Transmission Lines

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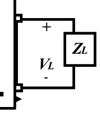
Intro to Transmission Lines (T.L.)

- Hi-frequency or Hi-power requires T.L.
- TEM waves propagate thru T.L.
- We will develop T.L. theory to see how waves propagate thru them

Seem to be in parallel, but these V are not equal!

- A 40-m long TL has V_g=15 V_{rms}, Z_o=30+j60 Ω, and $V_L=5e^{-j48^o}$ V_{rms}. If the ... input impedance ... propagation constant

Don't worry about the details, I'll teach you about solving this type of problems pretty soon.

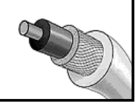


- Answers:

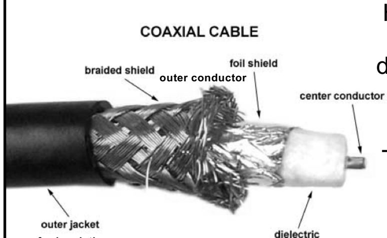
$$Z_{in} = 30 + j60\Omega, \quad I_{in} = 0.112\angle -63.4^o A,$$
$$V_{in} = 7.5\angle 0^o V_{rms}, \quad \gamma = 0.0101 + j0.02094\ m^{-1}$$
$$7.45\angle -111^o e^{-\gamma 40} = 0.112\angle -63^o$$

Transmission Lines

- **I. TL parameters (R',L',C', G')**
- **II. TL Equations (for *V* and *I*)**
- **III. 3 Concepts:**
  - **I. Input Impedance,**
  - **II. Reflection Coefficient and**
  - **III. Characteristic Impedance**
- **IV. SWR, Power**
- **V. Smith Chart**



Transmission Lines (TL)

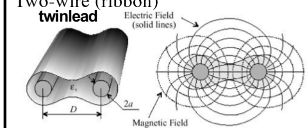


Have 2 conductors in parallel with a dielectric separating them

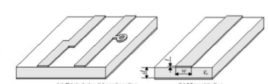
They transmit TEM waves inside the lines

Common Transmission Lines

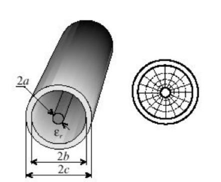
Two-wire (ribbon)
twinlead



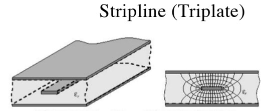
Microstrip



Coaxial



Stripline (Triplate)



Other TL (higher order)
[Chapter 12]

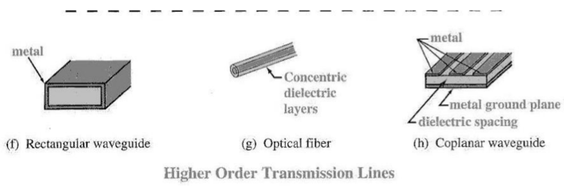


Figure 2-4: A few examples of transverse electromagnetic (TEM) and higher-order transmission lines.

Fields inside the TL

$V = -\int E \cdot dl$

$I = \oint H \cdot dl$

- V proportional to E,
- I proportional to H

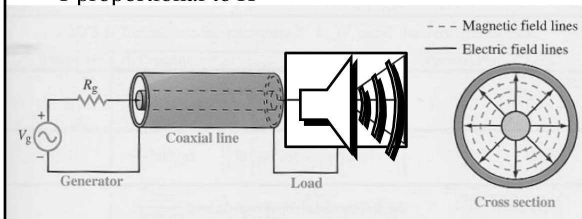


Figure 2-5: In a coaxial line, the electric field lines are in the radial direction between the inner and outer conductors, and the magnetic field forms circles around the inner conductor.

Distributed parameters

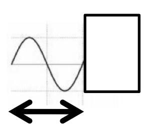
The parameters that characterize the TL are given in terms of per length.

- R = ohms/meter
- L = Henries/ m
- C = Farads/m
- G = mhos/m

At high frequencies we're dealing with wavelengths comparable to the size of the circuits.

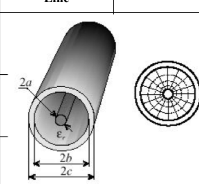
$\lambda_{60Hz} = c/60 = 5{,}000km$

$\lambda_{2GHz} = c/2000{,}000{,}000 = 15cm$



Common Transmission Lines

R', L', G', and C' depend on the particular transmission line structure and the material properties. R, L, G, and C can be calculated using fundamental EMAG techniques.

Parameter	Two-Wire Line	Coaxial Line	Parallel-Plate Line	Unit
R'	$\frac{1}{\pi a \sigma_{cond} \delta}$	$\frac{1}{2\pi\sigma_{cond}\delta}\left(\frac{1}{a}+\frac{1}{b}\right)$		
L'	$\frac{\mu}{\pi}\text{acosh}\left(\frac{D}{2a}\right)$	$\frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$		
G'	$\frac{\pi\sigma_{diel}}{\text{acosh}(D/(2a))}$	$\frac{2\pi\sigma_{diel}}{\ln(b/a)}$		
C'	$\frac{\pi\varepsilon}{\text{acosh}(D/(2a))}$	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\varepsilon\frac{w}{d}$	F/m

