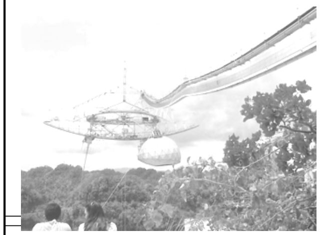
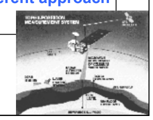


Radiative Transfer

Dr. X-Pol
Microwave Remote Sensing INEL 6669
Dept. of Electrical & Computer Engineering,
UPRM, Mayagüez, PR

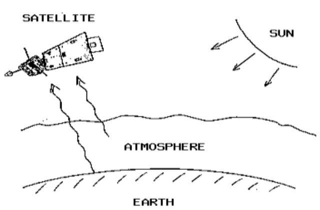
Outline

- **Theory of Radiative Transfer**
  - Extinction &Emission
  - Equation of Transfer
- **T_{AP} of absorbing/scattering Media**
- **T_{AP} of atmosphere & Terrain:**
  - upwelling and downwelling
- **Emission and Scattering by Terrain**
- **Homogeneous terrain medium with**
  - uniform T profile
  - non-uniform T&ε_r profile: coherent & incoherent approach
- **Emissivity of dielectric Slab**
- **Emissivity of Rough surface**



Radiative Transfer

- **Some energy is transmitted some is scattered, some is absorbed**



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Radiative Transfer :Extinction

Interaction between radiation and matter

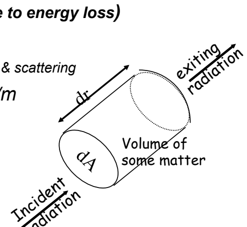
- **Extinction** ***(Change due to energy loss)***

$dI_{extin} = \boxed{\kappa_e I(r,r')dr}$

☞ *two processes: absorbtion & scattering*

$\kappa_e = \kappa_a + \kappa_s$ *in Nepers/m*

where κ_e is the *extinction or power attenuation coefficient*, it's due to absorption and scattering away in other direction.



Radiative Transfer : Emission

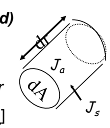
Interaction between radiation and matter

- **Emission** ***(Change due to energy gained)***

$dI_{emit} = (\kappa_a J_a + \kappa_s J_s)dr$

☞ *two processes or mechanisms: emit & scatter*

[J_a & J_s account for thermal emission & scattering]



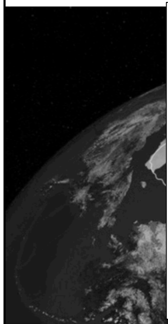
- **Introduce the scattering albedo**, $a = \kappa_s/\kappa_e$

$dI_{emit} = [(\kappa_e - \kappa_s)J_a + \kappa_s J_s)]$

$dI_{emit} = \kappa_e [(1-a)J_a + aJ_s)] = \boxed{\kappa_e J dr}$

where $J = (1-a)J_a + aJ_s$

http://www.sciencedaily.com/releases/2014/02/140219115110.htm



The retreat of sea ice in the Arctic Ocean is diminishing Earth's albedo, or reflectivity, by an amount considerably larger than previously estimated, according to a new study that uses data from instruments that fly aboard several NASA satellites.

The study, conducted by researchers at Scripps Institution of Oceanography, at the University of California, San Diego, uses data from the Clouds and Earth's Radiant Energy System, or CERES, instrument. There are CERES instruments aboard NASA's Tropical Rainfall Measurement Mission, or TRMM, satellite, Terra, Aqua and NASA-NOAA's Suomi National Polar-orbiting Partnership (Suomi NPP) satellites. The first CERES instrument was launched in December of 1997 aboard TRMM.

As the sea ice melts, its white reflective surface is replaced by a relatively dark ocean surface. This diminishes the amount of sunlight being reflected back to space, causing Earth to absorb an increasing amount of solar energy.

The Arctic has warmed by 3.6 F (2 C) since the 1970s.

This image shows a visualization of Arctic sea ice cover on Sept. 12, 2013, with a yellow line showing the 30-year average minimum extent. A new study shows that the magnitude of surface darkening in the Arctic (due to the retreat of sea ice) is twice as large as that found in previous studies.

