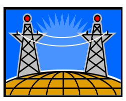



Electromagnetism
INEL 4151



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In summary

- Stationary Charges
  - Q
- Steady currents
  - I
- Time-varying currents
  - $I(t)$

- Electrostatic fields\ E
- Magnetostatic fields H
- Electromagnetic (waves!)
  - $E(t)$ & $H(t)$

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Outline

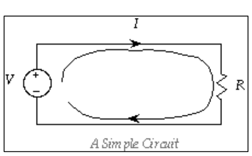
- Faraday's Law & Origin of emag
- Transformer and Motional EMF
- Displacement Current & Maxwell Equations
- Review: Phasors and Time Harmonic fields

9.2

Faraday's Law

Electricity => Magnetism

- In 1820 Oersted discovered that a steady current produces a magnetic field while teaching a physics class.



This is what Oersted discovered accidentally:

$$\oint_L \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot d\vec{S}$$

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Would magnetism would produce electricity?

- Eleven years later, and at the same time, (Mike) Faraday in London & (Joe) Henry in New York discovered that a <u>time-varying</u> magnetic field would produce an electric current!

$$V_{emf} = -N\frac{d\Psi}{dt}$$

$$\oint_L E\cdot dl = -N\int_s \frac{\partial}{\partial t} B\cdot dS$$

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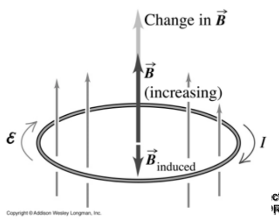
Len's Law = (-)

- If N=1 (1 loop)

$$V_{emf} = -\frac{d\Psi}{dt}$$



- The time change

$$V_{emf} = \oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{S}$$

can refer to ***B*** or ***S***

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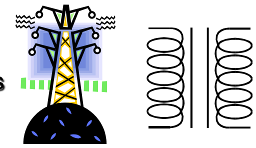
Electromagnetics was born!

- This is Faraday's Law - the principle of motors, hydro-electric generators and transformers operation.



$$Faraday's\,Law \oint_L \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$$Ampere's\,Law \oint_L \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot d\vec{S}$$

****Mention some examples of em waves***

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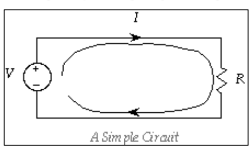
Faraday's Law

- For *N*=1 and *B*=0



$$V_{emf} = -N\frac{d\Psi}{dt}$$

$$V_{emf} = \oint_L E\cdot dl = IR$$

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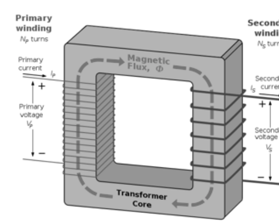
Example PE 9.3 A magnetic core of uniform cross-section 4 cm² is connected to a 120V, 60Hz generator.
Calculate the induced emf V_2 in the secondary coil.
N_1= 500, N_2=300

- **Use Faraday's Law**

$$V_1 = -N_1\frac{d\Psi}{dt}$$

$$V_2 = -N_2\frac{d\Psi}{dt}$$

$$V_2 = N_2\frac{V_1}{N_1}$$



Answer: 72 cos(120πt) V

9.3

Transformer & Motional EMF

Two cases of $V_{emf} = \oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{S}$

- ***B*** **changes**

$$\oint_L \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

Stoke's theorem

$$\int_S \left(\nabla\times\vec{E}\right)\cdot d\vec{S} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- ***S*** **(area) changes**

$$\oint_L \vec{E}\cdot dl = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{S}$$

$$\vec{E}_m = \vec{F}_m / Q = \vec{u}\times\vec{B}$$

$$\int_S (\nabla\times E)\cdot dS = \oint_L \left(u\times\vec{B}\right)\cdot dl$$

$$\nabla\times\vec{E} = \nabla\times\vec{u}\times\vec{B}$$

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3 cases:

