Transmission Lines

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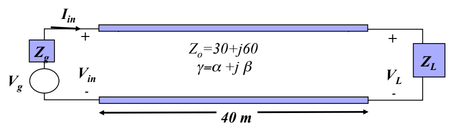
Derive the wave equation

- Maxwell predicted em waves
- wavelength, frequency, period

Exercise 11.3

- A 40-m long TL has V_g=15 V_{rms}, Z_o=30+j60 Ω, and V_L=5$e^{-j48°}$ V_{rms}. If the line is matched to the load and the generator, find: the input impedance Z_{in}, the sending-end current I_{in} and Voltage V_{in}, the propagation constant γ.



- Answers:

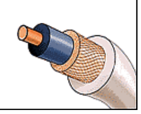
$$Z_{in} = 30 + j60\Omega, \quad I_{in} = 0.112\angle -63.4^o\, A,$$

$$V_{in} = 7.5\angle 0^o\, V_{rms}, \quad \gamma = 0.0101 + j0.02094\, m^{-1}$$

$$7.45\angle -111^o e^{-\gamma 40} = 0.112\angle -63^o$$

Transmission Lines

I. **TL parameters**
II. **TL Equations**
III. **Input Impedance, SWR, power**
IV. **Smith Chart**
V. **Applications**
   - **Quarter-wave transformer**
   - **Slotted line**
   - **Single stub**
VI. **Microstrips**

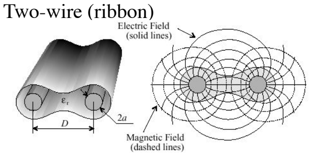


Transmission Lines (TL)

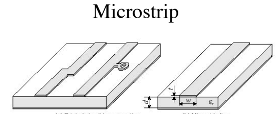
- TL have two conductors in parallel with a dielectric separating them
- They transmit TEM waves inside the lines

Common Transmission Lines

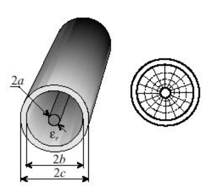
Two-wire (ribbon)



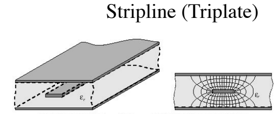
Microstrip



Coaxial



Stripline (Triplate)



Other TL (higher order)
[Chapter 11]

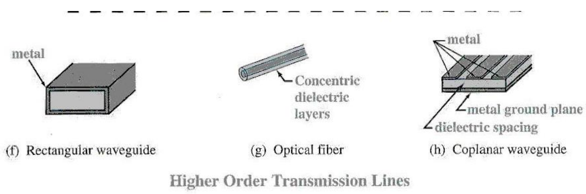


Figure 2-4: A few examples of transverse electromagnetic (TEM) and higher-order transmission lines.

Fields inside the TL

$$V = -\int E \cdot dl$$
$$I = \oint H \cdot dl$$

- V proportional to E,
- I proportional to H

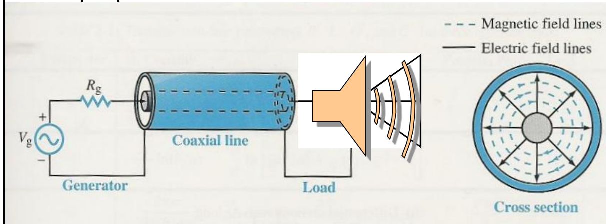


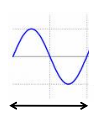
Figure 2-5: In a coaxial line, the electric field lines are in the radial direction between the inner and outer conductors, and the magnetic field forms circles around the inner conductor.

Distributed parameters

The parameters that characterize the TL are given in terms of per length.

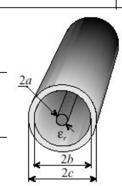
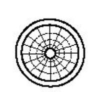
- R = ohms/meter
- L = Henries/ m
- C = Farads/m
- G = mhos/m

$$\lambda_{60Hz} = c/60 = 5{,}000km$$
$$\lambda_{2GHz} = c/2000{,}000{,}000 = 15cm$$



Common Transmission Lines

R', *L'*, *G'*, and *C'* depend on the particular transmission line structure and the material properties. *R*, *L*, *G*, and *C* can be calculated using fundamental EMAG techniques.

Parameter	Two-Wire Line	Coaxial Line	Parallel-Plate Line	Unit
R'	$\frac{1}{\pi a \sigma_{cond}\delta}$	$\frac{1}{2\pi\sigma_{cond}\delta}\left(\frac{1}{a}+\frac{1}{b}\right)$		
L'	$\frac{\mu}{\pi}\text{acosh}\left(\frac{D}{2a}\right)$	$\frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$		
G'	$\frac{\pi\sigma_{diel}}{\text{acosh}(D/(2a))}$	$\frac{2\pi\sigma_{diel}}{\ln(b/a)}$		
C'	$\frac{\pi\varepsilon}{\text{acosh}(D/(2a))}$	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\varepsilon\frac{w}{d}$	F/m

TL representation

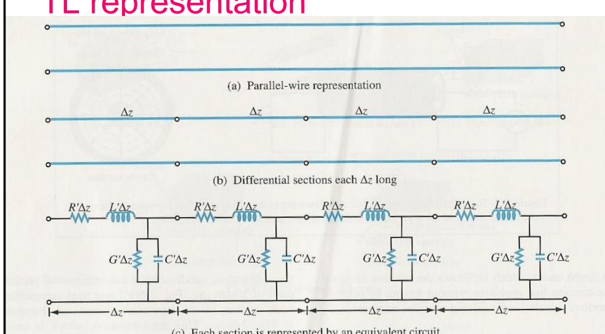


Figure 2-6: Regardless of its actual shape, a TEM transmission line is represented by the parallel-wire configuration shown in (a). To analyze the voltage and current relations, the line is subdivided into small differential sections (b), each of which is then represented by an equivalent circuit (c).

