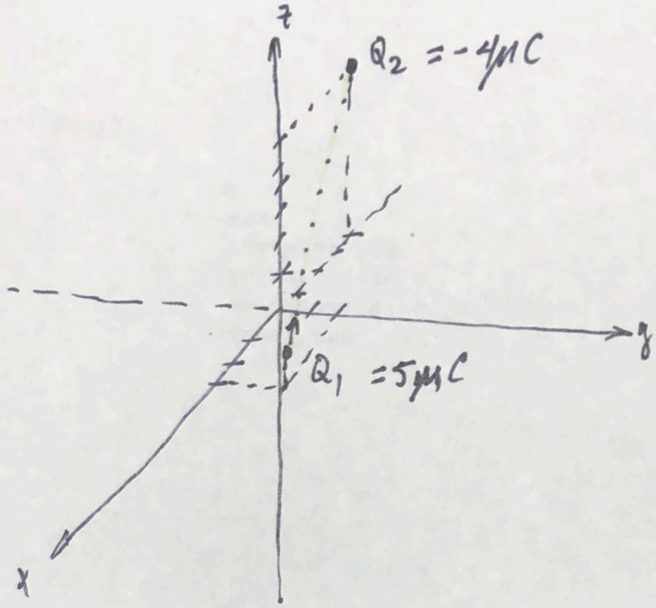
#1



$$\underline{r} = (3,2,1) - (-4,0,6) = (7,2,-5)$$

$$\underline{F} = \frac{kQ_1Q_2}{r^2}\hat{r} = \frac{kQ_1Q_2}{r^3}\underline{r} = \frac{(9\times10^9)(5\mu)(-4\mu)}{(7^2+2^2+5^2)^{3/2}}(7,2,-5)$$

$*r$

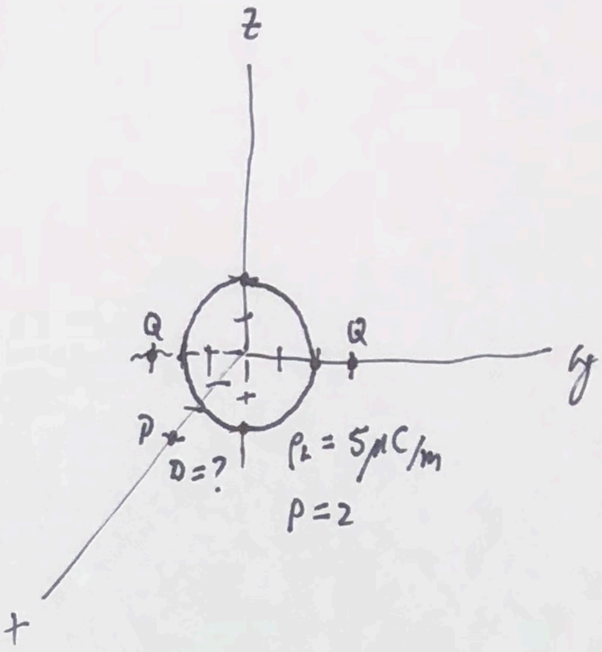
$r = 8.83$

$$= -\frac{180\times10^{-3}}{689}(7,2,-5) = +2.31\,(-0.793, -0.227, 0.566)\,mN$$

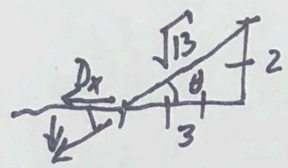
$\frac{\underline{r}}{r}$

$= 2.31\times10^{-3}\,N$ ó $2.31\,mN$

#2



a) $\underline{D}(\underline{P}) = ?$ $d\underline{D} = \epsilon_0 d\underline{E}$ por simetría $D = D_x$



