6. Capacitances in parallel are combined in the same manner as resistances in series.

7. The capacitance of a parallel-plate capacitor is given by

\[ C = \frac{\epsilon A}{d} \]

For vacuum, the dielectric constant is \( \epsilon = \epsilon_0 \approx 8.85 \times 10^{-12} \text{ F/m} \). For other materials, the dielectric constant is \( \epsilon = \epsilon_r \epsilon_0 \), where \( \epsilon_r \) is the relative dielectric constant.

8. Real capacitors have several parasitic effects.

9. Inductance accounts for magnetic-field effects. The units of inductance are henries (H).

10. The relationships between current and voltage for an inductance are

\[ v(t) = L \frac{di}{dt} \]

and

\[ i(t) = \frac{1}{L} \int_{t_0}^{t} v(t) \, dt + i(t_0) \]

11. The energy stored in an inductance is given by

\[ w(t) = \frac{1}{2} L i^2(t) \]

12. Inductances in series or parallel are combined in the same manner as resistances.

13. Real inductors have several parasitic effects.

14. Mutual inductance accounts for mutual coupling of magnetic fields between coils.

15. MATLAB is a powerful tool for symbolic integration, differentiation, and plotting of functions.

**Problems**

**Section 3.1: Capacitance**

P3.1. Describe the internal construction of capacitors.

P3.2. What current flows through an ideal capacitor if the voltage across the capacitor is constant with time? To what circuit element is an ideal capacitor equivalent in circuits for which the currents and voltages are constant with time?

P3.3. Explain what a dielectric material is and give two examples.

P3.4. How can electrical current flow “through” a capacitor even though a nonconducting layer separates the metallic parts?

P3.5. The voltage across a 10-\( \mu \text{F} \) capacitor is given by \( v(t) = 100\sin(1000t) \). Assume that the argument of the sin function is in radians. Find expressions for the current, power, and stored energy. Sketch the waveforms to scale versus time for time ranging from 0 to \( 2\pi \) ms.

P3.6. A 2000-\( \mu \text{F} \) capacitor, initially charged to 100 V, is discharged by a steady current of 100 \( \mu \text{A} \). How long does it take to discharge the capacitor to 0 V?

P3.7. A constant (dc) current \( i(t) = 3 \text{ mA} \) flows into a 50-\( \mu \text{F} \) capacitor. The voltage at \( t = 0 \) is \( v(0) = -20 \text{ V} \). The references for \( v(t) \) and \( i(t) \) have the passive configuration. Find the power at \( t = 0 \) and state whether the power flow is into or out of the capacitor. Repeat for \( t = 1 \) s.

*P3.8. We want to store sufficient energy in a 0.01-F capacitor to supply 5 horsepower (hp) for 1 hour. To what voltage must the capacitor be charged? (Note: One horsepower is equivalent to 745.7 watts.) Does this seem to be a practical method for storing this amount of energy? Do you think that an electric automobile design based on capacitive energy storage is feasible?

P3.9. Suppose that we have a 5-\( \mu \text{F} \) capacitor with 100 V between its terminals. Determine the magnitude of the net charge stored on each plate and the total net charge on both the plates.

P3.10. Suppose that a current given by \( i(t) = I_0 \cos(\omega t) \) flows through a capacitance \( C \) and the voltage is zero at \( t = 0 \). Assume that \( \omega t \) has units of radians. Furthermore, \( \omega \) is very large, ideally approaching infinity. For this current, does the capacitance approximate either an open or a short circuit? Explain.

\* Denotes that answers are contained in the Student Solutions files. See Appendix E for more information about accessing the Student Solutions.
P3.11. The voltage across a 1-\( \mu \)F capacitor is given by \( v(t) = 100e^{-100t} \). Find expressions for the current, power, and stored energy. Sketch the waveforms to scale versus time.

P3.12. We are given a 5-\( \mu \)F capacitor that is charged to 200 V. Determine the initial stored charge and energy. If this capacitor is discharged to 0 V in a time interval of 1 \( \mu \)s, find the average power delivered by the capacitor during the discharge interval.