- Stationary loop in time-varying *B* field

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

- Moving loop in static *B* field

$$\nabla\times\vec{E}=\nabla\times\vec{u}\times\vec{B}$$

- Moving loop in time-varying *B* field

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}+\nabla\times\vec{u}\times\vec{B}$$

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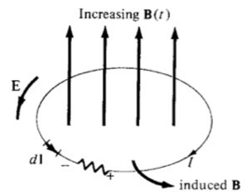
1. Stationary loop in a time-varying B field



$$V_{emf}=\oint_L\vec{E}\cdot d\vec{l}=-\int_s\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

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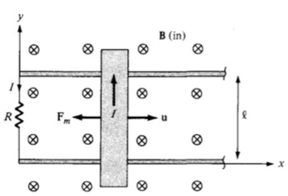
2. Time-varying loop area in a static B field

$$\mathbf{E}_m=\frac{\mathbf{F}_m}{Q}=\mathbf{u}\times\mathbf{B}$$



$$V_{emf}=\oint_L\mathbf{E}_m\cdot d\mathbf{l}=\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$\nabla\times\mathbf{E}_m=\nabla\times(\mathbf{u}\times\mathbf{B})$$

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3. A t-varying loop area in a t-varying B field

$$V_{emf}=\oint_L\mathbf{E}\cdot d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}+\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}+\nabla\times(\mathbf{u}\times\mathbf{B})$$

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PE 9.1

A conducting bar can slide freely over two conducting rails as shown in Figure 9.6. Calcu late the induced voltage in the bar

(a) If the bar is stationed at $y = 8$ cm and $\mathbf{B} = 4\cos 10^6 t\,\mathbf{a}_z$ mWb/m^2

(b) If the bar slides at a velocity $\mathbf{u} = 20\mathbf{a}_y$ m/s and $\mathbf{B} = 4\mathbf{a}_z$ mWb/m^2

(c) If the bar slides at a velocity $\mathbf{u} = 20\mathbf{a}_y$ m/s and $\mathbf{B} = 4\cos(10^6 t - y)\,\mathbf{a}_z$ mWb/m^2

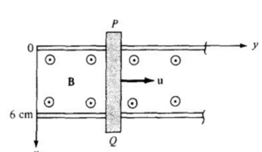


Figure 9.6 For Example 9.1.

$$V_{emf}=-\int_s\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}=4\cdot10^6\sin(10^6t)(.08).06)=19.2\sin(10^6t)kV$$

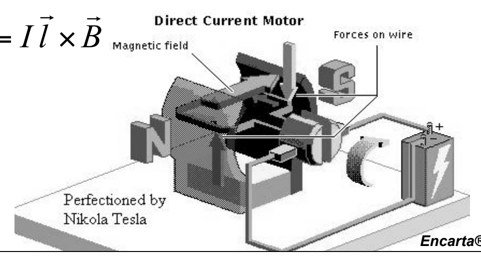
$$V_{emf}=\oint_L\vec{E}\cdot d\vec{l}=\oint_L\vec{u}\times\vec{B}\cdot dl$$

Moving loop in static B field

- When a conducting loop is moving inside a magnet (static *B* field), the force on a charge is:

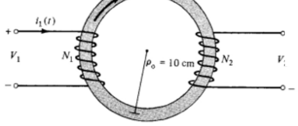
$$\vec{F}=Q\vec{u}\times\vec{B}$$

$$\vec{F}=I\,\vec{l}\times\vec{B}$$



Transformer Example

The magnetic circuit of Figure 9.8 has a uniform cross section of 10^{-3} m^2. If the circuit is energized by a current $i_1(t) = 3 \sin 100\pi t$ A in the coil of $N_1 = 200$ turns, find the emf induced in the coil of $N_2 = 100$ turns. Assume that $\mu = 500\,\mu_o$.



$$\Psi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{N_1 i_1}{\ell/\mu S} = \frac{N_1 i_1 \mu S}{2\pi\rho_o}$$

- **Find reluctance and use Faraday's Law**

$$V_2 = -N_2 \frac{d\Psi}{dt} = -\frac{N_1 N_2 \mu S}{2\pi\rho_o}\frac{di_1}{dt}$$

