Wave Incidence

[Chapter 10 cont, Sadiku]

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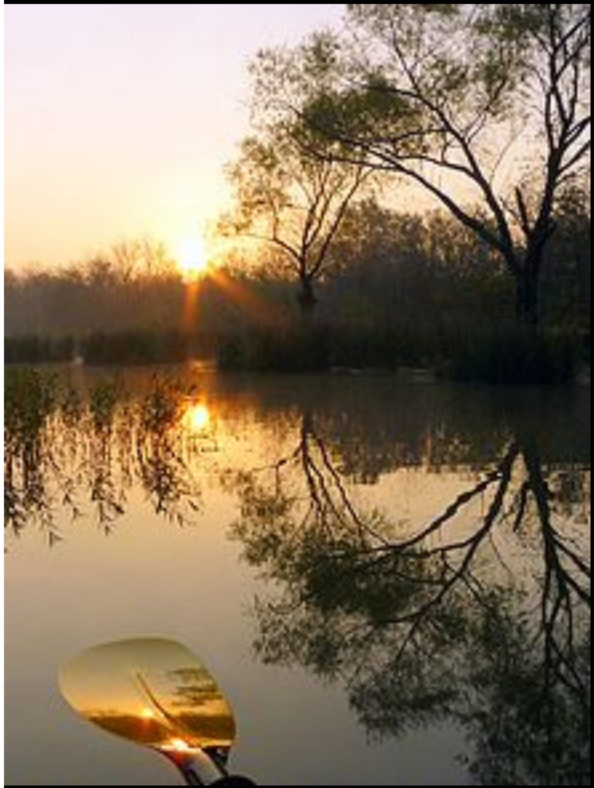
Ex. Light traveling in air encounters the water; another medium.



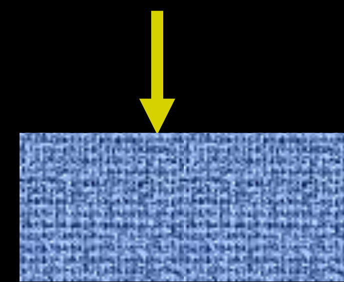
Wave incidence

- For many applications, [such as fiber optics, line power transmission], it's necessary to know what happens to a wave when it meets a different medium.
  - How much is *transmitted*?
  - How much is *reflected* back?

SEE Java Applets
http://physics.usask.ca/~hirose/ep225/anim.htm

We will look at…

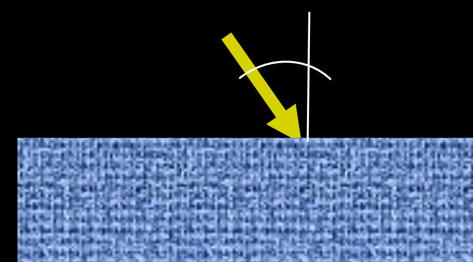
I. Normal incidence



Wave arrives at 0º from normal

- "Standing waves"

