NOISE

- Interference noise - caused by unwanted interaction between the circuit and
  the outside
- Inherent noise - Internal to the devices. Due to random phenomena in
  devices.
- Signal-to-noise ratio (SNR)
  \[ SNR = 20 \log \left( \frac{X_s}{X_n} \right) \]
  where \( X_s \) is the rms value of the signal, \( X_n \) is the rms value of noise.

1. Noise Statistics

- rms value of noise
  \[ X_n = \frac{1}{T} \sqrt{\int_0^T x_n^2(t)dt} \]
  where \( T \) is the averaging interval and \( x_n \) is the instantaneous noise signal
  - i.e. a voltage or a current.
- mean square value - average power dissipated in a 1Ω resistor. Equal to \( X_n^2 \).
- crest factor: ratio of peak value to rms value. Since noise is random, peak
  value can’t be predicted. The probability of the peak value exceeding a
  particular value can however be estimated. For a gaussian distribution:

<table>
<thead>
<tr>
<th>CF</th>
<th>Probability</th>
</tr>
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<tbody>
<tr>
<td>&gt;1</td>
<td>32%</td>
</tr>
<tr>
<td>&gt;2</td>
<td>4.6%</td>
</tr>
<tr>
<td>&gt;3</td>
<td>0.27%</td>
</tr>
<tr>
<td>&gt;3.3</td>
<td>0.1%</td>
</tr>
<tr>
<td>&gt;4</td>
<td>0.0063%</td>
</tr>
</tbody>
</table>

2. Noise Summation

\[ X_n^2 = \frac{1}{T} \int_0^T (x_{n1}(t) + x_{n2}(t))^2 dt = X_{n1}^2 + X_{n2}^2 + \frac{2}{T} \int_0^T x_{n1}(t)x_{n2}(t)dt \]

The integral is zero for uncorrelated signals. Thus in this case

\[ X_n = \sqrt{X_{n1}^2 + X_{n2}^2} \]
3. Noise Spectra

- Noise power $X_n^2$ is spread over a band of frequencies.
- Noise power depends on the frequency band width and location.
- Voltage noise power density: $e_n^2(f) = \frac{dE_n^2}{df}$, where $E_n^2$ is the mean square value of noise.
- Current noise power density: $i_n^2(f) = \frac{dI_n^2}{df}$, where $I_n^2$ is the mean square value of noise.
- Noise power densities: noise power over a 1Hz frequency band. Plot versus frequency to get a visual description of noise power distribution over a frequency band.
- In the frequency domain,
  \[ X_n = \sqrt{\int_{f_L}^{f_H} x_n^2(f) df} \]
  where $x_n$ represents voltage or current.

3.1. White Noise.

- Uniform spectral density. For white noise
  \[ X_n = x_{nw} \sqrt{f_H - f_L} \]
  and
  \[ X_n^2 = x_{nw}^2 (f_H - f_L) \]
- Noise power is proportional to bandwidth.

3.2. 1/f Noise.

- Power density is inversely proportional to frequency.
  \[ x_n^2 = \frac{K^2}{f} \]
  where $K$ is a constant.
- $x_n = K/\sqrt{f}$ → bode plot slope will be -10dB/dec.
- Noise rms is
  \[ X_n = K \sqrt{\ln(f_H/f_L)} \]
  and noise power is
  \[ X_n^2 = K^2 \ln(f_H/f_L) \]
  This means that 1/f noise will have the same power content in each decade.

3.3. IC Noise.

- Mixture of white and 1/f noise.
• $x$ is either voltage or current. $e_n^2 = e_{nw}^2 \left( \frac{f_H}{f_L} + 1 \right)$; $i_n^2 = i_{nw}^2 \left( \frac{f_H}{f_L} + 1 \right)$

• For voltage

$$E_n = e_{nw} \sqrt{E_c \ln \left( \frac{f_H}{f_L} \right)} + f_H - f_L$$

• Similar expression for current.
• To minimize noise limit bandwidth as necessary.

4. Noise Dynamics

• Noise analysis: find total rms noise at the output of a circuit given the noise density at its input and its frequency response.

