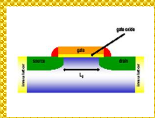


ELECTRIC FIELDS IN MATERIAL SPACE

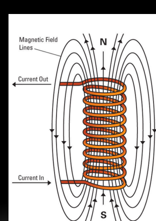
Sandra Cruz-Pol, Ph. D.
INEL 4151 ch 5
Electromagnetics I
ECE UPRM
Mayagüez, PR

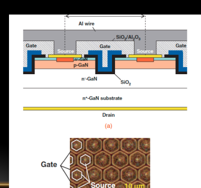
LAST CHAPTER: free space

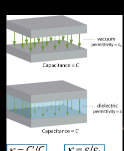
NOW: different materials

SOME APPLICATIONS

- *Superconductors*
- *High permittivity dielectrics*
- *Transistors*
- *Electromagnets*
- *Remote Sensing*







WE WILL STUDY ELECTRIC CHARGES:

- Conductors or Insulators
  - Depend on Frequency and Temperature…
- Boundary conditions

Conductors ~(metals)

Insulators (dielectrics)

Semiconductors

Material @ 20°C & Low frequency	Conductivity [S/m]
Silver	6.1×10^7
Copper	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Carbon	3×10^4
Sea water	4
Silicon	4.4×10^{-4}
Pure water	10^{-4}
Dry Earth	10^{-5}
Glass, Quartz	$10^{-12}, 10^{-17}$

Appendix B

Conductors- have many free electrons available.

Colder metals conduct better.

(superconductivity)

semiconductor

Insulators at most lower frequencies.

CURRENT

Units: Amperes [A]

Definition: is the electric charge passing through an area per unit time.

$$I = \frac{dQ}{dt}$$

Current Density, [A/m^2]

Is the current thru a perpendicular surface

$$J_n = \frac{\Delta I}{\Delta S}$$

$$I = \int_S \vec{J} \cdot d\vec{S}$$

DEPENDING ON HOW I IS PRODUCED:

There are different types of currents.

- Convection- I flows thru isolator: liquid, gas, vacuum.
  - Does not involve conductors,
  - Does not satisfy Ohm's Law
- Conduction- flows thru a conductor
- Displacement (ch9)

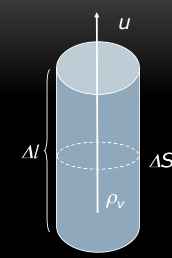
CURRENT IN A FILAMENT

- Convection current, [A]

$$I = \frac{\Delta Q}{\Delta t} =$$

- Convection density, A/m^2

$$J = \frac{\Delta I}{\Delta S} = \rho_v \vec{u}$$



CONDUCTION CURRENT

- Requires free electrons, it's inside conductor.

$$\vec{F} = -e\vec{E}$$

- Suffers collisions, drifts from atom to atom

Newton's Law

$$\frac{m\vec{u}}{\tau} = e\vec{E}$$

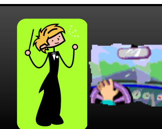
m mass of electron
τ time between collisions
$\vec{u}$ drift velocity

- Conduction current density is:

$$J = \rho_v \vec{u} = \frac{ne^2\tau}{m}\vec{E} = \sigma\vec{E}$$

where $\rho_v = ne$

A PERFECT CONDUCTOR



Has many *charges* that are free to move.

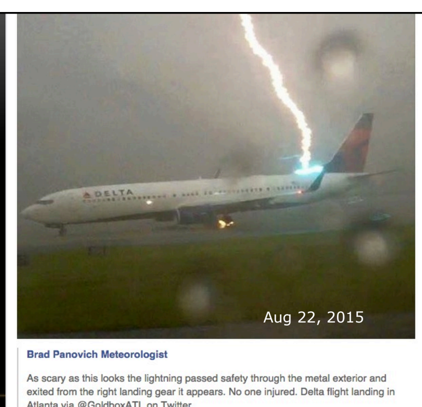
- Therefore it cannot have an E field inside which would not let the charges move freely.
- So, inside a conductor:

$$\vec{E} = 0$$

$$\rho_v = 0$$

$$V_{ab} = 0$$

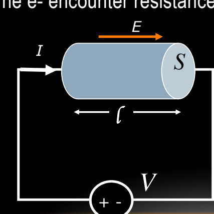
Charges move to the surface to make E=0



http://www.hngn.com/articles/121982/20150822/lightning-bolt-strikes-delta-air-plane-bound-las-vegas-video.htm