Credit: NASA Goddard's Scientific Visualization Studio/Cindy Starr

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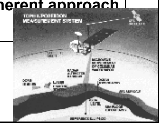


Radiative Transfer : Equation of Transfer I

$dI = I(r+dr) - I(r)$

$= dI_{emit} - dI_{extin}$

$= \kappa_e (J-I)dr$

$= (J-I)d\tau$

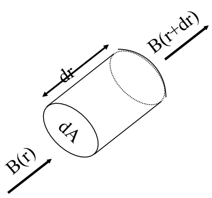
where

$d\tau = \kappa_e\, dr$ is the optical depth,

therefore,

$$\tau(r_1, r_2) = \int_{r_1}^{r_2} \kappa_e\, dr$$

and τ is the optical thickness or opacity in Np.



Radiative Transfer : Equation of Transfer II

$dI=(J-I)d\tau$ or $dI/d\tau + I = J$

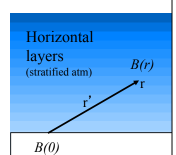
- **Multiplying by $e^{\tau(0,r')}$**

$$\frac{dI(r')}{d\tau}e^{\tau(0,r')} + I(r')e^{\tau(0,r')} = J(r')e^{\tau(0,r')}$$

$$\frac{d}{d\tau}[I(r')e^{\tau(0,r')}] = J(r')e^{\tau(0,r')}$$

- **and integrating from *0* to *r*.**

$$I(r) = I(0)e^{-\tau(0,r)} + \int_0^r \kappa_e J(r')e^{-\tau(r',r)}dr'$$



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T_{AP} of absorbing/scattering Media (1)

$$I(r) = I(0)e^{-\tau(0,r)} + \int_0^r \kappa_e J(r')e^{-\tau(r',r)}dr'$$

In the microwave region, where R-J applies, there's a $T \propto$ to I

$$I(r) = \frac{2k}{\lambda^2}T_B(r)B$$

Similarly, under thermodynamic equilibrium (emission = absorption) the absorption source function is [Recall $J = (1-a)J_a + aJ_s$]

$$J_a(r) = \frac{2k}{\lambda^2}T(r)B$$

where T is the physical temperature of the medium.

T_{AP} of absorbing/scattering Media (2)

Similarly, for the scattering source function, J_s

$$J_s(r) = \frac{1}{4\pi}\iint_{4\pi} \psi(r : r_i) I(r_i) d\Omega_i = \boxed{\frac{2k}{\lambda^2} T_{sc}(r) B}$$

where Ψ is the phase function and accounts for the portion of incident radiation scattered from direction $\mathbf{r}_i$ into direction $\mathbf{r}$,

where we have defined a scattered radiometric temperature as,

$$T_{sc}(r) = \frac{1}{4\pi}\iint_{4\pi} \psi(r : r_i) T_{AP}(r_i) d\Omega_i$$

In eq. 6.24 Ulaby & Long, the multiply by ½

T_{AP} of absorbing/scattering Media (3)

$$I(r) = I(0)e^{-\tau(0,r)} + \int_0^r \kappa_e J(r')e^{-\tau(r',r)} dr'$$

In terms of temperature, $J = (1-a)J_a + aJ_s$

$\kappa_e = \kappa_a + \kappa_s$

$$T_B(r) = T_B(0)e^{-\tau(0,r)} + \int_0^r \kappa_e[(1-a)T(r') + aT_{sc}(r')]e^{-\tau(r',r)} dr'$$

For scatter-free medium:

- $\kappa_s (= a\kappa_e) = 0$
- then $a = 0$ and $\kappa_e = \kappa_a$
- and the opacity is

$$\tau(r_1, r_2) = \int_{r_1}^{r_2} \kappa_a dr$$

$$d\tau = \kappa_a \sec\theta dz$$

Brightness temperature

$$T_B(z) = T_B(0)e^{-\tau(0,z)} + \int_0^z \kappa_e[(1-a)T(z') + aT_{sc}(z')]e^{-\tau(z',z)} \sec\theta dz'$$

Define the 1-way atmospheric transmissivity: Υ_a

$$\Upsilon_a = e^{-\tau(0,H)\sec\theta} = e^{-\int_0^H \kappa_e \sec\theta dz}$$

T_{AP} of absorbing/scattering Media III

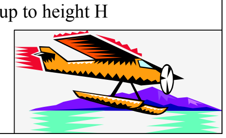
Ex. For scatter-free medium: An airborne radiometer measuring ice.

$$T_{AP}(H) = T_{AP}(0)e^{-\tau(0,H)} + \int_0^r \kappa_a T(z')e^{-\tau(z',H)} dz'$$

