Ch.12 Solutions

12.14 $a = 4\lambda$

$b = 3\lambda$

a. From Apopendix I

$$\frac{kb}{2}\sin\theta_s \cong 7.7 \Rightarrow \theta_s = \sin^{-1}\left[\frac{2(7.7)}{kb}\right] = \sin^{-1}\left[\frac{2(7.7)}{2\pi(3)}\right] = 54.785^o$$

$$\Theta_s = 2\theta_s = 2(54.785) = 109.57^o$$

b. From Appendix I, at

$$\frac{kb}{2}\sin\theta_s = 7.7 \Rightarrow E_\theta = 0.12833$$

$$or \quad E_\theta = -1`7.8dB$$

c.

$$\Theta_h(E - plane) \cong 114.6 \sin^{-1}\left[\frac{0.443}{3}\right] = 16.98^o$$

$$\Theta_h(H - plane) \cong 114.6 \sin^{-1}\left[\frac{0.443}{4}\right] = 12.72^o$$

From Table 12.1,

$$D_o = 10.2\left[\frac{ab}{\lambda^2}\right] = 122.4 = 20.88dB$$

12.22 From Fig. 12.15, for a 90% efficiency $\mu = \frac{ka}{2}\sin\theta_1 \cong 3.18 = \frac{kb}{2}\sin\theta_1$

$$\theta_1 = 37/2 = 18.5^o$$

For $a = b = \frac{2(3.18)}{k\sin(18.5^o)} = 3.19\lambda$

$$\lambda = \frac{3\times10^{10}\,cm/s}{10\times10^{9}\,Hz} = 3cm$$

12.26 a. $a = 0.9'' = 2.286cm = 0.762\lambda$

$b = 0.4'' = 1.016cm = 0.339\lambda$

Power density for isotropic source:

$$W_o = \frac{P_{rad}}{4\pi R^2} = \frac{1Watt}{4\pi\left(10\times10^3\right)^2} = 7.96\times10^{-10}\,W/m^2$$

Directivity from Table 12.1 , 12.2

$$D_o = \frac{8}{\pi^2}\left[\frac{4\pi}{\lambda^2}ab\right] = \frac{32}{\pi}(0.762)(0.339) = 2.63$$

Incident Power density

$$W_i = W_o D_o = \left(7.96\times10^{-10}\right)(2.63) = 2.09\times10^{-9} W/m^2$$

b. the maximum power that can be delivered to a matched load.

$$A_{em} = \varepsilon_{ap} A_p = 0.81ab = 1.88\times10^{-4} m^2$$

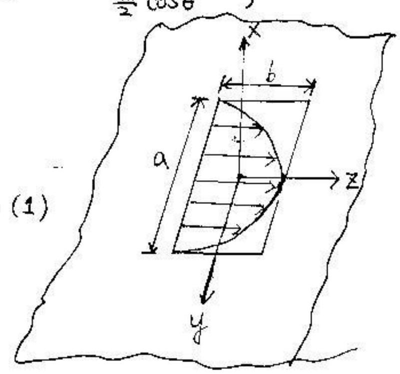
$$P_{\max} = W_i A_{em} = \left(2.09\times10^{-9} W/m^2\right)\left(1.88\times10^{-4} m^2\right) = 3.94\times10^{-13} W$$

12.27

$$\underline{E} = \hat{a}_\theta\, j\,\frac{\omega\mu b I_o e^{-jkr}}{4\pi r}\,\frac{\sin\left(\frac{kb}{2}\cos\theta\right)}{\frac{kb}{2}\cos\theta} = -j\frac{\omega\mu I_o e^{-jkr}}{4\pi r}\left\{-\hat{a}_\theta\, b\,\frac{\sin\left(\frac{kb}{2}\cos\theta\right)}{\frac{kb}{2}\cos\theta}\right\}$$

a. $\underline{E} = -j\,\dfrac{\omega\mu I_o e^{-jkr}}{4\pi r}\cdot \underline{h}_e(\theta)$

$$\underline{h}_e(\theta) = -\hat{a}_\theta\, b\,\frac{\sin\left(\frac{kb}{2}\cos\theta\right)}{\frac{kb}{2}\cos\theta}$$



b. $\underline{h}_e(\theta)\Big|_{max} = -\hat{a}_\theta\, b\,\dfrac{\sin\left(\frac{kb}{2}\cos\theta\right)}{\frac{kb}{2}\cos\theta}\Bigg|_{max} = -\hat{a}_\theta\, b\,(1)$

when $\frac{kb}{2}\cos\theta = 0 \Rightarrow \theta = 90°$

c. $p = \dfrac{|\underline{h}_e(\theta)\cdot\underline{E}^{inc}|^2}{|\underline{h}_e(\theta)|^2|\underline{E}^{inc}|^2}$

$$\underline{E}^{inc} = \hat{a}_\theta\, j\eta\,\frac{kI_o\ell e^{-jkr}}{8\pi r}\sin\theta$$

$$|\underline{h}_e(\theta)\cdot\underline{E}^{inc}|^2 = \left|(-\hat{a}_\theta b)\cdot\left[\hat{a}_\theta\, j\eta\,\frac{kI_o\ell e^{-jkr}}{8\pi r}\sin\theta\right]_{\theta=\frac{\pi}{2}}\right|^2 = \left|-j\eta b\frac{kI_o\ell e^{-jkr}}{8\pi r}\right|^2 = \left|\eta\frac{bkI_o\ell}{8\pi r}\right|^2$$

$$|\underline{h}_e(\theta)|^2 = |b|^2$$

$$|\underline{E}^{inc}|^2 = \left|j\eta\frac{kI_o\ell e^{-jkr}}{8\pi r}\sin\theta\Big|_{\theta=\frac{\pi}{2}}\right|^2 = \left|\eta\frac{kI_o\ell}{8\pi r}\right|^2$$

$$p = \frac{\left|\eta\frac{kbI_o\ell}{8\pi r}\right|^2}{|b|^2\left|\eta\frac{kI_o\ell}{8\pi r}\right|^2} = 1 = 0\,\mathrm{dB}$$