Distributed line parameters

Using KVL:

$$V(z,t) = R\,\Delta z\, I(z,t) + L\,\Delta z\frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

or

$$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = R\,I(z,t) + L\frac{\partial I(z,t)}{\partial t} \qquad (11.3)$$

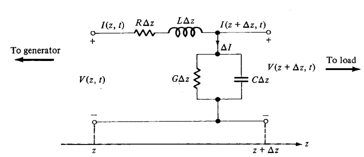


Figure 11.5 *L*-type equivalent circuit model of a differential length Δz of a two-conductor transmission line.

Distributed parameters

- Taking the limit as Δz tends to 0 leads to

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

- Similarly, applying KCL to the main node gives

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

Wave equation

- Using phasors

$$V(z,t) = \mathrm{Re}[V_s(z)e^{j\omega t}]$$
$$I(z,t) = \mathrm{Re}[I_s(z)e^{j\omega t}]$$

- The two expressions reduce to

$$-\frac{\partial V_s}{\partial z} = (R + j\omega L)I_s$$
$$-\frac{\partial I_s}{\partial z} = (G + j\omega C)V_s$$

$$\frac{\partial^2 V_s}{\partial z^2} - \gamma^2 V_s = 0$$

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

Wave Equation for voltage

TL Equations

- Note that these are the wave eq. for voltage and current inside the lines.

$$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0 \qquad \frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0$$

- The propagation constant is γ and the wavelength and velocity are

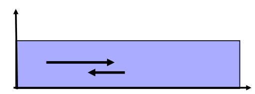
$$\gamma = \alpha + \beta j = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\lambda = 2\pi/\beta \qquad u = \omega/\beta = f\lambda$$

Waves moves through line

- The general solution is

$$V_s = V^+e^{-\gamma z} + V^-e^{\gamma z}$$



- In time domain is

$$V(z,t) = \mathrm{Re}[V_s(z)e^{j\omega t}]$$
$$= V^+e^{-\alpha z}\cos(\omega t - \beta z) + V^-e^{+\alpha z}\cos(\omega t + \beta z)$$

- Similarly for current, I

$$I(z,t) = I^+e^{-\alpha z}\cos(\omega t - \beta z) + I^-e^{\alpha z}\cos(\omega t + \beta z)$$

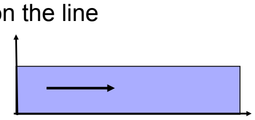
Characteristic Impedance of a Line, Z_o

- Is the ratio of positively traveling voltage wave to current wave at any point on the line

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z)$$

substituting

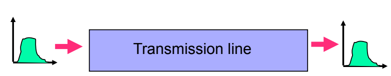
$$V(z) = V^+e^{-\gamma z}$$
$$I(z) = I^+e^{-\gamma z}$$
$$-(-\gamma V^+e^{-\gamma z}) = (R + j\omega L)I^+e^{-\gamma z}$$



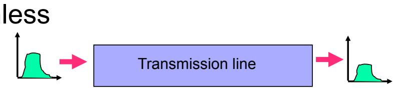
$$Z_o = \frac{V^+}{I^+} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o = -\frac{V^-}{I^-}$$

Different cases of TL

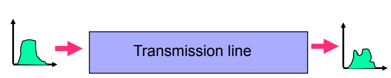
- Lossless



- Distortionless



- Lossy



Lossless Lines ($R=0=G$)

Has perfect conductors and perfect dielectric medium between them.

- Propagation: $\alpha = 0, \quad \gamma = j\beta, \quad \beta = \omega\sqrt{LC}$
- Velocity: $u = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}} = f\lambda, \qquad \lambda = \dfrac{2\pi}{\beta}$
- Impedance $X_o = 0 \qquad Z_o = R_o = \sqrt{\dfrac{L}{C}}$

Distortionless line ($R/L = G/C$)

Is one in which the attenuation is independent on frequency.

- Propagation: $\gamma = \alpha + j\beta$
  $\alpha = \sqrt{RG} \quad \beta = \omega\sqrt{LC}$
- Velocity: $u = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}} = f\lambda$
- Impedance $X_o = 0 \quad Z_o = R_o = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{R}{G}}$

Summary

	$\gamma = \alpha + j\beta$ Propagation Constant	Z_o Characteristic Impedance
=General	$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$	$Z_o = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$
Lossless	$\gamma = 0 + j\omega\sqrt{LC}$	$Z_o = R_o = \sqrt{\dfrac{L}{C}}$
Distortionless $RC = GL$	$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$	$Z_o = R_o = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{R}{G}}$

P.E. 11.2

A telephone line has R=30 Ω/km, L=100 mH/km, G=0, and C= 20μF/km. At 1kHz, obtain: the characteristic impedance of the line, the propagation constant, the phase velocity.

- Is it distortionless?
- Solution:

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{30+j2\pi(1k)(100m)}{0+j2\pi(1k)20\cdot 10^{-6}}} = 70.75\angle -1.37^o\,\Omega$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$= \sqrt{(30+j2\pi(100))(0+j2\pi 20\cdot 10^{-3})}$$
$$= 0.21 + j8.88\,/\,km$$

$$u = \frac{\omega}{\beta} = 707 \quad km/s$$

Define reflection coefficient at the load, Γ_L

Load is usually taken at $z=0$ and generator at $z=-l$

$$V_s(z) = V^+e^{-\gamma z} + V^-e^{+\gamma z}$$

$$\Gamma_L = \frac{V^-}{V^+}$$

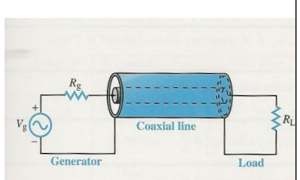


Figure 2-5: In a coaxial line, the electric field lines are in the outer conductors, and the magnetic field forms circles around

Terminated, Lossless TL

Then, $V_s(z) = V^+\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)$

Similarly, $I_s(z) = \dfrac{V^+}{Z_o}\left(e^{-\gamma z} - \Gamma_L e^{+\gamma z}\right)$

The impedance anywhere along the line is given by

$$Z(z) = \frac{V_s(z)}{I_s(z)} = Z_o\frac{\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)}{\left(e^{-\gamma z} - \Gamma_L e^{+\gamma z}\right)}$$

The impedance at the load end, Z_L, is given by

$$Z(l=0) = Z_L = Z_o\frac{(1+\Gamma_L)}{(1-\Gamma_L)}$$

Terminated, Lossless TL

Then, $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$

Conclusion: The reflection coefficient is a function of the load impedance and the characteristic impedance.