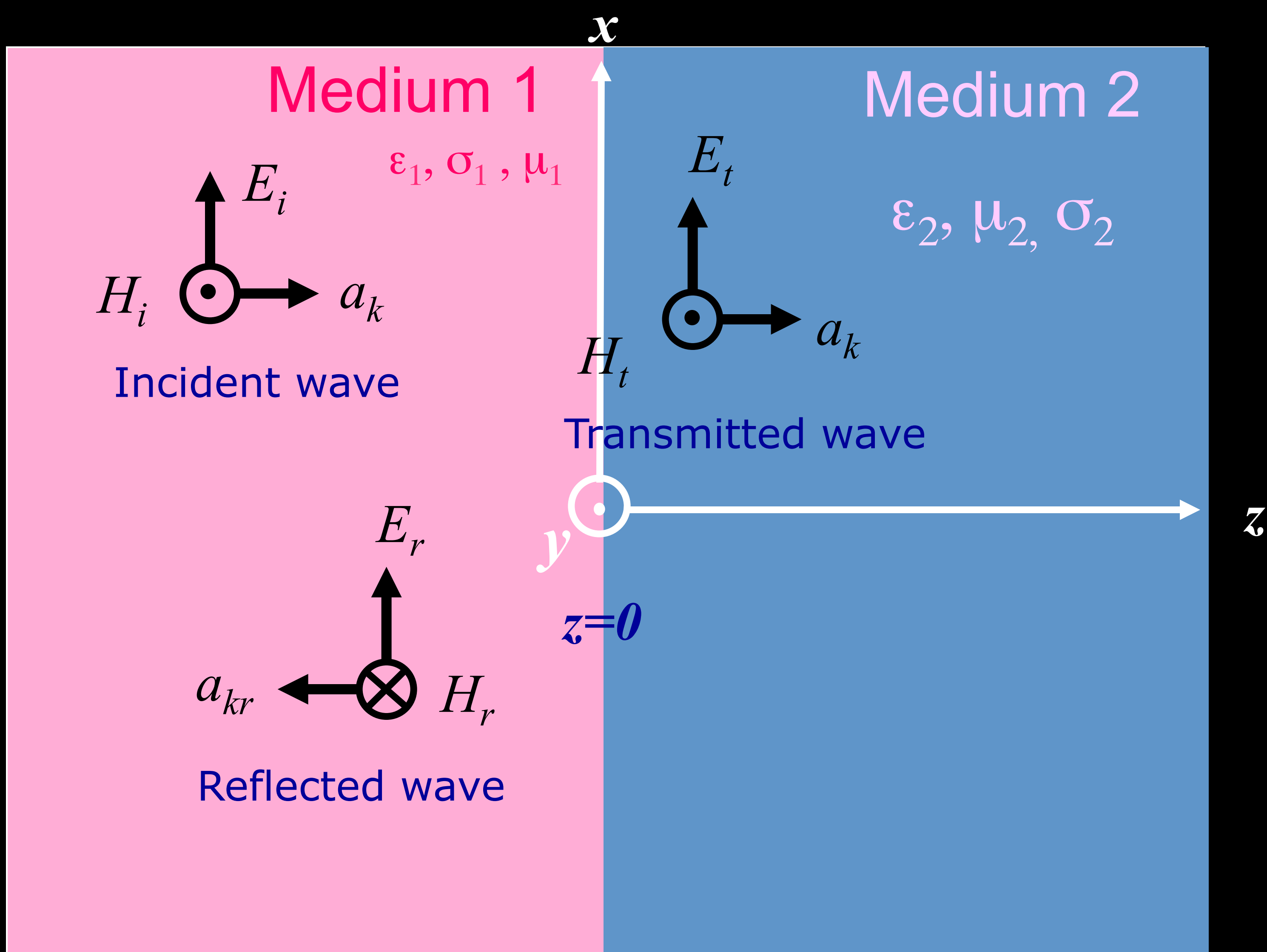
II. Oblique incidence



Wave arrives at another angle

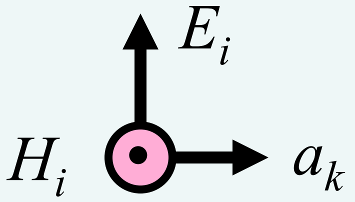
- Snell's Law and Critical angle
- Parallel or Perpendicular polarization
- Brewster angle

Reflection at Normal Incidence



Now in terms of equations …

- Incident wave

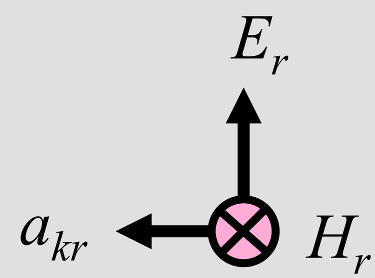


$$\vec{E}_{is}(z) = E_{io} e^{-\gamma_1 z} \hat{x}$$

$$\vec{H}_{is}(z) = H_{io} e^{-\gamma_1 z} \hat{y} = \frac{E_{io}}{\eta_1} e^{-\gamma_1 z} \hat{y}$$