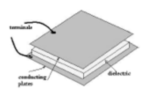
$$V_{emf} = -100(200)(3\omega\cos\omega t)\frac{500\mu_o 10^{-3}}{2\pi(0.10)} = -6\pi\cos 100\pi t$$

9.4

Displacement Current, J_d

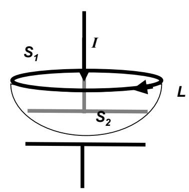
Maxwell noticed something was missing…

- And added J_d, the displacement current



$$\oint_L H\cdot dl = \int_{S_1} J\cdot dS = I_{enc} = I$$

$$\oint_L H\cdot dl = \int_{S_2} J\cdot dS = 0$$



$$\oint_L H\cdot dl = \int_{S_2} J_d\cdot dS = \frac{d}{dt}\int_{S_2} D\cdot dS = \frac{dQ}{dt} = I$$

At low frequencies J>>J_d, but at radio frequencies both terms are comparable in magnitude.

9.4

Maxwell's Equation in Final Form

Summary of Terms

- ***E*** = electric field intensity [V/m]
- ***D*** = electric field density [C/m^2]
- ***H*** = magnetic field intensity, [A/m]
- ***B*** = magnetic field density, [Teslas]
- ***J*** = current density [A/m^2]

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Maxwell Equations in General Form

Differential form	Integral Form	
$\nabla\cdot D = \rho_v$	$\oint_s D\cdot dS = \int_v \rho_v dv$	Gauss's Law for *E* field.
$\nabla\cdot B = 0$	$\oint_s B\cdot dS = 0$	Gauss's Law for *H* field. Nonexistence of monopole
$\nabla\times E = -\frac{\partial B}{\partial t}$	$\oint_L E\cdot dl = -\frac{\partial}{\partial t}\int_s B\cdot dS$	Faraday's Law
$\nabla\times H = J + \frac{\partial D}{\partial t}$	$\oint_L H\cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right)\cdot dS$	Ampere's Circuit Law

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Maxwell's Eqs.

- Also the equation of continuity $\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$
- Maxwell added the term $\frac{\partial D}{\partial t}$ to Ampere's Law so that it not only works for **static** conditions but also for **time-varying** situations.
  - This added term is called the displacement current density, while J is the conduction current.

Relations & B.C.

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_o(\mathbf{H} + \mathbf{M})$$

$$\mathbf{J} = \sigma \mathbf{E} + \rho_v \mathbf{u}$$

$$E_{1t} = E_{2t} \quad \text{or} \quad (\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{a}_{n12} = 0$$

$$H_{1t} - H_{2t} = K \quad \text{or} \quad (\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$$

$$D_{1n} - D_{2n} = \rho_s \quad \text{or} \quad (\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{a}_{n12} = \rho_s$$

$$B_{1n} - B_{2n} = 0 \quad \text{or} \quad (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{a}_{n12} = 0$$

9.6

Time Varying Potentials

We had defined

- Electric & Magnetic potentials:

$$V = \int_v \frac{\rho_v \, dv}{4\pi\varepsilon R} \qquad \mathbf{A} = \int_v \frac{\mu J \, dv}{4\pi R}$$

- Related to B as: $\mathbf{B} = \nabla \times \mathbf{A}$

Substituting into Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) \qquad \boxed{\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}}$$

Electric & Magnetic potentials:

- If we take the divergence of E:

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon} = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A})$$

- Or $$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\frac{\rho_v}{\varepsilon}$$

- Taking the curl of: $\mathbf{B} = \nabla \times \mathbf{A}$ & add Ampere's we get

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J} + \varepsilon\mu \frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial \mathbf{A}}{\partial t}\right)$$

$$= \mu \mathbf{J} - \mu\varepsilon \, \nabla\left(\frac{\partial V}{\partial t}\right) - \mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2}$$

Electric & Magnetic potentials:

- If we apply this vector identity

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

- We end up with

$$\nabla^2 \mathbf{A} - \nabla(\nabla \cdot \mathbf{A}) = -\mu \mathbf{J} + \mu\varepsilon \, \nabla\left(\frac{\partial V}{\partial t}\right) + \mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2}$$

Electric & Magnetic potentials:

- We use the Lorentz condition:

$$\nabla \cdot \mathbf{A} = -\mu\varepsilon \frac{\partial V}{\partial t}$$

To get:

$$\nabla^2 \mathbf{A} - \mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$$

and:

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon}$$

Which are both wave equations.