$T_{AP}(0) = T_{ice}$

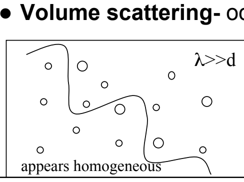
$e^{-\tau(0,H)}$ = attenuation of atmosphere up to height H

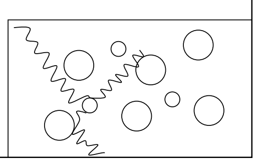
$$\tau(z_1, z_2) = \int_{z_1}^{z_2} \kappa_a \sec\theta dz$$



Scattering

- Rain and clouds produce a bit @ microwaves.
- Can be neglected for *f* under 10 GHz.
- **Surface scattering -** depends on the interface, dielectric properties, geometry.
- **Volume scattering-** occurs for λ ~dia & dist.





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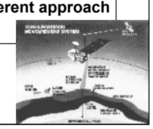


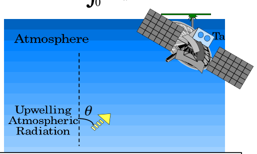
T_{AP} of atmosphere & Terrain: Upwelling

$r'=z'\sec\theta$

- **Upwelling (no ground)**

$$T_{UP}(r)=\sec\theta\int_0^H \kappa_a(z')T(z')e^{-\int_{z'}^{H}\kappa_a(z,f)\sec\theta\, dz}dz'$$

All the upward radiation emitted by the entire atmospheric path between the ground and the observation point.



If H>20km

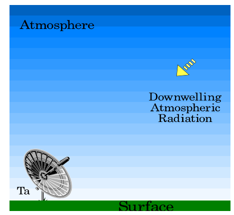
No ground contribution

T_{AP} of atmosphere & Terrain: Downwelling

- **Downwelling**

$r'=z'\sec\theta$

$$T_{DN}(r)=\sec\theta\int_0^\infty \kappa_a(z')T(z')e^{-\tau(0,z')\sec\theta}dz'$$



T_{AP} of atmosphere & Terrain: downwelling

- Special case: plane homogeneous atmosphere (or cloud) with $T(z)=T_o$ and $\kappa_a=\kappa_{ao}$ over the range $z=0$ to $z=H$.

$$T_{DN}(H)=\sec\theta\int_0^H \kappa_{ao}T_o e^{-\kappa_{ao}z'\sec\theta}dz'$$
$$=T_o[1-e^{-\tau(0,H)\sec\theta}]$$

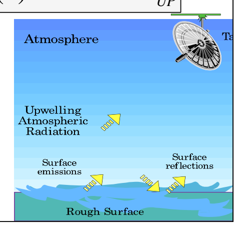
*Used in programming codes.

T_{AP} of Atmosphere & Terrain

- **Upwelling Radiation (with ground)**

$$T_B(\theta,H)=T_B(0)e^{-\tau(0,H)\sec\theta}+T_{UP}$$

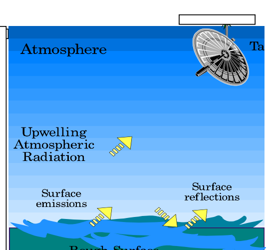
Where $T_B(0)$ is the contribution from the surface emissions and reflections (from downwelling and cosmic radiation) which is treated in more detail in the following chapter.



Brightness Temperature

Self-emitted radiation from

- the surface (e.g. terrain, ice, ocean)
- upward radiation from the atmosphere
- downward-emitted atmospheric radiation that is reflected by the surface in the antenna direction
- virtually no solar contamination



Radiative Transfer Theory

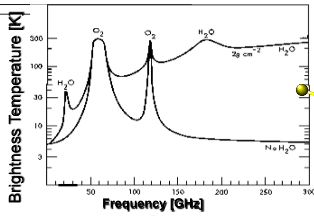
- **Interaction between radiation and matter : processes => emission & extinction (s & a)**

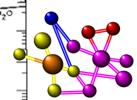
$$T_A(r)=T(0)e^{-\tau(0,r)}+$$
$$\int_0^\infty[(1-a)T(z)+aT_{sc}]\kappa_e(f,z)e^{-\int_z^r\kappa_e(f,z')dz'}dz$$

- **Under clear sky conditions - no scattering**

$$T_B(r)=T(0)e^{-\tau(0,r)}+\int_0^\infty T(z)\kappa_a(f,z)e^{-\int_z^r\kappa_a(f,z')dz'}dz$$

Example: Upward-looking radiometer like Arecibo looks at...





$$T_B = \int_0^{\infty} T(z)\kappa_a(f,z) e^{-\int_0^z \kappa_a(f,z')dz'} dz \; + \; T_C e^{-\tau(0,\infty)}$$

where T_C is the cosmic radiation

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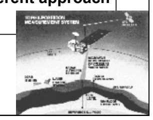


Emission and Scattering by Terrain: relate T_B & T_{SC} to the medium properties.

