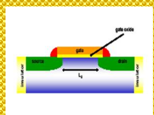

Electric fields in Material Space

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INEL 4151 ch 5
Electromagnetics I
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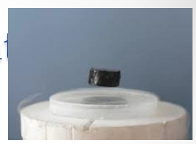
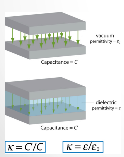
Last Chapter: free space

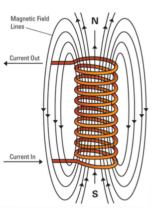
NOW: different materials

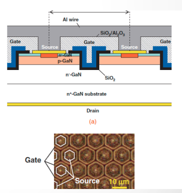
Some applica[tions]



- *Superconductors*
- *High permittivity dielectrics*
- *Transistors*
- *Electromagnets*







We will study Electric charges:

- Conductors or Insulators
  - Depend on Frequency and Temperature…
- Boundary conditions

Conductors ~(metals)

Insulators (dielectrics)

Semiconductors

Material @ 20°C & Low frequency	Conductivity [S/m]
Silver	6.1×10^7
Copper	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Carbon	3×10^4
Sea water	4
Silicon	4.4×10^{-4}
Pure water	10^{-4}
Dry Earth	10^{-5}
Glass, Quartz	$10^{-12}, 10^{-17}$

Appendix B

Conductors- have many free electrons available.

Colder metals conduct better.

(superconductivity)

semiconductor

Insulators at most lower frequencies.

Current

Units: Amperes [A]

Definition: is the electric charge passing through an area p[er unit t]ime.

$$I = \frac{dQ}{dt}$$

Current Density, [A/m²]

Is the current thru a perpendicular s[urface]

$$J_n = \frac{\Delta I}{\Delta S}$$

$$I = \int_S \vec{J} \cdot d\vec{S}$$

Depending on how I is produced:

There are different types of currents.

- Convection- I flows thru isolator: liquid, gas, vacuum.
  - Does not involve conductors,
  - Does not satisfy Ohm's Law
- Conduction- flows thru a conductor
- Displacement (ch9)

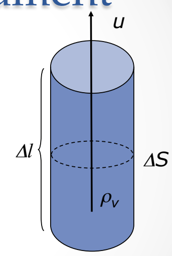
Current in a filament

- Convection current, [A]

$$I = \frac{\Delta Q}{\Delta t} =$$

- Convection density, A/m²

$$J = \frac{\Delta I}{\Delta S} = \rho_v \vec{u}$$



Conduction Current

- Requires free electrons, it's inside conductor.

$$\vec{F} = -e\vec{E}$$

- Suffers collisions, drifts from atom to atom

Newton's Law

$$\frac{m\vec{u}}{\tau} = e\vec{E}$$

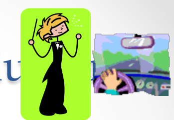
m mass of electron
τ time between collisions
$\vec{u}$ drift velocity

- Conduction current density is:

$$J = \rho_v \vec{u} = \frac{ne^2\tau}{m}\vec{E} = \sigma\vec{E}$$

where $\rho_v = ne$

A Perfect condu



Has many *charges* that are free to move.

- Therefore it cannot have an E field inside which would not let the charges move freely.
- So, inside a conductor:

$$\vec{E} = 0$$
$$\rho_v = 0$$
$$V_{ab} = 0$$

Charges move to the surface to make E=0

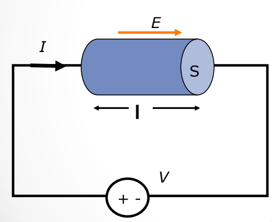
Brad Panovich Meteorologist

As scary as this looks the lightning passed safety through the metal exterior and exited from the right landing gear it appears. No one injured. Delta flight landing in Atlanta via @GoldboxATL on Twitter

http://www.hngn.com/articles/121982/20150822/lightning-bolt-strikes-delta-air-plane-bound-las-vegas-video.htm