P3.13. Prior to \( t = 0 \), a 100-\( \mu \)F capacitance is uncharged. Starting at \( t = 0 \), the voltage across the capacitor is increased linearly with time to 100 V in 2 s. Then, the voltage remains constant at 100 V. Sketch the voltage, current, power, and stored energy to scale versus time.

P3.14. The current through a 0.5-\( \mu \)F capacitor is shown in Figure P3.14. At \( t = 0 \), the voltage is zero. Sketch the voltage, power, and stored energy to scale versus time.

P3.15. Find the voltage, power, and stored energy at \( t = 10 \) ms for the capacitance in the circuit of Figure P3.15.

P3.16. A capacitance and the current through it are shown in Figure P3.16. At \( t = 0 \), the voltage is \( v_C(0) = 10 \) V. Sketch the voltage, power, and stored energy to scale versus time.

P3.17. At \( t = 5 \) s, the energy stored in a 10-\( \mu \)F capacitor is 200 J and is decreasing at 500 J/s. Determine the voltage magnitude and current magnitude at \( t = 5 \) s. Does the current enter or leave the positive terminal of the capacitor?

P3.18. Consider a very large capacitance (ideally, infinite) charged to 10 V. What other circuit element has the same current–voltage relationship? Explain your answer.

P3.19. A certain parallel-plate capacitor, which has one plate rotating so the overlap of the plates is a function of time, has a capacitance given by

\[
C = 2 + \sin(200t) \, \mu \text{F}
\]

in which the argument of the sine function is in radians. A constant voltage of 100 V is applied to this capacitor. Determine the current as a function of time.

P3.20. At \( t = t_0 \) the voltage across a certain capacitance \( C \) is zero. A pulse of current flows through the capacitance between \( t_0 \) and \( t_0 + \Delta t \), and the voltage across the capacitance increases to \( V_f \). What can you say about the peak amplitude \( I_m \) and area under the pulse waveform (i.e., current versus time)? What are the units and physical significance of the area under the pulse? What must happen to the peak amplitude and area under the pulse as \( \Delta t \) approaches zero, assuming that \( V_f \) remains the same?

P3.22. A 20-\(\mu\)F capacitor has a voltage given by

\[ v(t) = 3 \cos(10^5 t) + 2 \sin(10^5 t) \]

Assume that the arguments of the sine and cosine functions are in radians. Find the power at \(t = 0\) and state whether the power flow is into or out of the capacitor. Repeat for \(t_2 = (\pi/2) \times 10^{-5}\) s.

**Section 3.2: Capacitances in Series and Parallel**

P3.23. Describe how capacitances are combined in series and in parallel. Compare with how resistances are combined.

*P3.24. Find the equivalent capacitance for each of the circuits shown in Figure P3.24.

*Figure P3.24*

\[ C_{eq} \rightarrow 2 \mu F \]
\[ \begin{array}{c}
1 \mu F \\
\hline
2 \mu F
\end{array} \]

\[ C_{eq} \rightarrow 5 \mu F \]
\[ \begin{array}{c}
2 \mu F \\
\hline
4 \mu F \\
\hline
12 \mu F
\end{array} \]

P3.25. Suppose that we are designing a cardiac pacemaker circuit. The circuit is required to deliver pulses of 1-ms duration to the heart, which can be modeled as a 500-\(\Omega\) resistance. The peak amplitude of the pulses is required to be 5 V. However, the battery delivers only 2.5 V. Therefore, we decide to charge two equal-value capacitors in parallel from the 2.5-V battery and then switch the capacitors in series with the heart during the 1-ms pulse. What is the minimum value of the capacitances required so the output pulse amplitude remains between 4.9 V and 5.0 V throughout its 1-ms duration? If the pulses occur once every second, what is the average current drain from the battery? Use approximate calculations, assuming constant current during the output pulse. Find the ampere-hour rating of the battery so it lasts for five years.

P3.26. Find the equivalent capacitance between terminals \(x\) and \(y\) for each of the circuits shown in Figure P3.26.

