$\frac{123}{100}$

Nombre: Clave

INEL 3105: Análisis de Circuitos Eléctricos I

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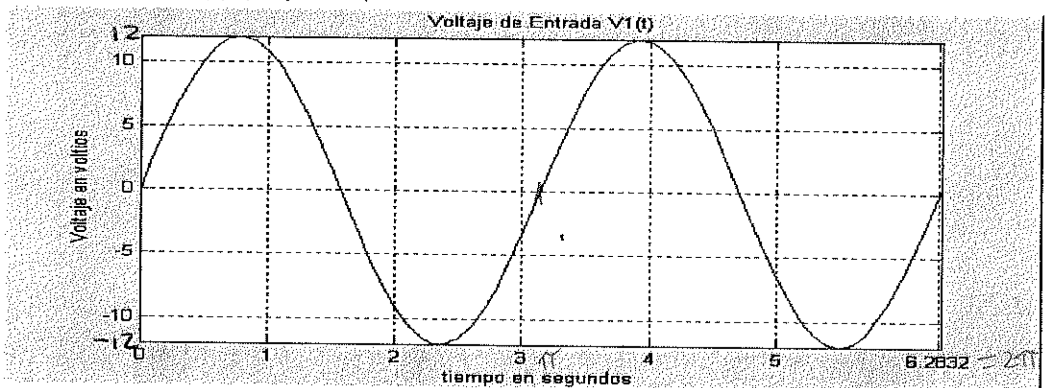
Examen # 3

I. Dado el voltaje $V_1(t)$: (25 puntos)

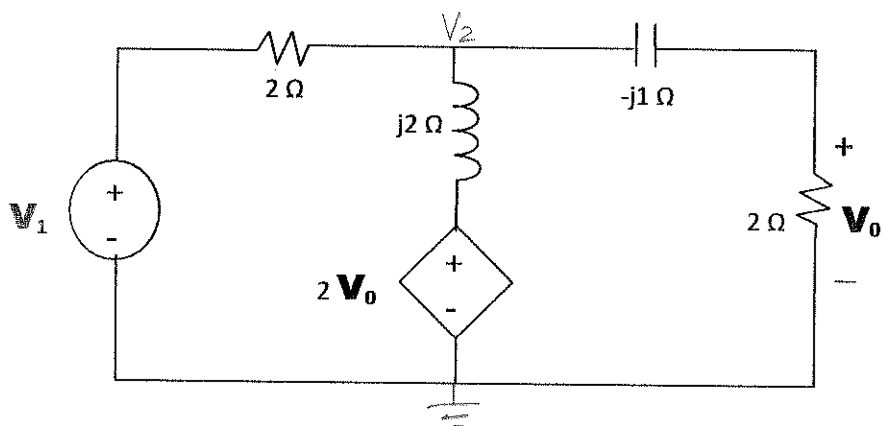


a. Halle la capacitancia del capacitor en el circuito

b. Halle la inductancia del inductor en ele circuito

c. Halle $V_0(t)$.

Mezclar Dominio t y f det. V_1 (-2)



(2) (1)

$$\omega \pi = 2\pi \Rightarrow \omega = 2 \Rightarrow V_1(t) = \sin(2t)(V) \Rightarrow V_1 = 12\angle 0^\circ \ (V)$$

a) $\frac{-j}{\omega C} = \frac{-j}{2C} = -j1 \Rightarrow C = \frac{1}{2} F = 0.5 F$ (2)

b) $j\omega L = j2L = j2 \Rightarrow L = 1H$ (2)

c) (7) $\frac{V_2 - V_1}{2} + \frac{V_2 - 2V_0}{j2} + \frac{V_2 - V_0}{-j1} = 0 \Rightarrow V_2(j+1-2) + V_0(-2+2) = jV_1$

(7) $V_0 = V_2\left(\frac{2}{2-j}\right) = \left(\frac{jV_1}{-1+j}\right)\left(\frac{2}{2-j}\right) = \frac{+2V_1}{(+1+j)(2-j)} = \frac{2V_1}{3+j}$