TL representation

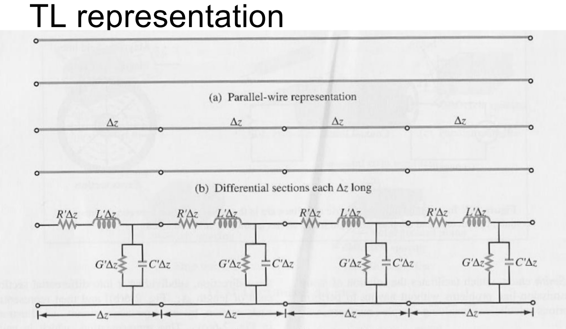
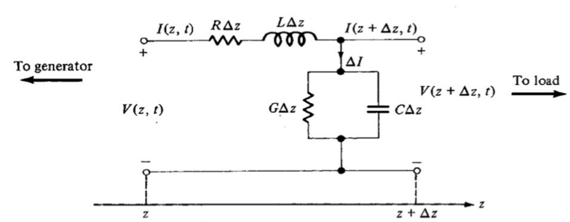


Figure 2-6: Regardless of its actual shape, a TEM transmission line is represented by the parallel-wire configuration shown in (a). To analyze the voltage and current relations, the line is subdivided into small differential sections (b), each of which is then represented by an equivalent circuit (c).

Distributed line parameters

Using KVL:



$$-\frac{V(z+\Delta z, t) - V(z, t)}{\Delta z} = R\, I(z, t) + L\frac{\partial I(z, t)}{\partial t}$$

Distributed parameters

- Taking the limit as Δz tends to 0 leads to

$$-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t}$$

- Similarly, applying KCL to the main node gives

$$-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t}$$

Wave equation

- Using phasors

$$V(z,t) = \text{Re}[V_s(z)e^{j\omega t}]$$
$$I(z,t) = \text{Re}[I_s(z)e^{j\omega t}]$$

- The two expressions reduce to

$$-\frac{\partial V_s}{\partial z} = (R + j\omega L)I_s$$
$$-\frac{\partial I_s}{\partial z} = (G + j\omega C)V_s$$

$$\frac{\partial^2 V_s}{\partial z^2} - \gamma^2 V_s = 0$$
$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

Wave Equation for voltage

Transmission Lines

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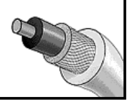
   II. **Reflection Coefficient and**
   III. **Characteristic Impedance**
IV. **SWR, Power**
V. **Smith Chart**



TL Equations

- Note that these are the wave eq. for voltage and current inside the lines.

$$\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0 \qquad \frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0$$

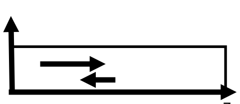
- The propagation constant is γ and the wavelength and velocity are

$$\gamma = \alpha + \beta j = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$\lambda = 2\pi/\beta \qquad u = \omega/\beta = f\lambda$$

Waves move through line

- The general solution is

$$V_s = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$



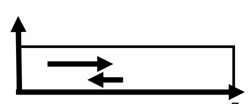
- In time domain is

$$V(z,t) = \text{Re}[V_s(z)e^{j\omega t}]$$
$$= V^+ e^{-\alpha z}\cos(\omega t - \beta z) + V^- e^{+\alpha z}\cos(\omega t + \beta z)$$

Waves move through line

- For Current

$$I_s = I^+ e^{-\gamma z} + I^- e^{\gamma z}$$



- Similarly for time-domain, I

$$I(z,t) = I^+ e^{-\alpha z}\cos(\omega t - \beta z) + I^- e^{\alpha z}\cos(\omega t + \beta z)$$

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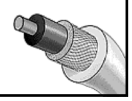
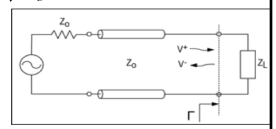
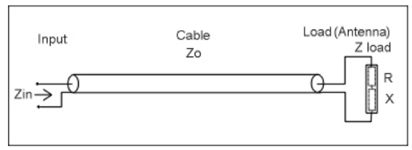


We will define 3 concepts:

- Characteristic impedance, Z_o
- Reflection Coefficient, Γ_L
- Input Impedance, Z_{in}





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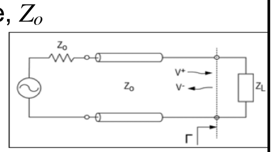
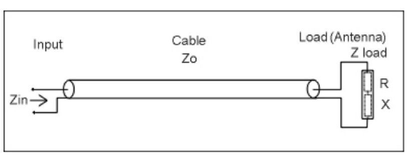




Characteristic Impedance of a Line, Z_o

- Is the ratio of positively traveling voltage wave to current wave at any point on the line

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z)$$

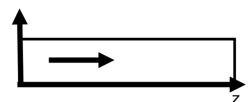
substituting

$$V(z) = V^+ e^{-\gamma z}$$

$$I(z) = I^+ e^{-\gamma z}$$

$$-(-\gamma V^+ e^{-\gamma z}) = (R + j\omega L)I^+ e^{-\gamma z}$$

$$Z_o = \frac{V^+}{I^+} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$



Characteristic Impedance, Z_o

$$Z_o = \frac{V^+}{I^+} = -\frac{V^-}{I^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

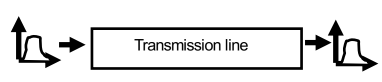
$$I_s = I^+ e^{-\gamma z} \quad + \quad I^- e^{\gamma z}$$

$$I_s = \frac{V^+}{Z_o} e^{-\gamma z} \quad + \quad \frac{V^-}{-Z_o} e^{\gamma z}$$

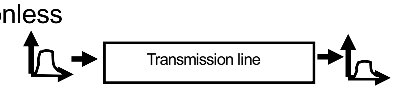
Different cases of TL

Find the characteristic impedance and propagation constant for each:

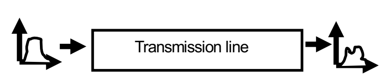
- Lossless



- Distortionless



- Lossy



Lossless Lines (*R=0=G*)

Have perfect conductors and a perfect dielectric medium between them.