$\cos\theta = \frac{3}{\sqrt{13}}$, $r = \sqrt{13}$

$D_x = D\cos\theta$ $\qquad L = 2\pi\rho = 4\pi$

$E = \frac{kQ}{r^2} = \frac{k\rho_L L}{r^2}$

$D = \frac{\rho_L L}{4\pi r^2} = \frac{\rho_L}{r^2}$

$$D_x = \frac{\rho_L \cos\theta}{r^2} = \frac{(5\mu)(3/\sqrt{13})}{(\sqrt{13})^2} = \frac{15\mu}{(13)^{3/2}} \; C/m^2$$

$= 3.2 \times 10^{-7} \; C/m^2$

b) $D_x = \frac{2Q\cos 45^\circ}{4\pi(3\sqrt{2})^2}$ $\qquad Q = \frac{4\pi(18)(3.2\times10^{-7})}{2\left(\frac{\sqrt{2}}{2}\right)} = 5.12\times10^{-5}\,C$

$Q = -5.12\times10^{-5}\,C$ debe tener signo opuesto

#3

$$\rho_v = 5xyz \ \text{nC/m}^3$$

$$0 \le x \le 2, \ -1 \le y \le 3, \ 0 \le z \le 4$$

$$Q = \int_{x=0}^{2}\int_{y=-1}^{3}\int_{z=0}^{4} \rho_v \, dz\, dy\, dx$$

$$= \int_{x=0}^{2}\int_{y=-1}^{3} 5xy\left[\tfrac{1}{2}z^2\right|_0^4 dy\, dx$$

$$= 5(8)\int_{x=0}^{2} x\left[\tfrac{1}{2}y^2\right]_{-1}^{3} dx$$

$$= 5(8)(4)\left[\tfrac{1}{2}x^2\right]_0^2$$

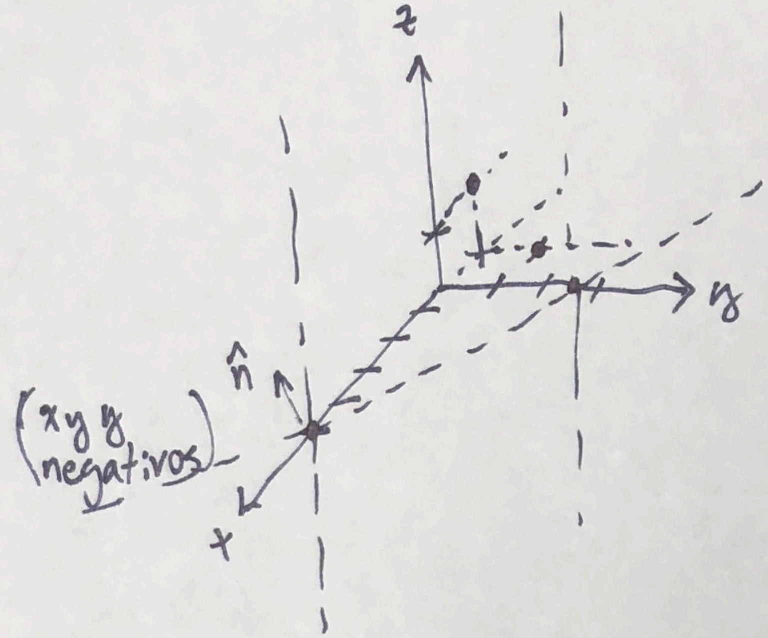
$$= 5(8)(4)(2)$$

$$= 320 \ \text{nC}$$

#4 plano: $x + 2y = 5$ $\quad \rho_s = 6\,nC/m^2$

$\underline{E}(-1,0,1) = ?$

para plano $\underline{D} = \frac{\rho_s}{2}\hat{n}$ ind. de posición.



