Electromagnetic Theorems & Principles

INEL 6216
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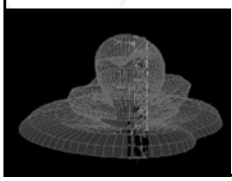


Theoremas que ayudan a resolver problemas de EM

- Dualidad
- Ser Único ("uniqueness")
- Imágenes
- Reciprocidad
- Equivalencia de Volumen
- Equivalencia de superficie:
  - Principio de Huygen
- Inducción

Dualidad

Electric source (J)	Magnetic source (M)
$\nabla\times\vec{E}_A=-j\omega\mu\vec{H}_A$	$\nabla\times\vec{H}_F=j\omega\varepsilon\vec{E}_F$
$\nabla\times\vec{H}_A=\vec{J}+j\omega\varepsilon\vec{E}_A$	$-\nabla\times\vec{E}_F=\vec{M}+j\omega\mu\vec{H}_F$
$\vec{H}_A=\frac{1}{\mu}\left(\nabla\times\vec{A}\right)$	$\vec{E}_F=-\frac{1}{\varepsilon}\left(\nabla\times\vec{F}\right)$

Same mathematical form
Systematic interchange of symbols

$\vec{E}_A$	$\vec{H}_F$
$\vec{H}_A$	$-\vec{E}_F$
$\vec{J}$	$\vec{M}$
ε	μ
$\vec{A}$	$\vec{F}$

Uniqueness theorem

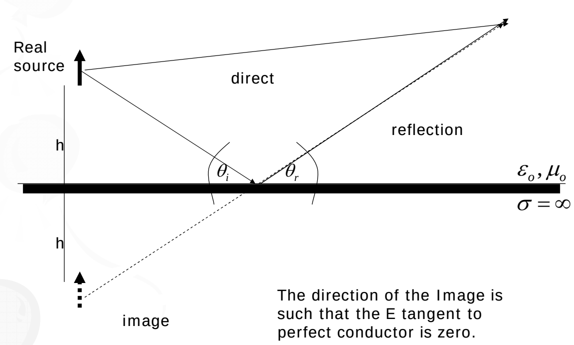
- Solution is unique given certain conditions: If you find <u>two</u> solutions, E_1 & E_2, H_1 & H_2, then over a surface S,

$$\hat{n}\times\left(\vec{E}_1-\vec{E}_2\right)=\hat{n}\times\left(\delta\vec{E}\right)=0$$
$$\hat{n}\times\left(\vec{H}_1-\vec{H}_2\right)=\hat{n}\times\left(\delta\vec{H}\right)=0=0$$

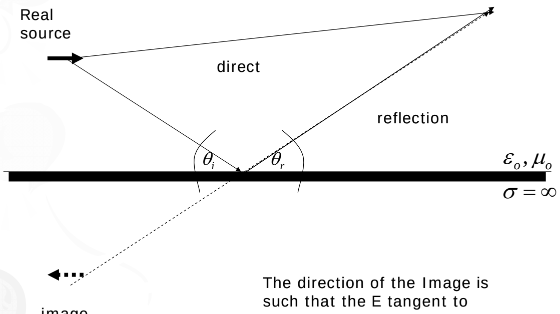
Image Theory

- When the ground, corner reflector or other obstacle is close to a radiating element, there will be reflections from it.
- This can be accounted for by image.
- Earth is more lossy at high frequency and moisture
- Assume ground is a perfect electric conductor, flat, and infinite in extent

Vertical (electric) Dipole

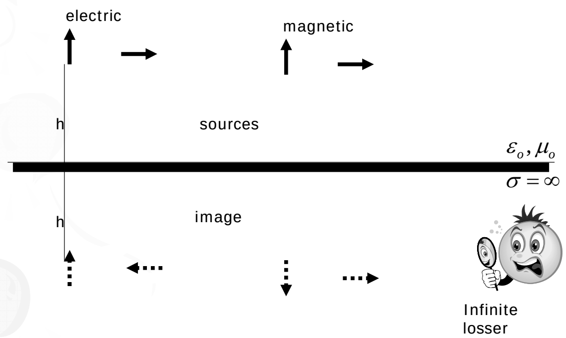


Horizontal (electric) Dipole

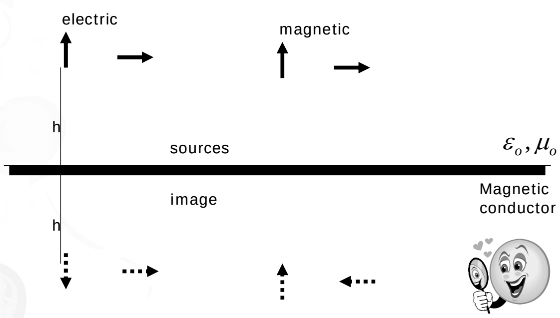


The direction of the Image is such that the E tangent to perfect conductor is zero.

Electric perfect conductor



Magnetic perfect conductor



Reciprocity Theorem

Related to transmitting and receiving properties of radiating systems.