• For a voltage amplifier, output noise = gain times input noise

$$e_{no}(f) = |A_n(jf)|e_{ni}(f)$$

• $E_{no}^2$ = total output rms noise = $\int_0^\infty e_{no}^2(f)df$, or

$$E_{no} = \sqrt{\int_0^\infty |A_n(jf)|^2e_{ni}^2(f)df}$$

• Similarly, for a transimpedance amplifier

$$E_{no} = \sqrt{\int_0^\infty |Z_n(jf)|^2e_{ni}^2(f)df}$$

• Noise equivalent bandwidth (NEB): For a gain with a dominant pole,

$$E_{no} = e_{nw} \sqrt{\int_0^\infty \frac{df}{1 + (f/f_o)^2}} = e_{nw} \sqrt{\pi f_o/2} = e_{nw} \sqrt{1.57f_o}$$

Works as a brick-wall filter with bandwidth $1.57f_o$. 

![Noise Diagram](image-url)
More generally,

\[ NEB = \frac{1}{A_{\text{rms}}^2} \int_0^\infty |A_n(jf)|^2 df \]

4.1. Example 7.3.

From 1 Hz to 1kHz:

\[ E_n = e_{n,v} \sqrt{f_{ce} \ln(f_H/f_L) + f_H - f_L} \]

with \( e_{n,v} = 20n \text{V}/\sqrt{\text{Hz}}, f_{ce} = 100 \text{Hz}, f_L = 1 \text{Hz} \) and \( f_H = 1kHz \). The result is \( E_{n,10} = 0.822 \mu\text{V} \).
From 1kHz to 10kHz, $e_{n_o}$ increases with $f$ at a rate of $1\text{dec/dec}$. So let $e_{n_o}(f) = \left(\frac{20\text{nV}}{\sqrt{Hz}}\right) \times (f/10^3) = 2 \times 10^{-11} f$

and

$$E_{n_{o2}} = 2 \times 10^{-11} \sqrt{\int_{10^3}^{10^8} f^2 \, df} = 11.5\mu V$$

For $f > 10^4 Hz$, we have white noise with $e_{nw} = 200nV/\sqrt{Hz}$ going through a low-pass filter with corner frequency $f_o = 100kHz$. Using

$$E_{n_{o3}} = e_{nw} \sqrt{1.57f_o}$$

$$E_{n_{o3}} = (200nV/\sqrt{Hz}) \times \sqrt{1.57 \times 10^5 - 10^4} = 76.7\mu V$$

The total is the root of the sum of the square of the three components

$$E_{n_o} = \sqrt{E_{n_{o1}}^2 + E_{n_{o2}}^2 + E_{n_{o3}}^2} = 77.5\mu V$$

5. SOURCES OF NOISE

5.1. Johnson (Thermal) Noise.

- Due to vibrations in solid
- White noise
- Present in all passive devices - resistors, inductors, capacitors.
- Modelled as a voltage source and Thevenin resistance:

5.2. Shot Noise.

- Caused by random charge crossing across potential barriers (like pn-junctions)
- Uniform power density: $i_n^2 = 2qI$, where $I$ is the current through the junction.

5.3. Flicker Noise.

- Due to traps caused by contamination and crystal dislocations on base-emitter junction.
- Also called contact noise
- $1/f$ noise:

$$i_n^2 = K\frac{I^a}{f}$$

where $K$ is a device constant, $I$ is the devices' dc current and $a$ is a device constant between $\frac{1}{2}$ and 2.
- Present in active devices.
- Also present in resistors, where it is called excess noise.
  - Wire-wound resistors are the quietest.
  - Carbon composition resistors can be an order of magnitude noisier.
– Carbon- and metal-film are intermediate.
– Important only if resistor’s d.c. current is large.

5.4. Avalanche Noise.
• Present in Zener diodes, where it is very intense and make them very noisy.

6. Transistor Noise

6.1. BJT.
• Exhibit all the above but Avalanche.
• \( e_n \) is thermal from \( r_b \) at the base + collector current shot noise reflected back to the base: \( e_n^2 = 4kT \left( r_b + \frac{1}{2g_m} \right) \)
• \( i_n \) is base current shot + flicker at base + collector current shot noise reflected to the base: \( i_n^2 = 2q \left( I_B + K_1 \frac{I_D}{f} + \frac{I_G}{|V_{DS}|} \right) \)
• \( r_b \) is the intrinsic base resistance, \( I_B \) and \( I_C \) are dc base and collector currents, \( g_m \) is the device transconductance, \( K \) and \( a \) are device constants and \( \beta \) is the current gain as a function of frequency.