$$\underline{n} = (1,2,0)$$

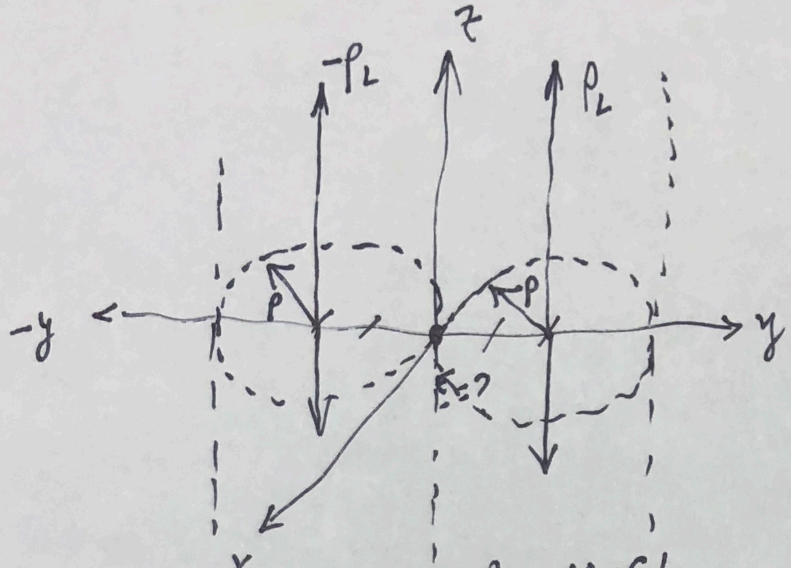
$$|\underline{n}| = \sqrt{1+4} = \sqrt{5}$$

$$\hat{n} = \left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, 0\right)$$

$$E = \frac{D}{\epsilon_0} = \frac{\rho_s}{2\epsilon_0} = \frac{6n}{2(8.85\times10^{-12})} = 1.13\times10^{3}\ V/m$$

$$\underline{E}(-1,0,1) = 1.13\left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, 0\right)\frac{kV}{m}$$

#5



$\rho_L = 10 nC/m$

$\rho = 2$

$Q_{ENC} = \Psi_{NETO}$

$\rho_L L = \epsilon_0 E(2\pi \rho L)$

$E = \dfrac{\rho_L}{2\pi \epsilon_0 \rho}$

$\underline{E}(0) = 2E(-\hat{y})$

$= \dfrac{\rho_L}{\pi \epsilon_0 \rho}(-\hat{y})$

$= \dfrac{10n}{\pi \epsilon_0 (2)} = \dfrac{10n}{(3.14)(8.85 \times 10^{-12})(2)} = 1.80 \times 10^2 \, (-\hat{y}) \, V/m$

- The divergence of a vector quantity $\vec{A}$ at a given point P is the outward flux per unit volume over a closed incremental surface as the volume shrinks about P.

$$div\vec{A} = \vec{\nabla}\cdot\vec{A} = \lim_{\delta v\to 0}\frac{\oint_S \vec{A}\cdot\overrightarrow{dS}}{\Delta v}$$

$\oint_S \vec{A}\cdot\overrightarrow{dS}$ is the net outflow of flux of a vector field $\vec{A}$ from a closed surface S

✓ **In Cartesian Coordinates:**

$$\boxed{\vec{\nabla}\cdot\vec{A} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)}$$

✓ **In Cylindrical Coordinates:**

$$\boxed{\vec{\nabla}\cdot\vec{A} = \left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right)}$$

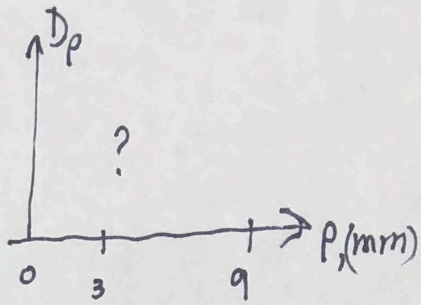
✓ **In Spherical Coordinates:**

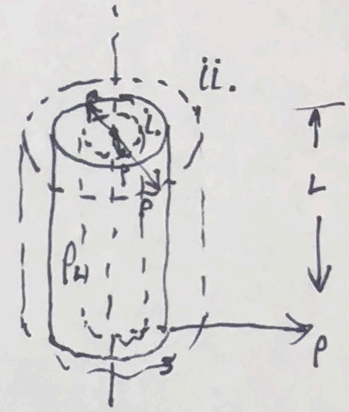
$$\boxed{\vec{\nabla}\cdot\vec{A} = \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right)}$$