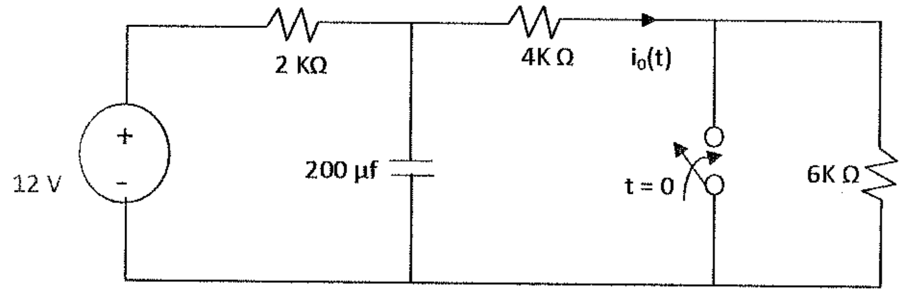
$$V_0 = \frac{2(12)}{3+j}(V) = \frac{24(3-j)}{10}(V) = \frac{12}{5}(3-j)(V) = \frac{36}{5} - j\frac{12}{5}$$

(2) $V_0 = \frac{12}{5}\sqrt{10} \angle \tan^{-1}\left(\frac{-1}{3}\right) V = \frac{12\sqrt{10}}{5}\angle -18.43^\circ = 7.589 \angle -0.322$ radianes

(2) $V_0(t) = \frac{12\sqrt{10}}{5}\sin\left(2t - \tan^{-1}\left(\frac{1}{3}\right)\right)$ Voltios

$= 7.58\cos(2t - 108.43^\circ)(V) = 7.58\cos(2t - 1.893)\ (V)$

II. Halle $i_0(t) = \left(K_1 + K_2 e^{\frac{-t}{\tau}}\right)u(t) + K_3[1 - u(t)]$ en el siguiente circuito. (25 puntos) + 3 bono



$i_0(0_-) = \frac{12}{2+4+6}\ mA = 1\ mA = k_3$ 3

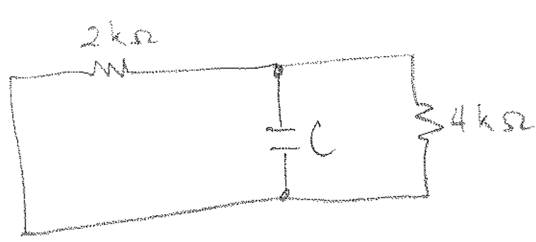
$V_c(0_-) = V_c(0_+) = 12\left(\frac{4+6}{12}\right) = 10\ V$ 6

$i_0(0_+) = \frac{10V}{4k\Omega} = \frac{5}{2}\ mA = k_1 + k_2$ 4

$i_0(\infty) = \frac{12V}{(2+4)k\Omega} = 2\ mA = k_1$ 4

$k_2 = \left(\frac{5}{2} - 2\right) mA = \frac{1}{2}\ mA$ 1

$\tau = R_{Th} C$



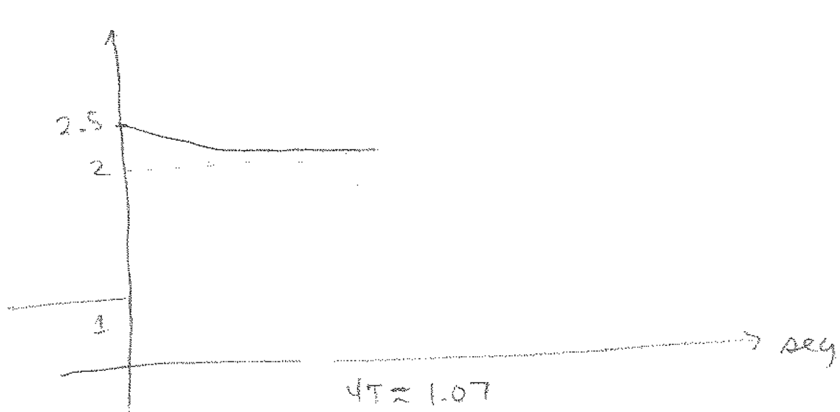
3 $R_{Th} = \frac{2(4)}{2+4} = \frac{4}{3}\ k\Omega$