Resistance

- If you force a Voltage across a conductor:
- Then E is not 0
- The e- encounter resistance to move



$$\vec{E} = V / l$$
$$J = I / S = \sigma E$$
$$R = \frac{V}{I} =$$

$\rho_c = 1/\sigma =$ resistivity of the material

Power in Watts

= Rate of change of energy or force x velocity

$$\int \rho_v dv \vec{E}\cdot\vec{u} = \int \vec{E}\cdot\rho_v \vec{u} dv$$

$$P = \int \vec{E}\cdot\vec{J} dv$$

$$P = \int_L \vec{E} dl \cdot \int_S \vec{J} dS$$

$$P = VI$$

Joule's Law

PE 5.1 Find the current thru the $\rho = 2, 1 \le z \le 5m$ surface

- For the current der $\vec{J} = 10 z \sin^2\phi\, \hat{a}_\rho \quad [mA/m^2]$

$$I = \int_S J\, dS = \int_1^5 10 z\, dz \int_0^{2\pi} \sin^2\phi\, \rho\big|_{\rho=2}\, d\phi$$

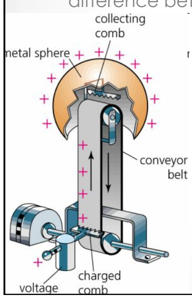
$$I = 20\frac{(25-1)}{2}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]_0^{2\pi}$$

$$I = 240\pi$$

$$= 754 mA$$

PE 5.2 In a Van de Graaff generator, w=0.1m, u=10m/s and the leakage paths have resistance 10^{14} Ω.

- If the belt carries charge 0.5 μC/m^2, find the potential difference between the dome and the base.



w= width of the belt

u= speed of the belt

$$I = \rho_S u w = 0.5\times10^{-6}(10)(.1)$$

$$V = IR = (.5)10^{-6}(10^{14}) = 50MV$$

PE 5.3 The free charge density in copper (Cu) is 1.81 x 10^{10}C/m^3.

- For a current density of 8 x 10^6 A/m^2, find the electric field intensity and the drift velocity.

$$J = \rho_v u = \sigma E$$

$$E = \frac{J}{\sigma} = \frac{8\times10^6}{5.8\times10^7} = .138\, V/m$$

$$u = \frac{J}{\rho_v} = \frac{8\times10^6}{1.81\times10^{10}} = 4.42\times10^{-4} \quad m/s$$

Polarization in dielectrics

The effect of polarization on a dielectric is to have a surface bound charge of:

and leave within it an accumulation of volume bound charge:

$$Q_b = \oint_v \rho_{pv} dv$$

$$\rho_{ps} = \vec{P}\cdot\hat{a}_n$$

$$\rho_{pv} = -\nabla\cdot\vec{P}$$

$$\vec{D} = \varepsilon_o\vec{E} + \vec{P}$$

$$P = \chi_e\varepsilon_o\vec{E}$$

ρ_{ps} and ρ_{pv} are the polarization (bounded) surface and volume charge densities

Permittivity and Strength

- Not really a constant!

$$\vec{D} = \varepsilon \vec{E}$$

$$\varepsilon = \varepsilon_o \varepsilon_r$$

$$\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_o}$$

Dielectric properties

- Linear = ε doesn't change with E
- Isotropic= ε doesn't change with direction
- Homogeneous= ε doesn't change from point to point.

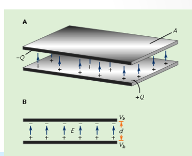
Coulomb's Law for any material:

$$F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_o\varepsilon_r R^2}\hat{a}_{12}$$

PE 5.6 A parallel plate capacitor with plate separation of 2mm has a 1kv voltage applied to its plane.