#7

$a = \rho = 3.00\,mm$

$\rho_L = 2\,C/m$





$Q_{ENC} = \Psi_{neto} =$

i. $0 < \rho < 3\,mm$ fracción de carga en cilindro: $\dfrac{\pi\rho^2}{\pi a^2}$

$$\rho_L = \frac{2\rho^2}{a^2}$$

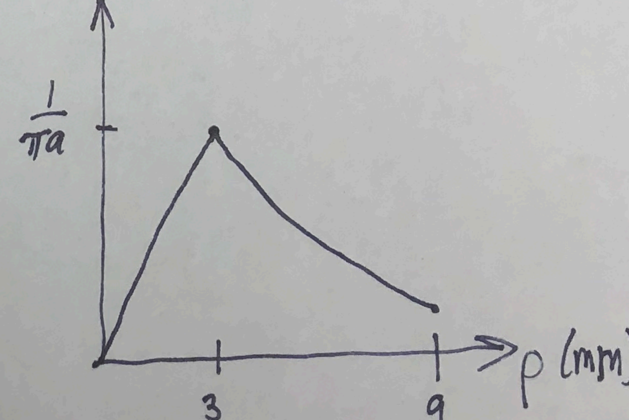
$$Q_{ENC} = \rho_L L = \frac{2\rho^2 L}{a^2} = \Psi_{neto} = D_\rho (2\pi\rho L)$$