Reflected wave

- It's traveling along $-z$ axis



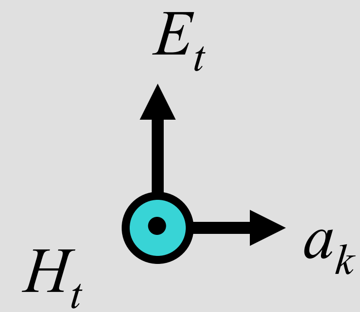
Reflected wave

$$\vec{E}_{rs}(z) = E_{ro} e^{\gamma_1 z} \hat{x}$$

$$\vec{H}_{rs}(z) = H_{ro} e^{\gamma_1 z}(-\hat{y}) = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z} \hat{y}$$

Transmitted wave

$$\vec{E}_{ts}(z) = E_{to} e^{-\gamma_2 z} \hat{x}$$



Transmitted wave

$$\vec{H}_{ts}(z) = H_{to} e^{-\gamma_2 z} \hat{y} = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z} \hat{y}$$

The total fields

- At medium 1 and medium 2

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \qquad \vec{E}_2 = \vec{E}_t$$

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r \qquad \vec{H}_2 = \vec{H}_t$$

- Tangential components must be continuous at the interface

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_t(0)$$

$$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0)$$

Define

- Reflection coefficient, Γ

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

- Transmission coefficient, τ

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Note:

- $1 + \Gamma = \tau$
- Both are dimensionless and may be complex
- $0 \leq |\Gamma| \leq 1$

PE 10.8

A 5GHz uniform plane wave E_{is} =10$e^{-j\beta z}$ a_x in free space is incident normally on a large plane, lossless dielectric slab (z>0) having $\varepsilon = 4\varepsilon_o$ and $\mu=\mu_o$.

Find:

- the reflected wave E_{rs} and
- the transmitted wave E_{ts}.

SEE http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html

Answer:

$-3.33\ e^{j\beta_1 z}\ \mathbf{x}$ V/m,

$6.67\ e^{-j\beta_2 z}\ \mathbf{x}$ V/m

where $\beta_2 = 2\beta_1 = 200\pi/3$

Case 1:

- Medium 1: perfect dielectric, $\sigma_1=0$
- Medium 2: perfect conductor, $\sigma_2=\infty$

Halla impedancias intrínseca.

