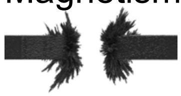



Magnetism

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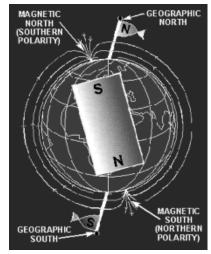
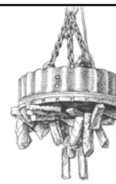
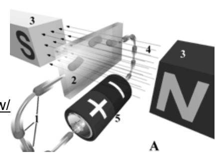
UPRM ch 7

http://www.treehugger.com/files/2008/10/spintronics-discover-could-lead-to-magnetic-batteries.php



Applications



Motors
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http://videos.howstuffworks.com/hsw/18034-electricity-and-magnetism-magnetic-levitation-video.htm



$$\vec{B} = \mu \vec{H}$$

H= magnetic field intensity [A/m]

B= magnetic field (or flux) density [Teslas]

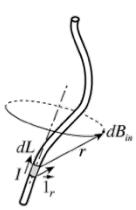
In free space the permeability is:

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

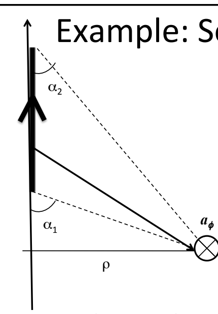
Magnetic Field
Biot-Savart Law

- States that:

$$d\vec{H} = \frac{I d\vec{l} \times \hat{r}}{4\pi R^2}$$



Example: Segment of current



$$d\vec{H} = \frac{I d\vec{l} \times \hat{r}}{4\pi R^2}$$

$$\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$

For an infinite line filament with current I (α_1=180° and α_2=0°):

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \qquad \hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$$

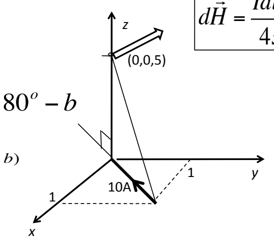
PE. 7.1 Find H at (0,0,5)

$$\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$

$$d\vec{H} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$

Due to 10A current in:

where α_2=90° and $\alpha_1 = 180^o - b$



$$\cos b = \frac{\sqrt{1^2+1^2}}{\sqrt{5^2+\sqrt{2}^2}} = -\cos(180^o - b)$$

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$$

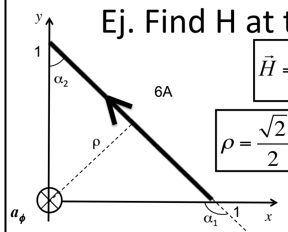
$$= \left(\frac{-\hat{a}_x - \hat{a}_y}{\sqrt{2}}\right) \times \hat{a}_z$$

$$= \frac{\hat{a}_y - \hat{a}_x}{\sqrt{2}}$$

$$\vec{H} = \frac{10}{4\pi(5)}\left(0 - \frac{-\sqrt{2}}{\sqrt{27}}\right)\left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)$$

$$\vec{H} = 30(-\hat{a}_x + \hat{a}_y)\frac{\text{mA}}{\text{m}}$$

Ej. Find H at the origin for:



$$\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$

$$\rho = \frac{\sqrt{2}}{2}$$

$$\alpha_2 = 45^o \quad \alpha_1 = 135^o$$

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$$

$$= \left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right) \times \left(\frac{-\hat{a}_x - \hat{a}_y}{\sqrt{2}}\right) = \hat{a}_z$$

$$\vec{H} = \frac{I\sqrt{2}}{4\pi}\left(\cos 45^o - \cos 135^o\right)\hat{a}_z$$

$$\vec{H} = 0.95\hat{a}_z \quad A/m$$

Circular loop of I

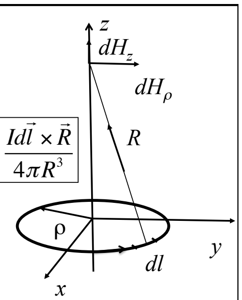
Defined by $x^2 + y^2 = 9, z = 0$

- Apply Biot-Savart:

$$d\vec{l} \times \vec{R} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & \rho d\phi & 0 \\ -\rho & 0 & h \end{vmatrix}$$

$$= \rho h d\phi \hat{a}_\rho + \rho^2 d\phi \hat{a}_z$$

$$d\vec{H} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$



- Only z-component of H survives due to symmetry:

$$\vec{H} = \int_0^{2\pi} \frac{I\rho^2 d\phi \hat{a}_z}{4\pi\left[\rho^2 + h^2\right]^{3/2}} = \frac{I\rho^2\hat{a}_z}{4\pi\left[\rho^2 + h^2\right]^{3/2}} \int_0^{2\pi} d\phi = \frac{I\rho^2\hat{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$$

Ampere's Law

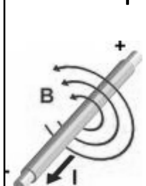
- Simpler
- Analogous to Gauss Law for Coulomb's
- For symmetrical current distributions

Recall

$$Q_{enc} = \int_v \rho_v \, dv = \oint_S \vec{D} \cdot d\vec{S}$$

Ampere's Law

$$\oint \vec{H} \cdot d\vec{l} = I_{enc} = \int \vec{J} \cdot d\vec{S}$$



$$\oint \vec{H} \cdot d\vec{l} = I_{enc} = I$$

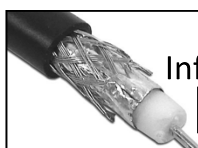
$$d\vec{l} = \rho d\phi \hat{a}_\phi$$

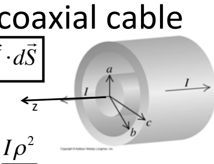
We define an Amperian path where H is constant.

$$\oint H_\phi \cdot \rho d\phi = H_\phi 2\pi\rho$$

$$\vec{H} = \frac{I}{2\pi\rho} a_\phi$$

Infinitely long coaxial cable





$$\oint \vec{H} \cdot d\vec{l} = I_{enc} = \int \vec{J} \cdot d\vec{S}$$

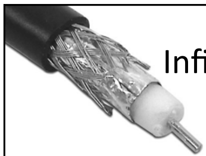
Four cases:
1) For ρ<a

$$I_{enc} = \int \frac{I}{\pi a^2} \cdot \rho \, d\phi \, d\rho \, \hat{a}_z = \frac{I\rho^2}{a^2}$$

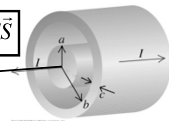
$$\oint H_\phi \cdot \rho d\phi = H_\phi 2\pi\rho$$

$$H_\phi = \frac{I\rho}{2\pi a^2}$$

Infinitely long coaxial cable



$$\oint \vec{H}\cdot d\vec{l} = I_{enc} = \int \vec{J}\cdot d\vec{S}$$



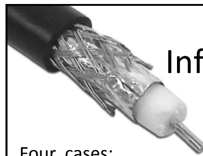
Four cases:
2) For a<ρ<b

$$I_{enc} = \int_0^a \int_0^{2\pi} \frac{I}{\pi a^2}\hat{a}_z \cdot \rho\, d\phi\, d\rho \hat{a}_z = I$$

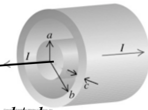
$$\oint H_\phi \cdot \rho d\phi = H_\phi 2\pi\rho$$

$$H_\phi = \frac{I}{2\pi\rho}$$

Infinitely long coaxial cable



$$\oint \vec{H}\cdot d\vec{l} = I_{enc} = \int \vec{J}\cdot d\vec{S}$$



Four cases:
3) For b<ρ<b+c

$$I_{enc} = \int \vec{J}\cdot \rho\, d\phi\, d\rho \hat{a}_z = I - \int_0^{2\pi}\int_b^{\rho} \frac{I\rho\, d\phi\, d\rho}{\pi\left((b+c)^2 - b^2\right)}$$

$$I_{enc} = I - \frac{I\left(\rho^2 - b^2\right)}{\left(2bc + c^2\right)}$$

$$\oint H_\phi \cdot \rho d\phi = H_\phi 2\pi\rho \qquad H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{c^2 + 2bc}\right]$$