9.7

Time Harmonic Fields Phasors Review

Time Harmonic Fields

- Definition: is a field that varies periodically with time.
  - Ex. Sinusoid
- Let's review Phasors!

Phasors & complex #'s

Working with harmonic fields is easier, but requires knowledge of phasor, let's review

- **complex numbers** and
- **phasors**

COMPLEX NUMBERS:

- Given a complex number *z*

$$z = x + jy = re^{j\phi} = r\angle\phi = r\cos\phi + jr\sin\phi$$

where $r = |z| = \sqrt{x^2 + y^2}$ is the magnitude

$$\phi = \tan^{-1}\frac{y}{x} \text{ is the angle}$$

Review:

- Addition,
- Subtraction,
- Multiplication,
- Division,
- Square Root,
- Complex Conjugate

examples:

$1/j =$

$1/\sqrt{j} =$

$e^{j45^o}/j =$

$e^{j45^o} - \sqrt{j} =$

$3e^{j90^o} + j =$

$\sqrt{2}e^{+j45^o} =$

$\sqrt{2}e^{+j45^o} + 10e^{+j90^o} =$

For a Time-varying phase

$$\phi = \omega t + \theta$$

Real and imaginary parts are:

$$\mathrm{Re}\{re^{j\phi}\} = r\cos(\omega t + \theta)$$

$$\mathrm{Im}\{re^{j\phi}\} = r\sin(\omega t + \theta)$$

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PHASORS

- For a sinusoidal current $I(t) = I_o \cos(\omega t + \theta)$ equals the real part of $I_o e^{j\theta} e^{j\omega t}$
- **The complex term $I_o e^{j\theta}$ which results from dropping the time factor $e^{j\omega t}$ is called the phasor current, denoted by I_s (*s* comes from sinusoidal)**

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To change back to time domain

The phasor is

1. multiplied by the time factor, $e^{j\omega t}$,
2. and taken the real part.

$$A = \mathrm{Re}\{A_s e^{j\omega t}\}$$

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Advantages of phasors

- **Time derivative** in time is equivalent to multiplying its phasor by $j\omega$

$$\frac{\partial A}{\partial t} \rightarrow j\omega A_s$$

- **Time integral** is equivalent to dividing by the same term.

$$\int A \partial t \rightarrow \frac{A_s}{j\omega}$$

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9.7

Time Harmonic Fields

Time-Harmonic fields (sines and cosines)

- The wave equation can be derived from Maxwell equations, indicating that the changes in the fields behave as a wave, called an electromagnetic wave or field.
- Since any periodic wave can be represented as a sum of sines and cosines (using Fourier), then we can deal only with harmonic fields to simplify the equations.

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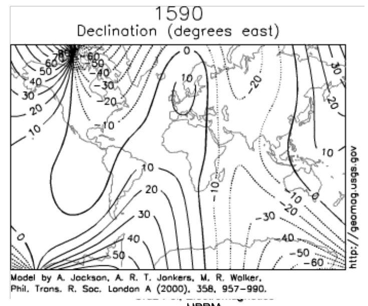
Maxwell Equations for Harmonic fields (phasors)

Differential form*	
$\nabla \cdot \varepsilon E = \rho_v$	Gauss's Law for E field.
$\nabla \cdot \mu H = 0$	Gauss's Law for H field. No monopole
$\nabla \times E = -j\omega\mu H$	Faraday's Law
$\nabla \times H = \sigma E + j\omega\varepsilon E$	Ampere's Circuit Law

* (substituting $D = \varepsilon E$ and $H = \mu B$)

Earth Magnetic Field Declination from 1590 to 1990



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