Reflección,

Transmisión

y campos

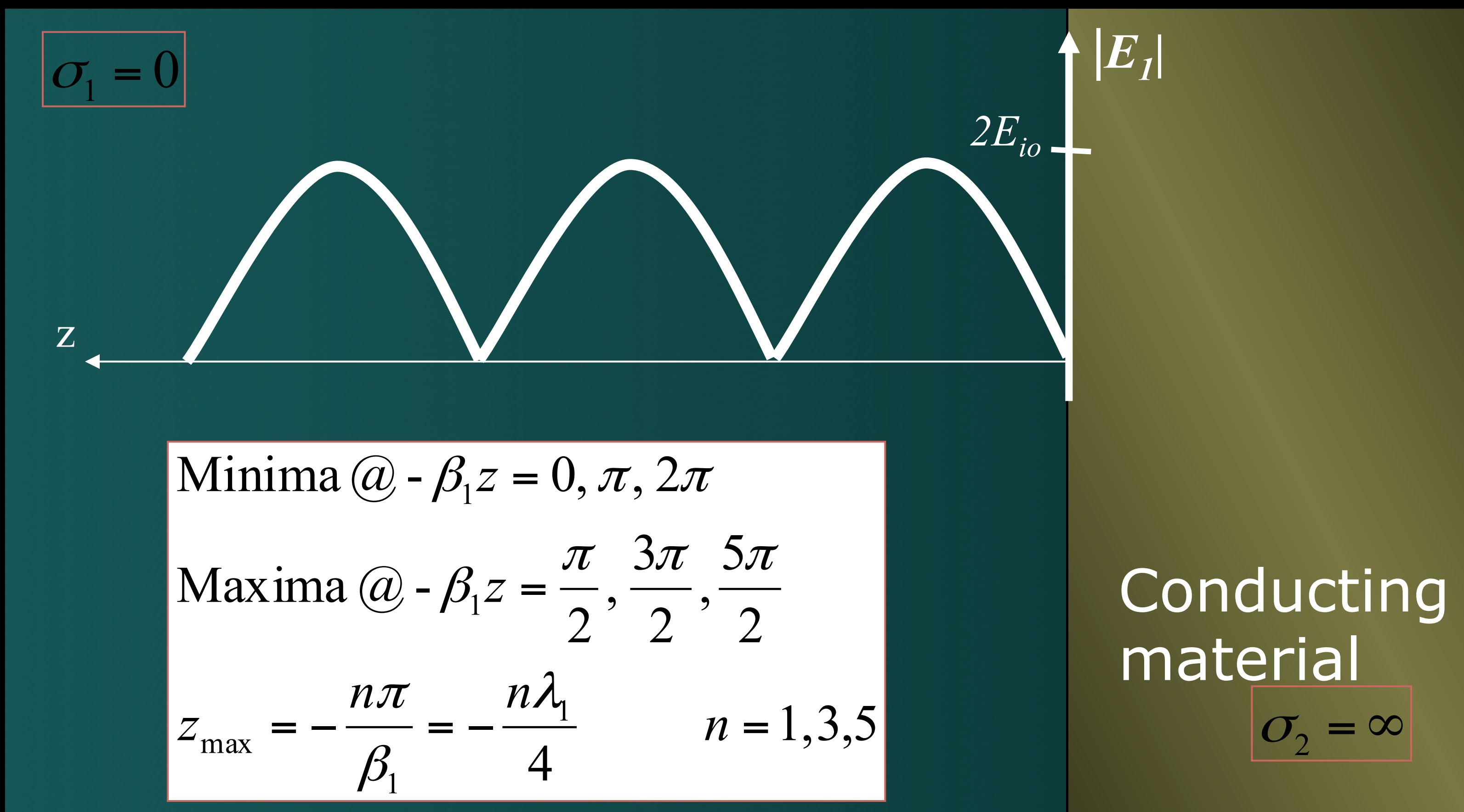
$$(1+j)\frac{\alpha}{\sigma}$$

$$\eta_2 = 0,$$
$$\Gamma = -1, \tau = 0$$
$$E_{1s} = -2jE_{io}\sin\beta_1 z\,\hat{x}\,(phasor)$$
$$E_1(z,t) = 2E_{io}\sin\beta_1 z\sin\omega t\,\hat{x}$$

http://www.phy.ntnu.edu.tw/java/waveSuperposition/waveSuperposition.html

The EM field forms a Standing Wave on medium 1

$$E_1 = 2E_{io} \sin \beta_1 z \sin \omega t \, \hat{x}$$



Minima @ $-\beta_1 z = 0, \pi, 2\pi$

Maxima @ $-\beta_1 z = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}$

$$z_{\max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{4} \qquad n = 1,3,5$$

Standing Wave Applets

- http://www.walter-fendt.de/ph14e/stwaverefl.htm

Case 2:

- Medium 1: perfect dielectric σ_1=0
- Medium 2: perfect dielectric σ_2=0

If $\eta_2 > \eta_1$,

$\Gamma > 0$,

τ and Γ are real.