*Figure P3.26*

\[ x \]
\[ 1 \mu F \]
\[ \begin{array}{c}
2 \mu F \\
\hline
2 \mu F
\end{array} \]
\[ \begin{array}{c}
1 \mu F \\
\hline
1 \mu F
\end{array} \]

\[ y \]
\[ 1 \mu F \]
\[ 3 \mu F \]
\[ \begin{array}{c}
1 \mu F \\
\hline
4 \mu F \\
\hline
2 \mu F
\end{array} \]

P3.27. What are the minimum and maximum values of capacitance that can be obtained by connecting two 1-\(\mu\)F capacitors in series and/or parallel? How should the capacitors be connected in each case?

P3.28. Consider two initially uncharged capacitors \(C_1 = 15 \mu F\) and \(C_2 = 10 \mu F\) connected in series. Then, a 50-V source is connected to the series combination, as shown in Figure P3.28. Find the voltages \(v_1\) and \(v_2\) after the source is applied. (Hint: The charges stored on the two capacitors must be equal, because the current is the same for both the capacitors.)

*Figure P3.28*
P3.29. We have capacitor \( C_1 = 200 \ \mu F \), which is charged to an initial voltage of 50 V, and capacitor \( C_2 = 200 \ \mu F \), which is charged to 100 V. If they are placed in series with the positive terminal of the first connected to the negative terminal of the second, determine the equivalent capacitance and its initial voltage. Now, compute the total energy stored in the two capacitors. Compute the energy stored in the equivalent capacitance. Why is it less than the total energy stored in the original capacitors?

P3.30. Suppose we have a 2-\( \mu F \) capacitance in parallel with the series combination of a 6-\( \mu F \) capacitance and a 3-\( \mu F \) capacitance. Sketch the circuit diagram and determine the equivalent capacitance of the combination.

Section 3.3: Physical Characteristics of Capacitors

*P3.31. Determine the capacitance of a parallel-plate capacitor having plates 10 cm by 30 cm separated by 0.01 mm. The dielectric has \( \varepsilon_r = 15 \).

P3.32. We have a 100-pF parallel-plate capacitor, with each plate having a width \( W \) and a length \( L \). The plates are separated by air with a distance \( d \). Assume that \( L \) and \( W \) are both much larger than \( d \). What is the new capacitance if: a. both \( L \) and \( W \) are doubled and the other parameters are unchanged? b. the separation \( d \) is halved and the other parameters are unchanged from their initial values? c. the air dielectric is replaced with oil having a relative dielectric constant of 35 and the other parameters are unchanged from their initial values?

*P3.33. Suppose that we have a 1000-pF parallel-plate capacitor with air dielectric charged to 1000 V. The capacitor terminals are open-circuited. Find the stored energy. If the plates are moved farther apart so that \( d \) is doubled, determine the new voltage on the capacitor and the new stored energy. Where did the extra energy come from?

P3.34. We have a parallel-plate capacitor, with each plate having a width \( W \) and a length \( L \). The plates are separated by air with a distance \( d \). Assume that \( L \) and \( W \) are both much larger than \( d \). The maximum voltage that can be applied is limited to \( V_{\text{max}} = Kd \), in which \( K \) is called the breakdown strength of the dielectric. Derive an expression for the maximum energy that can be stored in the capacitor in terms of \( K \) and the volume of the dielectric. If we want to store the maximum energy per unit volume, does it matter what values are chosen for \( L \), \( W \), and \( d \)? What parameters are important?

P3.35. One type of microphone is formed from a parallel-plate capacitor arranged so the acoustic pressure of the sound wave affects the distance between the plates. Suppose we have such a microphone in which the plates have an area of 10 cm\(^2\), the dielectric is air and the distance between the plates is a function of time given by

\[
d(t) = 100 + 0.3 \cos(1000t) \ \mu m
\]

A constant voltage of 200 V is applied to the plates. Determine the current through the capacitance as a function of time by using the approximation \( 1/(1+x) \cong 1-x \) for \( x \ll 1 \). (The argument of the sinusoid is in radians.)