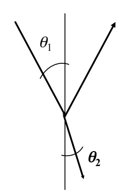
- **Flat surface vs. rough surface**

Flat surface
(Specular surface)
h<<λ
Height variations are much smaller than wavelength of radiation.
☞Snell's Law applies.

Rough surface
h≈λ
Lambertian surface is considered "perfectly" rough. Many times is a combination of both.

Properties of the Specular Surface

$\varepsilon_{c1} = \varepsilon' - j\varepsilon''$
μ_{r1}



ε_{c2}
μ_{r2}

The power transmissivity is,

$$T_{2,1} = 1 - \Gamma_{1,2}$$

Fresnel reflection gives the power reflectivity

$$\Gamma_{1,2} = \left|R_{1,2}\right|^2 = \left|\frac{Z_2 - Z_1}{Z_2 + Z_1}\right|^2$$

where,

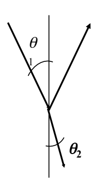
$$Z_i = \begin{cases} \eta_1 \cos\theta_i & \text{for VP} \\ \eta_1 \sec\theta_i & \text{for HP} \end{cases}$$

and,

$$\Gamma_{2,1} = \Gamma_{1,2} = \Gamma_1$$

Properties of the Specular Surface

ε_{c1}
μ_{r1}



ε_2
μ_2
κ_{a2}

Snell's Law relates the angles of the incidence and transmission

$$\sin\theta_2 = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \sin\theta_1$$

•The field attenuation coefficient is

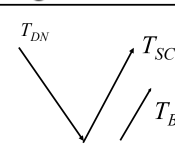
$$\alpha = \frac{2\pi}{\lambda_o}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1+(\varepsilon''/\varepsilon')^2} - 1\right]} \quad \text{[Np/m]}$$

•The power attenuation coefficient is

$$\kappa_a = 2\alpha \quad \text{[Np/m]}$$

Homogeneous terrain medium; Assuming uniform T profile, $T(z) = T_g$

ε_{c1}
μ_{r1}



$T(z) = T_g$

ε_2
μ_2
$K_{a2} = 2\alpha_2$

The scattered tempera

$$T_{SC} = \Gamma_1 T_{DN}$$

The brightness temperature is

$$T_B = (1 - \Gamma_1) T_g$$

The upwelling temperature of homogeneous terrain is

$$T_{APg} = \\ = \sec\theta_2 \int_{-\infty}^{0^-} \kappa_{a2} T(z') e^{-\tau(0^-,-z')\sec\theta_2} dz' \\ = T_g$$

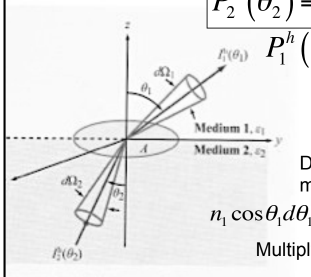
The emissivity of such isothermal medium is

$$e(\theta;p) = T_B/T_g = 1 - \Gamma_1$$

T_B transmission across Specular boundary

$$P_2^h(\theta_2) = I_2^h(\theta_2) A\cos\theta_2 d\Omega_2$$

$$P_1^h(\theta_1) = I_1^h(\theta_1) A\cos\theta_1 d\Omega$$



Snell's Law:

$$n_1\sin\theta_1 = n_2\sin\theta_2 \quad (1)$$

Differentiating wrt θ and multiplying by $d\phi$ on both sides

$$n_1\cos\theta_1 d\theta_1 d\phi_1 = n_2\cos\theta_2 d\theta_2 d\phi_2 \quad (2)$$

Multiplying (1) and (2)

$$\varepsilon_1\cos\theta_1 d\Omega_1 = \varepsilon_2\cos\theta_2 d\Omega_2$$

igure 6-16: Brightness transmission across a planar oundary.

T_B transmission for Specular

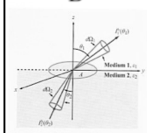


Figure 6-16: Brightness transmission across a planar boundary.