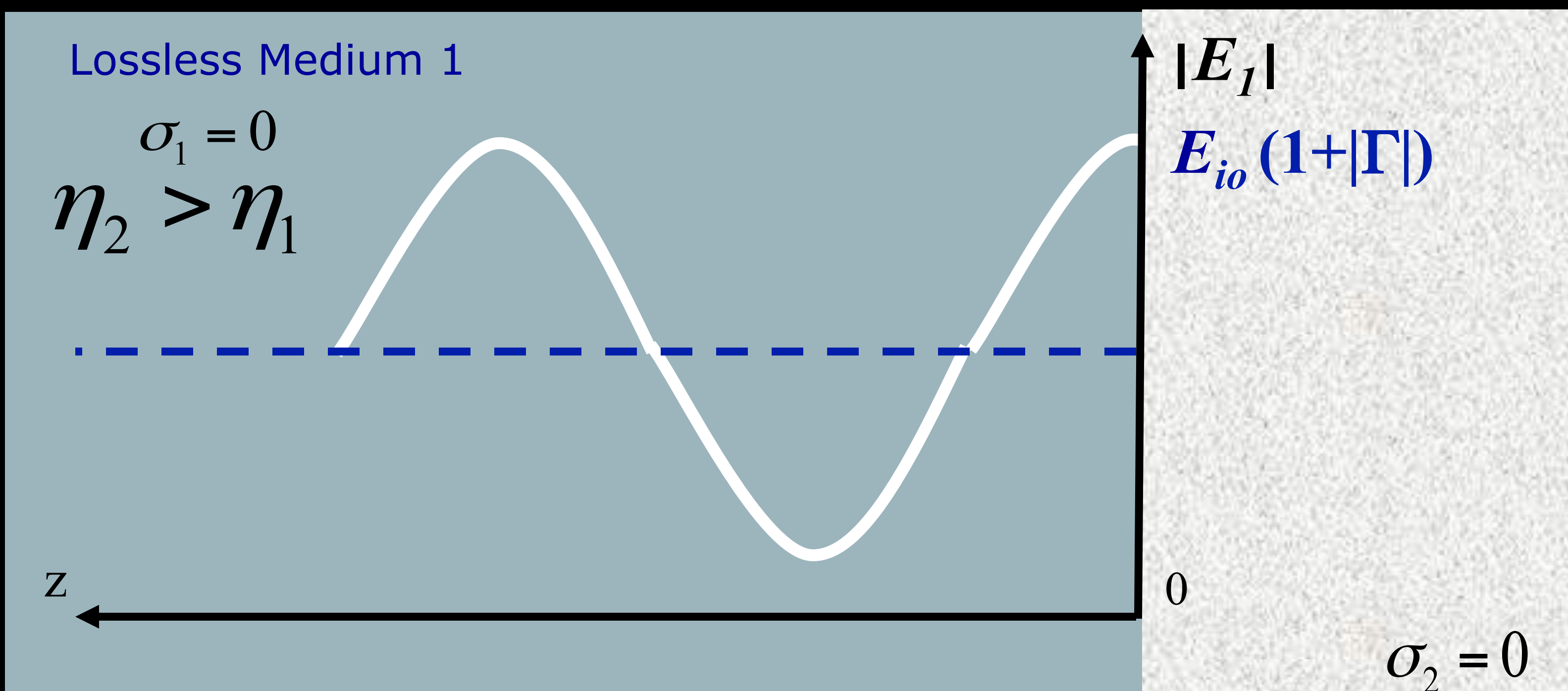
$$E_{1s} = E_{is} + E_{rs}$$
$$= E_{oi}(e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z})$$
$$= E_{oi}e^{-j\beta_1 z}(1 + \Gamma e^{+2j\beta_1 z})$$

$$-2\beta_1 z_{\max} = 0, 2\pi, 4\pi, 6\pi\ldots$$
$$\text{or } -\beta_1 z_{\max} = 0, \pi, 2\pi, 3\pi, \ldots$$

$$z_{\max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2} \qquad n = 0,1,2,3$$

Standing waves due to reflection

$$E_1 = E_i + E_r = E_{oi}(e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z}) = E_{oi}e^{-j\beta_1 z}(1 + \Gamma e^{+2j\beta_1 z})$$



$$z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2} \qquad n = 0,1,2,3$$

*****At every <u>half-wavelength</u>, everything repeats! *****

Case 3:

- Medium 1 = perfect dielectric σ_1=0
- Medium 2 = perfect dielectric σ_2=0

$$\text{If } \eta_2 < \eta_1,$$

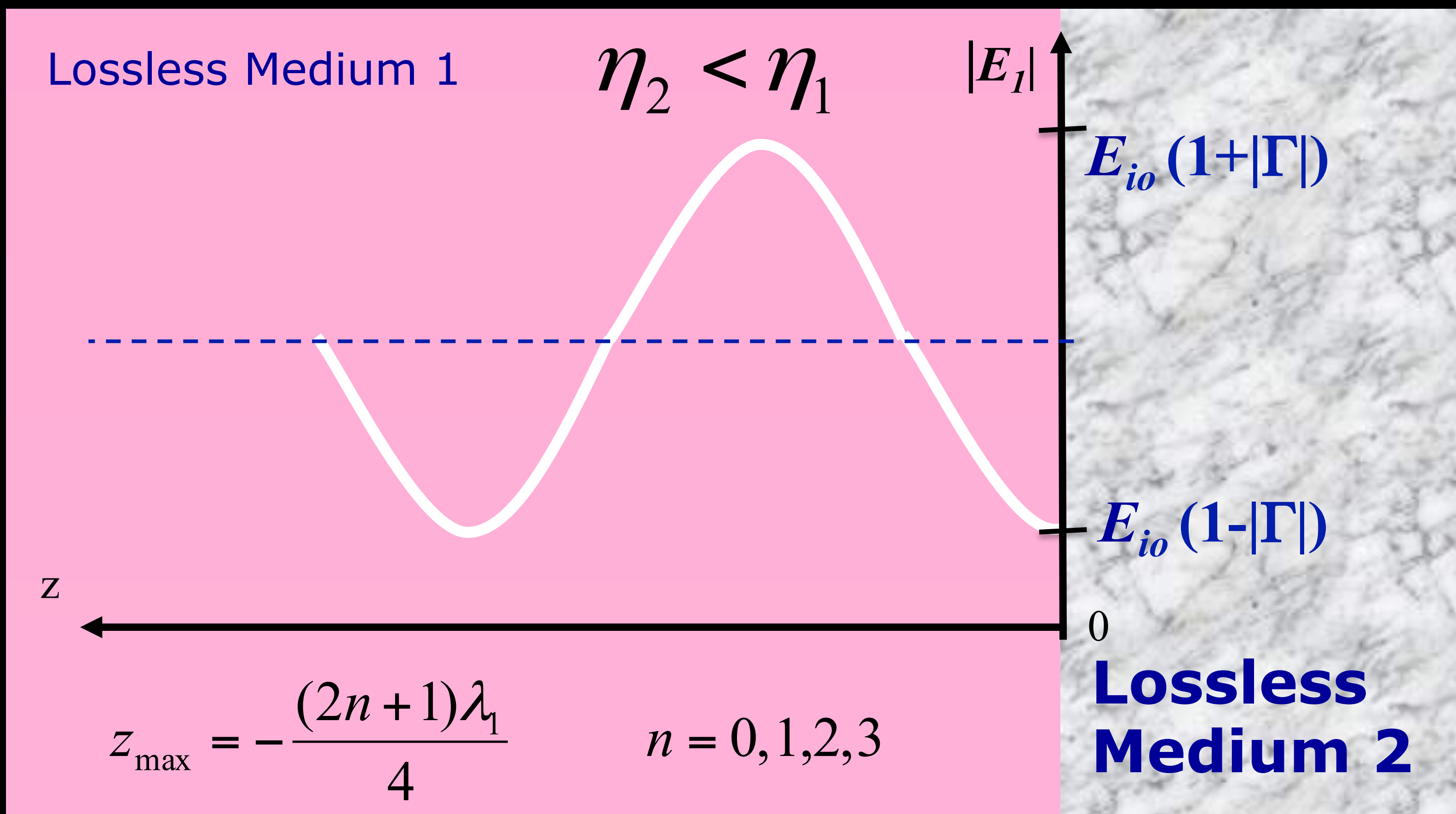
$$\Gamma < 0, \tau \text{ and } \Gamma \text{ are real.}$$

$$z_{\max} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)\lambda_1}{4} \qquad n = 1,2,3$$

$$z_{\min} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2} \qquad n = 0,1,2,3$$