P3.36. Consider a liquid-level sensor that consists of two parallel plates of conductor immersed in an insulating liquid, as illustrated in Figure P3.36. When the tank is empty (i.e., \( x = 0 \)), the capacitance of the plates is 200 pF. The relative dielectric constant of the liquid is 15. Determine an expression for the capacitance \( C \) as a function of the height \( x \) of the liquid.
P3.37. A parallel-plate capacitor like that shown in Figure P3.36 has a capacitance of 2500 pF when the tank is full so the plates are totally immersed in the insulating liquid. (The dielectric constant of the fluid and the plate dimensions are different for this problem than for Problem P3.36.) The capacitance is 100 pF when the tank is empty and the space between the plates is filled with air. Suppose that the tank is full and the capacitance is charged to 1000 V. Then, the capacitance is open circuited so the charge on the plates cannot change, and the tank is drained. Compute the voltage after the tank is drained and the electrical energy stored in the capacitor before and after the tank is drained. With the plates open circuited, there is no electrical source for the extra energy. Where could it have come from?

P3.38. A 0.1-μF capacitor has a parasitic series resistance of 10 Ω as shown in Figure P3.38. Suppose that the voltage across the capacitance is \(v_c(t) = 10 \cos(100t)\); find the voltage across the resistance. In this situation, to find the total voltage \(v(t) = v_r(t) + v_c(t)\) to within 1% accuracy, is it necessary to include the parasitic resistance? Repeat if \(v_c(t) = 0.1 \cos(10^3t)\). The argument of the cosine function is in radians.

P3.39. Suppose that a parallel-plate capacitor has a dielectric that breaks down if the electric field exceeds \(K \text{ V/m}\). Thus, the maximum voltage rating of the capacitor is \(V_{\text{max}} = Kd\), where \(d\) is the thickness of the dielectric. In working Problem P3.34, we find that the maximum energy that can be stored before breakdown is \(w_{\text{max}} = 1/2 \varepsilon_r \varepsilon_0 K^2 \text{Vol}\), in which Vol is the volume of the dielectric. Air has approximately \(K = 32 \times 10^5 \text{ V/m}\) and \(\varepsilon_r = 1\). Find the minimum volume of air (as a dielectric in a parallel-plate capacitor) needed to store the energy content of one U.S. gallon of gasoline, which is approximately 132 MJ. What thickness should the air dielectric have if we want the voltage for maximum energy storage to be 1000 V?

P3.40. As shown in Figure P3.40, two 10-μF capacitors have an initial voltage of 100 V before the switch is closed. Find the total stored energy before the switch is closed. Find the voltage across each capacitor and the total stored energy after the switch is closed. What could have happened to the energy?

![Figure P3.40](image)

Section 3.4: Inductance

P3.41. Explain, in a few sentences, the fluid-flow analogy for an inductor.

P3.42. How are inductors constructed?

*P3.43. The current flowing through a 2-H inductance is shown in Figure P3.43. Sketch the voltage, power, and stored energy versus time.

![Figure P3.43](image)

*P3.44. At \(t = 0\), the current flowing in a 0.5-H inductance is 4 A. What constant voltage must be applied to reduce the current to 0 at \(t = 0.2\) s?

*P3.45. A constant voltage of 10 V is applied to a 50-μH inductance, as shown in Figure P3.45. The current in the inductance at \(t = 0\) is \(-100\) mA. At what time \(t_e\) does the current reach +100 mA?
P3.46. A dc voltage of 10 V is applied to a 3-H inductor at \( t = 0 \), starting with an initial current of zero. Determine the current, power, and stored energy at \( t_1 = 6 \) s.

P3.47. The current flowing through an inductor is decreasing in magnitude. Is energy flowing into or out of the inductor? What if the current is constant with time?

P3.48. What is the value of the voltage across an ideal inductor if the current through it is constant with time? Comment. To what circuit element is an ideal inductor equivalent for circuits with constant currents and voltages?

P3.49. The current in a 100-mH inductance is given by \( 0.5 \sin(1000t) \) A. Find expressions and sketch the waveforms to scale for the voltage, power, and stored energy, allowing \( t \) to range from 0 to 3\( \pi \) ms. The argument of the sine function is in radians.

P3.50. The current flowing through a 300-mH inductance is given by \( 5\exp(-200t) \) A. Find expressions for the voltage, current, and stored energy. Sketch the waveforms to scale for \( 0 < t < 20 \) ms.

P3.51. The voltage across a 2-H inductance is shown in Figure P3.51. The initial current in the inductance is \( i(0) = 0 \). Sketch the current, power, and stored energy to scale versus time.