RESISTANCE

- If you force a Voltage across a conductor:
- Then E is not 0
- The e- encounter resistance to move



$$\vec{E} = V / l$$

$$J = I / S = \sigma E$$

$$R = \frac{V}{I}$$

$\rho_c = 1/\sigma$ = resistivity of the material

RESISTANCE

- In general, for conductor of nonuniform cross section:

$$R = \frac{V}{I} = \frac{\int \vec{E}\cdot d\vec{l}}{\int \sigma \vec{E}\cdot d\vec{S}}$$

POWER IN WATTS

= Rate of change of energy or force x velocity

$$\int \rho_v dv \vec{E}\cdot\vec{u} = \int \vec{E}\cdot \rho_v \vec{u} dv$$

$$P = \int \vec{E}\cdot\vec{J}dv$$

$$P = \int_L \vec{E}dl \cdot \int_S \vec{J}dS$$

$$P = VI$$

Joule's Law

PE 5.1 Find the current thru the cylindrical surface $\rho = 2, 1 \le z \le 5m$

- For the current density $\vec{J} = 10z\sin^2\phi\, \hat{a}_\rho \quad [mA/m^2]$

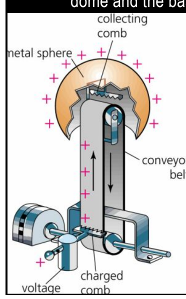
$$I = \int_S J\, dS = \int_1^5 10z\, dz \int_0^{2\pi} \sin^2\phi\, \rho\big|_{\rho=2}\, d\phi$$

$$I = 20\frac{(25-1)}{2}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right]_0^{2\pi}$$

$$I = 240\pi$$
$$= 754mA$$

PE 5.2 In a Van de Graaff generator, *w*=0.1m, *u*=10m/s and the leakage paths have resistance 10^{14} Ω.

- If the belt carries charge 0.5 μC/m², find the potential difference between the dome and the base.

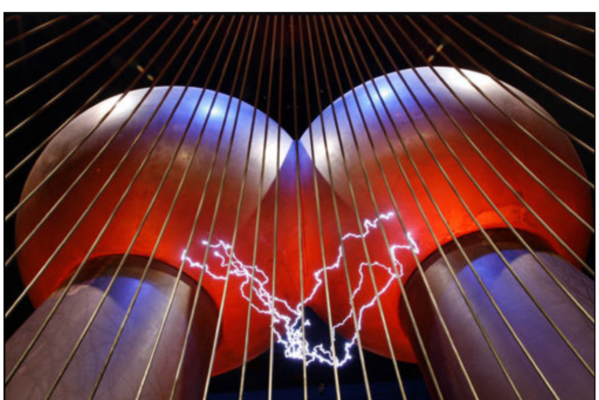


w= width of the belt

u= speed of the belt

$$I = \rho_S uw = 0.5\times10^{-6}(10)(.1)$$

$$V = IR = (.5)10^{-6}(10^{14}) = 50MV$$



PE 5.3 The free charge density in copper (Cu) is 1.81 x 10^{10}C/m³.

- For a current density of 8 x 10^6 A/m², find the electric field intensity and the drift velocity.

$$J = \rho_v u = \sigma E$$

$$[App: \sigma_{Cu} = 5.8\times10^7 S/m]$$

$$E = \frac{J}{\sigma} = \frac{8\times10^6}{5.8\times10^7} = 0.138\, V/m$$

$$u = \frac{J}{\rho_v} = \frac{8\times10^6}{1.81\times10^{10}} = 4.42\times10^{-4} \quad m/s$$

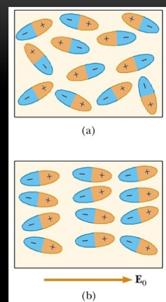
POLARIZATION IN DIELECTRICS

The effect of polarization on a dielectric is to have a surface bound charge aligned with the external field.

and leave within it an accumulation of volume bound charge which increases D.