2 $\tau = \frac{800}{3}\ ms \approx 266.7\ ms$

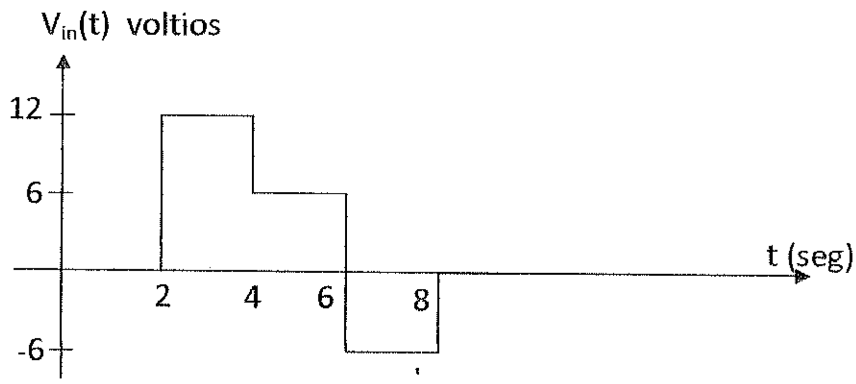
$\therefore$

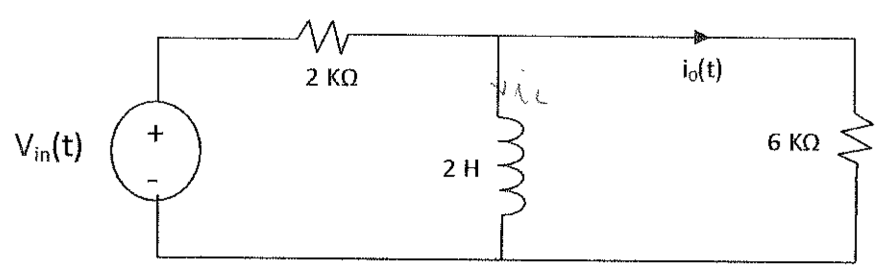
2 $i_0(t) = \left(2 + \frac{1}{2} e^{-3.75t}\right) u(t) + 1 - u(t)$ (mA) $t \to ms$

3



III. Dado Vin(t) y el siguiente circuito: (25 puntos) → (o bonos



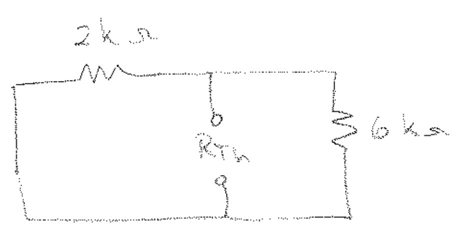


a. Exprese Vin(t) en función de u(t).
b. Halle $i_o(t)$. ¿Le ayuda en algo usar superposición y linealidad?

8 a) $V_{in}(t) = 12u(t-2) - 6u(t-4) - 12u(t-6) + 6u(t-8)$ (voltios)

b) Aprovechando linealidad y superposición asumimos $V_{in} = u(t)$ 1

$\tau = \frac{L}{R_{Th}}$



$\Rightarrow R_{Th} = \frac{12}{8} k\Omega = \frac{3}{2} k\Omega$ 2

$\tau = \frac{2}{(3/2)} = \frac{4}{3} ms$ 1

2 $i_L(0-) = \frac{V_{in}(0-)}{2k\Omega} = 0 = i_L(0+) = i_{os}(0-)$

2 $i_{os}(0+) = \frac{1}{8} mA$

2 $i_L(\infty) = \frac{V_{in}(\infty)}{2k\Omega} = \frac{1}{2} mA$

$\Rightarrow i_{os}(\infty) = 0$

2 $i_{os}(t) = \frac{1}{8} e^{-750t} u(t)$ si $V_{in}(t) = u(t)$

detrás

$\therefore$

$$i_o(t) = 12\left[\frac{1}{8}e^{-750(t-2)}\right]u(t-2) - 6\left[\frac{1}{8}e^{-750(t-4)}u(t-4)\right] +$$

$$-12\left[\frac{1}{8}e^{-750(t-6)}\right]u(t-6) + 6\left[\frac{1}{8}e^{-750(t-8)}\right]u(t-8) \text{ (mA)}$$