P3.52. The voltage across a 10-\( \mu \)H inductance is given by \( v_L(t) = 5 \sin(10^9 t) \) V. The argument of the sine function is in radians. The initial current is \( i_L(0) = -0.5 \) A. Find expressions for the current, power, and stored energy for \( t > 0 \). Sketch the waveforms to scale for time ranging from zero to 2\( \pi \) \( \mu \)s.

P3.53. What is the equivalent circuit element for a very large (ideally, infinite) inductance having an initial current of 10 A. Explain your answer.

P3.54. The energy stored in a 2-H inductor is \( 100 \) J and is increasing at 200 J/s at \( t = 4 \) s. Determine the voltage magnitude and current magnitude at \( t = 4 \) s. Does the current enter or leave the positive terminal of the inductor?


P3.56. Before \( t = 0 \), the current in a 2-H inductance is zero. Starting at \( t = 0 \), the current is increased linearly with time to 5 A in 1 s. Then, the current remains constant at 5 A. Sketch the voltage, current, power, and stored energy to scale versus time.

P3.57. The voltage across an inductance is given by \( v(t) = V_m \cos(\omega t) \) and the initial current in the inductor is zero. Suppose that \( \omega \) is very large—ideally, approaching infinity. For this voltage, does the inductance approximate either an open or a short circuit? Explain.

P3.58. Suppose that at \( t = t_0 \) the current through a certain inductance is zero. A voltage pulse is applied to the inductance between \( t_0 \) and \( t_0 + \Delta t \), and the current through the inductance increases to \( I_f \). What can you say about the peak amplitude \( V_m \) and the area under the pulse waveform (i.e., voltage versus time)? What are the units of the area under the pulse? What must happen to the peak amplitude and the area under the pulse as \( \Delta t \) approaches zero, assuming that \( I_f \) remains the same?

Section 3.5: Inductances in Series and Parallel

P3.59. Describe how inductances are combined in series and in parallel. Compare with how resistances are combined.

*P3.60. Determine the equivalent inductance for each of the series and parallel combinations shown in Figure P3.60.
P3.46. A dc voltage of 10 V is applied to a 3-H inductor at \( t = 0 \), starting with an initial current of zero. Determine the current, power, and stored energy at \( t_1 = 6 \) s.

P3.47. The current flowing through an inductor is decreasing in magnitude. Is energy flowing into or out of the inductor? What if the current is constant with time?

P3.48. What is the value of the voltage across an ideal inductor if the current through it is constant with time? Comment. To what circuit element is an ideal inductor equivalent for circuits with constant currents and voltages?

P3.49. The current in a 100-mH inductance is given by \( 0.5 \sin(1000t) \) A. Find expressions and sketch the waveforms to scale for the voltage, power, and stored energy, allowing \( t \) to range from 0 to 3\( \pi \) s. The argument of the sine function is in radians.

P3.50. The current flowing through a 300-mH inductance is given by \( 5\exp(-200t) \) A. Find expressions for the voltage, power, and stored energy. Sketch the waveforms to scale for \( 0 < t < 20 \) ms.

P3.51. The voltage across a 2-H inductance is shown in Figure P3.51. The initial current in the inductance is \( \dot{i}(0) = 0 \). Sketch the current, power, and stored energy to scale versus time.

P3.52. The voltage across a 10-\( \mu \)H inductance is given by \( v_L(t) = 5 \sin(100t) \) V. The argument of the sine function is in radians. The initial current is \( i_L(0) = -0.5 \) A. Find expressions for the current, power, and stored energy for \( t > 0 \). Sketch the waveforms to scale for time ranging from zero to \( 2\pi \) ms.

P3.53. What is the equivalent circuit element for a very large (ideally, infinite) inductance having an initial current of 10 A? Explain your answer.

P3.54. The energy stored in a 2-H inductor is \( 10 \) J and is increasing at 200 J/s at \( t = 4 \) s. Determine the voltage magnitude and current magnitude at \( t = 4 \) s. Does the current enter or leave the positive terminal of the inductor?


P3.56. Before \( t = 0 \), the current in a 2-H inductance is zero. Starting at \( t = 0 \), the current is increased linearly with time to 5 A in 1 s. Then, the current remains constant at 5 A. Sketch the voltage, current, power, and stored energy to scale versus time.