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P}$$

$$P = \chi_e \varepsilon_o \vec{E}$$



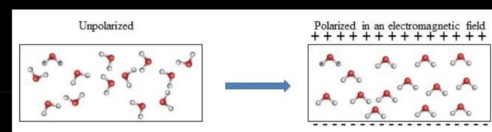
PERMITTIVITY AND STRENGTH

- Permittivity = "Dielectric Constant" = Not really a constant!

$$\vec{D} = \varepsilon \vec{E}$$

$$\varepsilon = \varepsilon_o \varepsilon_r$$

$$\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_o}$$



DIELECTRIC PROPERTIES

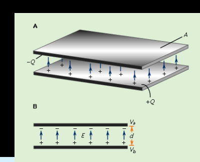
Coulomb's Law for any material

$$F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_o \varepsilon_r R^2} \hat{a}_{12}$$

PE 5.6 A parallel plate capacitor with plate separation of 2mm has a 1kv voltage applied to its plane.

- If the space between its plates is filled with polystyrene, find E and P.

$$\vec{E} = \frac{V}{d} = \frac{1000}{.002} = 500k\hat{a}_x V/m$$



$$\chi_e = \varepsilon_r - 1 = 1.55$$

$$\vec{P} = \chi_e \varepsilon_o \vec{E} = (1.55)(8.85 \times 10^{-12}) 5 \times 10^5 = 6.86\hat{a}_x \mu C/m^2$$

$$\vec{P} = \frac{1}{10\pi}\left(3\hat{a}_x - \hat{a}_y + 4\hat{a}_z\right) nC/m^2$$

PE 5.7 IN A DIELECTRIC MATERIAL, E_X= 5V/M AND

- Find: $\chi_e, \vec{E}, and\ \vec{D}$

$$\vec{P} = \varepsilon_o \chi_e \vec{E}$$

$$\chi_e = \frac{P_x}{\varepsilon_o E_x} = 2.16$$

$$\vec{E} = \frac{\vec{P}}{\varepsilon_o \chi_e} = 5\hat{a}_x - 1.67\hat{a}_y + 6.67\hat{a}_z$$

$$\vec{D} = \varepsilon_o \varepsilon_r \vec{E} = \frac{\varepsilon_r \vec{P}}{\chi_e} = 140\hat{a}_x - 477\hat{a}_y + 186\hat{a}_z$$

QUESTIONS?

CONTINUITY EQUATION

- Charge is conserved.

$$I_{out} = \oint \vec{J}\cdot d\vec{S} = \int \nabla\cdot\vec{J}\,dv$$

$$I_{in} = -\frac{dQ}{dt} = -\frac{d}{dt}\int_v \rho_v\,dv$$

$$\nabla\cdot\vec{J} = -\frac{d\rho_v}{dt}$$

FOR STEADY CURRENTS:

- Change= output current –input current = 0

$$\frac{d\rho_v}{dt} = 0$$

$$\nabla\cdot\vec{J} = 0$$

BOUNDARY CONDITIONS

- We have two materials
- How do the fields behave @ interface?

Evaluate *Maxwell's*:

$$\oint_l \vec{E}\cdot dl = 0$$

$$\oint_S \vec{D}\cdot dS = Q_{enc}$$

We look at the **tangential** and the **perpendicular** component of the fields.

$$\vec{E} = \vec{E}_t + \vec{E}_n$$

DIELECTRIC-DIELECTRIC B.C.

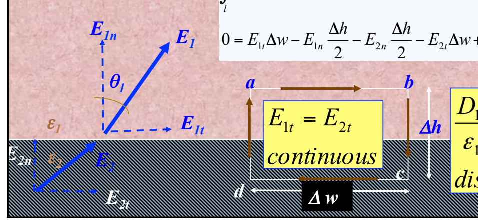
- Consider the figure below:

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$$

$$\oint_l \vec{E}\cdot dl = 0 =$$

$$0 = E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2}$$



$E_{1t} = E_{2t}$ *continuous*

$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ *discontinuous*

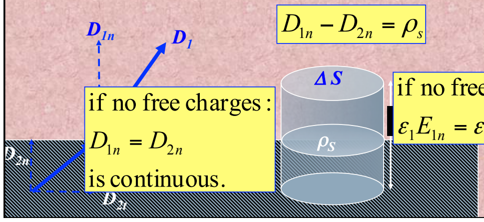
DIELECTRIC-DIELECTRIC B.C.

- Consider the figure below:

$$\Delta Q = \rho_s \Delta S = \oint_S \vec{D}\cdot dS =$$

$$D_{1n}\Delta S - D_{2n}\Delta S$$

$$D_{1n} - D_{2n} = \rho_s$$



if no free charges: $D_{1n} = D_{2n}$ is continuous.

if no free charges: $\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$