$$D_\rho = \frac{\rho}{\pi a^2}$$

ii. $\rho > 3\,mm$

$$Q_{ENC} = 2L = \Psi_{NETO} = D_\rho (2\pi\rho L)$$

$$D_\rho = \frac{1}{\pi\rho}$$

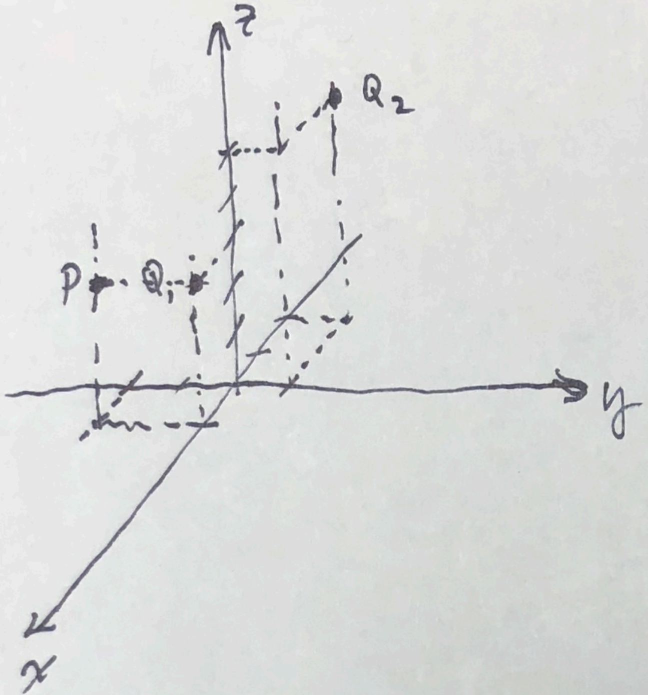


#10

$Q_1 = 2nC$ $\qquad$ $Q_2 = -4nC$

$(1,0,3)$ $\qquad$ $(-2,1,5)$

$P(1,-2,3)$



$$V = \frac{kq}{r}$$

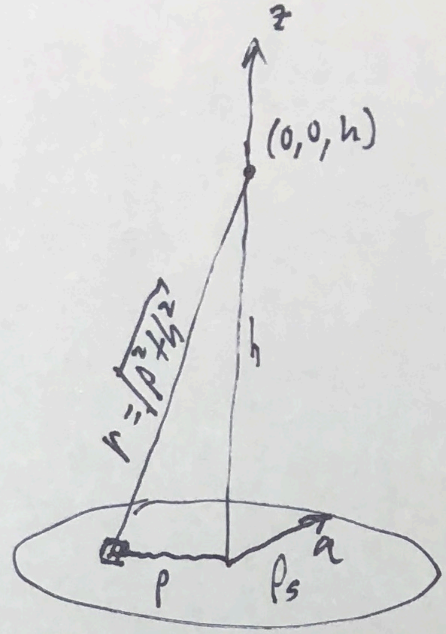
$$d_1 = |(1,-2,3) - (1,0,3)| = |(0,-2,0)| = 2$$

$$d_2 = |(1,-2,3) - (-2,1,5)| = |(3,-3,-2)| = \sqrt{22}$$

$$V(\underline{P}) = k\left(\frac{Q_1}{d_1} + \frac{Q_2}{d_2}\right) = (9)\left(\frac{2}{2} - \frac{4}{\sqrt{22}}\right) = 1.3\,V$$

#11

$\rho_s = \frac{1}{\rho}$ C/m²



$$V = \frac{kq}{r} \Rightarrow k\int_s \frac{dq}{r} \qquad dq = \rho_s\, dS = \rho_s\, \rho\, d\phi\, d\rho$$

$$= k\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \frac{\cancel{\rho_s}\, \rho\, d\phi\, d\rho}{\rho\sqrt{\rho^2+h^2}} = 2\pi k \cancel{\rho_s} \int_0^a \frac{d\rho}{\sqrt{\rho^2+h^2}}$$

$$\int \frac{dx}{\sqrt{x^2+a}} = \ln\left|x+\sqrt{x^2+a}\right| + C,\ a \neq 0$$

$x = \rho,\ a = h^2$

$$= 2\pi k \cancel{\rho_s} \left[\ln\left|\rho+\sqrt{\rho^2+h^2}\right|\right]_0^a$$

$$V = 2\pi k \cancel{\rho_s} \left[\ln\left|a+\sqrt{a^2+h^2}\right| - \ln|h|\right]$$

$$V = \frac{1}{2\epsilon_0} \ln\left|\frac{a+\sqrt{a^2+h^2}}{h}\right|$$

#12

$$W = \int \underline{E} \cdot d\underline{x} = -\int q\,\underline{E} \cdot d\underline{x} \qquad A(2,1,-1)\ \ B(5,1,2)$$

$$= -q \int_A^B \underline{E} \cdot d\underline{x} \qquad \int_A^B \rightarrow \left[\int_{x=2}^{5} E_x\,dx \Big|_{y=1,z=-1} \right] + \left[\int_{z=-1}^{2} E_z\,dz \Big|_{x=5,y=1} \right]$$

no path specified, use a simple one.

$$E_x\,dx = 2x(1)(-1)\,dx = -2x\,dx$$

$$E_z\,dz = (5)^2(1)\,dz = 25\,dz$$

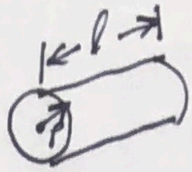
$$\int_2^5 -2x\,dx = -x^2 \Big|_2^5 = -19$$

$$\int_{-1}^{2} 25\,dz = 25z \Big|_{-1}^{2} = 75$$

$$W = -q\left[(-19)+(75)\right] = -2\mu\left[(-19)+(75)\right] = \quad = -112\,\mu J$$

resulta negativo porque la carga se mueve en esa dirección y se redujo la energía potencial de la carga al moverse.