Dividing P_1/P_2:

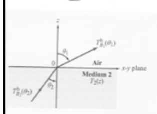
$$\frac{P_1^h(\theta_1) = I_1^h(\theta_1) A\cos\theta_1 d\Omega_1}{P_2^h(\theta_2) = I_2^h(\theta_2) A\cos\theta_2 d\Omega_2}$$

$$\varepsilon_1\cos\theta_1 d\Omega_1 = \varepsilon_2\cos\theta_2 d\Omega_2$$

$$\frac{P_1^h(\theta_1) = I_1^h(\theta_1)}{P_2^h(\theta_2) = I_2^h(\theta_2)}\frac{\varepsilon_2}{\varepsilon_1} = \Upsilon_{21}^h$$

This is the transmissivity

T_B transmission for Specular



igure 6-17: Medium 2 has a temperature-depth profile (z).

Since power is proportional to T_B:

$$\frac{I_1^h(\theta_1)}{I_2^h(\theta_2)}\frac{\varepsilon_2}{\varepsilon_1} = \Upsilon_{21}^h = \frac{\frac{kT_{B1}}{\lambda_1^2}B}{\frac{kT_{B2}}{\lambda_2^2}B}\frac{\varepsilon_2}{\varepsilon_1}$$

$$T_{B1}^h(\theta_1) = \left[\Upsilon_{21}^h\right] T_{B2}^h(\theta_1) = \left[1-\Gamma_{21}^h(\theta_2)\right] T_{B2}^h(\theta_1) = \left[1-\Gamma_{12}^h(\theta_1)\right] T_{B2}^h(\theta_1)$$

Where the reflectivities (ch.2):

$$\Gamma_{12}^h(\theta_1) = \left|\frac{\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2}\cos\theta_2}{\sqrt{\varepsilon_1}\cos\theta_1 + \sqrt{\varepsilon_2}\cos\theta_2}\right| \qquad \Gamma_{12}^v(\theta_1) = \left|\frac{\sqrt{\varepsilon_1}\cos\theta_2 - \sqrt{\varepsilon_2}\cos\theta_1}{\sqrt{\varepsilon_1}\cos\theta_2 + \sqrt{\varepsilon_2}\cos\theta_1}\right|$$

Emissivity at 10GHz for specular surface for H and V polarizations

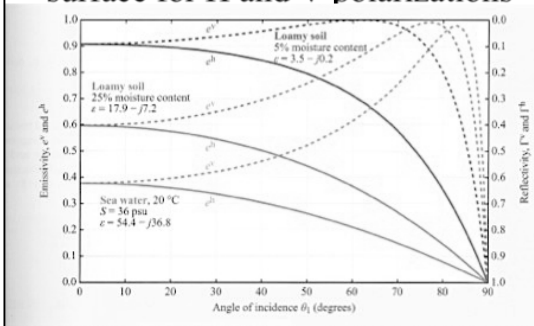


Figure 6-18: Calculated reflectivities and emissivities of a specular surface for h and v polarizations at 10 GHz.

Homogeneous terrain medium

- **The apparent temperature from specular surface is then**

$$T_{AP}(\theta,H) = T_{UP} + [(1-\Gamma_1)T_g + \Gamma_1 T_{DN}]e^{-\tau(0,H)\sec\theta}$$

Emission by the ground

Reflections from the atmosphere

Example

4.5. Consider a downward-looking, nadir-pointing radiometer observing the ocean surface from an airborne platform above a 2-km-thick cloud with a water content of 1.5 g m^{-3}. The absorption coefficient of a water cloud is given by the approximate expression

$$\kappa_a \simeq 2.4\times10^{-4} f^{1.95} m_v \ \mathrm{Np\,km^{-1}},$$

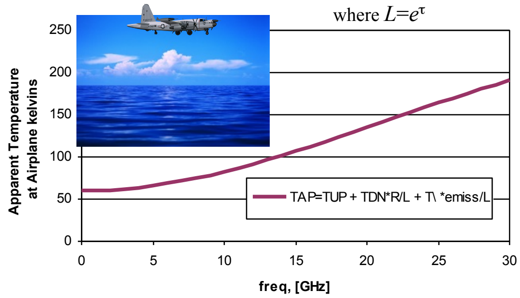
where f is in GHz and m_v and the water content in g m^{-3}. Assuming that the ocean has an apparent temperature $T_{AP}(0;0) = 150$ K, calculate and plot the apparent temperature observed by the radiometer as a function of frequency between 1 and 30 GHz. The cloud may be assumed to have a physical temperature of 275 K.