- Propagation: $\alpha = 0, \quad \gamma = j\beta, \quad \beta = \omega\sqrt{LC}$
- Wavelength & Velocity: $u = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}} = f\lambda, \qquad \lambda = \dfrac{2\pi}{\beta}$
- Impedance $\quad Z_o = R_o = \sqrt{\dfrac{L}{C}} \qquad X_o = 0$

Distortionless line (*R/L = G/C*)

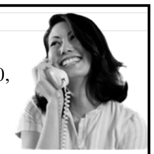
Is one in which the attenuation is independent on frequency. $\gamma = \alpha + j\beta$

- Propagation: $\alpha = \sqrt{RG} \quad \beta = \omega\sqrt{LC}$
- Velocity: $u = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}} = f\lambda$
- Impedance $\quad X_o = 0 \qquad Z_o = R_o = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{R}{G}}$

Summary

	$\gamma = \alpha + j\beta$ *Propagation Constant*	Z_o *Characteristic Impedance*
General (Lossy)	$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$	$Z_o = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$
Lossless	$\gamma = 0 + j\omega\sqrt{LC}$	$Z_o = R_o = \sqrt{\dfrac{L}{C}}$
Distortionless *RC = GL*	$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$	$Z_o = R_o = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{R}{G}}$

P.E. 11.2



A telephone line has R=30 Ω/km, L=100 mH/km, G=0, and C= 20μF/km. At 1kHz, FIND: the characteristic impedance of the line, the propagation constant, the phase velocity.

- Is it distortionless? **No**

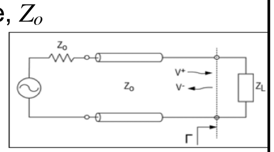
Solution:

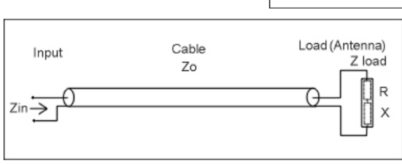
$Z_o = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}} =$

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

We will define 3 concepts:

- Characteristic impedance, Z_o
- Reflection Coefficient, Γ_L
- Input Impedance, Z_{in}





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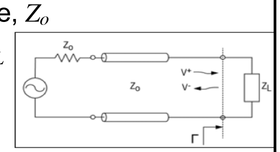
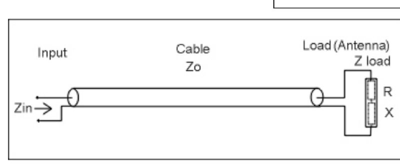




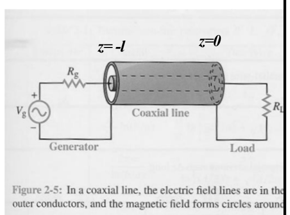
Reflection coefficient at the load, Γ_L

Load is usually taken at *z=0* and generator at *z= -l*

$$V_s(z) = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$

$$\Gamma_L = \frac{V^-}{V^+}$$

$$V_s(z) = V^+\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)$$



For a Lossless TL terminated with a load

Then, $V_s(z) = V^+\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)$

Similarly, $I_s(z) = \dfrac{V^+}{Z_o}\left(e^{-\gamma z} - \Gamma_L e^{+\gamma z}\right)$

The impedance **anywhere** along the line is given by

$$Z(z) = \frac{V_s(z)}{I_s(z)} = Z_o \frac{\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)}{\left(e^{-\gamma z} - \Gamma_L e^{+\gamma z}\right)}$$

The impedance at the load end, Z_L, is given by

$$Z(l=0) = Z_L = Z_o \frac{(1+\Gamma_L)}{(1-\Gamma_L)}$$

Terminated, Lossless TL

Solving for Γ_L

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Conclusion: The reflection coefficient is a function of the **load** impedance and the **characteristic** impedance.

Recall for the lossless case, $\gamma = 0 + j\beta = j\omega\sqrt{LC}$

Then

$$V_s(z) = V^+\left(e^{-j\beta z} + \Gamma_L e^{+j\beta z}\right)$$

$$I_s(z) = \frac{V^+}{Z_o}\left(e^{-j\beta z} - \Gamma_L e^{+j\beta z}\right)$$

Definition: Matched line

- Means that $Z_o = Z_L$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

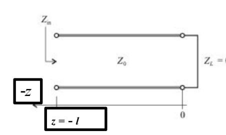
- Therefore there are no reflection!

$\Gamma_L = 0$

What happens when you connect the wrong TL to a speaker?

Terminated, Lossless TL

Using convention the coordinate system, $z = -l$, at input.



Rewriting the expressions for voltage and current, we have

$$V(-l) = V^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

$$I(-l) = \frac{V^+}{Z_o}\left(e^{+j\beta l} - \Gamma_L e^{-j\beta l}\right)$$

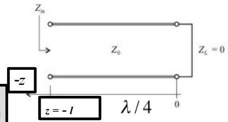
Rearranging,

$$V(-l) = V^+ e^{+j\beta l}\left(1 + \Gamma_L e^{-2j\beta l}\right)$$

$$I(-l) = \frac{V^+}{Z_o} e^{+j\beta l}\left(1 - \Gamma_L e^{-2j\beta l}\right)$$

Voltage anywhere on the Line

Recall,



$$V(l) = V^{+}\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

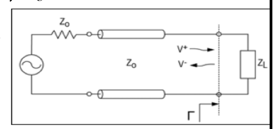
$V_{in} = V(-l)$ Input voltage= sending end,

