Junction Diode

- Build by forming adjacent p- and n-type regions. Interface between regions is called the *pn junction*.

- There are concentration gradients for free electrons (n-type side) and (p-type side). This gives place to diffusion currents across the junction.

- The holes that diffuse across the junction become excess minority carriers in the n-type region. Similarly, diffused electrons are excess minority carriers in the p-type region.

- Near the junction, device is depleted of majority carriers - *depletion region*. 
• fixed ions remain in the depletion region. These ions generate an electric potential called the build-in voltage.

• Diffusion across the junction continues until the build-in voltage is large enough to prevent.
Build-in voltage

- As carriers diffuse across the junction, fixed ions remain. The electric charge represented by these ions give place to an electric field that cause the energy bands to bent.

- The Fermi level is constant throughout a system in thermal equilibrium.
• The voltage drop across the depletion region is said to form a barrier. This internal voltage gives place to a drift current $I_S$ across the junction.

• An equilibrium is reached when the diffusion current due to the concentration gradient equals the drift current due to the build-in voltage. At equilibrium

$$I_D = I_S$$

• Since the Fermi level is the same at both sides of the junction, the bands will bent by $qV_0$. To find the build-in potential $V_0$, observe that

$$qV_0 = E_{pi} - E_{ni}$$

where $E_{pi}$ and $E_{ni}$ correspond to the middle of the forbidden band in the p- and n-type regions, respectively.
From chapter 1,

\[ p = N_V e^{-\frac{E_F - E_V}{kT}} \]

For intrinsic material, \( p = p_i \) and \( E_F = E_{pi} \). Thus

\[ p_i = N_V e^{-\frac{E_{pi} - E_V}{kT}} \]

\[ N_A = N_V e^{-\frac{E_F - E_V}{kT}} \]

\[ \frac{N_A}{p_i} = e^{-\frac{E_F - E_V}{kT}} \cdot e^{-\frac{E_{pi} - E_V}{kT}} \]

\[ = e^{E_{pi} - E_F} \]

\[ E_{pi} = E_F + kT \ln \left( \frac{N_A}{p_i} \right) \]

Similarly, for the n-type side,

\[ n = N_C e^{-\frac{E_C - E_F}{kT}} \]
For intrinsic material, $n = n_i$ and $E_F = E_{pi}$. Thus

$$n_i = N_C e^{-\frac{E_C - E_{ni}}{kT}}$$

$$N_D = N_C e^{-\frac{E_C - E_F}{kT}}$$

$$\frac{N_D}{n_i} = \frac{e^{-\frac{E_C - E_F}{kT}}}{e^{-\frac{E_C - E_{ni}}{kT}}} = e^{\frac{E_F - E_{ni}}{kT}}$$

$$E_{ni} = E_F - kT \ln \left( \frac{N_D}{n_i} \right)$$

- From these, after subtracting, dividing by $q$ and equating $\frac{kT}{q} = V_T$, the build-in voltage is found to be

$$V_O = V_T \ln \left( \frac{N_A N_D}{n_i^2} \right)$$
Width of the Depletion Region

- For charge equality $q x_p N_A = q x_n N_D$, where $x_p$ and $x_n$ represent the depletion region width in the p- and n-type regions, respectively. This can be expressed as

$$\frac{x_p}{x_n} = \frac{N_D}{N_A}$$

The total width of the depletion region, given by $x_p + x_n$, can be shown to be given by

$$W_{dep} = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{N_A + N_D}{N_A N_D} \right)} V_O$$
Biased Diodes

- An external voltage can force carriers to cross the junction. For that the positive terminal must be connected to the p-type region and the negative terminal to the n-type region. The diode is said to be forward biased. Because of this, the n- and p-type regions are called cathode and anode, respectively.

