll look at the case when the illumination at the rectangular aperture it's uniform.

Uniformly illuminated rectangular aperture

$$\mathbf{E}_a(x,y) = E_o\mathbf{x} \quad \text{for} \quad |x| \le a \qquad |y| \le b$$
$$= 0 \qquad \text{elsewhere}$$



$$\mathbf{f}_t = E_o\mathbf{x}\int_{-a}^{a}\int_{-b}^{b} e^{jk_x x + jk_y y}\,dxdy$$

$$= 4abE_o\mathbf{x}\frac{\sin k_x a}{k_x a}\frac{\sin k_y b}{k_y b}$$

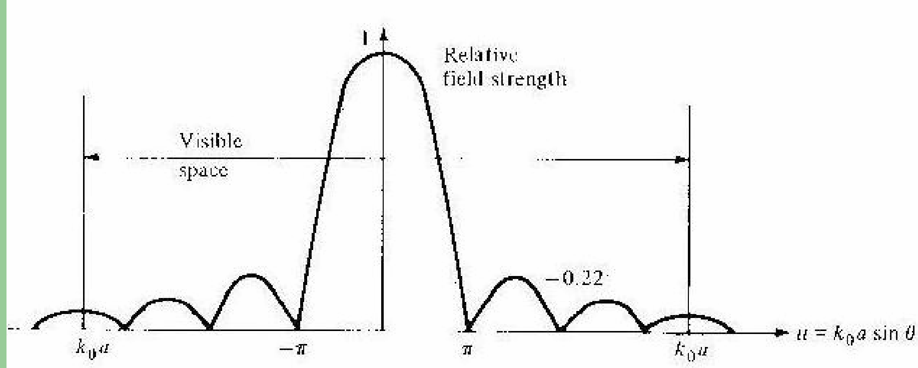
$$= 4abE_o\mathbf{x}\frac{\sin(k_o a\sin\theta\cos\phi)}{k_o a\sin\theta\cos\phi}\frac{\sin(k_o b\sin\theta\sin\phi)}{k_o b\sin\theta\sin\phi}$$

$$= 4abE_o\mathbf{x}\frac{\sin u}{u}\frac{\sin v}{v}$$

$$\mathbf{E}(r) = \frac{jk_o 4abE_o}{2\pi r}e^{-jk_o r}\frac{\sin u}{u}\frac{\sin v}{v}\left(\hat{\boldsymbol{\theta}}\sin\phi - \hat{\boldsymbol{\varphi}}\cos\phi\cos\theta\right)$$

***Note: in Balanis book, the aperture is axb, so no 4 factor on the eq. above.**

How does this pattern looks…

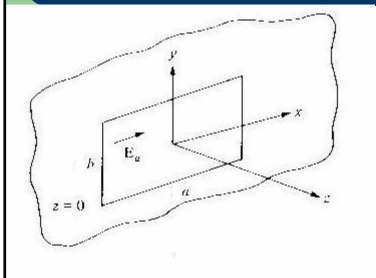


$$u = k_o a\sin\theta\cos\phi$$
$$v = k_o b\sin\theta\sin\phi$$

TE_{10} illuminated rectangular aperture

$$\mathbf{E}_a(x,y) = E_o \cos\left(\frac{\pi x'}{a}\right)\hat{y} \quad \text{for} \quad \begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array}$$

$$= 0 \qquad \text{elsewhere}$$



$$\mathbf{f}_t = E_o \hat{y} \int_{-a}^{a} \int_{-b}^{b} \cos\left(\frac{\pi x}{a}\right) e^{jk_x x + jk_y y} dx dy$$

$$X = \frac{u}{2} \qquad u = k_o a \sin\theta \cos\phi$$

$$Y = \frac{v}{2} \qquad v = k_o b \sin\theta \sin\phi$$

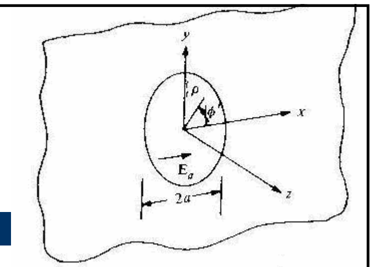
$$\mathbf{E}(r) = \frac{-jk_o ab E_o e^{-jk_o r}}{4r} \frac{\cos X}{X^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y} \left(\hat{\boldsymbol{\theta}} \sin\phi - \hat{\boldsymbol{\varphi}} \cos\phi \cos\theta\right)$$

Rectangular Aperture: Directivity

$$D_o = \epsilon_{ap}\, ab \left(\frac{4\pi}{\lambda^2}\right)$$

- For TE_{10} illuminated Rectangular Aperture the aperture efficiency is around 81%.
- For the uniform illumination, is 100% but in practice difficult to implement uniform illumination.

Circular Aperture



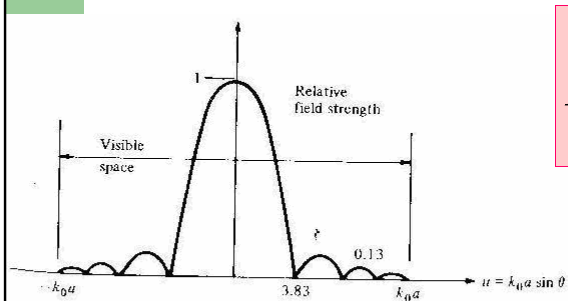
(Uniform illumination)

- In this case we use cylindrical coordinates

$$\mathbf{f}_t = E_o \mathbf{x} \int_0^a \int_0^{2\pi} e^{jk_o \sin\theta \cos(\phi-\phi')} \rho \, d\phi' \, d\rho$$

$$= 2\pi a^2 E_o \mathbf{x} \frac{J_1(k_o a \sin\theta)}{k_o a \sin\theta}$$

Circular Aperture w/ uniform illumination



$$D_o = \in_{ap} \left(\frac{C}{\lambda} \right)^2$$

- For TE_{11} illuminated Circular Aperture the aperture efficiency is around 84%.
- For the uniform illumination, is 100% but in practice difficult to implement uniformity