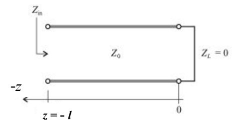
Recall for the lossless case, $\gamma = 0 + j\beta = j\omega\sqrt{LC}$

Then $V_s(z) = V^+\left(e^{-j\beta z} + \Gamma_L e^{+j\beta z}\right)$

$$I_s(z) = \frac{V^+}{Z_o}\left(e^{-j\beta z} - \Gamma_L e^{+j\beta z}\right)$$

Terminated, Lossless TL

It is customary to change to a new coordinate system, $z = -l$, at this point.



Rewriting the expressions for voltage and current, we have

$$V(-l) = V^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

$$I(-l) = \frac{V^+}{Z_o}\left(e^{+j\beta l} - \Gamma_L e^{-j\beta l}\right)$$

Rearranging,

$$V(-l) = V^+ e^{+j\beta l}\left(1 + \Gamma_L e^{-2j\beta l}\right)$$

$$I(-l) = \frac{V^+}{Z_o} e^{+j\beta l}\left(1 - \Gamma_L e^{-2j\beta l}\right)$$

Impedance (Lossy line)

The impedance **anywhere along the line** is given by

$$Z(l) = \frac{V(l)}{I(l)} = Z_o\frac{\left(1 + \Gamma_L e^{-2\gamma l}\right)}{\left(1 - \Gamma_L e^{-2\gamma l}\right)}$$

The reflection coefficient can be modified as follows

$$\Gamma(l) = \Gamma_L e^{-2\gamma l} = \Gamma_L (e^{-2\alpha l} e^{-2j\beta l})$$

Then, the impedance can be written as

$$Z(l) = Z_o\frac{\left(1 + \Gamma(l)\right)}{\left(1 - \Gamma(l)\right)}$$

After some algebra, an alternative expression for the impedance is given by

$$Z(l) = Z_{in} = Z_o\frac{\left(Z_L + Z_o \tanh \gamma l\right)}{\left(Z_o + Z_L \tanh \gamma l\right)}$$

Conclusion: The load impedance is "transformed" as we move away from the load.

Impedance (Lossless line)

The impedance anywhere along the line is given by

$$Z(l) = \frac{V(l)}{I(l)} = Z_o\frac{\left(1 + \Gamma_L e^{-2j\beta l}\right)}{\left(1 - \Gamma_L e^{-2j\beta l}\right)}$$

The reflection coefficient can be modified as follows

$$\Gamma(l) = \Gamma_L e^{-2j\beta l} = |\Gamma_L| e^{j\theta_\Gamma} e^{-2j\beta l}$$

Then, the impedance can be written as

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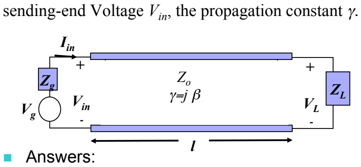
After some algebra, an alternative expression for the impedance is given by

$$Z(l) = Z_{in} = Z_o\frac{\left(Z_L + jZ_o \tan \beta l\right)}{\left(Z_o + jZ_L \tan \beta l\right)}$$

Conclusion: The load impedance is "transformed" as we move away from the load.

Example: Matched Case

A $\pi/8$ long TL has V_g=10 V_{rms}, Z_o=50Ω, and V_L=5e^{-j30^o} V_{rms}. If the line is matched to the load and the generator, find: the input impedance Z_{in}, the sending-end Voltage V_{in}, the propagation constant γ.



$$V(l) = V^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$Z_{in} = Z_o\frac{\left(Z_L + jZ_o \tan \beta l\right)}{\left(Z_o + jZ_L \tan \beta l\right)}$$

- Answers:

$Z_L = Z_0 \quad \Rightarrow Z_{in} = 50\Omega$

$\Gamma = 0$

$V_L = 5\angle 30^o = V|_{l=0} = V^+\left(e^{j\beta 0} + (0)e^{-j\beta 0}\right),$

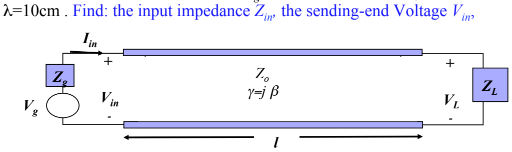
$Solve \quad for: \quad V^+ = 5\angle 30^o$

$$V(l = \pi/8) = 5\angle 30^o\left(e^{j\pi/4} + (0)e^{-j\pi/4}\right)$$

$$V_{in} = 5\angle 30^o\left(e^{j\pi/4}\right) = 5\angle 75^o$$

Exercise 1: using formulas

A 2cm lossless TL has V_L=10 e^{j30o}, Z_g=60 Ω, Z_L=50 Ω and Z_o=100Ω, λ=10cm . Find: the input impedance Z_{in}, the sending-end Voltage V_{in},



$$V(l) = V^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

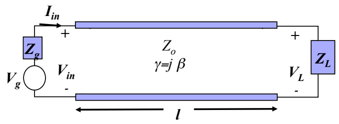
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

- Use this equation at load and at input, find V^+
- Find Z_{in} (at input)

$$Z_{in} = Z_o\frac{\left(Z_L + jZ_o \tan \beta l\right)}{\left(Z_o + jZ_L \tan \beta l\right)}$$

Exercise 2: using formulas

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},



$$Z_{in} = Z(2cm) = 40\frac{\left((100+j80)+j40\tan\frac{2\pi}{5}\right)}{\left(40+j(100+j80)\tan{2\pi}/{5}\right)}$$

$$Z_{in} = 12.2 - j21.17\,\Omega$$

Voltage Divider: $V_{in} = \dfrac{V_g Z_{in}}{Z_{in}+Z_g} = 3.30\angle -0.766 rad$

Example 3: Not matched to load

A generator with 10V_{rms} and R_g=50, is connected to a 75Ω load thru a 0.8λ, 50Ω-lossless line.