- If the external voltage makes the cathode more positive than the anode, then the diode is reversed biased.
Reverse Bias Junction Capacitance

Abrupt Profile

- Charge accumulated in each side of the depletion region is
  \[ q_p = q_n = q N_D x_n A \]

- Poisson’s Equation:
  \[ \frac{d^2 \varphi}{dx^2} = - \frac{\rho}{\epsilon_s} \]

  Must be integrated twice to find \( \varphi \).

\[
\frac{d^2 \varphi}{dx^2} = \begin{cases} 
0 & \text{if } x \leq -x_n \\
- \frac{q N_D}{\epsilon_S} & \text{if } x \leq -x_n \\
+ \frac{q N_A}{\epsilon_S} & \text{if } x \geq x_p \\
0 & \text{if } x \geq -x_p 
\end{cases}
\]

- Solving problem for equilibrium conditions.
• First integration:

\[
\frac{d\varphi}{dx} = \begin{cases} 
C_1 & \text{if } x \leq -x_n \\
-\frac{qN_D}{\varepsilon_S}x + C_2 & \text{if } x \leq -x_n \\
+\frac{qN_A}{\varepsilon_S}x + C_3 & \text{if } x \geq x_p \\
C_4 & \text{if } x \geq -x_p 
\end{cases}
\]

• Boundary conditions for first integration:
  - \( E \) must be continuous
  - \( E(-x_n) = -\frac{d\varphi}{dx} = 0 \)
  - \( E(-x_p) = -\frac{d\varphi}{dx} = 0 \)

• First integration after using b.c.
\[
\frac{d\varphi}{dx} = \begin{cases} 
0 & \text{if } x \leq -x_n \\
-\frac{q_{ND}}{\epsilon_S} (x + x_n) & \text{if } x \leq -x_n \\
+\frac{q_{NA}}{\epsilon_S} (x - x_p) & \text{if } x \geq x_p \\
0 & \text{if } x \geq -x_p
\end{cases}
\]

- Second integration:

\[
\varphi = \begin{cases} 
C_5 & \text{if } x \leq -x_n \\
-\frac{q_{ND}}{\epsilon_S} \left( \frac{x^2}{2} + x_n x \right) + C_6 & \text{if } x \leq -x_n \\
+\frac{q_{NA}}{\epsilon_S} \left( \frac{x^2}{2} - x_p x \right) + C_7 & \text{if } x \geq x_p \\
C_8 & \text{if } x \geq -x_p
\end{cases}
\]

- Boundary conditions for second integration:
  - \( \varphi \) is constant for \( x \leq -x_n \) and \( x \geq x_p \)
  - \( \varphi(-x_n) = V_0 \)
\[ \varphi(x_p) = 0 \text{ (defined as ground)} \]

- **Second integration after using b.c.:**

\[
\varphi = \begin{cases} 
V_0 & \text{if } x \leq -x_n \\
V_0 - \frac{qN_D}{2\varepsilon_S} (x + x_n)^2 & \text{if } -x_n \leq x \leq 0 \\
+ \frac{qN_A}{2\varepsilon_S} (x - x_p)^2 & \text{if } 0 \leq x \leq x_p \\
0 & \text{if } x \geq x_p 
\end{cases}
\]

- **Continuity at** \( x = 0 \)

\[
V_0 - \frac{qN_D}{2\varepsilon_S} x_n^2 = + \frac{qN_A}{2\varepsilon_S} x_p^2
\]

- **Expressing** \( x_p = \frac{N_D}{N_P} x_n \) **and**

\[
W_{dep} = x_n + x_p = (1 + \frac{N_D}{N_A})x_n = \frac{N_A + N_D}{N_A}x_n
\]
\begin{align*}
V_0 &= \frac{q}{2\epsilon_S} \left( N_D x_n^2 + N_A x_p^2 \right) \\
&= \frac{qN_D}{2\epsilon_S} \left( x_n^2 + \frac{N_A}{N_D} x_p^2 \right) \\
&= \frac{qN_D}{2\epsilon_S} \underline{x_n^2} \left( \frac{N_A + N_D}{N_A} \right) \\
\end{align*}

\begin{align*}
x_n &= \sqrt{\frac{2\epsilon_S}{qN_D} \frac{N_A}{N_D + N_A} V_0} \\
\end{align*}