Infinitely long coaxial cable

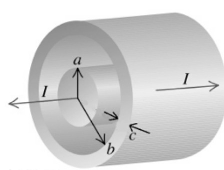
$$\oint \vec{H}\cdot d\vec{l} = I_{enc} = \int \vec{J}\cdot d\vec{S}$$

Four cases:
4) For ρ>b+c

$$I_{enc} = I - I = 0$$

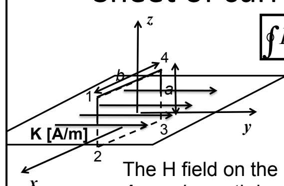
$$0 = H_\phi 2\pi\rho$$

$$H_\phi = 0$$



Recall we had ρ_v, ρ_s in C/m³, C/m²

Sheet of current distribution



$$\oint \vec{H}\cdot d\vec{l} = I_{enc} = K_y b$$

Cross section is a Line!

The H field is given by:

$$\vec{H} = \begin{cases} H_o \hat{a}_x & z > 0 \\ -H_o \hat{a}_x & z < 0 \end{cases}$$

The H field on the *Amperian* path is given by:

$$\oint \vec{H}\cdot d\vec{l} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1\right)\vec{H}\cdot d\vec{l}$$

$$= 0(-a) + (-H_o)(-b) + 0(a) + (H_o)(b)$$

$$= 2H_o b$$

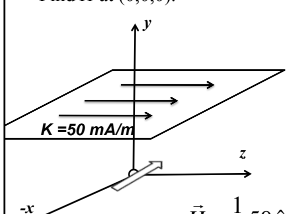
Despejando por H :

$$\vec{H} = \begin{cases} \frac{1}{2}\vec{K}_y \hat{a}_x & z > 0 \\ -\frac{1}{2}\vec{K}_y \hat{a}_x & z < 0 \end{cases}$$

In General: $\vec{H} = \frac{1}{2}\vec{K}\times\hat{a}_n$

PE. 7.5 Sheet of current

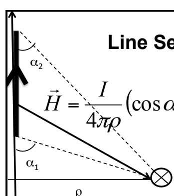
Plane *y* =1 carries a current *K*=50 **a**$_z$ mA/m.
Find *H* at (0,0,0).



$$\vec{H} = \frac{1}{2}\vec{K}\times\hat{a}_n$$

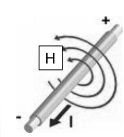
$$\vec{H} = \frac{1}{2}50\hat{a}_z \times \left(-\hat{a}_y\right) = 25\hat{a}_x mA/m$$

Line Segment



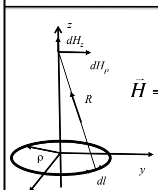
$$\vec{H} = \frac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right)\hat{a}_\phi$$

Infinite Line



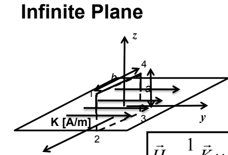
$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

Loop



$$\vec{H} = \frac{I\rho^2 \hat{a}_z}{2\left[\rho^2 + h^2\right]^{3/2}}$$

Infinite Plane



$$\vec{H} = \frac{1}{2}\vec{K}\times\hat{a}_n$$

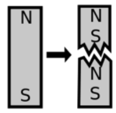
Magnetic Flux Density, B

- The magnetic flux is defined as:

$$\Psi = \int_S \vec{B}\cdot d\vec{S} \quad [\text{Wb}]$$

Webers = Teslas * m^2

which flows through a surface S.