13. $\ell = 5.0\,\text{cm}$



$2\rho = 0.812\,\text{mm}$

$Cu \rightarrow \sigma = 6 \times 10^{7}\ \text{S/m}$

a) $R = \dfrac{\ell}{\sigma A}$, $A = \pi \rho^2$

$$R = \frac{\ell}{\sigma \pi \rho^2} = \frac{5 \times 10^{-2}}{(6 \times 10^{7})\, \pi \left(\frac{0.812\text{m}}{2}\right)^2} = 1.6 \times 10^{-3}\ \Omega$$

$= 1.6\ \text{m}\Omega$

b) $I = 10\,\text{mA}$ $\quad P = I^2 R = (10^{-2})^2 (1.6 \times 10^{-3}) = 1.6 \times 10^{-7}$

$P = 1.6 \times 10^{-7}\ \text{W} = 0.16\ \mu\text{W} = 160\ \text{nW}$

14.

$\epsilon_r = 10.2$

$V = 12xy^2$ (V)

$\underline{E} = ?$

$\underline{P} = ?$

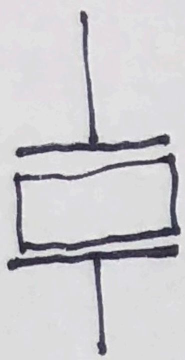
$\underline{D} = ?$

$\underline{E} = -\nabla V = -\left[12y^2\,\hat{x} + 24xy\,\hat{y}\right]$ (V/m) $= -12\left[y^2\hat{x} + 2xy\,\hat{y}\right]$ (V/m)

$\underline{P} = \epsilon_0(\epsilon_r - 1)\,\underline{E} = -9.77\times10^{-10}\left[y^2\hat{x} + 2xy\,\hat{y}\right]$ (C/m^2)

$\underline{D} = \epsilon_0\epsilon_r\,\underline{E} = -1.08\times10^{-9}\left[y^2\hat{x} + 2xy\,\hat{y}\right]$ (C/m^2)

15.



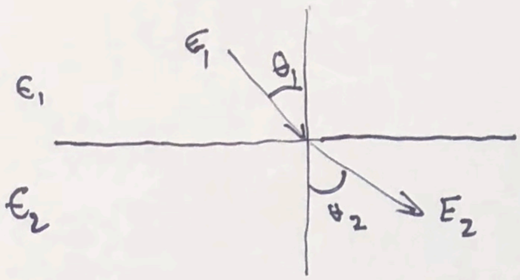
$$C = \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d}$$

$C_c = 48nF \qquad C_s = 12nF$

$$C_c = \frac{\epsilon_0 \epsilon_r A}{d} \qquad C_s = \frac{\epsilon_0 A}{d}$$

$$\frac{C_c}{C_s} = \epsilon_r = \frac{48}{12} = 4$$

#16



si ϵ_2 es el doble de ϵ_1, ¿cuánto será θ_2 si θ_1 es 30°?

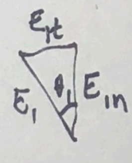
$$E_{1t} = E_{2t}$$

no dice nada de ρ_s, así que asumo que $\rho_s = 0$.

$$D_{1n} = D_{n2}$$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$\underline{E}_1 = E_{1t}\,\hat{t} + E_{1n}\,\hat{n}$$



$$\frac{E_{1t}}{E_{1n}} = \tan\theta_1$$

$$\frac{E_{2t}}{\frac{\epsilon_2}{\epsilon_1} E_{2n}} = \tan\theta_1$$

$$\frac{E_{2t}}{E_{2n}} = \tan\theta_2 = \frac{\epsilon_2}{\epsilon_1}\tan\theta_1$$

$$\theta_2 = \tan^{-1}\left[\left(\frac{1}{2}\right)^{-1}(\tan 30^\circ)\right]$$

$$\theta_2 = 49^\circ$$