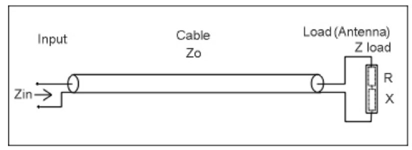
$V_L = V(0)$ Load Voltage = receiving end

$V(l = \lambda/4) = V^{+}\left(e^{j\pi/2} + (0)e^{-j\pi/2}\right)$ Voltage quaterwave from matchedload

We will define 3 concepts:

- Characteristic impedance, Z_o
- Reflection Coefficient, Γ_L
- Input Impedance, Z_{in}





Impedance (Lossless line)

The impedance **anywhere** along the line is given by

$$Z(l) = \frac{V(l)}{I(l)} = Z_o \frac{\left(1 + \Gamma_L e^{-2j\beta l}\right)}{\left(1 - \Gamma_L e^{-2j\beta l}\right)}$$

The reflection coefficient at any point along the line:

$$\Gamma(l) = \Gamma_L e^{-2j\beta l} = |\Gamma_L| e^{j\theta_\Gamma} e^{-2j\beta l}$$

Then, the impedance can be written as.

$$Z(l) = Z_o \frac{(1 + \Gamma(l))}{(1 - \Gamma(l))}$$

After some algebra, an alternative expression for the impedance is given by

$$Z(l) = Z_{in} = Z_o \frac{(Z_L + jZ_o \tan \beta l)}{(Z_o + jZ_L \tan \beta l)}$$

Conclusion: The load impedance is "transformed" as we move away from the load.

Impedance (Lossy line)

The impedance **anywhere along the line** is given by

$$Z(l) = \frac{V(l)}{I(l)} = Z_o \frac{\left(1 + \Gamma_L e^{-2\gamma l}\right)}{\left(1 - \Gamma_L e^{-2\gamma l}\right)}$$

The reflection coefficient can be modified as follows

$$\Gamma(l) = \Gamma_L e^{-2\gamma l} = \Gamma_L (e^{-2\alpha l} e^{-2j\beta l})$$

Then, the impedance can be written as

$$Z(l) = Z_o \frac{(1 + \Gamma(l))}{(1 - \Gamma(l))}$$

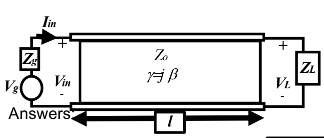
After some algebra, an alternative expression for the impedance is given by

$$Z(l) = Z_{in} = Z_o \frac{(Z_L + Z_o \tanh \gamma l)}{(Z_o + Z_L \tanh \gamma l)}$$

Conclusion: in lossy lines, we end up with the hyperbolic tangent.

Example: Matched Case

A TL has V_g=10 V_{rms}, Z_o=50Ω If the line is matched to the load and the generator, find: the input impedance Z_{in}, the sending-end Voltage V_{in}



$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0$$

- Answers

$Z_g = Z_L = Z_0 \quad \Rightarrow Z_{in} = 50\Omega$

$$Z_{in} = Z_o \frac{(Z_L + jZ_o \tan \beta l)}{(Z_o + jZ_L \tan \beta l)} = Z_o$$

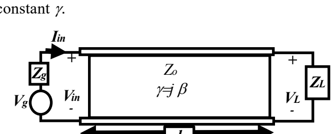
$\Gamma = 0$

VoltageDivider $V_{in} = 5\angle 0^o$

Example: Matched Case

A λ/8 long TL has V_L=5 < 30º, Z_o=50Ω If the line is matched to the load, find: the input impedance Z_{in}, the sending-end Voltage V_{in}, the propagation constant γ.

$$V(l) = V^{+}\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$



$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

- Answers:

$$Z_{in} = Z_o \frac{(Z_L + jZ_o \tan \beta l)}{(Z_o + jZ_L \tan \beta l)}$$

$Z_L = Z_0 \quad \Rightarrow Z_{in} = 50\Omega$

$\Gamma = 0$

$$V(l = \lambda/8) = 5\angle 30^o \left(e^{j\pi/4} + (0)e^{-j\pi/4}\right)$$

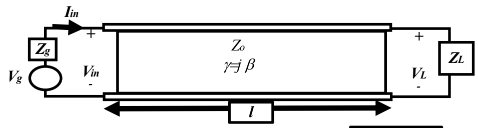
$V_L = 5\angle 30^o = V\big|_{l=0} = V^{+}\left(e^{j\beta 0} + (0)e^{-j\beta 0}\right),$

Solve for: $V^{+} = 5\angle 30^o$

$$V_{in} = 5\angle 30^o \left(e^{j\pi/4}\right) = 5\angle 75^o$$

Exercise 1: Not Matched

A 2cm lossless TL has V_L=10 e^{j30o}, Z_g=60 Ω, Z_L=50 Ω and Z_o=100Ω, λ=10cm . Find: the input impedance Z_{in}, the sending-end Voltage V_{in},



$$V(l) = V^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

- Use this equation at load and at input, find V^+
- Find V_{in}
- Find Z_{in} (at input)

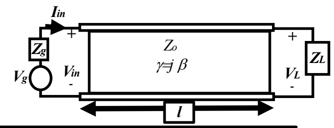
$$V^+ = 15\left(e^{j30}\right)$$

$$V_{in} = 39.9\angle 94^o$$

$$Z_{in} = Z_o \frac{(Z_L + jZ_o \tan\beta l)}{(Z_o + jZ_L \tan\beta l)} = 155 + j68\Omega$$