- The total flux thru a closed surface in a magnetic field is:

$$\oint_S \vec{B}\cdot d\vec{S} = 0$$

$$\oint_S \vec{B}\cdot d\vec{S} = \int_v \nabla\cdot\vec{B}\,dv = 0$$

$$\nabla\cdot\vec{B} = 0$$

Recall

$$\nabla\cdot\vec{D} = \rho_v$$

Monopole doesn't exist.

Maxwell's Equations for Static Fields

Differential form	Integral Form	
$\nabla\cdot\vec{D} = \rho_v$	$\oint_s \vec{D}\cdot d\vec{S} = \int_v \rho_v dv$	Gauss's Law for *E* field.
$\nabla\cdot\vec{B} = 0$	$\oint_s \vec{B}\cdot d\vec{S} = 0$	Gauss's Law for *H* field. Nonexistence of monopole
$\nabla\times\vec{E} = 0$	$\oint_L \vec{E}\cdot d\vec{l} = 0$	Faraday's Law; E field is conserved.
$\nabla\times\vec{H} = \vec{J}$	$\oint_L \vec{H}\cdot d\vec{l} = \int_s \vec{J}\cdot d\vec{S}$	Ampere's Law

Magnetic *Scalar and Vector* Potentials, V_m & A

$$\nabla\times\vec{H} = \vec{J}$$

When *J*=0, the curl of *H* is =0, then recalling the vector identity:

$$\nabla\cdot\left(\nabla\times\vec{H}\right) = 0 = \nabla\cdot\left(\nabla V\right)$$

- We can define a Magnetic Scalar Potential as:

$$\vec{H} = -\nabla V_m \quad \text{if } \vec{J} = 0$$

- The magnetic Vector Potential A is defined:

$$\vec{B} = \nabla\times\vec{A}$$

The magnetic vector potential, *A*, is defined from:

$$\vec{B} = \nabla\times\vec{A}$$

$$\text{where } \vec{B} = \mu_o\vec{H} = \int_L \frac{\mu_o I d\vec{l}\times\hat{a}_R}{4\pi R^2}$$

It can be shown that:

$$\vec{A} = \int_L \frac{\mu_o I dl}{4\pi R}$$

(we used this):

$$-\nabla\left(\frac{1}{R}\right) = \frac{1}{R^2}$$

The magnetic vector potential A is used in antenna theory.

Substituting into equation for Magnetic Flux:

$$\Psi = \int_S \vec{B}\cdot d\vec{S} = \int_S (\nabla\times A)\cdot d\vec{S} = \oint_L \vec{A}\cdot d\vec{l}$$

This is another way of finding magnetic flux.

$$\Psi = \oint_L \vec{A}\cdot d\vec{l}$$

P.E. 7.7 A current distribution causes a magnetic vector potential of:

$$\vec{A} = x^2 y\hat{x} + y^2 x\hat{y} - 4xyz\hat{z}$$

Find :

① *B* at (-1,2,5)

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & y^2 x & -4xyz \end{vmatrix}$$

Answer:

$$\vec{B} = 20\hat{x} + 40\hat{y} + 3\hat{z} \quad [\text{T}]$$

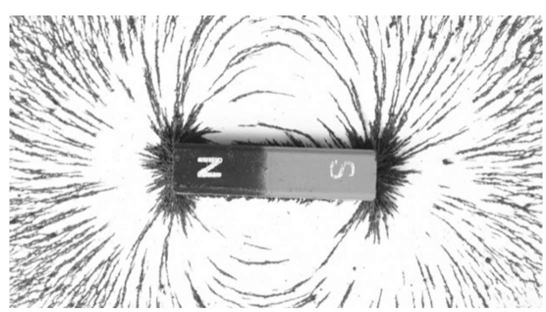
② Flux thru surface *z=1, 0≤ x ≤1, -1≤y ≤4*

$$\Psi = \int_S \vec{B}\cdot d\vec{S} = \oint_S \vec{A}\cdot d\vec{l}$$

Answer :

$$\Psi = 20 \quad [\text{Wb}]$$

Questions?



El Descubrimiento de Ondas