For the following problems refer to the semiconductor resistor designed in the example on section 1.1.2 of the lecture notes. It is acceptable to substitute finite differences for partial derivatives graphically.

1. Determine the surface donor concentration if the conductivity profile shown in figure 2 correspond to an N-type device. (10 points)

2. Estimate the temperature coefficient of the 100Ω resistor. The temperature coefficient is defined as:

   \[ \alpha = \frac{1}{R_0} \frac{\partial R}{\partial T} \]

   Assume room temperature (300K) and that the linear thermal expansion coefficient of Silicon is \( 3 \times 10^{-6} \) parts per degree Kelvin. (20 points)

3. Find the voltage that, when applied to the resistor, will increase the room-temperature incremental resistance to twice its nominal value. (20 points)
\[ \alpha = 5 \times 10^{-4} + \frac{3}{4} \frac{1}{\beta T} \frac{\partial}{\partial T} \left( \frac{1}{\beta T} \right) \]

\[ \sigma = q \mu n N_0 \]

Since \( q \) is a constant and all donors are ionized at room temperature,

\[ \sigma(T) = q \mu_n \left[ \mu(T) \right] \]

\[ \frac{2}{\beta T} \frac{\partial}{\partial T} \left( \frac{\partial}{\partial T} \right) \sigma = \frac{\partial}{\partial T} \left( \frac{1}{\beta T} \right) \frac{\partial}{\partial T} \sigma = -\frac{1}{\beta^2} \frac{\partial \sigma}{\partial T} \]

\[ \alpha = 3 \times 10^{-5} \text{ K}^{-1} - \beta \left( \frac{1}{\beta T} \right) \frac{\partial}{\partial T} \]

\[ = 3 \times 10^{-5} \text{ K}^{-1} - \frac{1}{q \mu_n \left[ \mu(T) \right]} \frac{\partial}{\partial T} \]

\[ \mu_n = 88 \text{ T}_n^{-0.59} + \frac{1250 \text{ T}_n^{-2.13}}{1 + \left( \frac{N}{1.24 \times 10^{14} \text{ cm}^{-3}} \right)^{0.88} \text{T}_n^{0.194}} \]

\( \mu_n \) = average donor conc.

\( \beta = 4.18 \text{ (cm}^2\text{v}^{-1}\text{s}^{-1}) \) \quad \text{(from example; see page 7 notes)}

\[ \beta = 4.18 \frac{1}{\text{cm}^2\text{v}^{-1}\text{s}^{-1}} \]

\[ \beta = \frac{4.18 \text{ (cm}^2\text{v}^{-1}\text{s}^{-1})}{1.6 \times 10^{19} \text{ cm}^{-3}} \times (1450 \text{ cm}^2/\text{v} \cdot \text{s}) \]

\[ = 1.8 \times 10^{16} \text{ cm}^{-3} \]

\[ \mu_n = 88 \text{ T}_n^{-0.59} + \frac{1250 \text{ T}_n^{-2.13}}{1 + \left( \frac{N}{1.24 \times 10^{14} \text{ cm}^{-3}} \right)^{0.88} \text{T}_n^{0.194}} \]

\[ \frac{\partial}{\partial T} \left( \frac{\partial}{\partial T} \right) \mu_n = \frac{0.143}{\text{T}_n^{-2.13}} - 0.88 \text{T}_n^{-0.59} \]

\( \text{Use} \)

\[ \frac{\partial \mu_n}{\partial T} = \frac{\Delta \mu_n}{\Delta T} \]

\[ \text{for} \; \text{T}_n = 1 \text{ k} \; \left( 300 \text{ K} \; \text{or} \; 30.3 \text{°C} \right) \]

\[ \frac{\partial \mu_n}{\partial T} = -7.16 \text{ cm}^{-2} \text{v}^{-1} \text{s}^{-1} \]

\[ \alpha = 3 \times 10^{-4} \text{ K}^{-1} - \frac{1}{\text{MFD}} \left( -7.16 \right) = 2 \times 10^{-4} \text{ K}^{-1} + 5 \times 10^{-5} \text{ K}^{-1} \]

\( \text{Wells} \rightarrow \text{Si chip resistivity} \; \alpha = 1.62 \text{ cm} \text{cm} / \text{°C} \)

\[ \alpha = \text{5.3} \times 10^{-4} \text{ K}^{-1} \]
$$R = \frac{1}{x_j w} \left( \frac{1}{g_{m_d}} \right) \frac{E}{V_d} = A \frac{E}{V_d}$$

The incremental resistance can be defined as
$$R' = \frac{\Delta V}{\Delta E}$$
(slope around some operating point)
$$= A \frac{\Delta E}{\Delta V_d} \Rightarrow \text{for the resistance to double,}
\text{the slope of } \Delta E/\Delta V_d \text{ must be half}$$

$$V_d = \frac{V_d}{E_c} \left( \frac{1}{1 + \left( \frac{E}{E_c} \right)^{1.11}} \right)$$

For \( E < E_c \), \( V_d \approx V_{d0} \frac{E}{E_c} \) \( \rightarrow \) nominal value (low field)

\[ \frac{\Delta V_d}{\Delta E} = \frac{V_d}{E_c} = \frac{1.07 \times 10^{-6} \text{ cm/s}}{6910 \text{ V/cm}} = 1548.5 \text{ V/cm}^2 \]
400 V/cm at low field

To numerically find the slope at a field \( E_1 \),
1. Determine \( V_{d1} \) at \( E_1 \)
2. \( \Rightarrow V_{d1} \) at \( E_1 + \Delta E_1 \) \( (\Delta E_1 \text{ can be } 1\% \text{ of } E_1) \)
3. Find slope \( \frac{V_{d1} - V_{d0}}{(E_1 + \Delta E_1) - E_1} = \frac{V_{d2} - V_{d1}}{\Delta E_1} \)

We want to find \( E_1 \) at which slope = \( \frac{1}{2} \) (1548.5) = 774.25
I get about \( E_1 = 3281 \text{ V/cm} \)
The voltage is \( V = \sqrt{3281 \times (1250 \times 10^{-6} \text{ cm})} (10^{-3} \text{ cm}) \)

\( V = 91 \text{ V} \)