Exercise 2: using formulas

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},



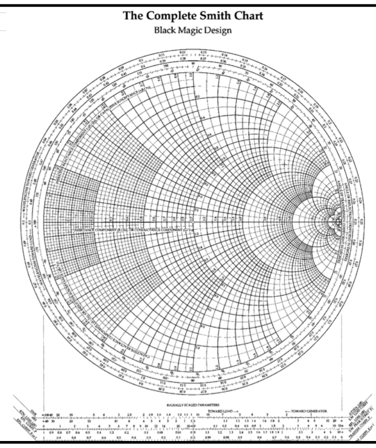
$$Z_{in} = Z(2cm) = 40\frac{\left((100 + j80) + j40\tan\frac{2\pi}{5}\right)}{\left(40 + j(100 + j80)\tan 2\pi/5\right)}$$

$$Z_{in} = 12.2 - j21.17\Omega$$

Voltage Divider:

$$V_{in} = \frac{V_g Z_{in}}{Z_{in} + Z_g} = 3.30\angle -0.766 rad$$

The Smith Chart



Example 3: Not matched to load

A generator with 10V_{rms} and R_g=50, is connected to a 75Ω load thru a 0.8λ, 50Ω-lossless line.

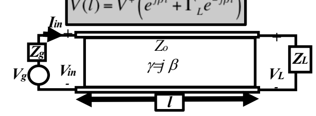
- Find V_L

$$Z_o = 50\Omega \quad Z_L = 75, \; l = .8\lambda$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad \Gamma_L = 0.2$$

$$Z_{in} = Z_o \frac{(Z_L + jZ_o \tan\beta l)}{(Z_o + jZ_L \tan\beta l)} \qquad Z_{in} = 35. + j8.75\Omega$$

$$V(l) = V^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$



PowerDivider

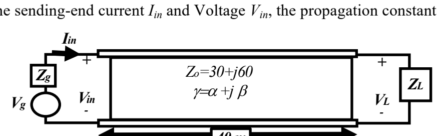
$V_{in} = \angle$

$V^+ = 5.006\angle 1.26 rad$

$V_L = V(l = 0)$

Exercise 11.3: Matched Case

A 40-m long TL has V_g=15 V_{rms}, Z_o=30+j60 Ω, and V_L=5e^{-j48o} V_{rms}. If the line is matched to the load and the generator, find: the input impedance Z_{in}, the sending-end current I_{in} and Voltage V_{in}, the propagation constant γ.



- Answers:

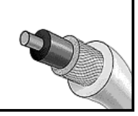
$$Z_{in} = 30 + j60\Omega, \quad I_{in} = 0.112\angle -63.4^o A,$$

$$V_{in} = 7.5\angle 0^o V_{rms}, \quad \gamma = 0.0101 + j0.02094\ m^{-1}$$

$$7.45\angle -111^o e^{-\gamma 40} = 0.112\angle -63^o$$

Transmission Lines

- **I. TL parameters (R',L',C', G')**
- **II. TL Equations (for *V* and *I*)**
- **III. 3 Concepts:**
  - **I. Input Impedance,**
  - **II. Reflection Coefficient and**
  - **III. Characteristic Impedance**
- **IV. SWR, Power**
- **V. Smith Chart**



Power

- The average input power at a distance l from the load is given by

$$P_{ave} = \frac{1}{2}\text{Re}\left[V(l)I^*(l)\right]$$

- which can be reduced to

$$P_{ave} = \frac{\left|V_o^+\right|^2}{2Z_o}\left(1-\left|\Gamma\right|^2\right)$$

- The first term is the incident power and the second is the reflected power. Maximum power is delivered to load if $\Gamma=0$

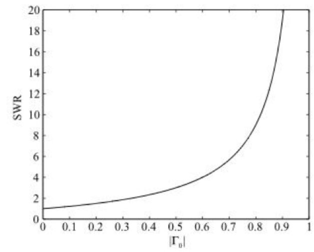
SWR or VSWR or s

Whenever there is a reflected wave, a standing wave will form out of the combination of incident and reflected waves.

The (Voltage) Standing Wave Ratio - SWR (or VSWR) is defined as

$$s = SWR = \frac{\left|V_{max}\right|}{\left|V_{min}\right|} = \frac{\left|I_{max}\right|}{\left|I_{min}\right|}$$

$$s = \frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}$$



Summary

- Input Impedance

$$Z_{in} = Z_o\frac{\left(Z_L + jZ_o\tan\beta l\right)}{\left(Z_o + jZ_L\tan\beta l\right)}$$

- Reflection Coef

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

- SWR

$$s = \frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}$$

Three (3) common Cases of line-load combinations:

- Shorted Line ($Z_L=0$)

$$Z_{in} = 0 + jZ_o\tan\beta l = jb \qquad \Gamma_L = -1, \quad s = \infty$$

- Open-circuited Line ($Z_L=\infty$)

$$Z_{in} = -jZ_o\cot\beta l \qquad \Gamma_L = 1, \quad s = \infty$$

- Matched Line ($Z_L = Z_o$)

$$Z_{in} = Z_o \qquad \Gamma_L = 0, \quad s = 1$$

Standing Waves –Short ($Z_L=0$)

$$Z_{in} = jZ_o\tan\beta l\,, \quad \Gamma_L = -1, \quad s = \infty$$

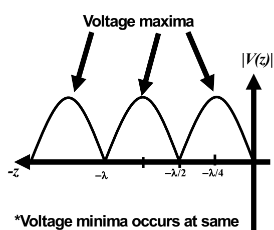
- So substituting in $V(z)$

$$V(z) = V^+[e^{j\beta l} + (-1)e^{-j\beta l}]$$

$$V(z) = V^+(2j\sin\beta l)$$

$$\left|V(z)\right| = \left|V^+\right|\left|2\sin(\beta l)\right|$$

$$\left|V(z)\right| = \left|V^+\right|\left|2\sin\left(\frac{2\pi}{\lambda}l\right)\right|$$