- Find V_L

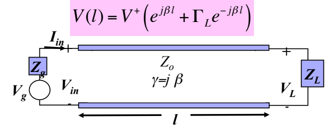
$Z_o = 50\Omega \quad Z_L = 75,\ l = .8\lambda$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$Z_{in} = Z_o\frac{(Z_L + jZ_o\tan\beta l)}{(Z_o + jZ_L\tan\beta l)}$$

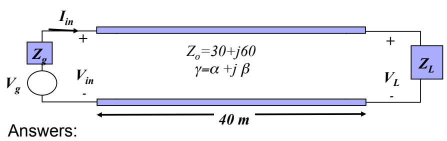
$$V(l) = V^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

$$\Gamma_L = 0.2$$
$$Z_{in} = 35. + j8.75\Omega$$
$$V_{in} = \angle$$
$$V^+ = 5.006\angle 1.26 rad$$
$$V_L = \angle^o$$



Exercise 11.3: Matched Case

A 40-m long TL has V_g=15 V_{rms}, Z_o=30+j60 Ω, and V_L=5e$^{-j48^o}$ V_{rms}. If the line is matched to the load and the generator, find: the input impedance Z_{in}, the sending-end current I_{in} and Voltage V_{in}, the propagation constant γ.



- Answers:

$$Z_{in} = 30 + j60\Omega, \quad I_{in} = 0.112\angle -63.4^o A,$$
$$V_{in} = 7.5\angle 0^o V_{rms}, \quad \gamma = 0.0101 + j0.02094\ m^{-1}$$
$$7.45\angle -111^o e^{-\gamma 40} = 0.112\angle -63^o$$

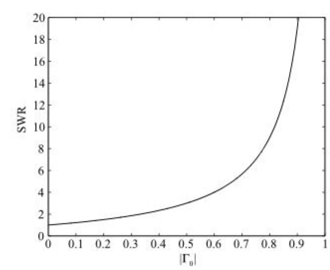
SWR or VSWR or s

Whenever there is a reflected wave, a standing wave will form out of the combination of incident and reflected waves.

The (Voltage) Standing Wave Ratio - SWR (or VSWR) is defined as

$$s = SWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|}$$

$$s = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$



Power

- The average input power at a distance l from the load is given by

$$P_{ave} = \frac{1}{2}\mathrm{Re}\left[V(l)I^*(l)\right]$$

- which can be reduced to

$$P_{ave} = \frac{|V_o^+|^2}{2Z_o}\left(1-|\Gamma|^2\right)$$

- The first term is the incident power and the second is the reflected power. Maximum power is delivered to load if Γ=0

Three (3) common Cases of line-load combinations:

- Shorted Line (Z_L=0)

$$Z_{in} = 0 + jZ_o\tan\beta l = jb \qquad \Gamma_L = -1, \quad s = \infty$$

- Open-circuited Line (Z_L=∞)

$$Z_{in} = -jZ_o\cot\beta l \qquad \Gamma_L = 1, \quad s = \infty$$

- Matched Line (Z_L = Z_o)

$$Z_{in} = Z_o \qquad \Gamma_L = 0, \quad s = 1$$

Standing Waves -Short

Shorted Line ($Z_L=0$), we had

$Z_{in} = jZ_o \tan \beta l\,,\quad \Gamma_L = -1,\quad s = \infty$

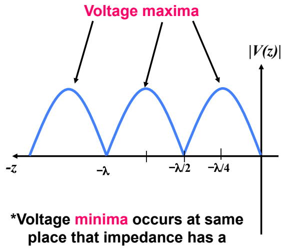
- So substituting in $V(z)$

$$V(z) = V^{+}[e^{j\beta l} + (-1)e^{-j\beta l}]$$

$$V(z) = V^{+}(2j\sin\beta l)$$

$$|V(z)| = |V^{+}||2\sin(\beta l)|$$

$$|V(z)| = |V^{+}|\left|2\sin\left(\frac{2\pi}{\lambda}l\right)\right|$$



*Voltage minima occurs at same place that impedance has a minimum on the line

Standing Waves -Open

Open Line ($Z_L=\infty$) ,we had

$Z_{in} = -jZ_o \cot \beta l\,,\quad \Gamma_L = +1,\quad s = \infty$

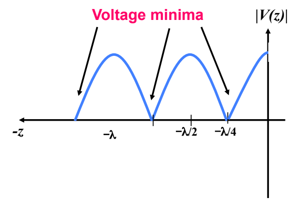
- So substituting in $V(z)$

$$V(z) = V^{+}[e^{j\beta l} + (+1)e^{-j\beta l}]$$

$$V(z) = V^{+}(2\cos\beta l)$$

$$|V(z)| = |V^{+}||2\cos(\beta l)|$$

$$|V(z)| = |V^{+}|\left|2\cos\left(\frac{2\pi}{\lambda}l\right)\right|$$



Standing Waves -Matched

Matched Line ($Z_L = Z_o$), we had

$Z_{in} = Z_o\,,\quad \Gamma_L = 0,\quad s = 1$

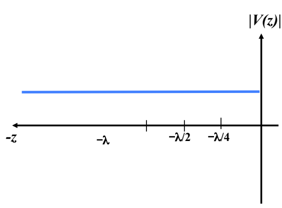
- So substituting in $V(z)$

$$V(z) = V^{+}[e^{j\beta l} + (0)e^{-j\beta l}]$$

$$V(z) = V^{+}e^{j\beta l}$$

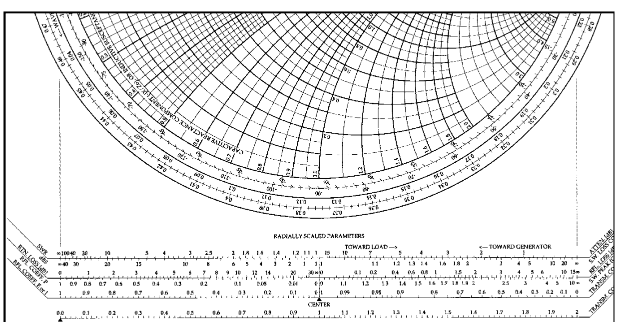
$$|V(z)| = |V^{+}||e^{j\beta l}|$$

$$|V(z)| = |V^{+}|$$



Java applets

- http://www.amanogawa.com/transmission.html
- http://physics.usask.ca/~hirose/ep225/
- http://www.educatorscorner.com/index.cgi?CONTENT_ID=2483