- Fields for sources J_1, M_1

$$\nabla \times \vec{E}_1 = -\vec{M}_1 - j\omega\mu\vec{H}_1$$
$$\nabla \times \vec{H}_1 = \vec{J}_1 + j\omega\varepsilon\vec{E}_1$$

- Fields for sources J_2, M_2

$$\nabla \times \vec{E}_2 = -\vec{M}_2 - j\omega\mu\vec{H}_2$$
$$\nabla \times \vec{H}_2 = \vec{J}_2 + j\omega\varepsilon\vec{E}_2$$

- *By reciprocity then,*

$$\iiint_V \left(\vec{E}_1 \cdot \vec{J}_2 - \vec{H}_1 \cdot \vec{M}_2\right) dv' = \iiint_V \left(\vec{E}_2 \cdot \vec{J}_1 - \vec{H}_2 \cdot \vec{M}_1\right) dv'$$

Volume Equivalence Th.

Used to find the fields scattered by dielectric obstacles.

If in <u>free-space</u> (ε_o, μ_o) we have:

$$\nabla \times \vec{E}_o = -\vec{M}_i - j\omega\mu_o\vec{H}_o$$
$$\nabla \times \vec{H}_o = \vec{J}_i + j\omega\varepsilon_o\vec{E}_o$$

But then, the same sources find <u>another medium</u> (ε, μ):

$$\nabla \times \vec{E} = -\vec{M}_i - j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = \vec{J}_i + j\omega\varepsilon\vec{E}$$

We define the <u>difference</u> as the <u>scattered</u> fields:

$$\vec{E}_s = \vec{E} - \vec{E}_o$$
$$\vec{H}_s = \vec{H} - \vec{H}_o$$

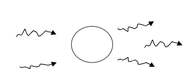
Volume Equivalence (cont.)

Using this, we can derive:

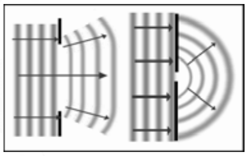
$$\nabla \times \vec{E}_s = -\vec{M}_{eq} - j\omega\mu_o\vec{H}_s$$
$$\nabla \times \vec{H}_s = \vec{J}_{eq} + j\omega\varepsilon_o\vec{E}_s$$

Where the volume equivalent current densities sources

$$\vec{M}_{eq} = j\omega(\mu - \mu_o)\vec{H}$$
$$\vec{J}_{eq} = j\omega(\varepsilon - \varepsilon_o)\vec{E}$$



Surface Equivalence [Huygen's Principle]

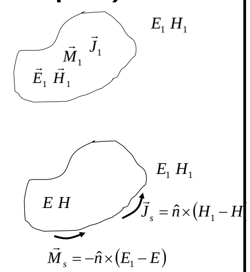


- predicts the future position of a wave when its earlier position is known.

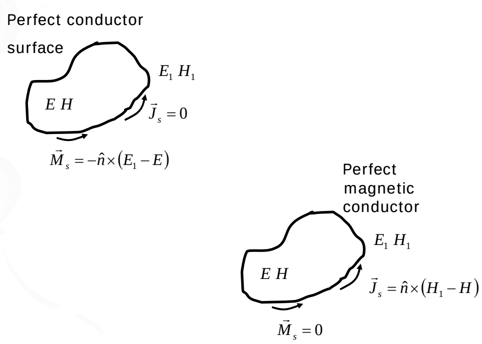
"Every point on a wave front can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The new wave front is the envelope of all the wavelets - that is, tangent to them."

Surface Equivalence (Huygen's Principle)

- Used mostly for aperture radiation
- Here actual sources are replaced by equivalent sources (J,M) within a region to simplify solution
- An imaginary closed surface is chosen (usually so that it coincides with conducting structure) but fields outside are the same.
- Chosen current densities source on the surface create the same fields outside.



$$\vec{J}_s = \hat{n}\times(H_1 - H)$$
$$\vec{M}_s = -\hat{n}\times(E_1 - E)$$

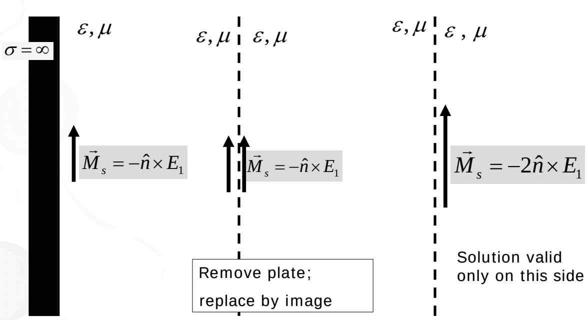


Love's Principle

- Choose the inside field to be zero

$$\vec{J}_s = \hat{n}\times(H_1)$$
$$\vec{M}_s = -\hat{n}\times(E_1)$$

Equivalent model for magnetic source radiating near perfect conductor



Induction Theorem
[similar to Huygen's]

- Used mostly for scattering from obstacles