*Voltage minima occurs at same place that impedance has a minimum on the line

Standing Waves –Open($Z_L=\infty$)

$$Z_{in} = -jZ_o\cot\beta l\,, \quad \Gamma_L = +1, \quad s = \infty$$

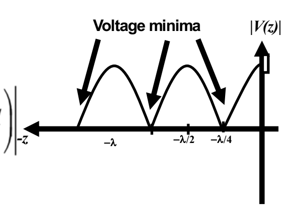
- So substituting in $V(z)$

$$V(z) = V^+[e^{j\beta l} + (+1)e^{-j\beta l}]$$

$$V(z) = V^+(2\cos\beta l)$$

$$\left|V(z)\right| = \left|V^+\right|\left|2\cos(\beta l)\right|$$

$$\left|V(z)\right| = \left|V^+\right|\left|2\cos\left(\frac{2\pi}{\lambda}l\right)\right|$$

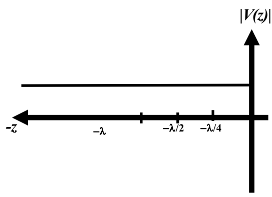


Standing Waves –Matched ($Z_L = Z_o$)

$$Z_{in} = Z_o\ , \quad \Gamma_L = 0, \quad s = 1$$

- So substituting in $V(z)$

$$V(z) = V^+[e^{j\beta l} + (0)e^{-j\beta l}]$$
$$V(z) = V^+ e^{j\beta l}$$
$$|V(z)| = |V^+||e^{j\beta l}|$$
$$|V(z)| = |V^+|$$



Java applets

- http://www.amanogawa.com/transmission.html
- http://physics.usask.ca/~hirose/ep225/
- http://www.educatorscorner.com/index.cgi?CONTENT_ID=2483

Transmission Lines

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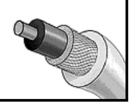
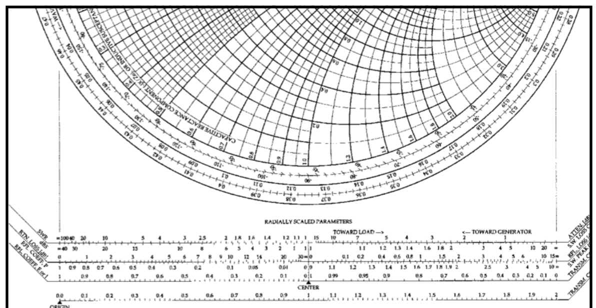
V. Smith Chart





Smith Chart

- Commonly used as graphical representation of a TL.
- Used in hi-tech equipment for <u>design</u> and <u>testing</u> of microwave circuits
- 1 turn (360^o) around the SC = to $\lambda/2$

Network Analyzer

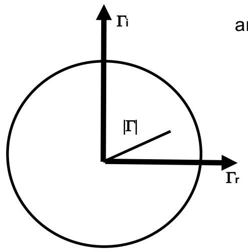
Smith Chart

$Z(l) = Z_o \frac{(1+\Gamma(l))}{(1-\Gamma(l))}$ $Z_L = Z_o \frac{(1+\Gamma_L)}{(1-\Gamma_L)}$

- Use the reflection coefficient real and imaginary parts .

$$\Gamma = |\Gamma| \angle \theta_\Gamma = \Gamma_r + j\Gamma_i = \frac{(Z_L - Z_o)}{(Z_L + Z_o)}$$

and define the normalized Z_L:



$$z_L = \frac{Z_L}{Z_o}$$

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

Now relating to $z = r + jx$

- After some algebra, we obtain two eqs.

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2$$ ***Circles of r***

$$[\Gamma_r - 1]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2$$ ***Circles of x***

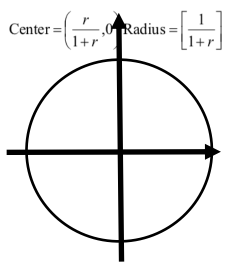
- Similar to general equation of a circle of radius a, center at (h,k)

$$(x-h)^2 + (y-k)^2 = a^2$$

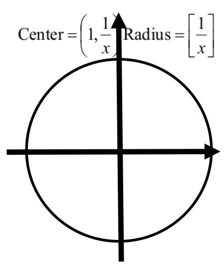
Examples of circles of r and x

$\text{Center} = \left(\frac{r}{1+r}, 0\right)$ $\text{Radius} = \left[\frac{1}{1+r}\right]$

$\text{Center} = \left(1, \frac{1}{x}\right)$ $\text{Radius} = \left[\frac{1}{x}\right]$