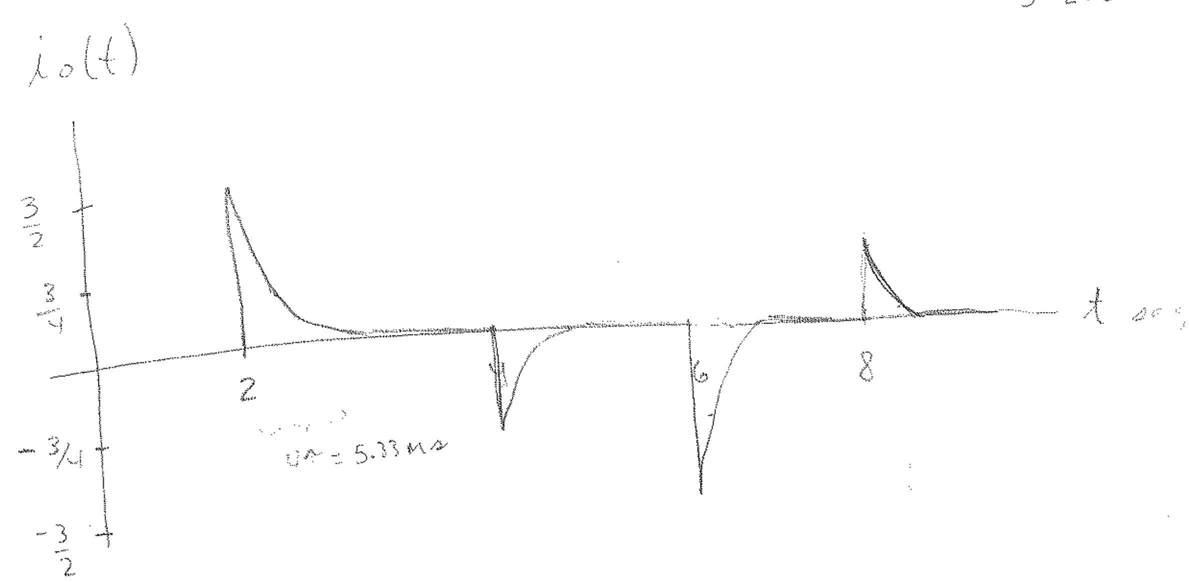
(4)

$$i_o(t) = \frac{3}{2}e^{-750(t-2)}u(t-2) - \frac{3}{4}e^{-750(t-4)}u(t-4) - \frac{3}{2}e^{-750(t-6)}u(t-6)$$

$$+ \frac{3}{4}e^{-750(t-8)}u(t-8)$$

$$i_o(t) = 6.72\,e^{-0.75t}u(t-2) - 15.064\,e^{-0.75t}u(t-4) - 135.026\,e^{-0.75t}u(t-6) + 302.572\,e^{-0.75t}u(t-8)$$

(5)

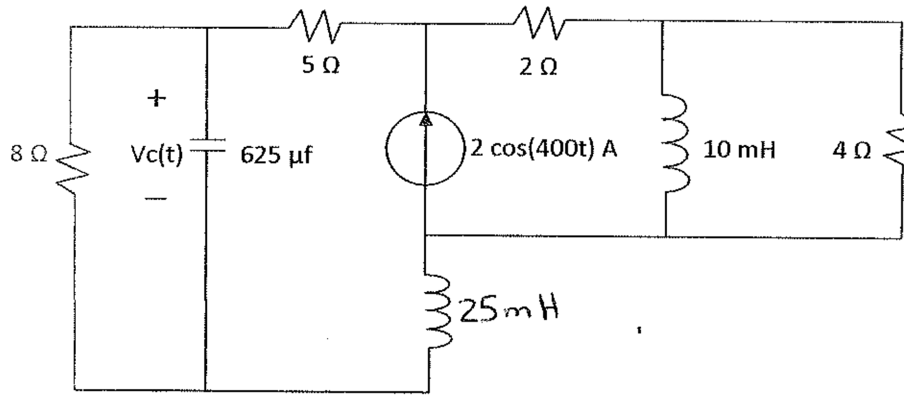


(5)

$4\tau = \frac{16}{3}$ ms que es un tiempo despreciable

$\therefore$ $i_o(t) \approx 0$ en la escala de tiempo dada.

