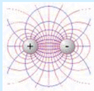

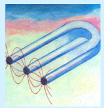

Electricity and Magnetism

INEL 4151

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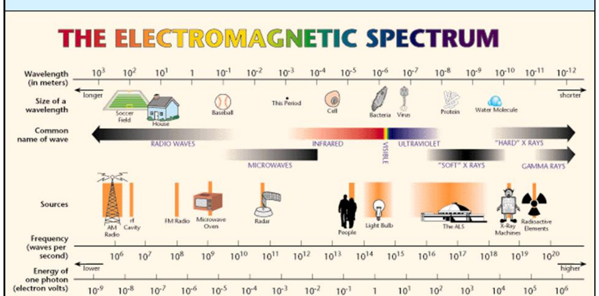
Topics

- **Electric** Fields, [Coulomb's Law], Gauss' Law, E, D, V)
  - Convection/conduction current, conductors, Polarization in dielectrics, Permittivity, conductors,[§5.3-5.5] resistance, capacitance [**§6.5**]
- **Magnetic** fields [Biot Savart Law], Ampere's Law,
  - Flux Density, Magnetic Potentials , [**§7.2-7.5, 7.7**]
  - Magnetic Force, torque, moment, dipole, inductors,
  - Magnetic circuits [**§8.2-8.3, 8.5-8.6, 8.8, 8.10**] Faradays Law, Transformer & Motional *emf*,
- **Electromagnetic Waves**: Maxwell Eqs., time varying potentials and Time Harmonic fields [**§9.2-9.7**]
  - waves in different media, power and Poynting vector,
  - **Incidence** at normal angles. [**§10.2-10.8**]
- **Transmission lines**: Parameters, equations, Input impedance, SWR, power,
  - **Smith** Chart [**§11.2-11.5**]



http://ece.uprm.edu/~pol/cursos

Some terms

- E = electric field intensity [V/m]
- D = electric field density or flux
- H = magnetic field intensity, [A/m]
- B = magnetic field density, [Teslas]

$$D = \varepsilon E$$
$$B = \mu H$$
$$\varepsilon_o = 8.85 \times 10^{-12} F/m = \frac{10^{-9}}{36\pi}$$
$$\mu_o = 4\pi \times 10^{-7} H/m$$

Vector Analysis Review:

- What is a vector?
- How to add them, multiply, etc,?
- Coordinate systems
  - Cartesian, cylindrical, spherical
- Vector Calculus review

Vector

- A vector has magnitude and direction.

$$\vec{A} = \hat{a}_A A$$

where $\hat{a}_A$ *is unit vector.*

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A} = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

- In Cartesian coordinates (*x,y,z*):

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

Vector operations

Commutative	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$ $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$	$k\vec{A}=\vec{A}k$
Associative	$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$	$k(l\vec{A})=(kl)\vec{A}$
Distributive	$k(\vec{A}+\vec{B})=k\vec{A}+k\vec{B}$ $\vec{C}\cdot(\vec{A}+\vec{B})=\vec{C}\cdot\vec{A}+\vec{C}\cdot\vec{B}$	

Example

Given vectors $A=a_x+3a_z$ and $B=5a_x+2a_y-6a_z$

- *(a)* $|A+B|$
- *(b)* $5A-B$

Answers: *(a)* 7 *(b)* (0,-2,21)

Vector Multiplications

- Dot product $\vec{A}\cdot\vec{B}=AB\cos\theta_{AB}$

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$

Note that: $\vec{A}\cdot\vec{A}=|\vec{A}|^2=A^2$

- Cross product $\vec{A}\times\vec{B}=AB\sin\theta_{AB}\hat{a}_n$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Also...

- Multiplying 3 vectors:

Scalar: $\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{B}\cdot(\vec{C}\times\vec{A})=\vec{C}\cdot(\vec{A}\times\vec{B})$

Vector: $\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B})$

- Projection of vector A along B:

$$\vec{A}_B=\vec{A}\cdot\hat{a}_B$$

Example

Given vectors $A=a_x+3a_z$ and $B=5a_x+2a_y-6a_z$

- *(c) the component of A along y*
- *(d) a unit vector parallel to 3A+B*

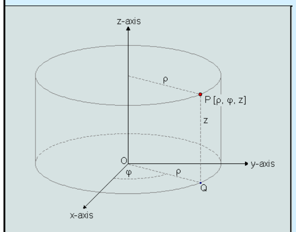
Answers: *(c)* 0 *(d)* ± (0.9117,.2279,0.3419)

Coordinates Systems

- Cartesian (x,y,z)
- Cylindrical (ρ,ϕ,z)
- Spherical (r,θ,ϕ)

Cylindrical coordinates (ρ,ϕ,z)

$$\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$



$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

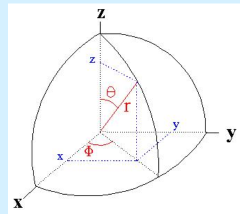
$$\rho = \sqrt{x^2 + y^2} \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$x = \rho\cos\phi \qquad y = \rho\sin\phi$$

$$\rho = A_\rho \hat{a}_\rho + A_\varphi \hat{a}_\varphi + A_z \hat{a}_z$$

Spherical coordinates, (r,θ,ϕ)

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$$



$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$r = \sqrt{x^2 + y^2 + z^2} \qquad \theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\sin\theta\cos\phi \qquad y = r\sin\theta\sin\phi \qquad z = r\cos\theta$$

Vector calculus review

Del (gradient)	$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$
Divergence	$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
Curl	$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$
Laplacian (del²)	$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

Cartesian Coordinates

Theorems

- Divergence

$$\oint_S \vec{A} \cdot d\vec{S} = \int_v \nabla \cdot \vec{A}\, dv$$

- Stokes'

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S \left(\nabla \times \vec{A}\right) \cdot d\vec{S}$$

- Laplacian

Scalar: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

Vector: $\nabla^2 \vec{A} = \nabla^2 A_x \hat{a}_x + \nabla^2 A_y \hat{a}_y + \nabla^2 A_z \hat{a}_z$

Vector calculus review

Del (gradient)	$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$
In Other Coordinate systems	$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z$
	$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$