4.6. Repeat Problem 5 for $\theta = 60°$ and $T_{AP}(60°;0) = 100$ K.

Probl. 4.5

$T_{AP}=T_{UP} + T_{DN}*\Gamma/L + T\ *emiss/L$

where $L=e^{\tau}$



Assigned Problems

Ulaby & Long 2013
6.1-7, 6.10, and 12

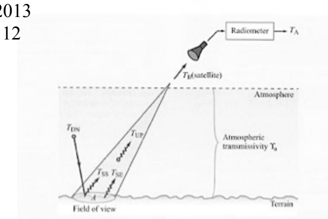
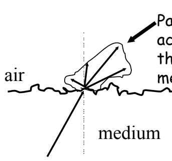


Figure 6-23: Satellite-borne downward-looking radiometer.

Emissivity of Rough surface

- When the surface is rough in terms of wavelength, there are small height irregularities on the surface that scatter power in many directions.
- There will be an angular dependency.



Rough surface emissivity

Medium (b) with small irregularities on the order of the wavelength.

Part of the scatter power is reflected in specular direction and it's mostly phase-coherent.

- The remainder is diffuse or phase-incoherent.
  - Part is the same polarization as incident
  - Rest is orthogonal pol

Law of thermodynamic equilibrium requires absorptivity=emissivity:

$$e^h(\theta,\phi)=a^h(\theta,\phi)$$

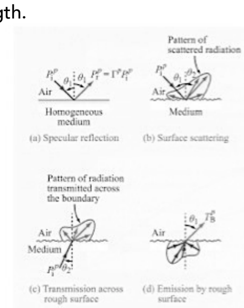


Figure 6-19: Specular- and rough-surface scattering and emission.

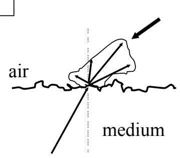
Rough surface emissivity

- For wave incident in medium 1 upon medium 2, the power absorbed by medium 2 $P^a = P^i - P^s$

$$S_h^i = \frac{1}{2\eta_1}\left|E_h^i\right|^2$$

$$P_h^i = S_h^i A\cos\theta = \frac{1}{2\eta_1}\left|E_h^i\right|^2 A\cos\theta$$

$$S^s = S_h^s + S_v^s = \frac{1}{2\eta_1}\left[\left|E_h^s\right|^2 + \left|E_v^s\right|^2\right]$$



Where the scattered fields at a distance R_r are related to the incident fields by:

$$E_h^s = \left(\frac{e^{-jkR_r}}{R_r}\right)\bar{S}_{hh}E_h^i$$

$$E_v^s = \left(\frac{e^{-jkR_r}}{R_r}\right)\bar{S}_{vh}E_h^i$$

- Same polarization as incident
- orthogonal polarization

Rough surface emissivity

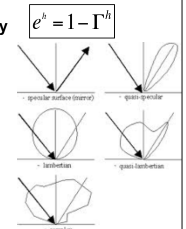
- **Find Reflectivity** $\Gamma^h = P_s / P_i$
- **Substitute and get emissivity = 1-reflectivity** $e^h = 1-\Gamma^h$

$$S^s = S_h^s + S_v^s = \frac{1}{2\eta_1 R_r^2}\left[\left|\bar{S}_{hh}\right|^2 + \left|\bar{S}_{vh}\right|^2\right]\left|E_h^i\right|^2$$

$$= \frac{A}{8\pi\eta_1 R_r^2}\left[\left|\sigma_{hh}^o\right|^2 + \left|\sigma_{vh}^o\right|^2\right]\left|E_h^i\right|^2$$

where S_{pq} are FSA scattering amplitudes & we used the backscattering coefficient defined by:

$$\sigma_{pq} = 4\pi\left|\bar{S}_{pq}\right|^2 \qquad \sigma_{pq}^o = \sigma_{pq}/A$$



Then the scattered power is given by:

$$P^s = \iint_{upper\ hemis} S^s R_r^2\, d\Omega = \frac{A}{8\pi\eta_1 R_r^2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left|\sigma_{hh}^o\right|^2 + \left|\sigma_{vh}^o\right|^2\right]\left|E_h^i\right|^2 R_r^2\, d\Omega$$