6.2. JFET.
• \( e_n \): channel thermal noise plus drain-current flicker noise
  \[ e_n^2 = 4kT \left( \frac{2}{3g_m} + K_2 \frac{I_D}{g_m} \right) \]
• \( i_n \) is negligible at room temperature but increases and might be significant at high temperatures.
  \[ i_n^2 = 2qI_G + \left( \frac{2\pi fC_{gs}}{g_m} \right)^2 \left( 4kT \frac{2}{3g_m} + K_3 \frac{I_D}{f} \right) \]
• \( g_m \) is the transconductance; \( I_D \) is the dc drain current; \( I_G \) is the gate leakage current; \( K_2, K_3 \) and \( a \) are device constants and \( C_{gs} \) is the gate-to-source capacitance.
• Due to low \( g_m \) FET’s \( e_n \) is usually higher then BJT’s, but \( i_n \) is smaller.

6.3. MOSFET.
• \( e_n \): thermal noise from channels resistance and drain-current Flicker noise
  \[ e_n^2 = 4kT \frac{2}{3g_m} + K_4 \frac{1}{W L f} \]
• Flicker noise is inversely proportional to transistor area \( W \times L \) so it is reduced by using large geometries.
• \( i_n^2 = 2qI_G \) is negligible at room temperature; increases with temperature.
• \( g_m \) is transconductance, \( K_4 \) is a device constant, \( W \) and \( L \) are channel width and length.
7. Op Amp Noise

- Modelled like offsets: with two current sources and one voltage source.

\[ e_{n_i}^2 = e_n^2 + i_{np}^2 R_3^2 + i_{R_3} R_3^2 + (i_{nn} + i_{R_1} + i_{R_3})(R_1 || R_2)^2 \]

using \( i_{R_3} = \frac{4kT}{R_3} \),

\[ e_{n_i}^2 = e_n^2 + i_{np}^2 R_3^2 + 4kTR_3 + (i_{nn} + \frac{4kT}{R_1} + \frac{4kT}{R_2})(R_1 || R_2)^2 \]

\[ e_{n_i}^2 = e_n^2 + i_{np}^2 R_3^2 + 4kTR_3 + i_{nn}^2 (R_1 || R_2)^2 + \frac{4kT(R_1 + R_2)}{R_1 R_2} \left( \frac{R_1 R_2}{R_1 + R_2} \right)^2 = e_n^2 + i_{np}^2 R_3^2 + i_{nn}^2 (R_1 || R_2)^2 + 4kT (R_3 + R_1 || R_2) \]

- Superposition:

- \( i_{np} = i_{nn} = i_n \),

\[ e_{n_i}^2 = e_n^2 + i_n^2 R_3^2 + 4kTR_3 \]

where \( R_s = R_3 + R_1 || R_2 \), \( R_{s2} = R_3^2 + (R_1 || R_2)^2 \).
• Setting $R_3 = 0$ reduces noise.
• $e_n$ dominates for low values of $R_i$: it is called the short-circuit noise.
• For $R_s \to \infty$, $e_{ni} \approx i_n^2 R_{2n}^2$: $i_n$ is called the open-circuit noise.
• The output noise is amplified by:

$$A_v = \frac{1 + \frac{R_3}{R_1}}{\sqrt{1 + (f/f_A)^2}} = \frac{A_0}{\sqrt{1 + (f/f_A)^2}}$$

• The total output rms noise is

$$E_{no} = A_0 \times \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2}$$

where

$$E_1^2 = e_{nw}^2 \left( f_{ce} \ln \frac{f_A}{f_L} + 1.57 f_A - f_L \right)$$

$$E_2^2 = R_2^2 \left( f_{cip} \ln \frac{f_A}{f_L} + 1.57 f_A - f_L \right)$$

$$E_3^2 = (R_1 || R_2)^2 i_{nw}^2 \left( f_{cep} \ln \frac{f_A}{f_L} + 1.57 f_A - f_L \right)$$

and

$$E_4^2 = 4kT (R_3 + R_1 || R_2) (1.57 f_A - f_L)$$

$f_L$ is $1/T_{obs}$ where $T_{obs}$ is the averaging time used to measure the output.

• For low-noise designs, use op amps with low $e_{nw}$ and low corner frequencies $f_{ce}$ and $f_{cn}$.

• The total rms input noise can be obtained by dividing the signal dc gain $A_{x0}$

$$E_{ni} = \frac{E_{no}}{|A_{x0}|}$$

and the signal-to-noise ratio from

$$SNR = 20 \log_{10} \frac{V_{(rms)}}{E_{ni}}$$