- If the space between its plates is filled with polystyrene, $\varepsilon_r = 2.55$ find E and P.

$$\vec{E} = \frac{V}{d} = \frac{1000}{.002} = 500k\hat{a}_x V/m$$



$$\chi_e = \varepsilon_r - 1 = 1.55$$

$$\vec{P} = \chi_e \varepsilon_o \vec{E} = (1.55)(8.85\times10^{-12})5\times10^5 = 6.86\hat{a}_x \mu C/m^2$$

PE 5.7 In a dielectric material, E_x= 5V/m and $\vec{P} = \frac{1}{10\pi}(3\hat{a}_x - \hat{a}_y + 4\hat{a}_z)nC/m^2$

- Find: $\chi_e, \vec{E}, and\ \vec{D}$

$$\vec{P} = \varepsilon_o \chi_e \vec{E} \qquad \chi_e = \frac{P_x}{\varepsilon_o E_x} = 2.16$$

$$\vec{E} = \frac{\vec{P}}{\varepsilon_o \chi_e} = 5\hat{a}_x - 1.67\hat{a}_y + 6.67\hat{a}_z$$

$$\vec{D} = \varepsilon_o \varepsilon_r \vec{E} = \frac{\varepsilon_r \vec{P}}{\chi_e} = 140\hat{a}_x - 477\hat{a}_y + 186\hat{a}_z$$

Questions?

Continuity Equation

- Charge is conserved.

$$I_{out} = \oint \vec{J}\cdot d\vec{S} = \int \nabla\cdot\vec{J}\, dv$$

$$I_{in} = -\frac{dQ}{dt} = -\frac{d}{dt}\int_v \rho_v\, dv$$

$$\nabla\cdot\vec{J} = -\frac{d\rho_v}{dt}$$

For steady currents:

- Change= output current –input current = 0

$$\frac{d\rho_v}{dt} = 0$$
$$\nabla \cdot \vec{J} = 0$$

Boundary Conditions

- We have two materials
- How do the fields behave @ interface?

Evaluate Maxwell's:

$$\oint_l \vec{E} \cdot dl = 0$$
$$\oint_S \vec{D} \cdot dS = Q_{enc}$$

We look at the **tangential** and the **perpendicular** component of the fields.

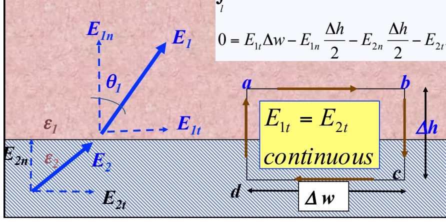
$$\vec{E} = \vec{E}_t + \vec{E}_n$$

Dielectric-dielectric B.C.

- Consider the figure below:

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$$
$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$$

$$\oint_l \vec{E} \cdot dl = 0 =$$
$$0 = E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2}$$



$E_{1t} = E_{2t}$ continuous

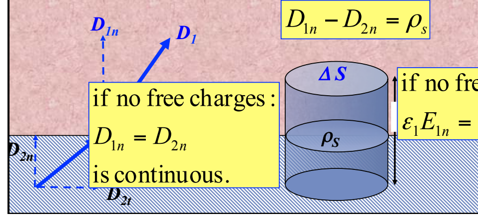
$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ discontinuous

Dielectric-dielectric B.C.

- Consider the figure below:

$$\Delta Q = \rho_s \Delta S = \oint_S \vec{D} \cdot dS = D_{1n}\Delta S - D_{2n}\Delta S$$

$$D_{1n} - D_{2n} = \rho_s$$



if no free charges: $D_{1n} = D_{2n}$ is continuous.

if no free charges: $\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$

5.13 In a slab of dieletric material for which $\varepsilon = 2.4\varepsilon_o$ and V=300z^2 V, Find $E = -\nabla V$ (a) D and ρ_v (b) P

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right] = -600z\hat{a}_z$$

$$\vec{D} = \varepsilon = 2.4\varepsilon_o\vec{E} = 12.7z nC/m^2\ \hat{z}$$

$$\rho_v = \nabla \cdot \vec{D} = 12.7nC/m^3$$

$$\vec{P} = \varepsilon_o\chi_e\vec{E} = (1.4)\varepsilon_o(-600z)\hat{z} = 7.43z nC/m^2\ \hat{z}$$