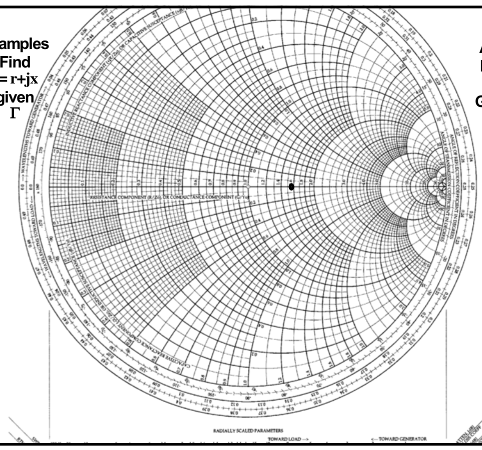


Circles of *r*



Circles of *x*

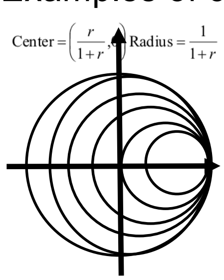
Examples Find z= r+jx given Γ



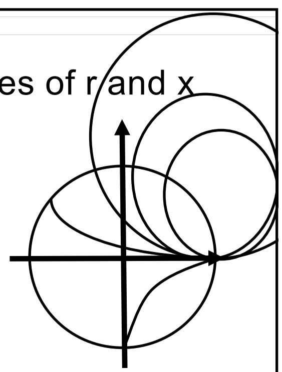
Also Find Γ Given Z

Examples of circles of r and x

$\text{Center} = \left(\frac{r}{1+r}, 0\right)$ $\text{Radius} = \frac{1}{1+r}$



Circles of *r*



Circles of *x*

Find SWR on the SC

- Numerically $s = r$ on the +axis of Γ_r in the SC

Proof:

$$\Gamma = \frac{z_L - 1}{z_L + 1} = (\text{when } z = r) = \frac{r-1}{r+1}$$

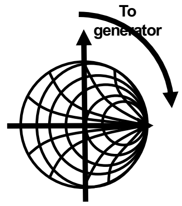
$$\text{but then } \Gamma = \Gamma_r + j0 \equiv \frac{s-1}{s+1}$$

Moving on the TL on the SmithC

- A lossless TL is represented as a circle of constant radius, $|\Gamma|$, or constant *s*

$$\Gamma(l) = \Gamma_L e^{-2j\beta l} = |\Gamma_L| e^{j\theta_\Gamma} e^{-2j\beta l}$$

- *Moving along the line from the load toward the generator, the phase decrease, therefore, in the SC equals to moves clockwisely.*



One-turn on the Smith Chart

- One turn (360º) around the SC = to λ/2 because in the formula below, if you substitute length for half-wavelength, the phase changes by 2π, which is one turn.

$$\Gamma(l) = \Gamma_L e^{-2j\beta l}$$

- Find the point in the SC where Γ=+1,-1, *j*, *-j*, 0, 0.5
- What is *r* and *x* for each case?

Fun facts : Admittance in the SC

- The admittance, $\boldsymbol{y=Y_L/Y_o}$ where $\boldsymbol{Y_o=1/Z_o}$, can be found by moving ½ turn (λ/4) on the TL circle

$$2\beta l = 2\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \pi$$

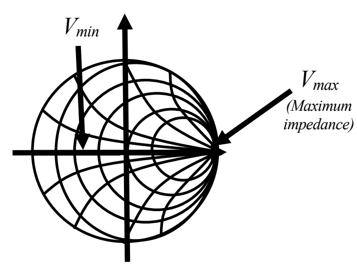
$$z(l=0) = \frac{Z_L}{Z_o} = \frac{1}{Z_o}\frac{V^+\left(1+\Gamma e^{2j\beta l}\right)}{V^+/Z_o\left(1-\Gamma e^{2j\beta l}\right)} = \frac{1+\Gamma e^{2j\beta l}}{1-\Gamma e^{2j\beta l}} = \frac{1+\Gamma e^{2j0}}{1-\Gamma e^{2j0}} = \frac{1+\Gamma}{1-\Gamma}$$

$$z(l=\lambda/4) = \frac{1+\Gamma e^{j\pi}}{1-\Gamma e^{j\pi}} = \frac{1+\Gamma(-1)}{1-\Gamma(-1)} = \frac{1-\Gamma}{1+\Gamma}$$

$$y(l=0) = \left(1/Y_o\right)\frac{V^+/Z_o\left(1-\Gamma e^{2j\beta l}\right)}{V^+\left(1+\Gamma e^{2j\beta l}\right)} = \frac{1-\Gamma e^{j0}}{1+\Gamma e^{j0}} = \frac{1-\Gamma}{1+\Gamma}$$

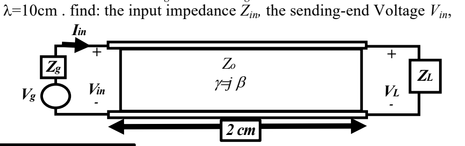
V_{max} and V_{min} on the SmithC

- The $\boldsymbol{\Gamma_r}$ +axis, where $\boldsymbol{r > 0}$ corresponds to $\boldsymbol{V_{max}}$
- The Γ_r -axis, where $\boldsymbol{r < 0}$ corresponds to V_{min}



Exercise: using S.C.

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},



$$z_L = \frac{100+j80}{40} = 2.5+j2$$

$$l_\lambda = .2\lambda$$

$$\Gamma_L = 0.622\angle 23.5^o$$

$$\Gamma(2cm) = 0.622\angle -120^o$$

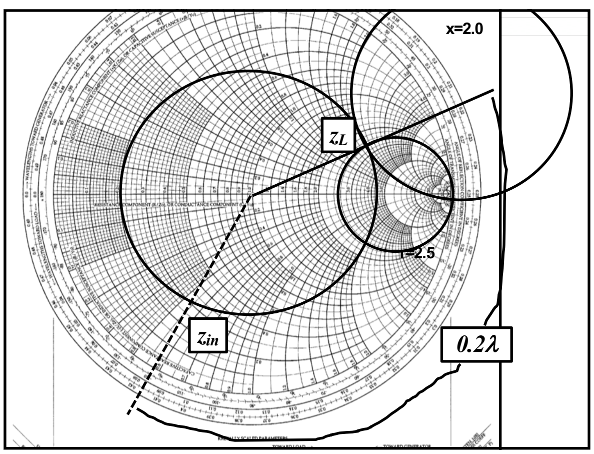
- Load is at .2179λ @ S.C.
- Move .2λ and arrive to .4179λ
- Read $z_{in} = .3 - j.55$

$$Z_{in} = 12 - j22\Omega$$

$$y_{in} = .76 - j1.4\Omega$$

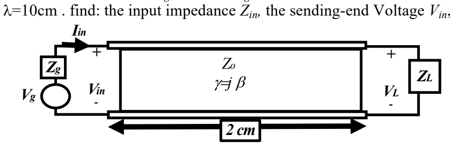
Voltage Divider:

$$V_{in} = \frac{V_g Z_{in}}{Z_{in}+Z_g} = 3.32\angle -0.775 rad$$



Exercise: cont….using S.C.