Smith Chart

- Commonly used as graphical representation of a TL.
- Used in hi-tech equipment for design and testing of microwave circuits
- One turn (360°) around the SC = to λ/2

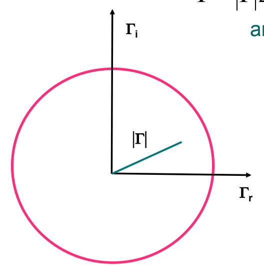
Network Analyzer

Smith Chart

- Use the reflection coefficient real and imaginary parts .

$$\Gamma = |\Gamma|\angle\theta_\Gamma = \Gamma_r + j\Gamma_i = \frac{(Z_L - Z_o)}{(Z_L + Z_o)}$$

and define the normalized Z_L:



$$z_L = \frac{Z_L}{Z_o} = \frac{(1+\Gamma_L)}{(1-\Gamma_L)} = r + jx$$

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$z_L = r + jx = \frac{1 + \Gamma_r + j\Gamma_i}{1 + \Gamma_r - j\Gamma_i}$$

Now relating to $z = r + jx$

- After some algebra, we obtain two eqs.

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2$$ *Circles of r*

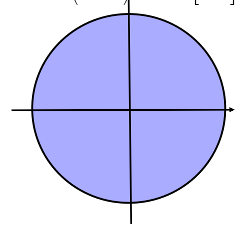
$$[\Gamma_r - 1]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2$$ *Circles of x*

- Similar to general equation of a circle of radius a, center at (h,k)

$$(x-h)^2 + (y-k)^2 = a^2$$

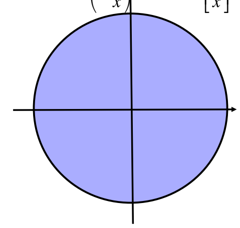
Examples of circles of r and x

$$\text{Center} = \left(\frac{r}{1+r}, 0\right) \quad \text{Radius} = \left[\frac{1}{1+r}\right]$$



Circles of *r*

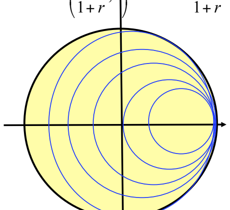
$$\text{Center} = \left(1, \frac{1}{x}\right) \quad \text{Radius} = \left[\frac{1}{x}\right]$$



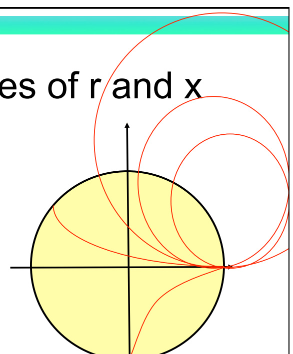
Circles of *x*

Examples of circles of r and x

$$\text{Center} = \left(\frac{r}{1+r}, 0\right) \quad \text{Radius} = \frac{1}{1+r}$$



Circles of *r*



Circles of *x*

The joy of the SC

- Numerically $s = r$ on the +axis of Γ_r in the SC

Proof:

$$\Gamma = \frac{z_L - 1}{z_L + 1} = (\text{when } z = r) = \frac{r-1}{r+1}$$

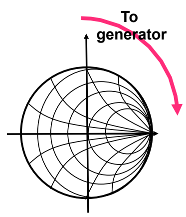
$$\text{but then} \quad \Gamma = \Gamma_r + j0 \equiv \frac{s-1}{s+1}$$

Fun facts about the Smith Chart

- A lossless TL is represented as a circle of constant radius, $|\Gamma|$, or constant s

$$\Gamma(l) = \Gamma_L e^{-2j\beta l} = |\Gamma_L| e^{j\theta_\Gamma} e^{-2j\beta l}$$

- *Moving along the line from the load toward the generator, the phase decrease, therefore, in the SC equals to moves clockwisely.*



Fun facts about the Smith Chart

- One turn (360°) around the SC = to $\lambda/2$ because in the formula below, if you substitute length for half-wavelength, the phase changes by 2π, which is one turn.

$$\Gamma(l) = \Gamma_L e^{-2j\beta l}$$

- Find the point in the SC where Γ=+1,-1, j, $-j$, 0, 0.5
- What is r and x for each case?

Fun facts : Admittance in the SC

- The admittance, $y=Y_L/Y_o$ where $Y_o=1/Z_o$, can be found by moving ½ turn ($\lambda/4$) on the TL circle

$$2\beta l = 2\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \pi$$

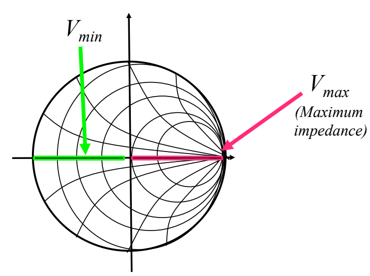
$$z(l=0) = \frac{Z_L}{Z_o} = \frac{1}{Z_o}\frac{V^+\left(1+\Gamma e^{2j\beta l}\right)}{V^+/Z_o\left(1-\Gamma e^{2j\beta l}\right)} = \frac{1+\Gamma e^{2j\beta l}}{1-\Gamma e^{2j\beta l}} = \frac{1+\Gamma e^{2j0}}{1-\Gamma e^{2j0}} = \frac{1+\Gamma}{1-\Gamma}$$

$$z(l=\lambda/4) = \frac{1+\Gamma e^{j\pi}}{1-\Gamma e^{j\pi}} = \frac{1+\Gamma(-1)}{1-\Gamma(-1)} = \frac{1-\Gamma}{1+\Gamma}$$

$$y(l=0) = \left(\frac{1}{Y_o}\right)\frac{V^+/Z_o\left(1-\Gamma e^{2j\beta l}\right)}{V^+\left(1+\Gamma e^{2j\beta l}\right)} = \frac{1-\Gamma e^{j0}}{1+\Gamma e^{j0}} = \frac{1-\Gamma}{1+\Gamma}$$

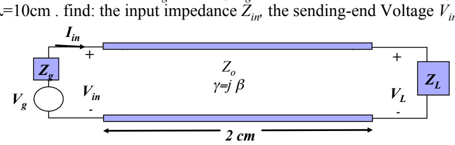
Fun facts about the Smith Chart

- The Γ_r +axis, where $r > 0$ corresponds to V_{max}
- The Γ_r -axis, where $r < 0$ corresponds to V_{min}



Exercise: using S.C.