IV. Halle Vc(t). (25 puntos)

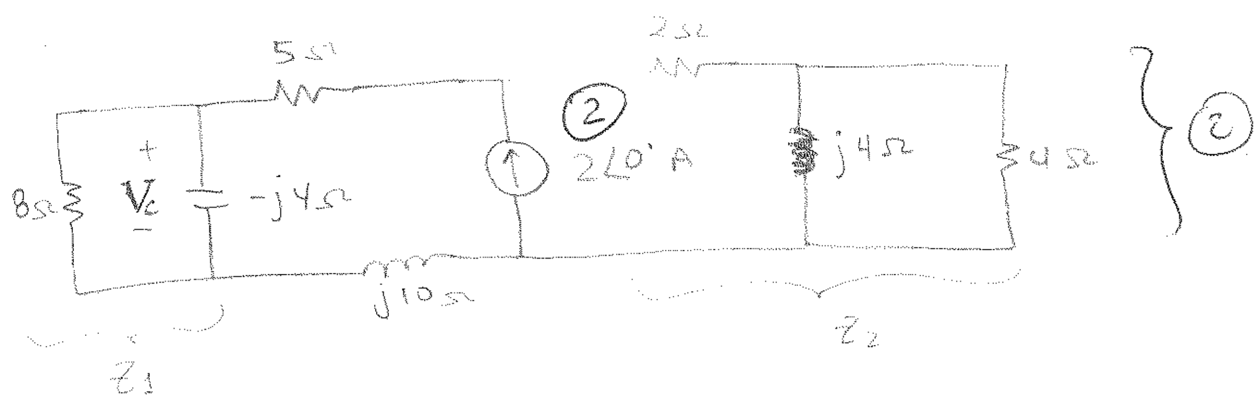


(4)

$V_c(t) = 2.323\,\cancel{\sin(400t)}$
$\sin(400t - 76.57)\,(V)$

(2) $Z_C = \dfrac{-j}{(400)625\mu} = \dfrac{-j}{4(0.0625)} = \dfrac{-j}{4(0.25)^2} = -j4\,\Omega$

(2) $Z_{L_{10}} = j4000\,m = j4\,\Omega \qquad Z_{L_{25}} = j(400)25m = j10\,\Omega$

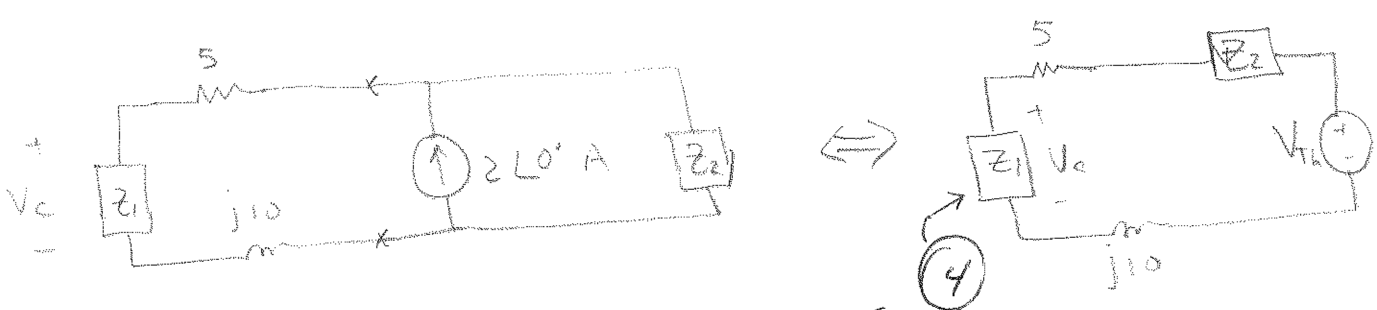


(3) $Z_1 = \dfrac{-j32}{8-j4}\,\Omega = \dfrac{-j32(8+j4)}{64+16} = \dfrac{-j2(8+j4)}{5} = \dfrac{8}{5}(1-j2)\,\Omega$