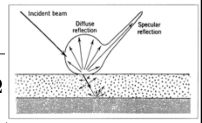
Rough surface emissivity

- Find Reflectivity $\Gamma^h = P_s / P_i$
- Substitute and get emissivity = 1-reflectivitty $e^h = 1 - \Gamma^h$

$$P^s = \frac{P_i}{4\pi\cos\theta_i}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left|\sigma^o_{hh}\right|^2 + \left|\sigma^o_{vh}\right|^2\right]d\Omega$$

$$\Gamma^h = P_s / P_i = \frac{1}{4\pi\cos\theta_i}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left|\sigma^o_{hh}\right|^2 + \left|\sigma^o_{vh}\right|^2\right]d\Omega$$

$$e^h = 1 - \frac{1}{4\pi\cos\theta_i}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left|\sigma^o_{hh}\right|^2 + \left|\sigma^o_{vh}\right|^2\right]d\Omega$$

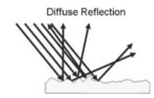


Where the backscattering coefficient consists of coherent component along the specular direction and incoherent component along all directions. (Chapter 5)

Rough surface emissivity

- Substitute and get emissivity = 1-reflectivit $e^h(\theta_1) = 1 - \Gamma^h(\theta_1)$

$$e^h = 1 - \frac{1}{4\pi\cos\theta_i}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left|\sigma^o_{hh}\right|^2 + \left|\sigma^o_{vh}\right|^2\right]d\Omega$$



Where the coherent component is given by: (Chapter 5)

$$\sigma^o_{hh,coh} = 4\pi\cos\theta_i\Gamma^h(\theta_i)e^{-4\psi^2}\delta(\cos\theta_i - \cos\theta_s)\delta(\phi_i - \phi_s)$$

where ψ is the roughness parameter, s=*rms* height given by $\psi = ks\cos\theta_1$

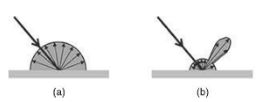
$$e^h = 1 - \Gamma^h e^{-4\psi^2} - \frac{1}{4\pi\cos\theta_i}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left|\sigma^o_{hh,inc}\right|^2 + \left|\sigma^o_{vh}\right|^2\right]d\Omega$$

Rough surface emissivity

- The eq. for Γ^h can also be used to relate surface scattered brightness temperature, T_{SS} to the unpolarized emitted atmospheric temperature T_{DN} incident from all directions in the upper hemisphere.

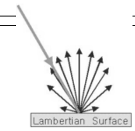
$$T^h_{SS} = \Gamma^h T_{DN} e^{-4\psi^2} + \frac{1}{4\pi\cos\theta_i}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left|\sigma^o_{hh,inc}\right|^2 + \left|\sigma^o_{vh}\right|^2\right]T_{DN}\,d\Omega$$

Accounts for the incoherent part of the scattering by the surface



Lambertian surface emissivity

- For Lambertian surface (perfectly rough):



$$e^h(\theta_i, \phi_i) = 1 - \frac{\sigma^o_o}{4}$$

Which is polarization and angle independent

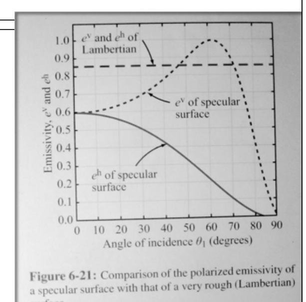


Figure 6-21: Comparison of the polarized emissivity of a specular surface with that of a very rough (Lambertian) surface.

Emissivity of 2-layer Composite

In thermodynamic equilibrium we have:

$$T^h_B(\theta_1) = e^h(\theta_1)T_o$$

Where the emissivity is related to the reflectivity as (for v or h):

$$e^h(\theta_1) = 1 - \Gamma^h(\theta_1)$$

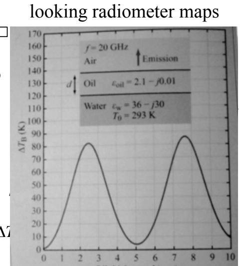
$$e^v(\theta_1) = 1 - \Gamma^v(\theta_1)$$