- The charge is given by

\begin{align*}
q_j &= N_D q A x_n \\
&= qA \sqrt{\frac{2\epsilon_S}{q} \frac{N_A N_D}{N_D + N_A} V_0}
\end{align*}
• For a reversed biased junction, replace $V_0$ with $V_0 + V_{bias}$

$$q_j = qA \sqrt{2\varepsilon_S N_A N_D (V_0 + V_{bias}) \frac{N_A N_D}{q (N_D + N_A)}}$$

• Apply the definition of capacitance,

$$C = \frac{dq}{dV}$$

to the junction charge using the applied voltage, $V_{bias}$ as the voltage. Thus,

$$C_j = \left. \frac{dq_j}{dV_{bias}} \right|_{V_{bias}=V_R}$$

where $V_R$ is the applied bias voltage.

$$C_j = \frac{qA}{2} \sqrt{2\varepsilon_S N_A N_D \frac{1}{q (N_D + N_A) (V_0 + V_R)}}$$
• in terms of

$$C_{j0} = A \sqrt{\frac{q \epsilon_s}{2}} \left( \frac{N_A N_D}{N_A + N_D} \right) \frac{1}{V_0}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

• From our previous expression for $C_j$,

$$C_j^2 = \frac{q A^2 \epsilon_S}{2} \frac{N_A N_D}{N_D + N_A} \frac{1}{V_0 + V_R}$$

$$V_R = \frac{q A^2 \epsilon_S}{2} \frac{N_A N_D}{N_D + N_A} \frac{1}{C_j^2} - V_0$$

If we measure $C_j$ as we change $V_R$ and plot $V_R$ versus $1/C_j^2$, 
we should get a straight line with intercept \(-V_0\) and slope

\[ \frac{qA^{2\varepsilon_S}}{2} \frac{N_A N_D}{N_D + N_A} \]

This only works if the junction is abrupt.
Linear Junction

- Charge density: $\rho = -ax$

$$\frac{d^2 \varphi}{dx^2} = \frac{a}{\epsilon_S} x$$

- First integration:

$$\frac{d\varphi}{dx} = \frac{a}{\epsilon_S} \frac{1}{2} x^2 + C_1$$

B.C.: $E = \frac{d\varphi}{dx}$ continuous.

At $x = x_p = + \frac{w}{2}$, $E = 0$. Thus

$$C_1 = - \frac{a}{\epsilon_S} \frac{1}{2} x_p^2$$

and

$$\frac{d\varphi}{dx} = \frac{a}{2\epsilon_S} (x^2 - x_p^2)$$
• Second integration:

\[
\varphi = \frac{a}{2\varepsilon_S} \left( \frac{1}{3} x^3 - x_p^2 x \right) + C_2
\]

B.C.: \( \varphi = 0 \) at \( x_p \)

\[
C_2 = -\frac{a}{2\varepsilon_S} \left( \frac{1}{3} x_p^3 - x_p^2 x_p \right)
\]

\[
= \frac{a}{3\varepsilon_S} x_p
\]

and

\[
\varphi = \frac{a}{2\varepsilon_S} \left( \frac{1}{3} x^3 - x_p^2 x + x_p \right)
\]

• At \( x = -x_n = -\frac{w}{2} \), \( \varphi = V_0 + V_{bias} \),

\[
V_0 + V_{bias} = \frac{a}{3\varepsilon_S} (x_n^3 + x_p^3)
\]

\[
= \frac{a}{12\varepsilon_S} w^3
\]
or

\[ w^3 = \frac{12\epsilon_S}{a} (V_0 + V_{bias}) \]

- The junction capacitance is given by

\[
C_j = \frac{A\epsilon_S}{w} = \frac{\sqrt[3]{\frac{a}{12\epsilon_S}} \frac{1}{\sqrt[3]{V_0 + V_{bias}}}}{A^{\frac{3}{2}} \sqrt{\frac{a\epsilon_S^2}{12V_0}} \frac{1}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}}} = \frac{C_{j0}}{\sqrt[3]{1 + \frac{V_{bias}}{V_0}}} \]
where

\[ C_{j_0} = A^3 \sqrt{\frac{a \varepsilon_s^2}{12V_0}} \]