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},



$$z_L = \frac{100+j80}{40} = 2.5 + j2$$

$$l_\lambda = .2\lambda$$

$$\Gamma_L = 0.622\angle 23.5^o$$

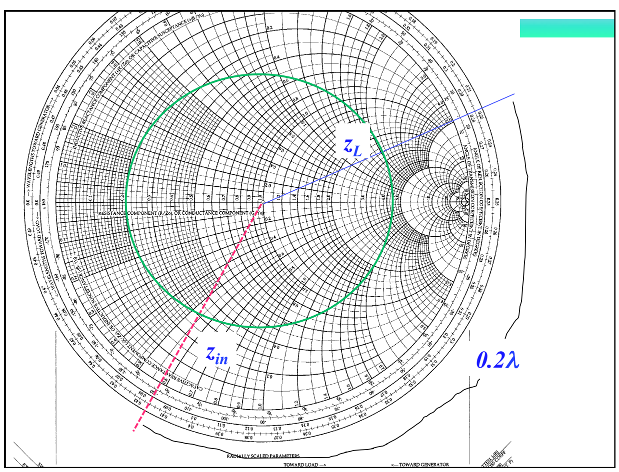
$$\Gamma(2cm) = 0.622\angle -120^o$$

- Load is at .2179λ @ S.C.
- Move .2λ and arrive to .4179λ
- Read $z_{in} = .3 - j.55$

$$Z_{in} = 12 - j22\Omega \qquad y_{in} = .76 - j1.4\Omega$$

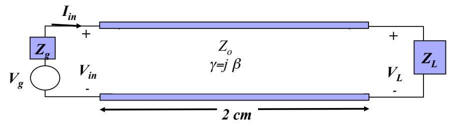
Voltage Divider:

$$V_{in} = \frac{V_g Z_{in}}{Z_{in} + Z_g} = 3.32\angle -0.775 rad$$



Exercise: cont….using S.C.

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},

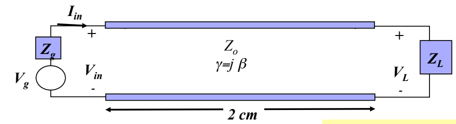


$z_L = 2.5 + j2$ $l_\lambda = .2\lambda$ $\Gamma_L = 0.622\angle 23.5^o$

- Distance from the load (.2179λ) to the nearest minimum & max
- Move to horizontal axis toward the generator and arrive to .5λ (V_{max}) and to .25λ for the V_{min}.
- Distance to min=.5-.2179=.282λ
- Distance to 2st voltage maximum is .282λ +.25λ=.482 See drawing

Exercise : using formulas

- A 2cm lossless TL has V_g=10 V_{rms}, Z_g=60 Ω, Z_L=100+j80 Ω and Z_o=40Ω, λ=10cm . find: the input impedance Z_{in}, the sending-end Voltage V_{in},



$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.62\angle 23.4^o$$

$$Z_{in} = Z(2cm) = 40\frac{\left(100 + j80\right) + j40\tan\frac{2\pi}{5}}{\left(40 + j(100 + j80)\tan 2\pi/5\right)}$$

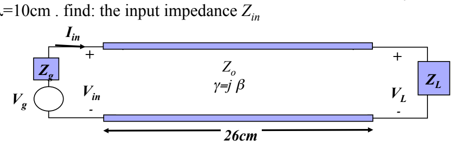
$$\Gamma(2cm) = \Gamma_L e^{-2j\beta l}$$
$$= \Gamma_L\angle -144^o = 0.62\angle -120.6^o$$

Voltage Divider: $Z_{in} = 12.2 - j21.17\,\Omega$

$$V_{in} = \frac{V_g Z_{in}}{Z_{in} + Z_g} = 3.30\angle -0.766rad$$

Another example:

- A 26cm lossless TL is connected to load Z_L=36-j44 Ω and Z_o=100Ω, λ=10cm . find: the input impedance Z_{in}



$z_L = .36 - j.44$ $l_\lambda = 2.6\lambda = 5(.5\lambda) + .1\lambda$ $\Gamma_L = 0.54\angle -127^o$

- Load is at .427λ @ S.C.
- Move .1λ and arrive to .527λ (=.027λ)
- Read $z_{in} = .31 + j.16$

$Z_{in} = 31 + j16\,\Omega$

Distance to first V_{max}:

$l_{min} = 0.5\lambda - .427\lambda = 0.028\lambda$

$l_{max} = 0.028\lambda + .25\lambda = .278\lambda$

Exercise 11.4

- A 70 Ω lossless line has s =1.6 and θ_Γ=300º. If the line is 0.6λ long, obtain Γ, Z_L, Z_{in} and the distance of the first minimum voltage from the load.
- Answer $\Gamma_L = 0.23\angle 300^o$

$$\Gamma = \frac{s-1}{s+1}$$

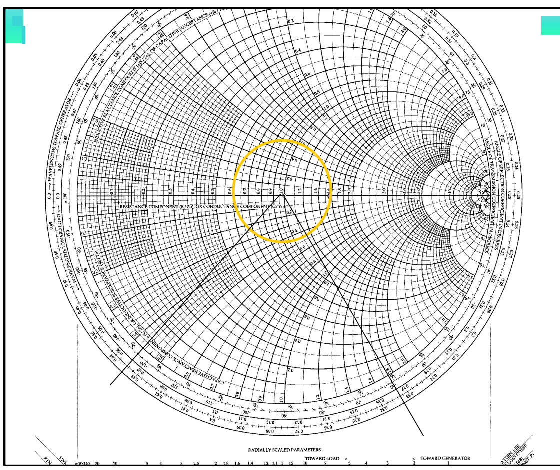
$z_L = 1.15 - j.48$

$Z_L = Z_o z_L = 80.5 - j33.6\Omega$

- The load is located at: $.3338\lambda$
- Move to .4338 λ and draw line from
- center to this place, then read where it crosses you TL circle.
- Distance to V_{min} in this case, l_{min}=.5λ-.3338λ=$\lambda/6$