P3.57. The voltage across an inductance is given by \( v(t) = V_m\cos(\omega t) \) and the initial current in the inductor is zero. Suppose that \( \omega \) is very large—ideally, approaching infinity. For this voltage, does the inductance approximate either an open or a short circuit? Explain.

P3.58. Suppose that at \( t = t_0 \) the current through a certain inductance is zero. A voltage pulse is applied to the inductance between \( t_0 \) and \( t_0 + \Delta t \), and the current through the inductance increases to \( I_f \). What can you say about the peak amplitude \( V_m \) and the area under the pulse waveform (i.e., voltage versus time)? What are the units of the area under the pulse? What must happen to the peak amplitude and the area under the pulse as \( \Delta t \) approaches zero, assuming that \( I_f \) remains the same?

Section 3.5: Inductances in Series and Parallel

P3.59. Describe how inductances are combined in series and in parallel. Compare with how resistances are combined.

P3.60. Determine the equivalent inductance for each of the series and parallel combinations shown in Figure P3.60.
P3.61. Find the equivalent inductance for each of the series and parallel combinations shown in Figure P3.61.

P3.62. We have four 3-H inductances. What is the maximum inductance that can be obtained by connecting all of the inductances in series and/or parallel? What is the minimum inductance?

P3.63. Two inductances $L_1 = 1 \text{ H}$ and $L_2 = 2 \text{ H}$ are connected in parallel as shown in Figure P3.63. The initial currents are $i_1(0) = 0$ and $i_2(0) = 0$. Find an expression for $i_1(t)$ in terms of $i(t), L_1$, and $L_2$. Repeat for $i_2(t)$. Comment.

P3.64. We need to combine (in series or in parallel) an unknown inductance $L$ with a second inductance of 4 H to attain an equivalent inductance of 3 H. Should $L$ be placed in series or in parallel with the original inductance? What value is required for $L$?

P3.65. We need to combine (in series or in parallel) an unknown inductance $L$ with a second inductance of 10 H to attain an equivalent inductance of 6 H. Should $L$ be placed in series or in parallel with the original inductance? What value is required for $L$?

Section 3.6: Practical Inductors

P3.66. Draw the equivalent circuit for a real inductor, including three parasitic effects.

P3.67. A 10-mH inductor has a parasitic series resistance of $R_s = 1 \Omega$ as shown in Figure P3.67. a. The current is given by $i(t) = 0.1 \cos(10^2t)$. The argument of the cosine function is in radians. Find $v_R(t)$, $v_L(t)$, and $v(t)$. In this case, for 1% accuracy in computing $v(t)$, could the resistance be neglected? b. Repeat if $i(t) = 0.1 \cos(10t)$.

P3.68. A constant (dc) current of 100 mA flows through a real inductor and the voltage across
its external terminals is 400 mV. Which of the circuit parameters of Figure 3.22 on page 146 can be deduced from this information and what is its value? Explain how you arrived at your answer.

P3.69. Find \(v(t), i_C(t), i(t)\), the energy stored in the capacitance, the energy stored in the inductance, and the total stored energy for the circuit of Figure P3.69, given that \(i_1(t) = \sin(10^6 t)\) A. (The argument of the sine function is in radians.) Show that the total stored energy is constant with time. Comment on the results.

![Figure P3.69](image)

P3.70. For the circuit of Figure P3.70, determine \(i(t), v_L(t), v(t)\), the energy stored in the capacitance, the energy stored in the inductance, and the total stored energy, given that \(v_C(t) = 40\cos(1000t)\) V. (The argument of the cosine function is in radians.) Show that the total stored energy is constant with time. Comment on the results.

![Figure P3.70](image)

**Section 3.7: Mutual Inductance**

P3.71. Describe briefly the physical basis for mutual inductance.

*P3.72. a. Derive an expression for the equivalent inductance for the circuit shown in Figure P3.72. b. Repeat if the dot for \(L_2\) is moved to the bottom end.

![Figure P3.72](image)

P3.73. The mutually coupled inductances in Figure P3.73 have \(L_1 = 2\) H, \(L_2 = 3\) H, and \(M = 1\) H. Furthermore, \(i_1(t) = \sin(20t)\) A and \(i_2(t) = 0.5\cos(30t)\) A. (The arguments of the sine and cosine functions are in radians.) a. Find expressions for \(v_1(t)\) and \(v_2(t)\). b. Repeat with the dot placed at the bottom of \(L_2\).

