Exercise Nº 4 12 pts.

The figure presents a flexible connection consisting of rubber pads (thickness t = 10 mm) bonded to steel plates. The pads are 200mm long and 100mm wide and the applied load is \( P = 15 \text{kN} \):

a) Determine the shear modulus (transversal elastic modulus) \( G \) of the rubber knowing that the relative horizontal displacement of the central steel plate is \( \delta = 1.90 \text{mm} \).

b) What would be the safety factor of the steel plates if their thickness is 5mm and the yield strength of this steel is \( 95 \text{ MPa} \).

\[ \tau = \frac{G}{2} \left( \frac{r}{t} \right) \delta \]

\[ \tau = \frac{G}{2} \left( \frac{10 \text{ mm}}{10 \text{ mm}} \right) \frac{1.9 \text{ mm}}{10 \text{ mm}} = 0.19 \text{ MPa} \]

\[ \tau = \frac{P/2}{b \cdot t} = \frac{15,000 \text{ N}}{2 \times 0.1 \text{ m} \times 0.2 \text{ m}} = 0.04 \text{ MPa} \]

\[ \tau_{\text{ave}} = 375 \text{ kPa} \]

\[ G = \frac{0.375 \text{kPa}}{0.19} = 1.97 \text{ MPa} \]

b) Consider the most affected plate: central one (load = \( P \))

\[ \sigma = \frac{P}{A} = \frac{15,000 \text{ N}}{0.1 \text{ m} \times 0.005 \text{ m}} = 30 \text{ MPa} \]

\[ n = \frac{\sigma_{y}}{\sigma} = \frac{95 \text{ MPa}}{30 \text{ MPa}} = 3.17 \]
The following arm is rotating at high speed around the axis A. The material in AB is made of a titanium alloy with a Young's modulus $E = 110$ GPa and a density of $4,500$ kg/m$^3$. The mass of the connecting rod AB is negligible and should be considered as a rigid body (it does not experience deformations). All measurements are in meters. Calculate the stress in the connecting rod AB if the rotating speed was 200 rpm ($1$ rpm $= \frac{2\pi}{60}$ rad/s).

\[ \ell_c = 5.25 \]
\[ \ell_B = 3.75 \]
\[ d = 0.25 \]
\[ d_{BC} = 0.45 \]

\[ dm = \rho A dx \]

\[ dF = dm \omega^2 x = \rho A \omega^2 x \, dx \]

\[ F = \int_{\ell_B}^{\ell_c} \rho A \omega^2 x \, dx = \rho A \omega^2 \left( \ell_c^2 - \ell_B^2 \right) / 2 \]

\[ \sigma = \frac{F}{A} = \rho \frac{\omega^2}{2} \left( \ell_c^2 - \ell_B^2 \right) = \frac{4500 \, kg/m^3 \times (200 \times \frac{2\pi}{60})^2}{2} \left( 5.25^2 - 3.75^2 \right) \, m^2 \]

\[ \sigma = 13.32 \times 10^6 \, kg/m^3 \times m^2/s^2 = 13.32 \times 10^6 \, N/m^2 \]

\[ \sigma = 13.32 \, MPa \]

\[ \delta = \sigma \times (\ell_B - \ell_c) = \frac{13.32 \, MPa \times (5.25 - 3.75)}{110,000 \, MPa} \times m \]

\[ \delta = 181.7 \times 10^{-6} \, m = 0.0018 \, mm \]
The change in diameter of large steel bolt is carefully measured as the nut (B) is tightened.
Knowing that \( E = 200 \text{ GPa} \) and \( \nu = 0.29 \), determine the internal force in the bolt, if the diameter is observed to decrease by 13\( \mu \text{m} \) (0.013 mm). Then calculate the bearing stress on the bolt head A.

\[
\varepsilon' = \frac{d\varepsilon}{d\varepsilon} = -\frac{0.013}{80} = -0.0001625
\]

\[
\varepsilon = -\nu\varepsilon
\]

or \( \varepsilon = -\frac{\varepsilon'}{\nu} \)

\[
\varepsilon = -\frac{0.0001625}{0.29} = 5.6 \times 10^{-4}
\]

\[
\varepsilon = \frac{P}{A} = \frac{E}{E} = \frac{P}{A} = \frac{A E E}{AA} = \frac{\pi d^2}{4} \times \varepsilon
\]

\[
P = \frac{\pi}{4} (0.08 \text{ m})^2 \times 200 \times 10^9 \frac{N}{m^2} \times 5.6 \times 10^{-4}
\]

\[
P = 563 \text{ kN}
\]

Area = \( AO - A_{\text{bolt}} \)

\[
AO = 6 \times 6 = 36 \frac{b \times b}{2} \times \frac{b \times b}{2} \times \frac{b \times b}{2} = \frac{3}{8} b^2
\]

\[
\text{Area} = \frac{3}{8} b^2 - \frac{\pi}{4} d^2 = \frac{1}{4} \left( \frac{3\sqrt{3}}{2} \times 0.139^2 - \pi \times 0.08^2 \right) m^2
\]

\[
\text{Area} = 7.52 \times 10^{-3} m^2
\]

\[
\sigma_{\text{bear}} = \frac{P}{\text{Area}} = \frac{563 \times 10^3 N}{7.52 \times 10^{-3} m^2} = 74.8 \times 10^6 \text{ Pa}
\]

\[
\sigma_{\text{bearing}} = 74.8 \text{ MPa}
\]