$z_{in} = 0.68 - j.25$

$Z_{in} = 47.6 - j17.5\,\Omega$



Java Applet : Smith Chart

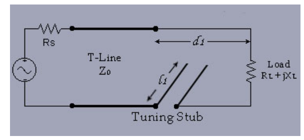
- http://education.tm.agilent.com/index.cgi?CONTENT_ID=5

Hasta aqui INEL 4151

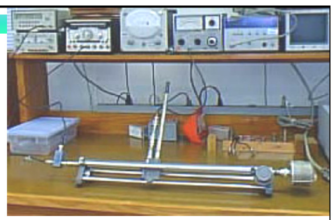
- Material extra sigue (INEL 4155)

Applications

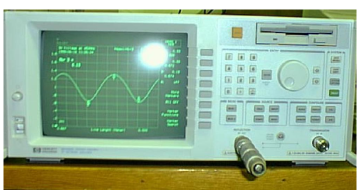
- Slotted line as a frequency meter
- Impedance Matching
  - If Z_L is Real: Quarter-wave Transformer ($\lambda/4$ X_{mer})
  - If Z_L is complex: Single-stub tuning (use admittance Y)
- Microstrip lines



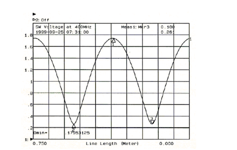
Slotted Line



- Used to measure frequency and load impedance



HP Network Analyzer in Standing Wave Display



Standing Wave on A Transmission Line for a Complex Load as Displayed on the HP8712C (d_{min} is referenced to marker 3).

http://www.ee.olemiss.edu/software/naswave/Stdwave.pdf

Slotted line example

Given s, the distance between adjacent minima, and l_{min} for an "air" 100Ω transmission line, Find f and Z_L

- $s=2.4$, $l_{min}=1.5$ cm, $l_{min\text{-}min}=1.75$ cm
- Solution: $f = \frac{c}{\lambda} = \frac{3\times10^8}{3.5cm} = 8.6GHz$

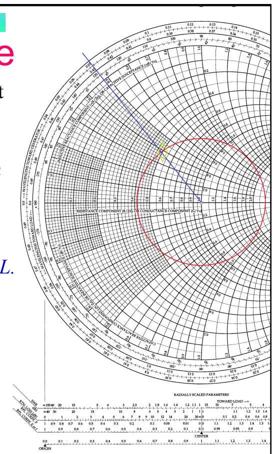
Draw a circle on $r=2.4$, that's your T.L.

move from V_{min} to z_L

$$l_{min} = \frac{1.5}{3.5/\lambda} = .429\lambda$$

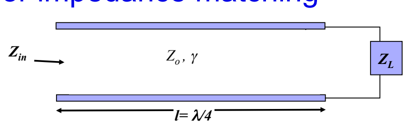
$$z_L = .5 + j.38$$

$$Z_L = Z_o z_L = 50 + j38\,\Omega$$



Quarter-wave transformer

…for impedance matching



$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\left[\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)\right]}{Z_o + jZ_L \tan\beta l}$$

$$\tan(\pi/2) = \infty$$

$$Z_{in} = \frac{Z_o^2}{Z_L}$$

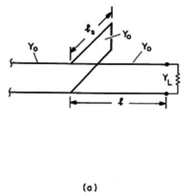
Conclusion: **A piece of line of $\lambda/4$ can be used to change the impedance to a desired value (e.g. for impedance matching)

Single Stub Tuning

…for impedance matching

- A stub is connected in parallel to sum the admittances
- Use a reactance from a short-circuited stub or open-circuited stub to cancel reactive part
- $Z_{in}=Z_o$ therefore $z=1$ or $y=1$ (this is our **goal**!)

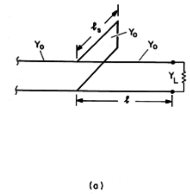
Single Stub Basics



- *We work with Y, because in <u>parallel</u> connections they <u>add</u>.*
- Y_L $(=1/Z_L)$ is to be matched to a TL having characteristic admittance Y_o by means of a "stub" consisting of a shorted (or open) section of line having the same characteristic admittance Y_o

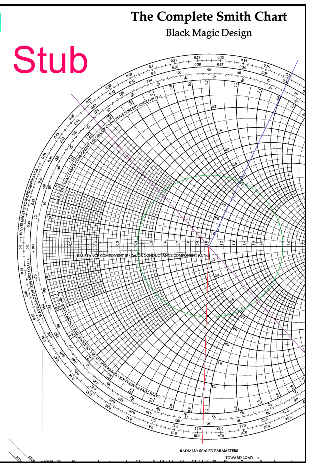
http://web.mit.edu/6.013_book/www/chapter14/14.6.html

Single Stub Steps



- First, the length l is adjusted so that the <u>real part</u> of the admittance at the position where the stub is attached is equal to Y_o or y_{line} = 1+jb
- Then the length of the shorted stub is adjusted so that it's susceptance cancels that of the line, or y_{stub}= -jb

Example: Single Stub



A 75Ω lossless line is to be matched to a 100-j80 Ω load with a shorted stub. Calculate the <u>distance from the load</u>, the <u>stub length</u>, and the necessary <u>stub admittance.</u>