![Figure P3.73](image)

P3.74. A pair of mutually coupled inductances has \(L_1 = 2\) H, \(L_2 = 1\) H, \(i_1 = 2\cos(1000t)\) A, \(i_2 = 0\), and \(v_2 = 2000\sin(1000t)\) V. (The arguments of the sine and cosine functions are in radians.) Find \(v_1(t)\) and the magnitude of the mutual inductance.

P3.75. Suppose we place a short circuit across the terminals of \(L_2\) in the mutually coupled inductors shown in Figure 3.23(a) on page 147. Derive an expression for the equivalent inductance seen looking into the terminals of \(L_1\).

P3.76. Consider the parallel inductors shown in Figure P3.63, with mutual coupling and the dots at the top end of \(L_1\) and the bottom end of \(L_2\). Derive an expression for the equivalent inductance seen by the source in terms of \(L_1, L_2\), and \(M\). [Hint: Write the circuit equations and manipulate them to obtain an expression of the form \(v(t) = L_{eq} \frac{di(t)}{dt}\) in which \(L_{eq}\) is a function of \(L_1, L_2\), and \(M\).]
Section 3.8: Symbolic Integration and Differentiation using MATLAB

P3.77. The current through a 200-mH inductance is given by \( i_L(t) = \exp(-2t)\sin(4\pi t) \) A in which the angle is in radians. Using your knowledge of calculus, find an expression for the voltage across the inductance. Then, use MATLAB to verify your answer for the voltage and to plot both the current and the voltage for \( 0 \leq t \leq 2 \text{ s} \).

P3.78. A 1-H inductance has \( i_L(0) = 0 \) and \( v_L(t) = t\exp(-t) \) for \( 0 \leq t \). Using your calculus skills, find an expression for \( i_L(t) \). Then, use MATLAB to verify your answer for \( i_L(t) \) and to plot \( v_L(t) \) and \( i_L(t) \) for \( 0 \leq t \leq 10 \text{ s} \).

T3.5. Determine the equivalent inductance \( L_{eq} \) between terminals \( a \) and \( b \) in Figure T3.5.

![Figure T3.5](image)

T3.6. Figure T3.6 has \( L_1 = 40 \text{ mH} \), \( M = 20 \text{ mH} \), and \( L_2 = 30 \text{ mH} \). Find expressions for \( v_1(t) \) and \( v_2(t) \).

![Figure T3.6](image)

T3.7. The current flowing through a 20-\( \mu \text{F} \) capacitor having terminals labeled \( a \) and \( b \) is \( i_{ab} = 3 \times 10^5 t^2 \exp(-2000t) \) A for \( t \geq 0 \). Given that \( v_{ab}(0) = 5 \text{ V} \), write a sequence of MATLAB commands to find the expression for \( v_{ab}(t) \) for \( t \geq 0 \) and to produce plots of the current and voltage for \( 0 \leq t \leq 5 \text{ ms} \).

Exercise 3.6. Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in Appendix D and complete solutions are included in the Student Solutions files. See Appendix E for more information about the Student Solutions.

T3.1. The current flowing through a 10-\( \mu \text{F} \) capacitor having terminals labeled \( a \) and \( b \) is \( i_{ab} = 0.3\exp(-2000t) \) A for \( t \geq 0 \). Given that \( v_{ab}(0) = 0 \), find an expression for \( v_{ab}(t) \) for \( t \geq 0 \). Then, find the energy stored in the capacitor for \( t = \infty \).

T3.2. Determine the equivalent capacitance \( C_{eq} \) for Figure T3.2.

![Figure T3.2](image)

T3.3. A certain parallel-plate capacitor has plate length of 2 cm and width of 3 cm. The dielectric has a thickness of 0.1 mm and a relative dielectric constant of 80. Determine the capacitance.

T3.4. A 2-mH inductance has \( i_{ab} = 0.3 \sin(2000t) \) A. Find an expression for \( v_{ab}(t) \). Then, find the peak energy stored in the inductance.