Standing waves due to reflection

$$E_1 = E_i + E_r = E_{oi}(e^{-j\beta_1 z} + \Gamma e^{+j\beta_1 z}) = E_{oi}e^{-j\beta_1 z}(1 + \Gamma e^{+2j\beta_1 z})$$



At every half-wavelength, all *em* properties repeat

Standing Wave Ratio, *s*

- Measures the amount of reflections, the more reflections, the larger the standing wave that is formed.
- The ratio of $|E_1|_{max}$ to $|E_1|_{min}$

$$s = \frac{|E_1|_{\max}}{|E_1|_{\min}} = \frac{|H_1|_{\max}}{|H_1|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Ideally
s=1 (0 dB)
No reflections

or

$$|\Gamma| = \frac{s-1}{s+1}$$

PE 10.9

- The plane wave E=50 sin (ωt – 5x) $\mathbf{a}_y$ V/m in a lossless medium ($\mu=4\mu_o$, $\varepsilon=\varepsilon_o$) encounters a lossy medium ($\mu=\mu_o$, $\varepsilon=4\varepsilon_o$, σ=0.1 mhos/m) normal to the x-axis at x=0. Find

←Answers:

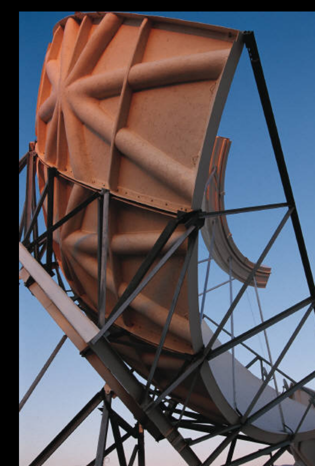
- Γ =0.8186 exp(j171°)
- τ =0.23 exp(j33.56°)
- s =10.03
- E_r = 40.93 sin (ωt +5x + *171*°) ***y***
- E_t = 11.5 e $^{-6.02x}$ sin (ωt - 7.83x + 33.6°) ***y*** V/m
- ***http://www.walter-fendt.de/ph14e/stwaverefl.htm***

Ex. Antenna Radome

A 10GHz aircraft radar uses a narrow-beam scanning antenna mounted on a gimbal behind a dielectric radome.

- Even though the radome shape is far from planar, it is approximately planar over the narrow extent of the radar beam.
- If the radome material is a lossless dielectric with μ_r=1 and ε_r=9, choose its thickness *d* such that the radome appears transparent to the radar beam.
- Mechanical integrity requires *d* to be greater that 2.3 cm.

Antenna with radome



Antenna with no radome

Answer:
$\lambda/2$=.5cm, d=2.5cm

Power Flow in Medium 1

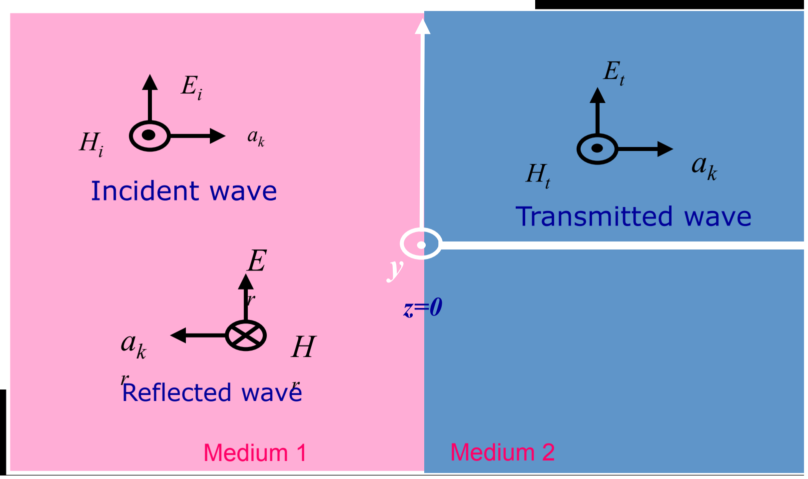
- The net average power density flowing in medium 1