(3) $Z_2 = 2 + \dfrac{j16}{4+j4} = 2 + \dfrac{j4}{1+j} = 2 + j2(1-j) = 4 + j2$

⇒

(1)



$V_{Th} = I_N Z_{Th} = 2Z_2\ (V)$

(6)

$$V_c = V_{Th}\left(\frac{Z_1}{Z_1+Z_2+5+j10}\right) = \frac{2Z_1Z_2}{Z_1+Z_2+5+j10}\ (V) = \frac{\frac{16}{5}(1-j2)(4+j2)}{\frac{53}{5}+j\frac{44}{5}}$$

$$V_c = \frac{32(1-j2)(2+j)}{53+j44} = \frac{32(4-3j)}{53+j44} = \frac{32(5)\angle -36.87}{68.89\angle 39.69} = 2.323\angle -76.57^\circ$$

①

$\therefore$

$V_c(t) = 2.323 \cos(400t - 76.57)$ (V) ③

V. (Bono de 10 puntos)

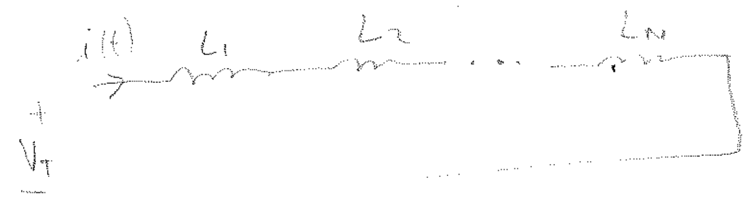
a. Demuestre que para inductores en serie, la inductancia equivalente es la suma de las inductancias de los inductores en serie.

b. Demuestre que para capacitores en paralelo, la capacitancia equivalente es la suma de las capacitancias delos capacitores en paralelo.

c. ¿Cuál es la capacitancia equivalente de dos capacitores en serie?

d. ¿Cuál es la inductancia equivalente de dos inductores en paralelo?

a)



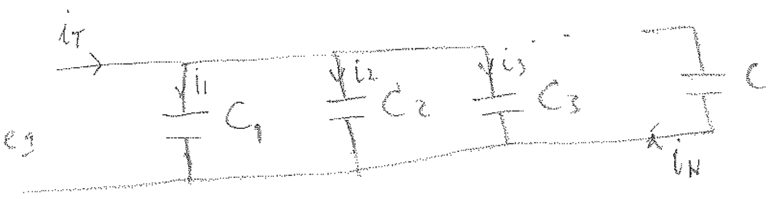
3

$$V_T = \sum_{K=1}^{N} V_{LK} = \sum_{K=1}^{N} L_K \frac{di(t)}{dt} = \frac{di(t)}{dt} \sum_{K=1}^{N} L_K = \frac{di(t)}{dt} L_{eq}$$

$$\therefore \quad L_{eq} = \sum_{k=1}^{N} L_k$$

b)

Ceq

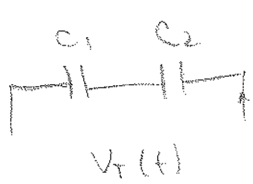


3

$$i_T = \sum_{K=1}^{N} i_K = \sum_{K=1}^{N} C_K \frac{dv_c}{dt} = \frac{dv_c}{dt} \sum_{K=1}^{N} C_K = \frac{dv_c}{dt} C_{eq}$$

$$\therefore \quad C_{eq} = \sum_{K=1}^{N} C_K$$

c)

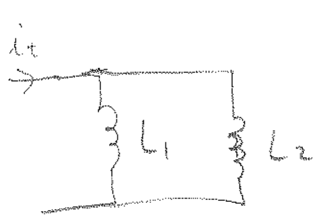


$$V_T = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

2

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

d)



$$\frac{di_t}{dt} = \frac{V_L}{L_{Eq}} = \frac{V_L}{L_1} + \frac{V_L}{L_2}$$

2

$$\Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$