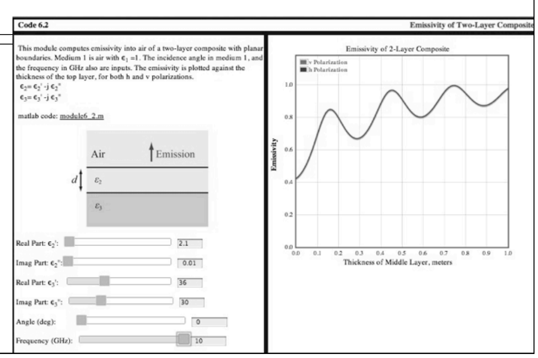
Ch.2 $$= 1 - \left|\frac{\rho_{12} + \rho_{23}e^{-2\gamma_2 d\cos\theta_2}}{1 + \rho_{12}\rho_{23}e^{-2\gamma_2 d\cos\theta_2}}\right|^2$$

Both medium 2 and 3 are lossy.

$$\gamma_2 = \alpha_2 + j\beta_2 \quad \alpha_2 = -\omega\sqrt{\mu\varepsilon_o}\,\mathrm{Im}\left\{\sqrt{\varepsilon_2}\right\} \quad \beta_2 = \omega\sqrt{\mu\varepsilon_o}\,\mathrm{Re}\left\{\sqrt{\varepsilon_2}\right\} \quad \cos\theta_2 = \sqrt{1 - \left(\frac{j2\pi\sqrt{\varepsilon_1}\sin\theta_1}{\alpha_2 + j\beta_2 \quad \lambda_o}\right)^2}$$

Example: a 20GHz nadir looking radiometer maps





```
*subroutine to compute Tdownwelling where ka= water + oxygen attenuations
* Begin computation of radiative transfer integral
*Integration of Tb_dn, Tb_up, opacity
DO 45 ifr = 1 ,3
                freq =fre(ifr)
**              Compute  the DOWNWELLING BRIGHTNESS TEMPERATURE      *********
                TBdn(ifr) = 0.
                opa = 0.0
                TAU = 1.0
 DO 130 J = 1, JJ
                        vap =v(j)
                        pre =p(j)
                        tem =t(j)
          absorp=VAPOR(freq,Vap,Pre,Tem,CL,CW,CC)+ OXYGEN(freq,Vap,Pre,Tem,CX)
                LTMP0 = EXP(-DZ*absorp)
                DTB = Tem*(1.0 - LTMP0)*TAU
                TAU = TAU*LTMP0
                TBdn(ifr) = TBdn(ifr) + DTB
 130            CONTINUE
          tauf(ifr) = tau
* Add on freq dependent cosmic background term:
   TBdn(ifr) = TBdn(ifr) + (2.757+0.00379*(freq-18))*TAU
*
*COMPUTE THE  UpWELLING BRIGHTNESS TEMPERATURE***********
   TBup(ifr) = 0.
   TAU = 1.0
          DO 135 J =  JJ,1,-1
                vap =v(j)
                pre =p(j)
                tem =t(j)
        LTMP0 = EXP(-DZ*(VAPOR(freq,Vap,Pre,Tem,CL,CW,CC)+ OXYGEN(freq,Vap,Pre,Tem,CX)) )
                DTB = Tem*(1.0 - LTMP0)*TAU
                TAU = TAU*LTMP0
                TBup(ifr) = TBup(ifr) + DTB
 135   CONTINUE
45          CONTINUE
```

Code for Tdn

```
**          Compute  the DOWNWELLING BRIGHTNESS
TEMPERATURE*********
                TBdn = 0.
                TAU = 1.0
                jj=1000
                dZ=30000/jj
 DO 130 J = 1, JJ
                vap =v(j); pre =p(j); tem =t(j)
        Ka=VAPOR(freq,Vap,Pre,Tem,CL,CW,CC)+
OXYGEN(freq,Vap,Pre,Tem,CX)
                LTMP0 = EXP(-dZ*Ka)
                DTB = Tem*(1.0 - LTMP0)*TAU
                TAU = TAU*LTMP0
                TBdn = TBdn + DTB
 130            CONTINUE
* Add on freq dependent cosmic background term:
   TBdn(ifr) = TBdn(ifr) + (2.757+0.00379*(freq-18))*TAU
```

Code for Tup

```
*COMPUTE THE  UpWELLING BRIGHTNESS
TEMPERATURE***********
   TBup(ifr) = 0.
   TAU = 1.0
      DO 135 J =  JJ,1,-1
          vap =v(j)
          pre =p(j)
          tem =t(j)
        LTMP0 = EXP(-DZ*ka(j))
          DTB = Tem*(1.0 - LTMP0)*TAU
          TAU = TAU*LTMP0
          TBup(ifr) = TBup(ifr) + DTB
 135   CONTINUE
45     CONTINUE
```