$$P_{ave1}(z) = \frac{1}{2}\mathrm{Re}[E_1 \times H_1^{\ *}]$$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{x}E_{io}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)\times \hat{y}\,\frac{E_{io}^{*}}{\eta_1}\left(e^{j\beta_1 z} + \Gamma^{*} e^{-j\beta_1 z}\right)\right]$$

$$= \hat{z}\,\frac{|E_{io}|^2}{2\eta_1}\left(1-|\Gamma|^2\right)$$

$$= P_{ave}^{i} + P_{ave}^{r}$$



Power Flow in Transmitted wave

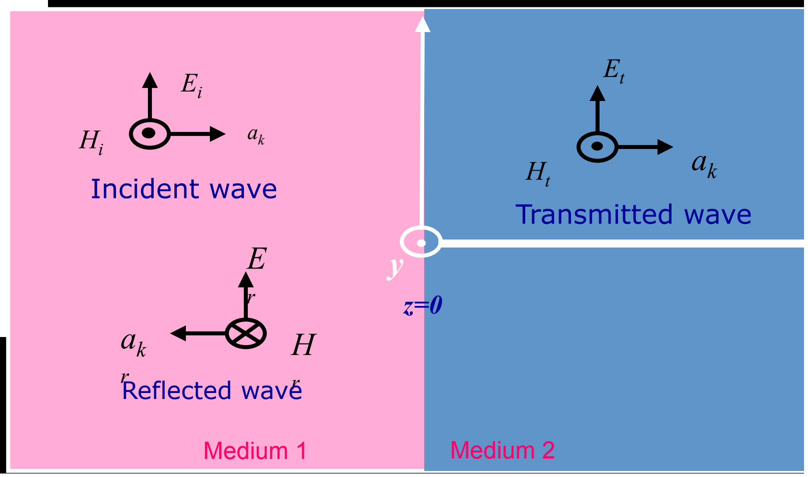
- The net average power density flowing in medium 2

$$P_{ave2}(z) = \frac{1}{2}\mathrm{Re}[E_2 \times H_2^{\,*}]$$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{x}\tau E_{io} e^{-j\beta_2 z} \times \hat{y}\tau^* \frac{E_{io}^*}{\eta_2} e^{j\beta_2 z}\right]$$

$$= \hat{z}|\tau|^2 \frac{|E_{io}|^2}{2\eta_2}$$

$$where \ \ \hat{a}_k = \hat{z}$$



Power in Lossy Media

$$P_{ave1}(z) = \hat{z}\,\frac{\left|E_o^i\right|^2}{2\eta_{c1}}\left(e^{-2\alpha_1 z} - \left|\Gamma\right|^2 e^{2\alpha_1 z}\right)$$

$$P_{ave2}(z) = \hat{z}\left|\boldsymbol{\tau}\right|^2 \frac{\left|E_o^i\right|^2}{2} e^{-2\alpha_2 z}\,\mathrm{Re}\left(\frac{1}{\eta_{c2}^*}\right)$$

where

$$\Gamma = \frac{\sqrt{\varepsilon_{c1}} - \sqrt{\varepsilon_{c2}}}{\sqrt{\varepsilon_{c1}} + \sqrt{\varepsilon_{c2}}} \quad \text{and} \quad \varepsilon_{c2} = \varepsilon_2 - j\frac{\sigma_2}{\omega_2}$$

quiz

The plane wave $\mathbf{E} = 30 \cos(\omega t - z)\mathbf{a}_x$ V/m in air normally hits a lossless mediu

($\mu = \mu_o, \varepsilon = 4\varepsilon_o$) at $z = 0$. (a) Find Γ, τ, and s.

We will look at…

I. Normal incidence

Wave arrives at 90º from the surface

– Standing waves

II. Oblique incidence (lossless)

Wave arrives at an angle

– Snell's Law and Critical angle

– Parallel or Perpendicular

– Brewster angle