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},

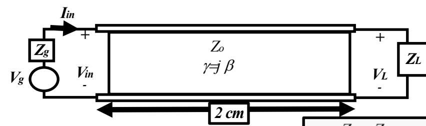


$z_L = 2.5 + j2$ $l_\lambda = .2\lambda$ $\Gamma_L = 0.622\angle 23.5^o$

- Distance from the load (.2179λ) to the nearest minimum & max
- Move to horizontal axis toward the generator and arrive to .5λ (V_{max}) and to .25λ for the $V_{min.}$
- Distance to min=.5-.2179=.282λ
- Distance to 2nd voltage maximum is .282λ +.25λ=.482 See drawing

Exercise : using formulas

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},



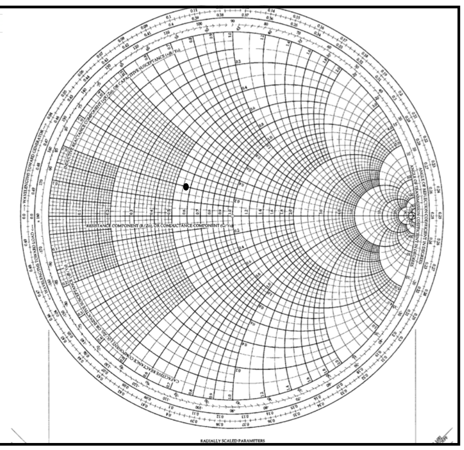
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.62\angle 23.4^o$$

$$Z_{in} = Z(2cm) = 40\frac{\left((100 + j80) + j40\tan\frac{2\pi}{5}\right)}{\left(40 + j(100 + j80)\tan 2\pi/5\right)}$$

$$\Gamma(2cm) = \Gamma_L e^{-2j\beta l} = \Gamma_L\angle -144^o = 0.62\angle -120.6^o$$

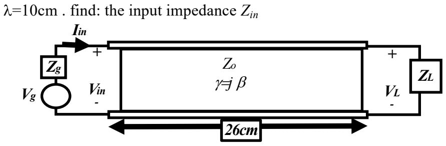
Voltage Divider: $Z_{in} = 12.2 - j21.17\,\Omega$

$$V_{in} = \frac{V_g Z_{in}}{Z_{in} + Z_g} = 3.30\angle -0.766 rad$$



Another example:

- A 26cm lossless TL is connected to load Z_L=36-j44 Ω and Z_o=100Ω, λ=10cm . find: the input impedance Z_{in}



$z_L = .36 - j.44$ $l_\lambda = 2.6\lambda = 5(.5\lambda) + .1\lambda$ $\Gamma_L = 0.54\angle -127^o$

- Load is at .427λ @ S.C.
- Move .1λ and arrive to .527λ (=.027λ)
- Read $z_{in} = .31 + j.16$

$Z_{in} = 31 + j16\Omega$

Distance to first V_{max}:

$l_{min} = 0.5\lambda - .427\lambda = 0.028\lambda$

$l_{max} = 0.028\lambda + .25\lambda = .278\lambda$

Exercise 11.4

- A 70 Ω lossless line has s =1.6 and at the load θ_Γ=300°. If the line is 0.6λ long, obtain Γ, Z_L, Z_{in} and the distance of the first minimum voltage from the load.
- Answer $\Gamma_L = 0.23\angle 300^o$

$$\Gamma = \frac{s-1}{s+1}$$

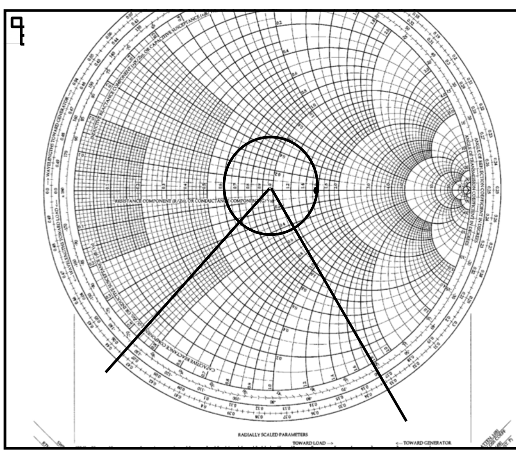
$z_L = 1.15 - j.48$

$Z_L = Z_o z_L = 80.5 - j33.6\Omega$

- The load is located at: $.3338\lambda$
- Move to .4338 λ and draw line from
- center to this place, then read where it crosses you TL circle.
- Distance to V_{min} in this case, l_{min}=.5λ-.3338λ= $\sim\lambda/6$

$z_{in} = 0.68 - j.25$

$Z_{in} = 47.6 - j17.5\,\Omega$



Java Applet : Smith Chart

- http://education.tm.agilent.com/index.cgi?CONTENT_ID=5

The end of INEL 4151

- Material extra for (INEL 4155)