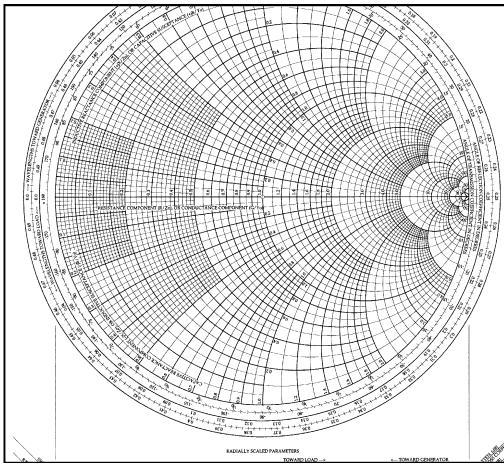
Answer: $z_L = 1.33 - j1.067$

Change to: $y_L = .457 + j.366$

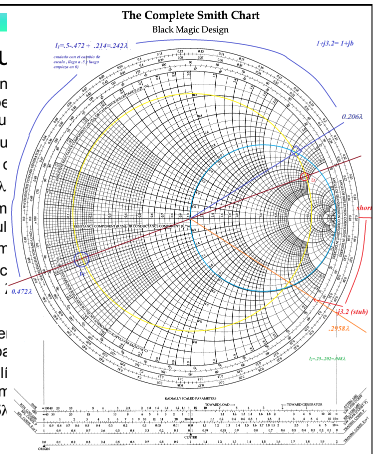
.4338- .3393=0.094λ (1+j.96) or next intersection:0.272λ,

Short stub:.25-.124=0.126λ

With y_{stub}= -j.96/75 =−j.0128 mhos



Example: Single Stu



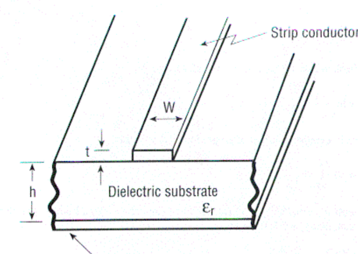
A 50Ω lossless T.L. is 20 m long an perfectly match , what should be line. Assume an operating frequ

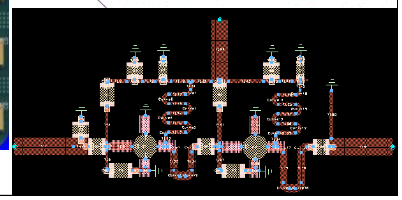
Answer: λ=30m, z_L= 2.4+j4.4 (circu

Trazo raya por el centro y leo y_L al c

- La y_L está en la posición 0.472λ
- Trazo círculo amarillo, esa es m interseca el circulo de r=1(circul
- Lo interseca en 1+j3.2 Ese es m
- Por tanto la distancia desde la c cambio de escala) +.214= .242
-
- Ahora miro el círculo del segme =1).y busco donde jb= -j3.2 (aba
- Para buscar su posición, trazo lí está en .2958λ. El segmento em roja que dice short) está en .25λ
- .25-.202= .0485λ

Microstrips





Microstrips analysis equations & Pattern of EM fields

$$\varepsilon_{eff} = \frac{(\varepsilon_r + 1)}{2} + \frac{(\varepsilon_r - 1)}{2\sqrt{1 + 12h/w}}$$

The characteristic impedance is given by the following approximate formulas:

$$Z_o = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{h}\right), & w/h \le \\ \dfrac{1}{\sqrt{\varepsilon_{eff}}} \dfrac{120\pi}{[w/h + 1.393 + 0.667 \ln(w/h + 1.444)]}, & w/h \ge \end{cases}$$

The characteristic impedance of a wide strip is often low while that of a nar high.

$$Z_o = \frac{1}{\sqrt{\varepsilon_{eff}}} \frac{120\pi}{w/h + 1.393 + 0.667 \ln(w/h + 1.444)} \text{[for } w/h$$

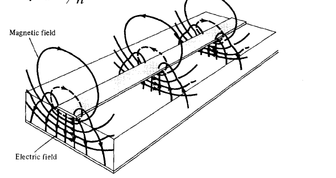


Figure 11.39 Pattern of the EM field of a microstrip line. Source: From

$$Z_o = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(\frac{8h}{w} + \frac{w}{h}\right) \quad w/h \le 1$$

Microstrip Design Equations

$$w/h = \frac{8e^A}{e^{2A} - 2} \qquad w/h < 2$$

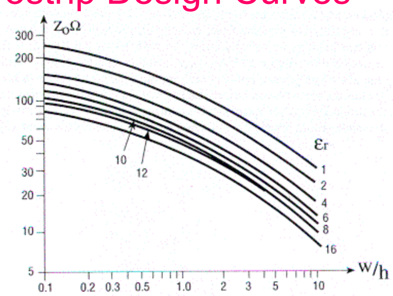
$$w/h = \frac{2}{\pi}\left\{ B - 1 - \ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B-1) + .39 - \frac{0.61}{\varepsilon_r}\right]\right\} \qquad w/h > 2$$

Falta un radical en ε_{eff}

$$u = \frac{c}{\sqrt{\varepsilon_{eff}}} \qquad A = \frac{Z_o}{60}\sqrt{\frac{\varepsilon_r - 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B = \frac{60\pi^2}{Z_o} \frac{1}{\sqrt{\varepsilon_r}}$$

Microstrip Design Curves



Variation of Z_o for Different Dielectric Constants and Aspect Ratio

Example

A microstrip with fused quartz (ε_r=3.8) as a substrate, and ratio of line width to substrate thickness is w/h=0.8, find:

- Effective relative permittivity of substrate
- Characteristic impedance of line
- Wavelength of the line at 10GHz

Answer: ε_{eff}=2.75, Z_o=86.03 Ω, λ=18.09 mm

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\sqrt{1 + 12h/w}} = \frac{4.8}{2} + \frac{2.8}{2\sqrt{1 + 12/0.8}} = 2.75$$

$$Z_o = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(\frac{8h}{w} + \frac{w}{h}\right) \text{ [for } w/h \le 1] = \frac{60}{\sqrt{2.75}} \ln\left(\frac{8}{.8} + .8\right) = 86.03$$

$$\lambda = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{3 \times 10^8}{10 \times 10^9 \sqrt{2.75}} = 18.1mm$$

Diseño de microcinta:

Dado (ε_r=4) para el substrato, y h=1mm halla w para Z_o=50 Ω y cuánto es ε_{eff}?

- Solución: Suponga que como Z es pequena w/h>2